Realtime Absolute Orientation with Outlier Rejection for Visual Odometry & Mesh Merging

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Abstract

This paper addresses a high level spatio-temporal problem, namely "absolute orientation", which arises in visual-odometry (using stereo), or registering two models created by different Structure from Motion (SFM) reconstructions. We compare the very popular method due to Horn[1] using quaternions and our own independently derive method using the orthonormal rotation matrix $\mathbf{R}$. We also introduce a *novel* approach for outlier rejection using spectral clustering[2].

Introduction: Absolute orientation, Horn[1], is a higher level spatio-temporal problem that arises in visual-odometery, computational graphics, or extending 3D models using SFM. The problem statement is simple: given two sets of matched points, $\mathbf{p}_1$ and $\mathbf{p}_2$ in $\mathbf{R}^3$, find the *similarity* transform that transforms $\mathbf{p}_1$ into $\mathbf{p}_2$ and reject any outliers that may be present. By similarity transform we mean the translation, rotation and scale such that: [i]

$$\mathbf{p}_1 = s * \mathbf{p}_2 * \mathbf{R} + \mathbf{t} \qquad (1)$$

where s is the scale ambiguity (arising if two sets of points are generated from an SFM output), $\mathbf{R}$ is the 3x3 orthonormal rotation matrix, and $\mathbf{t}$ is the translation vector.

Several authors have already addressed this problem. Perhaps the absolute reference on absolute orientation is Horn[1], where a unit quaternion (on SO^3) represents the rotation. Horn, Hilden and Negahdaripour[3] too address the absolute orientation problem, but use the rotation matrix $\mathbf{R}$. This approach is later rejected for reasons such as:

- Rotation matrices have too many parameters – i.e. 9 instead of 4 parameters for quaternions.
- Rotation matrices have "strange" constraints, i.e. $\mathbf{R} * \mathbf{R}^T$ has to be the 3x3 identity matrix, and det($\mathbf{R}$) = 1.
- Lastly, quaternions are ideal for interpolative purposes compared to the rotation matrix $\mathbf{R}$.[4]

The first two objections are rather mute. First, to solve for proper quaternion, Horn creates a 9x9 matrix and solves for variables bilinear in the elements of $\mathbf{q}$. Second, a unit quaternion $\mathbf{q}$ has to satisfy the constraint $\mathbf{q} \bullet \mathbf{q}^* = 1$ where $\mathbf{q}^*$ is the conjugate of $\mathbf{q}$, and $\bullet$ represents quaternion multiplication; Horn addresses this using a Lagrange multiplier. Regarding interpolation, Michaels and Boult are quite correct. While rotation matrices do not lend themselves, to interpolation, transforming them to Rodrigez representation is trivial.

In the next section we provide a fast and robust technique to address the absolute orientation problem and also introduce a new approach to outlier rejection based on spectral clustering[2] which has proven extremely robust.

Mathematical Derivation: Let us assume for now, that we have already solved for translation, $\mathbf{t}$, and scale, s, in equation 1. Now the two sets of 3D points after correction for these parameters are v and w, and we have $\mathbf{v}_i = \mathbf{R}\,\mathbf{w}_i$. We like to minimize the sum squared error: $\varepsilon^2 = \Sigma_i \mid \mathbf{v}_i - \mathbf{R}\,\mathbf{w}_i \mid^2$. Expanding ε^2 we see that it is equivalent to *maximizing*: $\sigma^2 = \Sigma_i \mathbf{v}_i^T \mathbf{R}\,\mathbf{w}_i$. Adding the orthonormality constraint for $\mathbf{R}$, i.e., $\mathbf{I} - \mathbf{R} * \mathbf{R}^T = 0$, where $\mathbf{I}$ is the identity matrix and using the symmetric Langrange multiplier $\mathbf{M}$, we need to maximize:

$$G(\mathbf{R}) = \Sigma_i \mathbf{v}_i^T \mathbf{R}\,\mathbf{w}_i - \tfrac{1}{2} \text{ trace } (\mathbf{M}\,(\mathbf{I} - \mathbf{R} * \mathbf{R}^T)) \qquad (2)$$

Umeyama[5] also provides a solution to this problem, adding in another Lagrange multiplier for the det($\mathbf{R}$)=1. As we will see shortly, our approach provides a simpler solution to the problem.[ii] Differentiating G($\mathbf{R}$) with respect to matrix $\mathbf{R}$, and using trace of scalar is the scalar, and trace($\mathbf{AB}$) = trace($\mathbf{BA}$) we can show that:

$$\mathbf{K} - \mathbf{M}\,\mathbf{R}^T = 0 \qquad (3)$$

where $\mathbf{K} = \Sigma_i \mathbf{w}_i \mathbf{v}_i^T$. Using $\mathbf{K} = \mathbf{M}\,\mathbf{R}^T$ and multiplying both sides by their own transpose we get $\mathbf{K}\,\mathbf{K}^T = \mathbf{M}^2$. That is $\mathbf{M}$ is the symmetric square root of the matrix $\mathbf{K}\,\mathbf{K}^T$. Now using singular value decomposition of $\mathbf{K} = \mathbf{U}\,\mathbf{D}\,\mathbf{V}^T$, where $\mathbf{D}$ is the diagonal matrix of singular values, $\mathbf{M} = \mathbf{U}\,\mathbf{D}\,\mathbf{U}^T$. But from equation 3, $\mathbf{R} = \mathbf{K}^{-1}\,\mathbf{M}$, therefore:

$$\mathbf{R} = \mathbf{V}\,\mathbf{U}^T \qquad (4)$$

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[i] We represent points by lower case bold, matrices by higher case bold, and scalars with lower case letters.
[ii] We received the reference to Umeyama's work after our derivation. Although outliers are not addressed in this work, the mathematical treatment is very elegant, and interested readers are encourage to examine this work

