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Paper Title: Deformations of Flat Unsymmetric Laminates Subjected to Inplane Loads

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ABSTRACT

The geometrically nonlinear deformation response of initially flat unsymmetric cross-ply laminates subjected to an inplane compressive load and two sets of boundary conditions is studied. Stability of the deformations is considered. At issue is whether or not the plate remains flat with increased compressive loading, and whether it buckles. A semi-infinite unsymmetric cross-ply laminate is used to show the combined effects of geometric nonlinearities and bending-stretch coupling. Finite element results for finite laminates are then presented, and it is shown that to a large degree the boundary conditions control the character of the deformation response. It appears that clamped boundary conditions support buckling behavior, in the classic sense of bifurcation, whereas simply-supported conditions do not.

KEYWORDS: unsymmetric laminates, bifurcation behavior, instability

INTRODUCTION

Throughout the development of composite materials and structures, there has been minimal interest in unsymmetric laminates. This lack of interest stems from the fact that with typical elevated temperature curing of epoxy matrix materials, initially flat unsymmetric laminates warp out of plane when cooled from the elevated cure temperature. The warpage is seen as a detriment. Additionally, when cooled there can often be multiple warped configurations. The laminate can be changed from one configuration to another by a simple snap-through action. This behavior is due to the fact that the warping leads to out-of-plane displacements that are many times the laminate thickness, and therefore, geometric nonlinearities are important. Nonlinear problems can often lead to multiple solutions, and an unsymmetric laminate is an example of this characteristic. Hyer [1-3], Hamamoto and Hyer [4], and Dano and Hyer [5] have explained the multiple-configuration behavior and the fact that stability of the cooled configuration is an important issue. Dano and Hyer [6, 7] have also studied the forces necessary to change the cooled laminate from one configuration to another. The existence of multiple configurations presents an uncertainty with unsymmetric laminates that is also considered a detriment.

Recently, to eliminate the need for an elevated temperature cure condition, and to provide increased flexibility with manufacturing, there has been an interest in using electron beams to cure epoxy matrix composites. Of course, epoxy resins have to be formulated for this form of curing, and though there is some degree of temperature rise with the electron beam approach, there is the potential for fabricating

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flat unsymmetric laminates. A flat unsymmetric laminate, or a laminate where the shape can be controlled by a tool plate as opposed to cooling effects, has significant potential for tailoring applications because of the inherent elastic coupling between inplane and out-of-plane effects. Specifically, the existence of elements of the B matrix provides the opportunity to tailor structural response in ways that are not possible with conventional unsymmetric laminates. Compared to the volumes of literature on the behavior of symmetric laminates to all varieties of loading, there is little literature for unsymmetric laminates. One important response that has been studied for unsymmetric laminates is buckling. However, Quta and Leissa [8] claim that much of this work is in error because the studies assume that an unsymmetric laminate remains flat under inplane compression, then buckles. With bending-stretching coupling present, an unsymmetric laminate will not generally remain flat when compressed inplane, and whether or not it actually buckles, in the classic sense of bifurcation, is open to debate. Leissa [9] argued that a flat general unsymmetric laminate will remain flat under inplane loading only if all four edges are clamped. For antisymmetric angle ply laminates, the four edges need only be simply supported. The key to the required support conditions lies with the moments needed at the edge to counter the moments produced by the inplane loads through the B matrix. Quta and Leissa [8] studied the geometrically linear response of four types of unsymmetric laminates subjected to uniform and linearly-varying inplane loads, including uniaxial, biaxial, and inplane shear loads. Twelve sets of boundary conditions were considered and the out-of-plane deflection, specifically the presence or lack of out-of-plane deflection, was investigated. To within the numerical accuracy of the finite-element analysis used, Quta and Leissa [8] confirmed the earlier assertions of Leissa [9].

