Turbulent concentration of chondrules:

size distribution and multifractal scaling

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Abstract

Size-selective concentration of particles in 3D turbulence may be related to collection of chondrules and other constituents into primitive bodies in a weakly turbulent protoplanetary nebula. In the terrestrial planet region, both the characteristic size and narrow size distribution of chondrules are explained, whereas "fluffier" particles would be concentrated in lower density, or more intensely turbulent, regions of the nebula. The spatial distribution of concentrated particle density obeys multifractal scaling, suggesting a close tie to the turbulent cascade process. This scaling behavior allows predictions of the concentration probabilities to be made in the protoplanetary nebula, which are so large ($> 10^3 - 10^4$) that further studies must be made of the role of mass loading.

Background and Introduction: Most chondritic meteorites are composed in large part of mm-sized, once-melted silicate particles (chondrules) and metallic grains which are narrowly sorted, most likely by aerodynamic cross section (1); various hypotheses have been advanced to explain these properties (2,3). In prior work (3), we noted that chondrule-sized particles are unable to settle to the nebula midplane unless turbulence is vanishingly small. Instead, we proposed that, following an undefined *formation* process in the nebula (4), chondrules pursue an extended free-floating existence until plausible conditions of nebula gas density and turbulent intensity in the terrestrial planet region concentrate them into much more massive, but still not solid, "clouds" which might have enough coherence to resist disruption by eddies and settle to the midplane for subsequent accumulation into planetesimals. We think of this process as "primary accretion". The early stages are easily generalized to a wide range of fluffier particles in lower density nebula regions. The final stage (settling of dense clouds) remains unstudied, and will require better understanding of the behavior of particle ensembles whose density is large enough to affect the gas flow properties.

We have defined the *concentration factor* C as the ratio of the local particle density to its global average; in numerical studies to date (3, 5, 6), Reynolds numbers of $10^2 - 10^3$ produce $C \approx 40 - 300$. While C is seen to increase systematically with increasing Reynolds number, estimates of concentrations at nebula Reynolds numbers had required sizeable extrapolations (3). Here, we utilize our recent discovery of the Reynolds-number-independent fractal properties of the particle density field (6) to provide a firmer basis for predictions under nebula conditions.

Turbulence and turbulent concentration (TC) in the nebula: Homogeneous, isotropic, 3D turbulence is characterized by a cascade of energy through a range of scales, known as the inertial range, from the largest (or integral) spatial scale L (with associated velocity V_L) to the smallest (or Kolmogorov) scale $\eta = LRe^{-3/4}$ where it is dissipated (7). The intensity of turbulence is often characterized by the Reynolds number $Re \equiv LV_L/\nu_m$, where ν_m is the molecular viscosity; Re is the ratio of transport by macroscopic motions to that by molecular motions. This definition expresses the velocity and length scales in terms of viscosity; however, TC is based more on turbulent kinetic energy k than on turbulent viscosity (8). Therefore, we carefully distinguish between these quantities. We redefine

$Re = \alpha_\nu cH/\nu_m$, where c and H are the sound speed and vertical scale height of the nebula, and α_ν is the familiar Shakura-Sunyaev parameter. Similarly, we define α_k by $k = V_L^2/2 \equiv \frac{1}{2}\alpha_k c^2$. With the stipulation that true turbulent eddies cannot have frequencies smaller than the local orbital frequency, we find $L \approx H\sqrt{\alpha_k}$, and $V_L \approx c\sqrt{\alpha_k}$.

There is current concern about how, or whether, turbulent kinetic energy is maintained in neutral nebula gas in the terrestrial planet region (9), which is related to the subtle distinction between α_ν and α_k. If $\alpha_\nu << \alpha_k$, for instance, the energy source for k would need to be something other than simple viscous evolution of a Keplerian disk. There are several energy sources (9) for which typical estimates of α_k are $\sim 10^{-4}$ - 10^{-2}, and thus $Re = 10^8$ - 10^{10} (10). Thus, in spite of pressing concerns, the issue of turbulence appears to remain open. Henceforth we neglect the distinction between α_k and α_ν and treat Re and its associated α as providing L and V_L, whether or not their product constitutes a net positive Reynolds stress.

We presume a Kolmogorov-type inertial range, within which each length scale l is characterized by velocity $v(l) = V_L(l/L)^{1/3}$ and eddy frequency $\omega(l) = v(l)/l = \Omega(l/L)^{-2/3}$, where in the nebula $\Omega = \omega(L) = V_L/L$ is the local orbital frequency under solar gravity (3,7). Because the most interesting scales for particle concentrations are on the order of $\eta << L$, deviations from isotropy due to rotation are not a major concern. For particles smaller than the gas mean free path (ie, smaller than several cm radius under nebula conditions), the stopping time due to gas drag is $t_s = r\rho_s/c\rho_g$, where r and ρ_s are particle radius and internal density and ρ_g is the gas density. The particle Stokes number $St_l \equiv t_s\omega(l)$ determines the particle response to eddies of a particular scale and frequency; previous studies have shown that the optimally concentrated particles have $St_\eta = t_s\omega(\eta) \approx 1$ (5), that is, their stopping time is comparable to the Kolmogorov eddy turnover time.

