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Improvements to the Two-Thickness Method for Deriving Acoustic Properties of Materials

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1. ABSTRACT

The characteristic impedance and other derivative acoustic properties of a material can be derived from impedance tube data using the specific impedance measured from samples with two different thicknesses. In practice, samples are chosen so that their respective thicknesses differ by a factor of 2. This simplifies the solution of the equations relating the properties of the two samples so that the computation of the characteristic impedance is straightforward. This approach has at least two drawbacks. One is that it is often difficult to acquire or produce samples with precisely a factor of 2 difference in thickness. A second drawback is that the phase information contained in the imaginary part of the propagation constant must be unwrapped before subsequent computations are performed. For well-behaved samples, this is not a problem. For ill behaved samples of unknown properties, the phase unwrapping process can be tedious and difficult to automate. Two alternative approaches have been evaluated which remove the factor-of-2 sample thickness requirement and directly compute unwrapped phase angles. One uses a Newton-Raphson approach to solve for the roots of the samples' simultaneous equations. The other produces a wave number space diagram in which the roots are clearly discernable and easily extracted. Results are presented which illustrate the flexibility of analysis provided by the new approaches and how this can be used to better understand the limitations of the impedance tube data.

2. INTRODUCTION

The two-thickness method is one of several methodologies available for measuring a material's acoustic properties in an impedance tube (see Song[1] for an overview). To obtain the characteristic impedance, two specific impedance measurements are made of two distinct thicknesses of material samples. To simplify derivation of the characteristic impedance, one sample thickness is constrained to be a factor of two of the other[2]. This can be a problem in that it is often difficult to obtain samples that meet the thickness constraint exactly, thus introducing an undetermined error into the analysis. The analysis is further hampered by a need to 'unwrap' the phase information contained in the solution before the propagation constant can be properly formed. Many of the measurement/analysis techniques described in the literature would have a similar requirement. In general, the phase unwrapping is difficult to automate and can obscure relevant data under certain conditions.

The two solution techniques described in the following sections can be used to analyze data taken from material samples of any two thicknesses as long as the thickness difference is sufficient to produce numerically meaningful data. In addition, the solutions produce the propagation constant directly so that phase unwrapping of intermediate terms is not necessary. Finally, the quality of the data can judged before the solution is extracted by viewing the wave number spectrum produced by the analyses. One technique applies the Newton-Raphson method to approximate the roots of the solution. The second technique produces a wave number/frequency space diagram from which the solution is easily extracted. One benefit realized by the relaxation of the sample thickness constraint is that several samples of various thicknesses can be measured and cross analyzed. This capability provides a means of verifying results and better understanding the limitations of the impedance tube data. Before these results are presented, the applicable theory and experimental setup are described.

2. THEORY

The expression relating specific impedance, $\mathbf{Z}_n$, to characteristic impedance, $\mathbf{Z}_c$, is

$$\mathbf{Z}_n = \mathbf{Z}_c \coth(\gamma d_n), \tag{1}$$

where γ is the propagation constant and d_n is the sample thickness. The propagation constant is a complex quantity whose real part is the absorption coefficient, α, and whose imaginary part is the wave number, k. If the two samples are constructed such that $d_2 = 2d_1$, expressions for the characteristic impedance and propagation constant can be derived as follows.

$$\mathbf{Z}_c = [\mathbf{Z}_1(2\mathbf{Z}_2 - \mathbf{Z}_1)]^{1/2}$$

$$\gamma = \left(\frac{1}{2d_1}\right)\ln\left(\frac{1+\mathbf{a}}{1-\mathbf{a}}\right) \tag{2}$$

where

$$\mathbf{a} = \left(\frac{2\mathbf{Z}_2 - \mathbf{Z}_1}{\mathbf{Z}_1}\right)^{1/2}$$

The specific impedance values for the two samples are obtained in an impedance tube as explained in the following section. If the two sample depths do not differ by exactly d_1, i.e., if $d_2 = 2d_1 \pm \varepsilon$, then some undetermined amount of error is introduced into the result. Also, the imaginary part of the propagation constant, the wave number, contains phase information that is bounded by $\pm\pi$ and is typically unwrapped before it is used in subsequent calculations. If the specific impedance data are well behaved, this process is straightforward and easily automated. However, it is often the case that the quality of the data degrades in both the low and high frequency limits. This obscures the phase information and introduces a degree of subjectivity into the analysis.