The present work is an extension of the work of Quta and Leissa [8]. The paper considers only uniaxial compressive loading, only cross-ply laminates, and fewer boundary conditions. However, geometric nonlinearities are included and stability is studied. The following section is a brief tutorial, based on a semi-infinite laminate, that demonstrates the importance of including geometric nonlinearities when studying unsymmetric laminates. The section following that then describes the specific problem of interest, namely, a laminate with finite dimensions. Since the commercial finite-element code ABAQUS [10] is used, finite element considerations are discussed.

A ONE-DIMENSIONAL PROBLEM

To demonstrate the importance of geometric nonlinearities, consider, as in fig. 1, an initially flat unsymmetric cross-ply laminate, semi-infinite in the y-direction, and loaded by a uniform inplane applied load in the x-direction, N^{+}, at the boundary $x=+L/2$. To study the response of this plate, simple Kirchhoff plate theory will be used, as well as the well-known von Kármán nonlinear strain-displacement relations. Since the plate is semi-infinite in the y-direction, the partial derivative with respect to y of most variables is zero. As a further simplification, it will be assumed that the displacement of the geometric midplane in the y-direction, v, is zero. As a result of these assumptions, the governing partial differential equations reduce to a rather simple set of ordinary differential equations, which can be solved in closed form. To complete the problem, the inplane displacement, u, at $x=-L/2$ is restrained to be zero, and the out-of-plane displacement, w, at both boundaries is restrained to be zero. For simple support conditions $M_x = 0$ at both boundaries, and for a clamped conditions $dw/dx = 0$ at both boundaries. With the simplifying assumptions, the three governing equilibrium equations reduce to

$$\frac{dN_x}{dx} = 0 \qquad \frac{dN_{xy}}{dx} = 0 \qquad \frac{d^2M_x}{dx^2} + N_x\frac{d^2w}{dx^2} = 0 \tag{1}$$

and the constitutive equations become

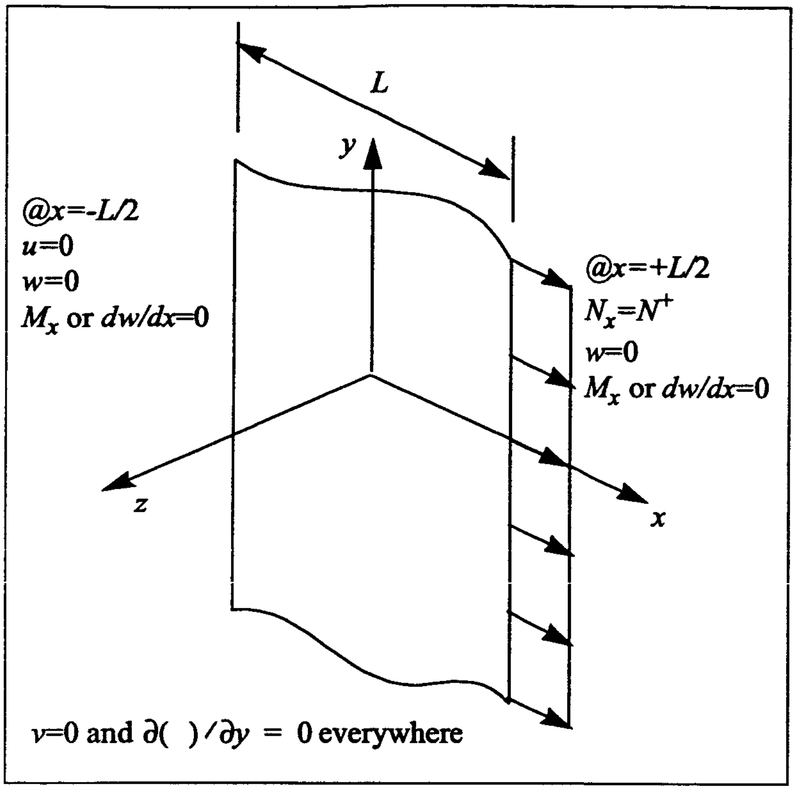

Figure 1 Geometry and nomenclature for semi-infinite laminate

$$N_x = A_{11}\left(\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right)-B_{11}\frac{d^2w}{dx^2} \qquad M_x = B_{11}\left(\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right)-D_{11}\frac{d^2w}{dx^2} \tag{2}$$