Our original work (3) concluded that the optimally concentrated particles in the terrestrial planet region of the nebula have radius and density comparable to those of chondrules; furthermore, TC is easily generalized to a wide range of nebula conditions. The expression from (3) for optimally concentrated particle size may be rewritten as $r\rho_s = (m_{H_2}/4\sigma_{H_2})^{0.5}(\Sigma/\alpha)^{0.5} \approx 1.4 \times 10^{-3} R_{\rm AU}^{-0.75}\alpha^{-0.5}$ g cm^{-2}, where Σ is surface mass density, m_{H_2} and σ_{H_2} are the mass and cross-section of H_2, and properties of the central scale height in a Hayashi-type "minimum mass" nebula are assumed (11). This relationship is shown in **figure 1** for several different typical locations. Note that porous aggregates (PA), having considerably lower radius-density product than chondrules, are optimally concentrated at the low gas densities which characterize the outer planet region or low density regions high above the nebula midplane. Such porous objects are easily produced because the low relative velocities in turbulence of low-density grains and their constituent monomers lead to large sticking efficiency and minimal disruption (13). TC may thus have been ubiquitous, initiating the formation of "cometesimals" from porous grain aggregates at 10-30 AU. However, textural evidence might be difficult to obtain from this regime; subsequent compaction would obliterate evidence for any preferred size or density of easily squashed fluffy constituents. Chondrules and their parent meteorites, by nature of their availability and unique, persistent textures, provide the most obvious initial testing ground for turbulent concentration.

Concentrated particle size distribution:

We have recently determined the properties of turbulent concentration from direct numerical simulations of homogeneous, isotropic, incompressible 3D turbulence at three Reynolds numbers Re = 110, 426, and 1300 (6,14). The particles can be given arbitrary aerodynamic