An alternative approach is to use the simultaneous equations in (1) to eliminate $\mathbf{Z}_c$, forming an expression in γ.

$$0 = \mathbf{Z}_1 \coth(\gamma d_2) - \mathbf{Z}_2 \coth(\gamma d_1) \tag{3}$$

The roots of (3) are valid solutions for the propagation constant and can be found using values for the specific impedances measured for material of any two thicknesses. The following two methods have been used to find the roots of (3).

A. Newton-Raphson Method

The Newton-Raphson method uses an algorithm of successive approximation based on a first order Taylor Series expansion of the function to which the roots are sought. If f(x) is the function and x_i the current approximation for a root of f(x) near x_i, then the next approximation for the root is given by

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}. \qquad (4)$$

Figure 1a shows an example of the algorithm converging to the root of sin(x) at π from a starting point at x=2. The algorithm may not find the nearest root as illustrated in Figure 1b where the starting point is x=1.8 and the converged root is 2π. To effectively use this approach, the set of x_i must be chosen carefully so that all roots are discovered, the inevitable redundant roots being removed from the solution set. An example of wave number solutions derived using the Newton-Raphson method is shown in Figure 2a. Several solution sets are found due to the periodic nature of the coth function. The valid set is the one that approaches (0,0).

B. Wave Number Space Method

In the wave number space method, eq. (3) is evaluated over the complex wave number/frequency range of interest. An example of the resulting data is illustrated in Figure 2b. The solution sets are the minima that are displayed as dark areas in the graph. Notice the similarity between these solutions and those derived with Newton-Raphson. As in the Newton-Raphson method the correct solution must be identified as that set which intercepts (0,0). These points are then extracted from the complex wave number/frequency matrix.

3. EXPERIMENTAL SETUP

The impedance measurements described in this paper were conducted using the NASA Langley Vertical Impedance Tube (VIT). This normal incidence impedance tube is mounted in a vertical configuration such that bulk materials can easily be tested. By using a vertical arrangement, the test specimen is held in place by gravity, thus eliminating the need for a restraining layer (e.g., a wire mesh screen). A diagram of the VIT, along with the instrumentation used, is shown in Figure 3. The tube is approximately 0.7 m long, with a square cross-section (51 mm by 51 mm). Two 120-watt, phase-matched acoustic drivers generate acoustic plane waves in the tube for frequencies up to 3.0 kHz, with sound pressure levels up to 140 dB at the test specimen surface. The test material is installed in an enclosure designed to achieve an airtight seal, then aligned and clamped to the tube exit.

Acoustic plane waves propagate down the tube to the test specimen face, and a portion of the acoustic wave is reflected. The combination of incident and reflected waves creates a standing wave pattern that uniquely characterizes the normal incidence acoustic impedance of the specimen. A 6.4 mm condenser-type microphone, flush mounted in the wall 6.4 mm from the face of the test specimen, is used as a reference microphone to measure the sound pressure level near the face of the specimen. Two additional 6.4-mm microphones are flush mounted in the side-wall of the VIT, 73 and 105 mm from the test specimen surface. As shown in Figure 3, these microphones are mounted in a rotating plug, such that their locations can be accurately and conveniently switched. A variation of the two-microphone method[4] as implemented at Langley[3] is used to measure the normal incidence impedance of each test specimen. For the current study, broadband noise tests were conducted at overall sound pressure levels (integrated over 0.5 to 3.0 kHz frequency range) of 120 and 140 dB, such that potential nonlinearities of the test specimen could be determined. A test specimen was considered to be linear if the results (acoustic impedance) were the same for both sound pressure levels.

Figure 3 also shows a block diagram of the signal conditioning instrumentation and data processing system for the acoustic signals. A random noise generator connected through a power amplifier to the acoustic drivers was used to supply broadband noise to the VIT. Transfer functions between the two measurement-microphone responses were recorded with a spectrum analyzer at frequencies of 0.4 to 3.0 kHz, in steps of 0.025 kHz. The two microphones locations were then switched (by rotating the rotating plug) and the measurement was repeated. By appropriate averaging of the two sets of data, the relative amplitude and phase at each of the two measurement locations was determined. These data were recorded for subsequent analysis[3], which was used to determine the normal incidence acoustic impedance of the test specimen.