Of course, by the first equilibrium equation and the boundary condition at $x=+L/2$, $N_x(x) = N^+$ for all x. If it assumed that the plate response is symmetric with respect to the $x=0$ location, then the out-of-plane solution to the above problem is

$$w(x) = A_1 + A_2\cosh\left(\sqrt{\frac{N^+}{D_{11}^R}}x\right) \qquad \text{with} \qquad D_{11}^R = D_{11}\left(1-\frac{B_{11}^2}{A_{11}D_{11}}\right) \tag{3}$$

where the reduced bending stiffness D_{11}^R is defined. For simple supports, and positive and negative values of N^+, respectively, the solutions for $w(x)$ become

$$w(x) = \frac{B_{11}}{A_{11}}\left(\frac{\cosh\left(2\sqrt{\lambda}\frac{x}{L}\right)}{\cosh\sqrt{\lambda}}-1\right); \; w(x) = \frac{B_{11}}{A_{11}}\left(\frac{\cos\left(2\sqrt{\lambda}\frac{x}{L}\right)}{\cos\sqrt{\lambda}}-1\right), \qquad \text{with} \quad \lambda = \frac{N^+L^2}{4D_{11}^R} \tag{4}$$

where the non-dimensional loading parameter λ has been introduced. The non-dimensional out-of-plane displacement at the center of the plate, $w(0)$, can be defined as

$$\Delta = w(0)/\left(\frac{B_{11}}{A_{11}}\right) \tag{5}$$

Figure 2 illustrates the relation between the non-dimensionalized out-of-plane deflection at the center of the plate and the loading parameter. The geometrically linear solution for the deflection of the center of the plate,

$$\Delta = -\frac{\lambda}{2} \tag{6}$$

is also shown on the figure. It can be seen that when the applied load is tensile, the deflection predicted by the geometrically nonlinear solution is less, in magnitude, than the linear solution. When the applied load is compressive, the deflection predicted by the nonlinear solution is greater than the linear prediction. This occurs because as the load increases in tension, the coupling of the inplane load with the out-of-plane deflections through geometric nonlinearities, a coupling which tends to flatten the plate, overpowers the coupling of the inplane load and out-of-plane deflections due to material effects, i.e., the B_{11} term, a coupling which is linear and causes the plate to deflect out-of-plane. When the applied load is compressive, the two coupling effects are additive, and the deflection is more than just the linear material coupling effect alone. Thus, for this simply-supported case, the plate does not remain flat when subjected to an inplane loading. For compressive loads, the deflection becomes asymptotically large at $\sqrt{\lambda} = \pi/2$.

For clamped boundaries, an interesting situation arises, namely, the application of the boundary condition results in an eigenvalue problem. That is, for arbitrary values of the loading parameter λ, the only solution is for the plate to remain flat. If the loading parameter represents a compressive load, then there are nonzero deflection solutions, with undetermined amplitude, if

$$\sqrt{\lambda} = \pi/2,\ 3\pi/2,\ 5\pi/2, \text{etc.} \tag{7}$$

This represents a classic bifurcation problem, and provides an example of bifurcation of an initially flat unsymmetric laminate subjected to inplane compression. Though Leissa [9] did not address semi-infinite plates, the contrast here between the simple-support and clamped boundary conditions are in line with those findings.