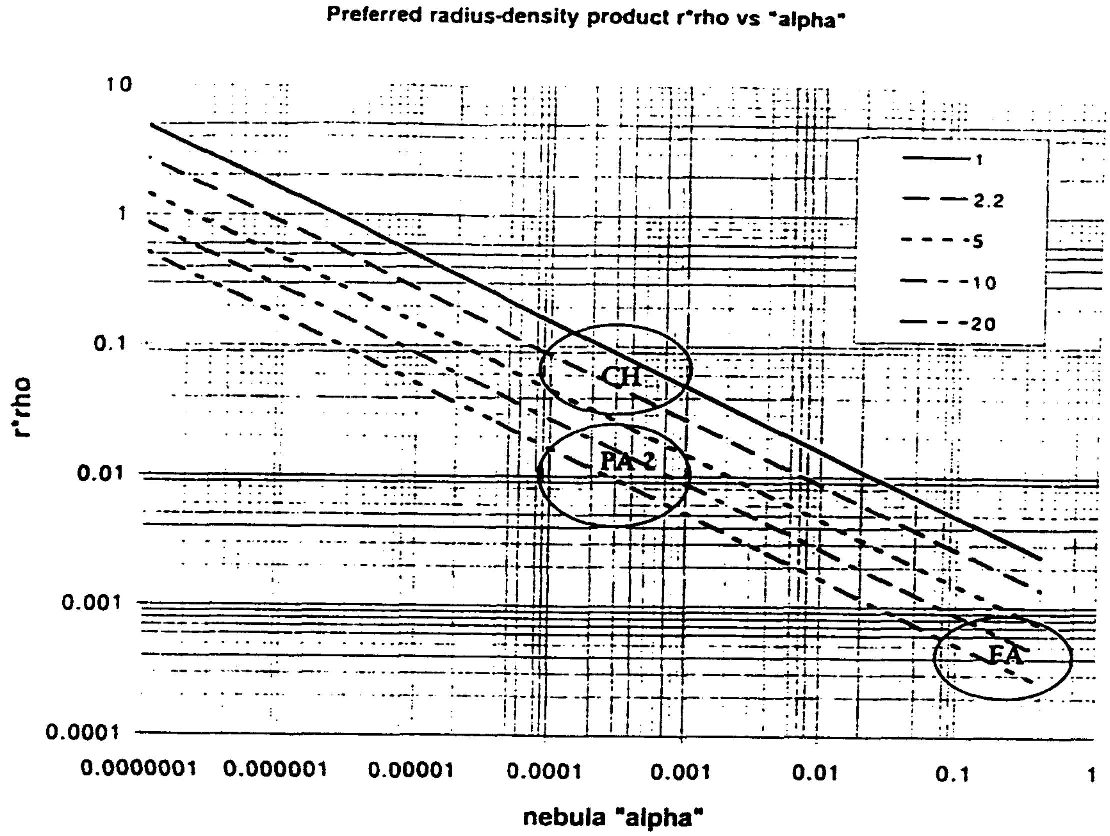

Figure 1: The optimal Stokes number for turbulent concentration, converted to the product of particle radius and density by adopting nominal nebula gas density and local rotation rate at various locations in a typical solar nebula, as functions of α. Solid silicate particles with chondrule sizes (CH) are favored in the terrestrial planet region for nominal α (3). Smaller gas densities at larger distances from the sun concentrate smaller $r\rho_p$ products (porous aggregates, or PA) for any α. It has also been suggested that extremely low density, intensely turbulent regions may concentrate tiny grains or very fluffy aggregates (FA) (12).

stopping times t_s: their motions respond only to gas drag and are integrated in the spatial domain. The computationally intensive calculations are run on 16 C'90 cpus at Ames Research Center.

One important finding relates to the detailed form of the size distribution of selectively concentrated particles. If dense clouds of particles are precursors of primitive bodies, the distribution with Stokes number of particles in dense regions should correspond to the distributions found in chondrites. Simulations were initiated with uniform spatial distributions of particles, themselves uniformly distributed in stopping time over the range $St_\eta = 0.1$ - 6 (14), and the relative equilibrium abundance of particles was studied as a function of St_η and C. In the large-C limit of interest, the shape of the distribution of relative abundance *vs* St_η was found to be *independent* of both C and *Re* (14). Thus, the numerical results should be valid as a prediction of the size distribution in clouds under nebula conditions.

In **figure 2** we compare a histogram of relative particle numbers as a function of St from our numerical simulations, with binned data for 253 disaggregated chondrules from the L4 ordinary chondrite (OC) ALH85033. While a few ostensibly similar data sets exist in the literature (15), data for this and several other meteorites imply that merely measuring the chondrule radius distribution and assuming some mean density misrepresents the $r\rho_s$ distribution because of chondrule-to-chondrule density variations (16). The meteorite histogram was aligned horizontally with the predicted histogram by assuming that the optimally concentrated particle size lies at $St_\eta = 1$. The *shape* of the theoretical profile contains no adjustable parameters, and the data and model profiles are in excellent agreement. The implication is that TC by itself can explain the very narrow chondrule size distribution, whatever the chondrule *formation* process may have produced. Larger or smaller "chondrules"