4. RESULTS

To illustrate the use of the three methods (two-thickness, Newton-Raphson and wave number space), data taken from samples of polyimide microspheres are analyzed. The first data set is used to compare the results of the three methods and establish the validity of the two new methods. Subsequent examples illustrate the pros and cons of the two new methods. Finally, results from sample sets of several different thicknesses are used to point out the limitations of impedance tube data and how areas of data degradation can be recognized.

A. Example 1

These data were taken from 1 and 2 inch samples (2.54 and 5.08 cm) of microspheres with diameters between 425 and 600 microns. This dataset was chosen because it was well behaved, providing a clean example of phase wrap and how the three methods approach the solution. The phase wrap can be observed in Figure 4. This wave number is data derived from the imaginary part of γ, eq. (2). The phase wrap that occurs around 1400 Hz is easily unwrapped to yield the correct result in that the jump occurs in one sample interval and is equivalent to 2π radians.

The Newton-Raphson method solves for all the roots of eq. (3) in the given wave number/frequency domain. The user must select both the resolution and extent of this domain. A subset of the results returned for this dataset is shown in Figure 5. A second set of roots is shown in the higher wave numbers that illustrates the redundant solutions that are derived from the periodic function. This same behavior is observed in the Wave Number Space surface, Figure 6, where the dark areas represent local minima, i.e., the roots of eq. (3). The correct solution set is traced by the white line and can be seen to be equivalent to the solutions found using Newton-Raphson and Two-Thickness methods. As in the Newton-Raphson method, the correct solution set must be identified and extracted which may not be as straightforward as implied here. A more challenging case is presented in the next example.

B. Example 2

The data for this example are taken from foam made from the polyimide microspheres. The samples were 2.3 and 4.8 inches (5.8 and 12.2 cm). This material turned out to be not so well behaved as can be seen in the wave number plot in Figure 7. The low frequency wave number data are extremely erratic making the unwrap process somewhat hit and miss as is shown by example in Figure 8. Here, two sets of unwrapped data are displayed, one with the analysis starting at 300 Hz, the other at 325 Hz, one bin higher in frequency. As can be seen, the change in starting frequency causes a large shift in the wave number values. While it is obvious that the negative wave number data is erroneous (i.e., the data starting at 325 Hz), it is not clear that the positive data set is correct either as it doesn't seem to approach (0,0). The Newton-Raphson method, Figure 9, produces clear solution sets above 1500 Hz, but provides no clear association between the data points below 1000 Hz, leaving the open question of which solution is the valid set. The Wave Number Space method, Figure 10, produces a clearer picture of the solution sets and indentifies a solution similar to the 300 Hz solution produced by the two-thickness method, but it is still not clear that this is the correct solution as it appears the next higher set of roots would be more likely to intercept (0,0).

This behavior can be better understood if a region of valid data is defined. The wave number data extracted using the wave number space method is plotted in Figure 11. The 3 horizontal lines are associated with wavelength dependent behavior in the material. The top line is the wave number at which a half wavelength can exist in the material across the 2 inch (5.08 cm) diameter of the tube. At this point, about 60 rad/m, cross modes will begin to exist in the material. The center line is the wave number at which a 1/4 wavelength will exist across the thickness of the thinner sample and maximum particle velocities will be approached in the material. This corresponds to the point at which maximum absorption is achieved. At frequencies below this point, average velocity in the sample will decrease, lowering the sample material's influence on the reflected wave. The bottom line is the wave number at which one-tenth wavelength exists across the material thickness of the thinner sample. The sample cannot be expected to produce detectable effects below this point.

The 60 rad/m cut-off point can be used to select the valid root set. In the discussion above, some doubt is raised as to whether the selected set is indeed the correct one, as the data do not seem to trend toward (0,0). Inspection of Figure 10 indicates that most of the roots for the next higher set of data are above the cut-off point and, thus, not to be trusted. Confidence in the selected set is also increased as its behavior as shown in Figure 11 seems to fit the valid data region, i.e., the data become ill-behaved above 60 rad/m and below 10 rad/m. The low frequency data could be improved by using thicker samples, i.e., by lowering the 1/4 and 1/10 wavelength limits. Although, thicker samples would have this effect, the thicker samples would also increase absorption, decreasing signal to noise and possibly corrupting the data at the higher frequencies. These effects will be demonstrated in the next example.