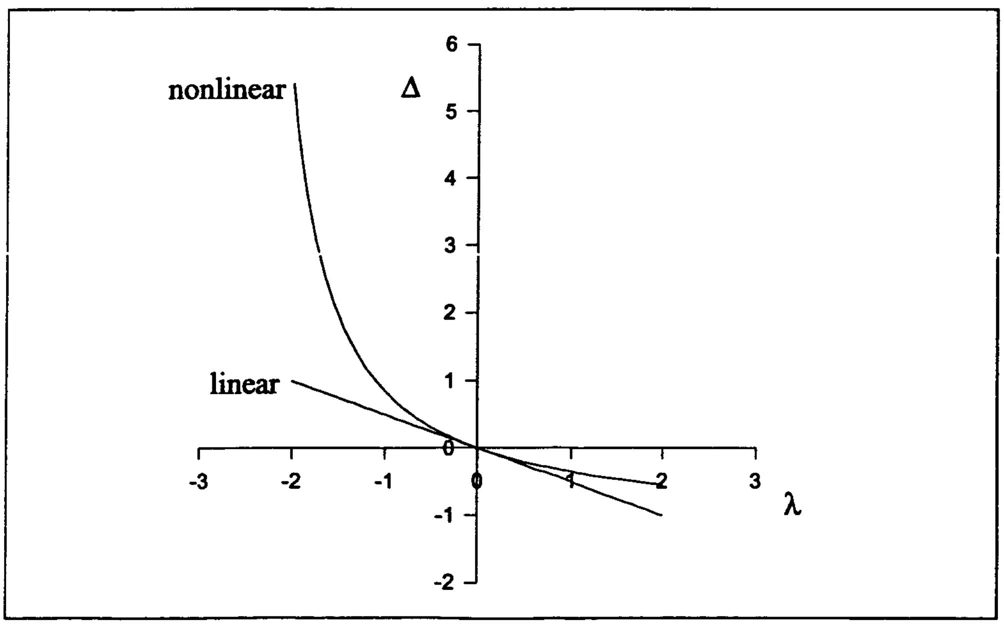

Figure 2 Out-of-plane deflection behavior for a simply-supported semi-infinite laminate

PROBLEM DESCIPTION AND ANALYSIS DETAILS FOR A FINITE LAMINATE

To study finite plates, rectangular laminates of dimension *L* by *W* shown in fig. 3 will be considered. The long dimensions of the plate are referred to as the sides and the short dimensions the ends. The structural *x-y-z* and principal material 1-2-3 coordinate systems, and the displacements and rotations, as defined in ABAQUS, are shown. Interest here is in the compressive loading and that is applied by displacing the right end of the plate a known amount *U* to shorten the plate. The associated load is denoted as *P*. Such a loading can be produced in the laboratory, and since it makes sense to begin any experimental investigation of the deformation response of unsymmetric laminates by considering small-scale laboratory size specimens, the plates considered are 0.508-m long by 0.406-m wide (20.0 in. by 16.0 in.). The plates are assumed to be constructed of eight-layers of graphite-epoxy and measure 1.016-mm (0.04 in.) thick. The properties of a layer are assumed to be

$$E_1=130.0 \text{ GPa } (18.85 \text{ Msi});\ E_2=9.70 \text{ GPa } (1.407 \text{ Msi});\ G_{12}=5.00 \text{ GPa } (0.725 \text{ Msi})$$
$$\nu_{12}=0.300;\ h=0.1270 \text{ mm } (0.005 \text{ in})$$

Out-of-plane displacement vs. endshortening results will be presented for the three locations labeled *I*, *II*, and *III* in fig. 3. Locations *I* and *III* are quarter locations, while location *II* is at the center. Plots of the deformed geometry will be presented for specific endshortening levels. Additionally, the relationship between endshortening and the associated load *P* will be presented. Two combinations of boundary conditions will be prescribed along the ends and sides, namely, simply supported on all four boundaries, SS-SS, and clamped on all four boundaries, CL-CL. To provide some commonality with the semi-infinite plate discussed in the previous section, the plates are constrained so that v=0 along the sides. Practically speaking, this condition has merit because components of a structure are generally joined to other components, resulting in some degree of constraint along the edges. Rather than look at a range of elastic restraints, adding another variable to the problem, the v=0 condition represents one end of the range. This issue is important because, through Poisson effects, inplane compression in the *x*-direction can cause the plate to expand in the *y*-direction. With v=0 along the sides, Poisson expansion is restricted by an inplane compression force in the *y*-direction. With the bending-stretching coupling present in an unsymmetric laminate, this inplane force, along with the applied inplane displacement, are factors.