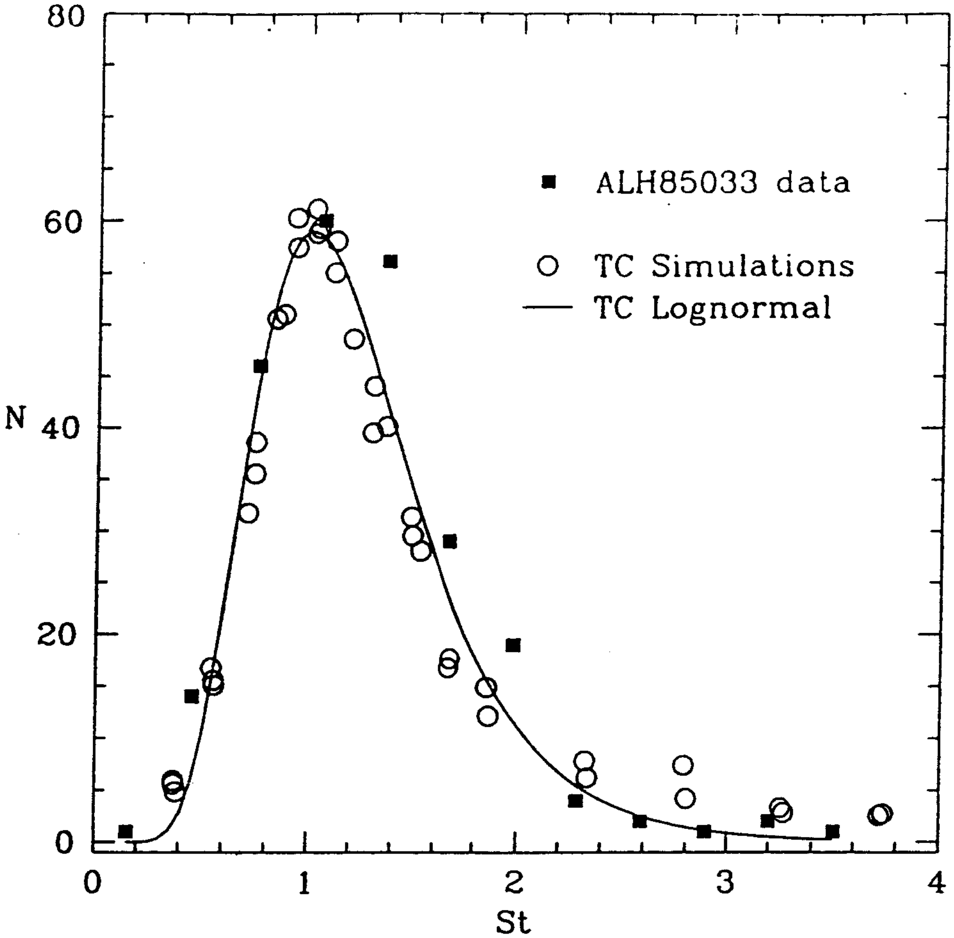

Figure 2: Comparison of the size-density distribution of 253 chondrules disaggregated from L4 ALH 85033 (solid symbols) with the theoretically determined shape distribution (open symbols) for the relative abundance of particles concentrated in turbulence (14,16). The theoretical profile is independent of C and Re, close to lognormal in shape, and a good match to the shape of the data. Four different meteorites now exhibit agreement this good (Cuzzi et al 1999)

might have existed, but would simply not be concentrated to such a significant extent for delivery to the midplane unless swept up into chondrule rims or broken to the proper size. Of course, some particles of arbitrary size will always appear just by accident, and parent body processes will confuse the situation further.

Particle concentration is a multifractal with Re-independent properties: A nonspecialist's definition of a fractal is a structure which is generated by sequential application of a rule on regularly decreasing spatial scales. Simple fractals with constant (but non-integer) dimension result from rules which produce a binary distribution of the local density (say, either 0 or 1) - and are invariant to changes in scale. Examples of these are the Cantor set or the Sierpinski gasket, in which segments of a line, or portions of a plane (17), are simply removed without changing the surrounding values. Their average density, as a function of scale ϵ, may be written as $\rho(\epsilon) = \rho_0 \epsilon^{-D}$ with dimension $D = 0.63$ and $D = 1.52$ respectively.

For comparison, *multifractals* result from application of rules in which the local measure is changed while *conserving* the total measure - for example, by unequal (but invariant) repartitioning of the content of a bin into sub-bins at each of the many subsequent stages of a cascade process (17). Such quantities have no well defined local value in the limit of diminishing bin size; that is, their local values are spatially spiky or "singular" (17). Considerable study has been devoted to multifractals in turbulence, because turbulence (and specifically its inertial range) is the archetype of a cascade process (7,17). For instance, *dissipation of turbulent kinetic energy,* which occurs on the Kolmogorov scale, is not spatially uniform but has the spatial distribution of a mutifractal (18). The spatial distributions of multifractals are predictable in a statistical sense, using probability distribution functions (PDFs) which are derived directly from their dimensions.

In multifractals, the dimension *varies* with the value of the *measure*: in our case, the measure is particle concentration C which is defined in terms of ϵ^a, where a is a scaling index which can be regarded as a local dimension for C. The key statistical descriptor of a multifractal is its *singularity spectrum* $f(a)$, where $f(a)$ plays the role of a dimension of the probability distribution function for C (19,18,6) and is the key element in the PDF's (discussed further below).

To make predictions at values of Re very different from directly accessible values, thus, the Re–dependence of $f(a)$ is crucial. Using the methodology of (18), we have shown that the spatial distribution of optimally concentrated ($St_\eta = 1$) particles is a multifractal, with $f(a)$ invariant over more than an order of magnitude in Re (Re=100 - 1300) (6). This Re-invariance for $f(a)$ is *only* seen if binning is tied to some fundamental flow-relative scale such as the Kolmogorov scale η, which varies with Re in a known way. Dissipation has also been shown to have an Re-independent $f(a)$ - from numerical work at $Re \sim 100$, including our own, through laboratory experiments with $Re \sim 10^4$, to experimental studies of the atmospheric boundary layer with $Re \sim 10^6$ (20,18). Particle concentration and dissipation are physically connected through their preference for the Kolmogorov scale, and the shapes of the singularity spectra for dissipation, and for particle concentration binned at close to the Kolmogorov scale, are similar (6). These facts suggest that the physics which leads to turbulent concentration is closely related to the turbulent cascade process - as for dissipation. The turbulent cascade is known to have statistically predictable, Re-independent properties in the inertial range. Based on these arguments, we believe and presume that the particle concentration singularity spectrum $f(a)$ remains Re-independent to far larger values than those of our numerical experiments. Given this invariance, we can predict nebula conditions more confidently than from extrapolation alone, as was done in (3).