C. Example 3

These data were generated using polyimide microspheres with diameters in the range of 425 to 600 microns. The sample thicknesses were 0.5, 1 and 2 inches (1.27, 2.54 and 5.08 cm). Two sets of data are presented, data generated using 0.5 and 1 inch samples and data generated using 1 and 2 inch samples. The data have been analyzed using the Wave Number Space method. The wave number data are shown in Figure 12. The solid lines are associated with data taken with the 1 and 2 inch samples, the dashed lines with the 0.5 and 1 inch samples. The horizontal lines are the 1/10 wavelength lower limit and 1/4 wavelength max absorption lines. It can be seen from the data that the results diverge above 1000 Hz. The data associated with the thicker samples takes on a curved slope due to increased absorption and cross mode interference that starts at the 1/2 wavelength wave number (about 60 rad/m for both sample sets). These effects appear minimal for the wave number, yet have a large effect on the absorption coefficient, Figure 13. The vertical lines in Figure 13 correspond to the frequencies at which the 1/10 wavelength lower limit and 1/4 wavelength max absorption lines intersect the wave number lines in Figure 12. Here it can be seen that the thinner samples produce more credible results in the higher frequency ranges. This is due to the reduced attenuation of the wave through the sample and the better signal to noise ratio with respect to the cross modes. It is expected that these data sets can be combined at around 1000 Hz to produce a single propagation constant for the entire frequency range.

D. Example 4

To demonstrate the ability to analyze impedance data from samples whose thicknesses are not a ratio of two, impedance data taken from 2, 3 and 4 inch (5.08, 7.62 and 10.16 cm) microsphere samples were analyzed. The wave number results for 2”-4”, 3”-4” and 2”-3” analyses are shown in Figure 14. As can be seen, the data are identical up to the 60 rad/m half wavelength cross mode limit. With this capability, if n samples are acquired, n **combination** 2 analyses can be performed. For example 6 analyses can be performed on 4 samples.

5. CONCLUSION

The Newton-Raphson and Wave Number Space methods of finding solutions for the propagation constant when given specific impedance data derived from samples of different thicknesses are shown to produce results comparable to the two-thickness method. The two new approaches do not require that the sample thicknesses be a ratio of 2, thus allowing more analyses to be performed on available sample data. Understanding regions of valid data improves interpretation of results and provides a means of combining the valid regions of results associated with data from different sample thicknesses.

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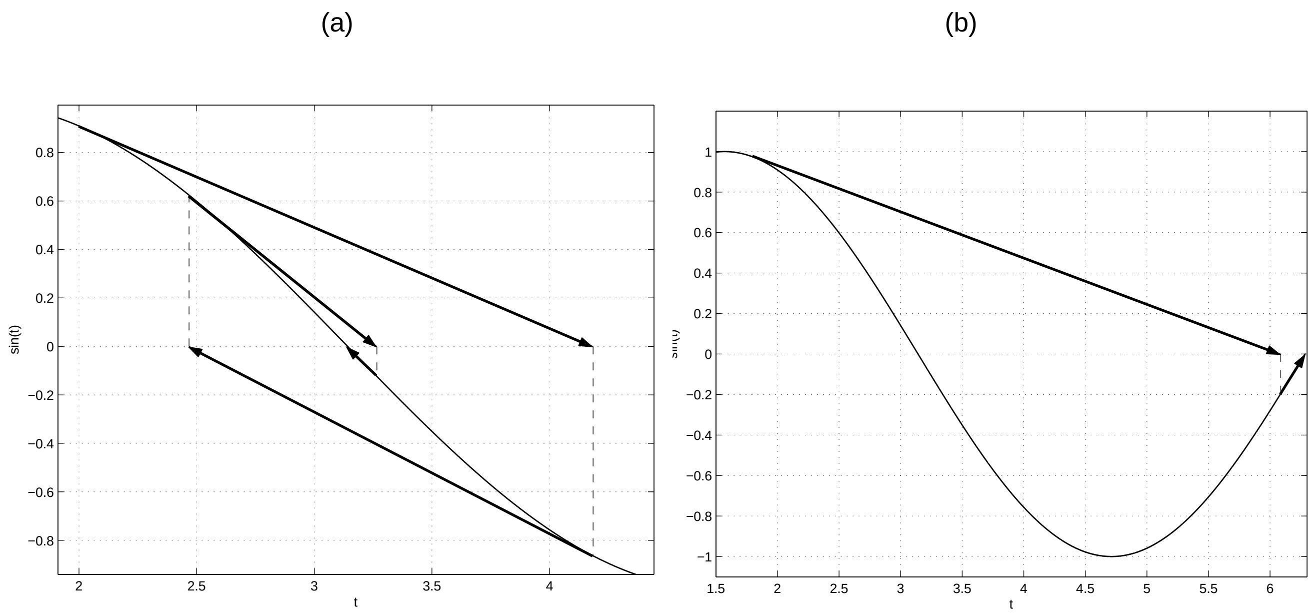