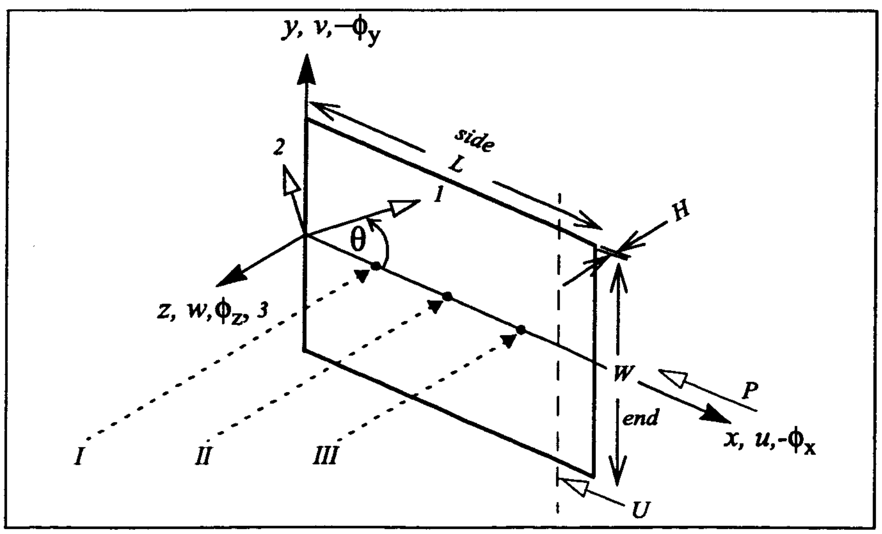

Figure 3 Description and nomenclature of plate with finite geometry

Nine-noded, two-dimensional, geometrically nonlinear, S9R5 shell elements in the ABAQUS finite-element code are used to model the plates. The plates are compressed statically by increasing the endshortening using very small increments of U. At each value of U the static stability of the plate is checked by examining the eigenvalues of the tangent stiffness matrix, a zero or negative eigenvalue indicating a statically unstable laminate configuration. At the value of U where the static solution ceases to be stable, some iteration is then done to adjust U so that the value of U just at the onset of the instability can be closely determined. For a value of U slightly greater than this onset value, and for this value of U held fixed, a dynamic analysis, using a small amount of damping, is then initiated. For a statically unstable condition, simply initiating a dynamic analysis should be enough to allow the plate to begin to move toward a stable equilibrium configuration. However, for some unstable conditions, this requires integrating forward in time for long periods, as the plate may move slowly from its unstable configuration. Accordingly, as an alternative to simply initiating the dynamic analysis, the plate is given a very small pressure pulse perpendicular to the plate at time zero of the dynamic analysis. The dynamic analysis is carried out for small pressure pulses in both the $+z$ and $-z$ directions (see fig. 3). For all cases studied here, if pressure pulses are used, the plate moves dynamically to the same configuration independent of the sign of the pressure pulse. As the plate motion is decaying around this unique configuration, a new static analysis is initiated using a decayed dynamic configuration as a starting point. The static analysis then converges within a reasonable number of iterations to what is interpreted to be the statically stable configuration for that value of endshortening. This numerical scenario corresponds to a displacement-controlled compression test in the laboratory. Ten elements in each direction, for a total of 100 elements, are used to model the plates. To test the robustness of this mesh, for several specific test cases the mesh was doubled in each direction, for a total of 400 elements. All aspects of plate response were identical for the two meshes, including the dynamic stability analysis. Of course, the dynamic stability analysis took considerably more time with 400 elements than with 100 elements, so the 100 element mesh was used.