The scaling index a is related to C as follows: the fractional probability P_i of a particle lying in a bin which contains N_i particles out of N_p total particles is defined as $P_i = N_i/N_p \equiv \epsilon^{a_i}$, where $\epsilon \equiv$ bin size/domain size. The associated concentration factor $C_i \equiv \frac{N_i/v_i}{N_p/v}$, where v_i is the volume of a bin and v is the total volume of the domain; thus $C_i = \frac{N_i/N_p}{v_i/v} = (\epsilon^{a_i}/\epsilon^3) = \epsilon^{a_i - 3}$. As mentioned above, $Re-$invariance of $f(a)$ is only expected when the bin size is some multiple J of η. The domain normalized bin size is then $\epsilon = J\eta/ML = \frac{J}{M}Re^{-3/4} \equiv 1/\mathcal{R}$, where ML is the domain size (18) expressed in units of the integral scale L, and we use the inertial range relationship $\eta = LRe^{-3/4}$. Thus, $C = \mathcal{R}^{3-a}$, or $a = 3 - \frac{\ln C}{\ln \mathcal{R}}$. For a nebula characterized by some turbulent α, with bin size 2η, integral scale $L = H\sqrt{\alpha}$, and a domain size of $2H$ (21), one obtains

$$\mathcal{R} = \alpha^{1/4}(cH/\nu_m)^{3/4}. \tag{1}$$

The normalized PDF for a is usually written as $F(a) = \rho(a)\epsilon^{3-f(a)}$, where the prefactor $\rho(a)$ can be approximated as $\sqrt{\ln \mathcal{R}}$ (18). We define the PDF $F_v(C)$ as the volume fraction occupied by bins having concentration factor C, with $\int_{C_{min}}^{C_{max}} F_v(C)dC = \int_{a_{max}}^{a_{min}} F(a)da \equiv 1$. Transforming variables and their PDFs, we get

$$F_v(C) = F(a) \mid \frac{da}{dC} \mid = \frac{\rho(a)}{C\ln\mathcal{R}}\mathcal{R}^{f(a)-3} \approx \frac{1}{C\sqrt{\ln\mathcal{R}}}\mathcal{R}^{f(a)-3}. \tag{2}$$

Note how the function $f(a)$ assumes the role of a dimension for $F_v(C)$. The fraction of *particles* occupying bins with C is $F_p(C) = CF(C)$, and both $F_v(C)$ and $F_p(C)$ have cumulative versions $F_v(> C) = \int_C^\infty F_v(C)dC$, and $F_p(> C) = \int_C^\infty F_p(C)dC$. We have obtained numerical results which validate the (ergodic) assumption that the fraction $F_p(> C)$

(spatially averaged over all particles at several snapshots in time) is equal to the fraction of *time* $F_t(>C)$ spent by a *given* particle in regions denser than C (temporally averaged over extended trajectories for a few particles).

If $f(a)$ is indeed a Re-independent, universal function for optimally concentrated ($St_\eta = 1$) particles, we can then predict PDF's for any given nebula $\alpha(=\alpha_k)$ using equations (1) and (2) derived above. As a check on the method (there are subtle normalization issues). an average $f(a)$, obtained from our numerical calculations at all three Re values (22), was used to calculate $F_p(>C)$ $(=F_t(>C))$ for comparison with the distributions directly determined from numerical results at our three Re values (6). As shown in **figure 3**, the single $f(a)$ does quite well at predicting the PDFs at all three Re. Also shown in **figure 3** are associated predictions of $F_p(>C)$ for four values of nebula α, using equations (1) and (2). Based on these predictions, $St = 1$ particles spend 10 percent of their time in regions with $C > 10^3$, and 1 percent of their time in regions with $C > 10^4$, under nebula conditions. These results might help us understand why some chondrule types seem to have been formed under highly oxidizing conditions (1).

Encounters with dense clouds, and entrapment: The PDF's can then be combined with the particle velocity through space V_p to calculate the "encounter time" T_{enc} of a chondrule with a region of arbitrary C, using a duty cycle argument: $F_t(>C) \approx \frac{t_{in}(>C)}{t_{out}} \approx \frac{t_{in}(>C)}{T_{enc}}$, where $t_{in}(>C) << T_{enc}$ is the time spent traversing a bin with concentration greater than C. Thus, for bins of dimension 2η and particle velocity V_p, $t_{in}(>C) = 2\eta/V_p$, so $T_{enc} \approx \frac{t_{in}(>C)}{F_t(>C)} = \frac{t_{in}(>C)}{F_p(>C)} = \frac{2\eta}{V_p}\frac{1}{F_p(>C)}$, and the encounter rate is $\frac{1}{T_{enc}} = \frac{V_p}{2\eta}F_p(>C)$. We have verified numerically that, as expected for particles with stopping times t_s much shorter than the overturn time of the largest eddies Ω^{-1} (23,11), V_p is nearly identical to the typical gas