Figure 1. Convergence of Newton-Raphson Method for x1=2.0, (a), and, x1=1.8, (b)

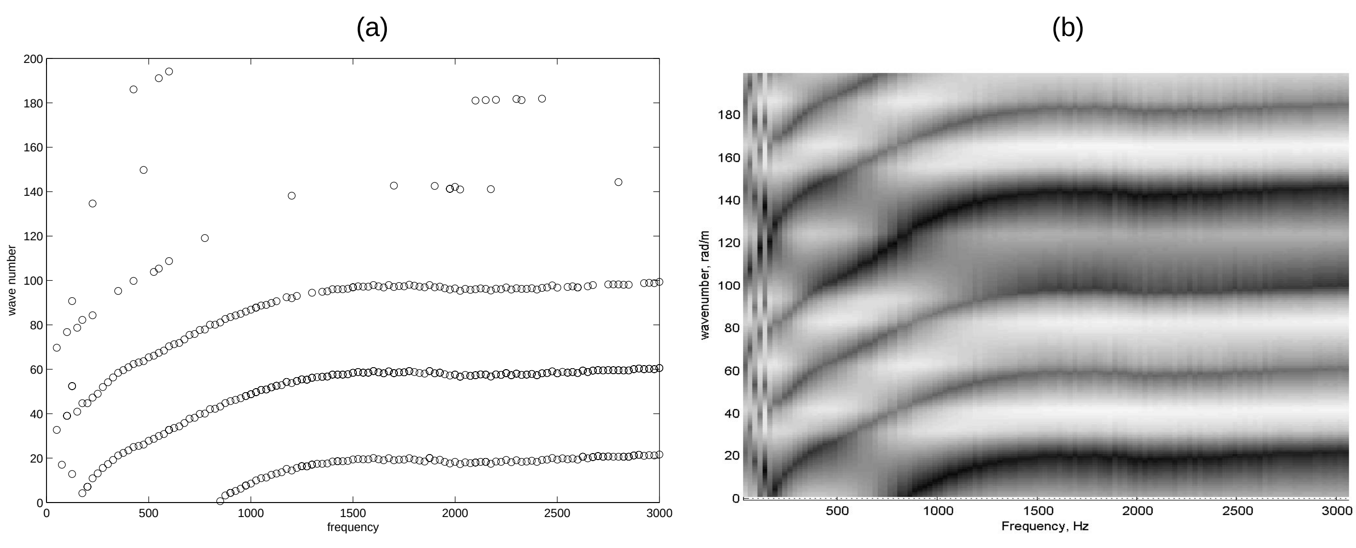

Figure 2. Example Solution Methods, Newton-Raphson, (a), and Wave Number Space, (b)

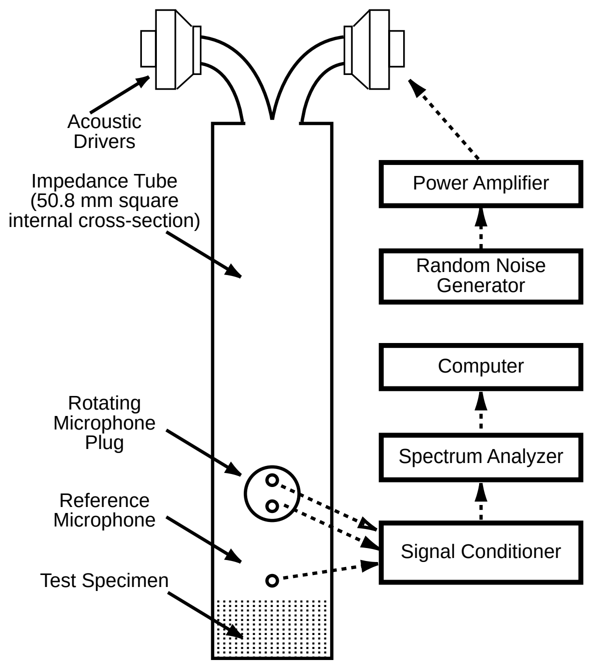

Figure 3. Sketch of Vertical Impedance Tube (VIT)