NUMERICAL RESULTS

Figure 4 presents the normalized load vs. endshortening relation, the normalized out-of-plane displacement vs. endshortening relations, and deformed plate configurations at selected values of endshortening for a SS-SS $[0_4/90_4]_T$ unsymmetric cross-ply laminate as the endshortening increases from zero to a normalized value of 10. The normalizing factor Ucr is the classic buckling value of endshortening for a SS-SS $[0_2/90_2]_S$ symmetric cross-ply laminate. The term 'classic' implies that the boundaries are free to move inplane ($v\neq0$) and are only constrained from out-of-plane deformation (w=0). This normalizing factor was chosen simply because it is easily calculated. Strain levels in the plate were not particularly high. The normalizing factor Pcr used to normalize P is the load associated with Ucr.

The three parts of fig. 4 for the out-of-plane displacements at locations I, II, and III show that as the endshortening is increased from zero, point A, the plate deforms out of plane a significant amount in a half-wave configuration. At point B, the load vs. endshortening relation decreases slope slightly. As the endshortening is increased, the plate configuration begins to change. When the endshortening reaches the level of point C, the plate appears to have two half waves in the loading direction. A close look at the out-of-plane displacements at location *II*, however, reveals that the displacement is not quite zero in the center of the plate. At point C the plate becomes unstable, and a dynamic analysis is used to affect the transition to point D. At point D the deformed configuration looks like that at point C, but the out-of-plane displacement at location *II* is positive, while the magnitudes of the out-of-plane displacements at locations *I* and *III* have decreased relative to point C. A further increase in endshortening to point E does not produce any configuration changes. It is clear there is nothing that resembles bifurcation, or buckling, behavior in fig. 4. Rather, the plate makes a smooth continuous shape change as the endshortening increases from A to B to C. The transition from point C to point D could be referred to as a secondary instability. However, since there is not a primary instability, i.e., buckling, the nomenclature is not quite applicable. None-the-less, the transition from C to D is due to an instability.

If instead of enforcing simply-support boundary conditions on all four sides, clamped conditions are enforced, the response illustrated in fig. 5 occurs. As the endshortening is increased from zero, the plate appears to remain flat. According to Leissa [8], this should be the case, though the finite-element results here indicate there are very small out-of-plane displacements. As they are many orders of magnitude less than the out-of-plane displacements along path AB in fig. 4, these are felt to be a result of numerical round-off in the finite-element analysis. Increasing the endshortening past point B results in a statically unstable condition. The dynamic analysis leads to a statically stable equilibrium configuration whereby the plate is deformed out of plane in what appears to be a two half-wave configuration, as shown by the inset of the deformed geometry of point C. However, the configuration is not exactly a two half-wave one, rather there is a small out-of-plane displacement at the center of the plate, location *II*, as seen in the upper portion of fig. 5. With increasing endshortening past point B, the displacement at location *II* is at first negative, then positive, though it remains small. At locations *I* and *III* the displacements are much larger and have opposite signs. This pseudo-half-wave configuration remains throughout the range of endshortening studied. There is no secondary buckling behavior.

From the results presented, it can be concluded that bifurcation behavior is possible for flat unsymmetric cross-ply laminates with four clamped boundaries. Simple supports do not lead to bifurcation behavior. However, there appears to be an instability in the response at high levels of endshortening. Of course, neither the changing configuration with increasing endshortening for the simply-supported case nor the instability would be predicted by a geometrically linear analysis. For the clamped case, a linear analysis would predict the laminate would remain flat (to within the numerical tolerance of zero) for all endshortening levels.