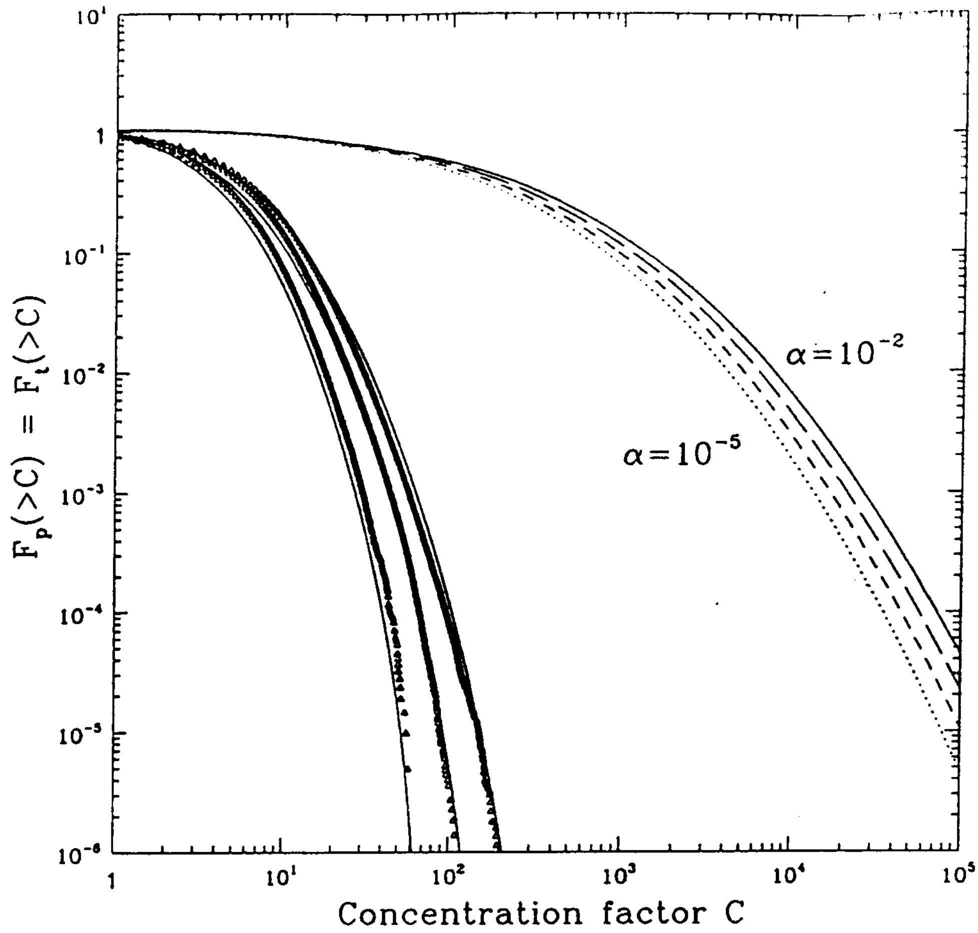

Figure 3: Probability Distribution Functions (PDF's) for the fraction of particles lying in regions with concentration factor greater than C ($F_p(> C)$), or, equivalently, the fraction of time spent by any particle in such regions ($F_t(> C)$) as discussed in the text. The three sets of points are binned directly from our numerical simulations; the associated curves are calculated from a single averaged $f(a)$ obtained from all three values of *Re*. The curves without points use the *same* $f(a)$ to predict PDF's at the larger *Re* corresponding to four plausible nebula α values: 10^{-2} (solid line), 10^{-3} (long dashed line), 10^{-4} (short dashed line), and 10^{-5} (dotted line).

velocity V_g, where $V_g \approx V_L \approx \sqrt{2k} \approx c\sqrt{\alpha}$ (23).

Specifically, we may calculate the encounter rate (and time) with a cloud so dense that a particle becomes entrapped with its neighbors and removed from further free circulation. Normally, particles traverse dense regions without incident, as their gas drag stopping time is longer than their transit time (3). An entrapment threshold occurs when interparticle collisions prevent particles from passing through a cloud; this implies a critical cloud optical depth τ_{coll} of unity, defining a critical C_{coll}. For small, dense 2η-sized clumps, $\tau_{coll} = 1 = 2\eta C_{coll}(\rho_{ch}/m)\pi r^2$, where ρ_{ch} is the average (unconcentrated) chondrule mass density, and m and r are chondrule mass and radius. Then $C_{coll} \approx 300 r \rho_s / \eta \rho_g f_{ch} \approx 3 \times 10^5 / f_{ch}$, where f_{ch} is the fraction of all solid mass in chondrule form, $\rho_s \sim 3$ is the density of a chondrule, and nominal parameters at 2.2 AU are assumed (11). Such clouds are orders of magnitude denser than the gas ($\rho_{cl}/\rho_g \sim 1500$ for $f_{ch} = 0.3$ and $C_{coll} \sim 10^6$). For these parameters, and the PDF's we have in hand, T_{enc} is on the order of 10^4 years - short compared to nebula evolution timescales. This quantifies the first part of our earlier suggestion (3) that such an encounter at $T_{enc}(C_{coll})$ ends the freely wandering life of a chondrule. A second stage, in which the chondrule and its cluster of neighbors might settle toward the midplane under the vertical component of solar gravity as a dense, but still far from solid, unit (3), remains more speculative and is under study. Alternately, TC might merely provide a large increase in the collisional aggregation rate of optimally sized particles.