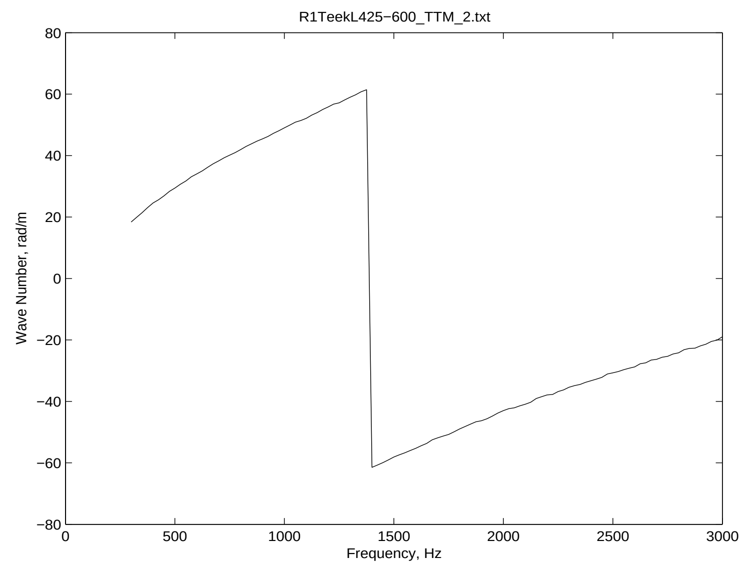

Figure 4. Example 1, Two-Thickness Method

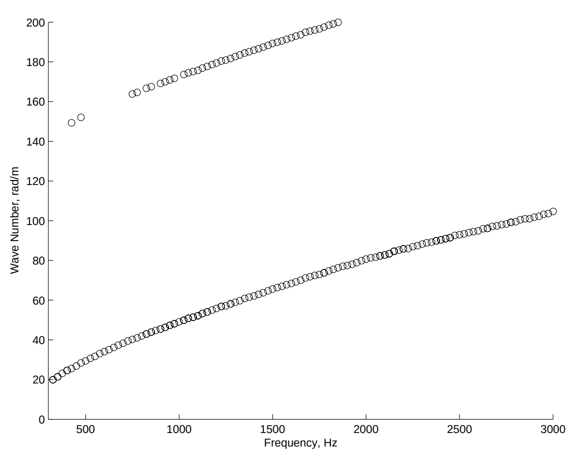

Figure 5. Example 1, Newton-Raphson Method

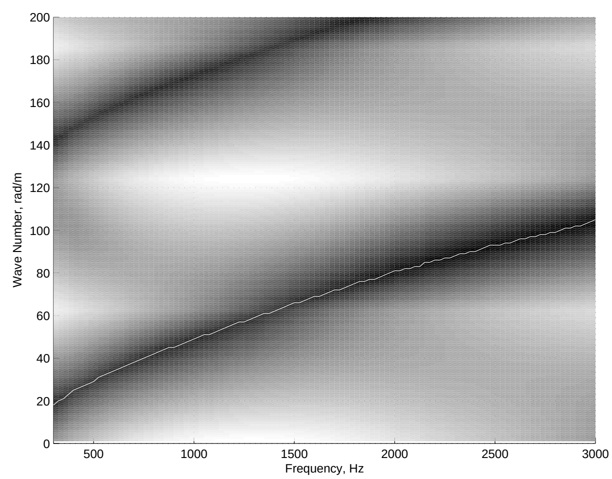

Figure 6. Example 1, Wave Number Space Method

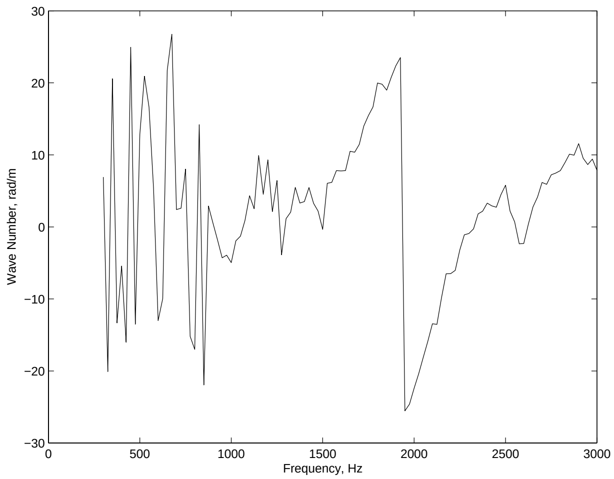

Figure 7. Example 2, Raw Wave Number Data