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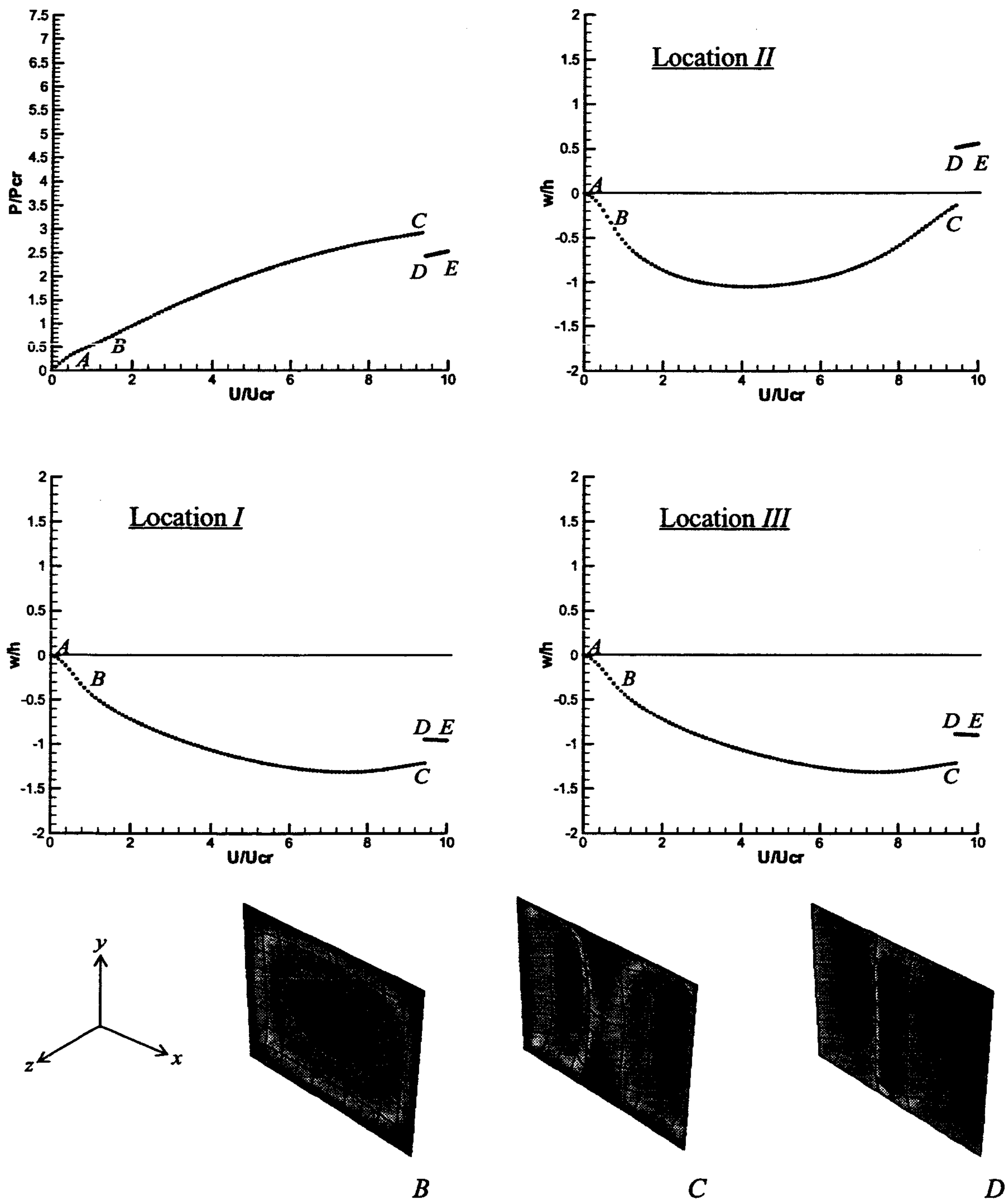