Discussion:

Limitations due to mass loading: Even with this approach being clear in principle, there is one important unknown factor remaining. The cascade process model is only valid as long as no "new" physics emerges at some step in the cascade to change the "rule", or

$f(a)$, applicable to subsequent steps in the cascade. However, the above predictions imply that particle collisions trap particles in small, dense clouds only when $\rho_p/\rho_g \geq 10^3$. While these high concentrations literally relate to regions comparable to η in size, where there is no turbulence to damp, the cascade that *produced* such a cloud from its "penumbra", must have extended to larger sizes within which turbulent motions *may* be damped by *lower* concentrations. We have made some preliminary calculations of $f(a)$ for mass loading $\rho_p/\rho_g \sim 1$. Turbulent concentration persists, but its $f(a)$ is altered in the sense that high C values have a lower probability. Clearly, this effect must be quantified before more specific predictions of T_{enc} and other accumulation timescales can be made.

Complications due to cloud evolution and parent body processing: The subsequent evolution of dense clouds in the vertical component of solar gravity must still be studied. Do they retain their identity or disperse? In subsequent stages, solids must be compacted to orders of magnitude higher density than the dense clouds produced by turbulent concentration - possibly due initially to collisions between dense clouds *en route* to, or within the midplane, followed by midplane sweepup of the dense clouds or their constituents by objects which can begin as small as ten meters (*ie* (11), section 5.4; also (13)). The fact that most primitive meteorites contain ample evidence for abrasion, fragmentation, and other mechanical processes which may well have continued long after the nebula gas vanished and aerodynamic processes became irrelevant, warns us that we should not expect turbulent concentration to explain all properties of even "primitive" meteorites.

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Bibliography

1) Aerodynamic sorting discussed by Dodd, R.T. (1976) E. P. S. L. 30, 281-291; Skinner W. R. and J. M. Leenhouts (1991) 24th L.P.S.C.; Keubler, K., H. McSween, and W. D. Carlson (1997) 28th LPSC Abstracts p. 773. For general discussions of chondrules see Grossman, J., A. E. Rubin, H. Nagahara, and E. A. King (1989) in "Meteorites and the early solar system", J. F. Kerridge and M. S. Matthews, Univ. of Arizona Press, the book "Chondrules and the Protoplanetary Disk", edited by R. H. Hewins, R. H. Jones, and E. R. D. Scott, Cambridge University Press, Cambridge, UK, 1996, and R. Jones et al (2000) in "Protostars and Planets IV", V. Mannings, A. Boss, and S. Russell, eds; Univ. of Arizona Press, in press.

2) Whipple, F. (1966) Science 153, 54-65; Liffman, K. and M. Brown (1995) Icarus 116, 275; Shu, F. H., H. Shang, A. Glassgold, and T. Lee (1997) Science 271, 1545; Akridge, G., P. Benoit, and D. Sears (1998) Icarus 132, 185.

3) Cuzzi, J. N., Dobrovolskis, A. R. , and Hogan, R. C.. in Hewins et al (1996), cited in (1) above.

4) Grossman, J. (1989) in Kerridge and Matthews (1989) cited in (1); Boss, A. in Hewins

et al (1996) cited in (1); also Connolly and Love (1998) Science 280, 62-67

5) Squires, K. and J. K. Eaton (1990) Phys. Fluids A2, 1191; Wang, L. and M. R. Maxey (1993) J. Flu. Mech. 256, 27-42; Fessler, J. R., J. D. Kulick, and J. K. Eaton (1994) Phys. Fluids 6, 3742-3749; For a recent review see Eaton, J. K. and J. R. Fessler, (1994) Int. J. Multiphase Flow 20, Suppl., 169-209.

6) Hogan, R. C., Cuzzi, J. N., and Dobrovolskis, A. R. (1999) Phys Rev E, in press

7) Tennekes, H., and Lumley, J. L. (1972) A First Course in Turbulence, MIT Press, Cambridge Mass.