Equation 4 is both simple and extremely easy to calculate. The matrix K is 3x3 and its SVD is trivial. To make sure the calculated R is not a reflection matrix, simply take its determinant. If the result is -1, multiply the last row by -1.[iii]

Lastly, note that if we subtract the centeroid of the points v_i and w_i from these points, equation (2) still holds, but the translation vector, t, is removed. As a result the only remaining issues are how to preprocess the points p_1 and p_2 to remove the outliers, and how to find the scale factor s. Once the scale, s, parameter is determined we can rescale the remaining points, subtract their respective centeroids, and find the rotation matrix R using equation 4. With the scale, s, and rotation matrix R in hand, finding the least square translation vector, t, using equation 1 is trivial.

Determining the Scale and Outlier Rejection: Let D_1 and D_2 be the symmetric matrices that contain the distance between every two points in p_1 and in p_2 respectively. If W is the element by element division of D_1 by D_2 except for the diagonals which should remain 0, then the elements of W are the scale factor, s, plus added noise. The exception being the rows and columns of W corresponding to outliers (i.e., mismatched points). One popular method of rejecting the outliers is RANSAC[6]. But using spectral clustering, it can be shown that the Eigen vector of W, e_1, corresponding to W's largest Eigen values, is almost blockwise constant. That is, for all elements of e_1 only those that correspond to the rows (or columns) of W containing the outliers have a value different from the rest. The remaining values of e_1 are almost all constant (with very minor differences due to noise). This gives us the following simple technique to reject the outliers: 1) construct W. 2) find the vector e_1 for W's largest Eigen value. 3) sort e_1, keeping track of the changes in its rows. 4) create the permutation matrix Q by rearranging the columns of the identity matrix, I, according the changes in the e_1. 5) create the matrix $X = Q^T W Q$. This matrix is diagonally block structured and s can easily be derived. It is noteworthy, that one can use RANSAC on the block diagonal elements of X matrix thus increasing the speed of RANSAC by filtering out obvious outliers.

Experimental Results: Figure 1a shows the results of our Monte-Carlo simulations. Three techniques, Horn, our SVD based derivation and QR decomposition of the matrix R derived using least squares were compared. While the QR method was the fastest –see figure 1.a – it also generated the most unreliable solutions. Our approach was faster than Horn's technique (or its equivalents) , with the sum square error of the two methods being indistinguishable. Figures 1.b and 1.c show the structure of the X matrix for 40% outliers and 96% outliers!
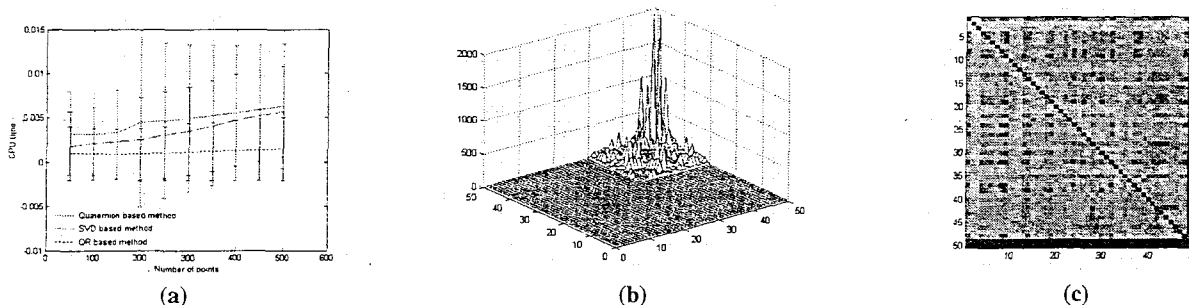

| (a) | (b) | (c) |

Figure 1. (a) The results of Monte Carlo simulation for Horn, QR, and our SVD based method. (b) and (c) the block diagonal structure of the spectral clustering matrix X for 40% and 96% outliers.

References:
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[5] Umeyama, S. et al. "Least-squares estimation of transformation parameters between two point patterns" IEEE PAMI, April 1991. vol.13, no.4, p. 376-80
[6] Fischler M. and Bolles, R.: 1981, "RANSAC: *random sample consensus: A paradigm for model fitting with applications to image analysis and automated cartography*" Communications of the ACM 26, 381-395.

[iii] This is equivalent to reflecting around the Z axis and arises from the properties of minimum Eigen values.