Figure 4 Response characteristics of SS-SS $[0_4/90_4]_T$ plate

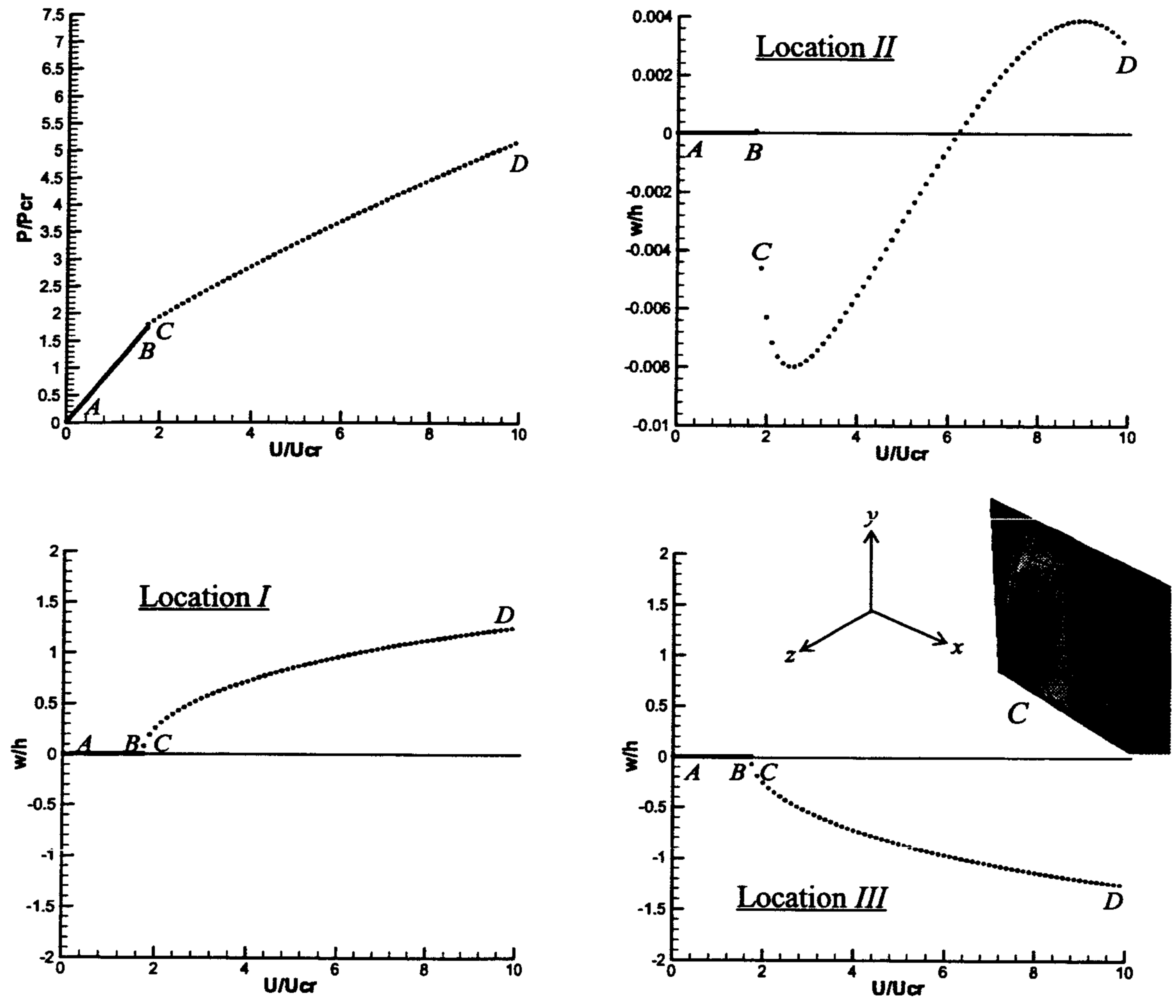

Figure 5 Response characteristics of CL-CL $[0_4/90_4]_T$ plate