8) The distinction between turbulent viscosity and turbulent diffusivity is discussed by Prinn, R. J. (1990) Ap. J. 348, 725-729; for a recent model of how they differ in keplerian disks see Kato and Yoshizawa (1997) Pub. Ast. Soc. Jap. 49, 213-220

9) Balbus, S. A., J. F. Hawley, and J. M. Stone (1996, Astrophys. J 467, 76) have questioned differential rotation (suggested by Dubrulle 1993, Icarus 106, 59) as a source of turbulence. Richard and Zahn (1999; Astron. Astrophys. submitted) have reopened the issue of differential rotation as a source. However, the results of Kato and Yoshizawa (1997, *loc. cit*) seem to support the Balbus et al. position. Other possible sources for turbulence in the nebula midplane at the stage in question include thermal convection (Cabot et al 1987 Icarus 69, 387-422; Goldman and Wandel 1994 Ap. J. 443, 187-198), and ongoing supersonic gas infall (Prinn, R. 1990, Ap. J. 348, 725-729).

10) Here and in subsequent expressions we adopt for R =2.2 AU: $r_{ch} = 150\mu$, $\rho_g = 1.6 \times 10^{-10}$ g cm^{-3}, $\nu_m = 6 \times 10^{-5}$ cm^{2}sec^{-1}, $c = 8 \times 10^4$cm sec^{-1}, and $H = 0.05R$; G is the gravitational constant. Then $\eta = LRe^{3/4} = (H/\alpha)^{1/4}(\nu_m/c)^{3/4} \sim 0.5$km $(10^{-4}/\alpha)^{1/4}$.

11) Cuzzi, J. N., A. R. Dobrovolskis, and J. M. Champney (1993) Icarus, 106, 102

12) Wood, J. A. (1998) Astrophys. J. 503, L101-L104

13) Weidenschilling, S. J. and J. N. Cuzzi (1993) in "Protostars and Planets III, E. Levy and J. Lunine, eds, Univ. of Az. Press; also Weidenschilling, S. J. (1996) Icarus 127, 290-306; Chokshi et al (1993) Ap. J. 407, 806-819; Cuzzi, J. N. et al (1998) 29th LPSC Abstracts.

14) Hogan, R. C., and Cuzzi, J. N. in preparation

15) Hughes, D. W. (1978, EPSL, 38, 391-400), and Eisenhour, D. (1996, Meteoritics, 31, 243-248) treat all chondrules as identical except for radius; Rubin, A. and J. Grossman (1987, Meteoritics 22, 237-251) note how chondrules of different mineralogy do have different size distributions but don't correlate them quantitatively.

16) Cuzzi, J. N., R. C. Hogan, and J. M. Paque (1999) 30th LPSC

17) Frisch, U. and Parisi, G. (1985) in "Turbulence and predictability in Geophysical Fluid Dynamics and Climate dynamics", M. Ghil, ed. (N. Holland-Amsterdam); Meneveau, C. and Sreenivasan, K. R. (1987) Phys. Rev. Lett. 59, 1424-1427; Mandelbrot, B. (1989) Pure and Appl. Geophys, 131, 5-42; Meneveau, C. and Sreenivasan, K. R. (1991) J. Fluid Mech. 224, 429-484

18) Chhabra, A. B., Meneveau. C., Jensen, V. R. and Sreenivasan K. R. (1989), Physical Review A 40, 5284-5293

19) Halsey, T. C., M. H. Jensen, L. P. Kadanoff, I. Procaccia, and B. I. Shraiman (1986); Phys. Rev. A. 33, 1141-1151

20) Hosogawa, I. and K. Yamamoto (1990) Phys. Fluids A, 2, 889-892

21) The size of the domain is a critical parameter in the theory, because it influences the value of the PDF's for C. There seems to be no current formal theory for determining the size of the domain or its Re-dependence (K. R. Sreenivasan, personal communication,

1999). However, we have developed arguments from a simple cascade model which we believe show that for large nebula Re, and relevant values of C, the formal domain is much larger than the vertical thickness of the nebula. That is, the domain is *physically limited* rather than mathematically defined. In this case, a cube $2H$ on a side is the appropriate domain, supporting equation (1) and the results of figure 3. An annulus of cubes this size can also be handled as $2\pi R/2H \approx 60$ "realizations", much as we use multiple realizations of our own computational domain to build up statistics. A full exposition of these arguments is beyond the scope of this short paper.

22) It is important to use the $f(a)$ derived from "annealed" *vs* from "quenched" averaging because we are interested in low-probability events; the distinction is discussed by Chhabra, A. B. and K. R. Sreenivasan (1991) Phys. Rev. A. 43, 1114-1117. Our $f(a)$ is well fit by the function $f(a) = -12.414 + 89.659/a - 132.01/a^2$.

23) Völk, H. J., F. C. Jones, G. E. Morfill, and S. Röser (1980) Astron. Astrophys. 85, 316; also Markievicz, W. J., H. Mizuno, and H. J. Völk (1991) Astron. Astrophys. 242, 286-289