A Well-Balanced Central-Upwind Scheme for the 2D Shallow Water Equations on Triangular Meshes

Steve Bryson[1]
NASA Ames Research Center/SCCM Program, Stanford University, USA
bryson@nas.nasa.gov
Doron Levy
Department of Mathematics, Stanford University, USA
dlevy@math.stanford.edu

We are interested in approximating solutions of the two-dimensional shallow water equations with a bottom topography on triangular meshes

$$\begin{pmatrix} h \\ hu \\ hv \end{pmatrix}_t + \begin{pmatrix} hu \\ hu^2 + \frac{1}{2}h^2 \\ huv \end{pmatrix}_x + \begin{pmatrix} hv \\ huv \\ hv^2 + \frac{1}{2}h^2 \end{pmatrix}_y = \begin{pmatrix} 0 \\ -hB_x \\ -hB_y \end{pmatrix}.$$

Here, (u, v) are the components of the velocity vector and h is the height of the water above the bottom $B(x, y)$. A Central-upwind scheme on unstructured grids for hyperbolic systems of conservation laws was recently introduced by Kurganov and Petrova in [2]. At first sight, it seems to be straightforward to extend this scheme to balance laws through a suitable discretization of the source term. However, we prove that for general triangulations it is impossible to discretize the source term in such away that the method of [2] is well-balanced.

We show that there is a certain flexibility in choosing the numerical fluxes in the design of semi-discrete Godunov-type central schemes. We take advantage of this fact to generate a new second-order, central-upwind method for the two-dimensional shallow water equations that is well-balanced. Our construction is inspired by the ideas in [1]. We demonstrate the accuracy of our method as well as its balance properties in a variety of examples.

References

[1] A. Kurganov and D. Levy, *Central-upwind schemes for the Saint-Venant system, Mathematical Modelling and Numerical Analysis*, 36 (2002), 397-425.

[2] A. Kurganov and G. Petrova, *Central-upwind schemes on unstructured grids for hyperbolic systems of conservation laws*, submitted to *SIAM Journal on Scientific Computing*, 2003.

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