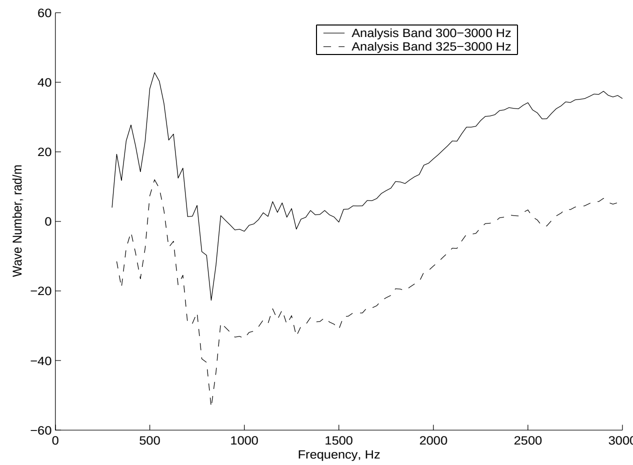

Figure 8. Example 2, Two-Thickness Method

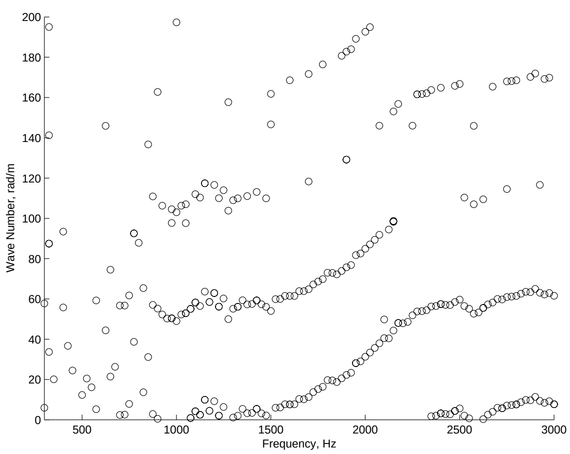

Figure 9. Example 2, Newton-Raphson Method

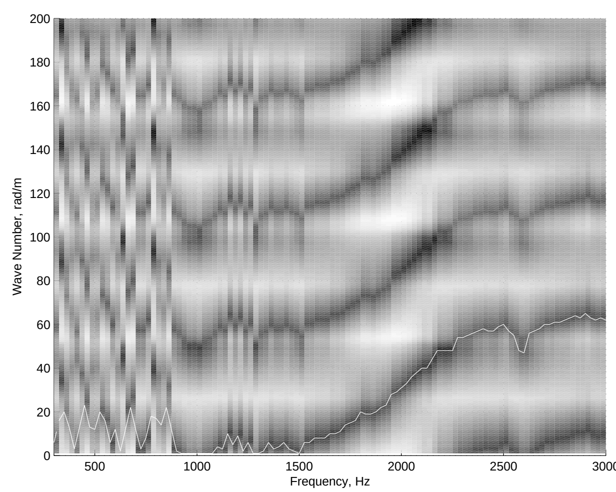

Figure 10. Example 2, Wave Number Space Method

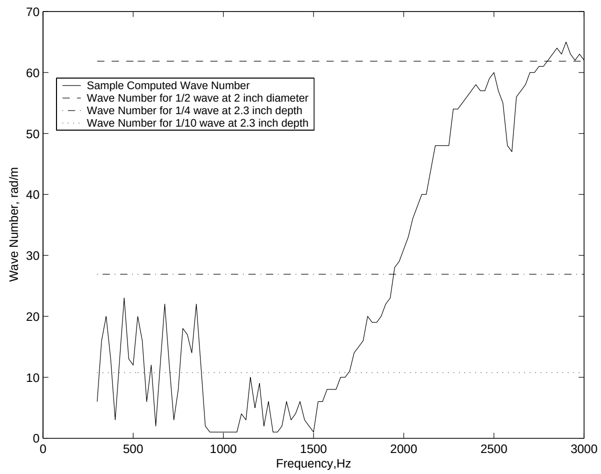

Figure 11. Example 2, Data Quality Limits

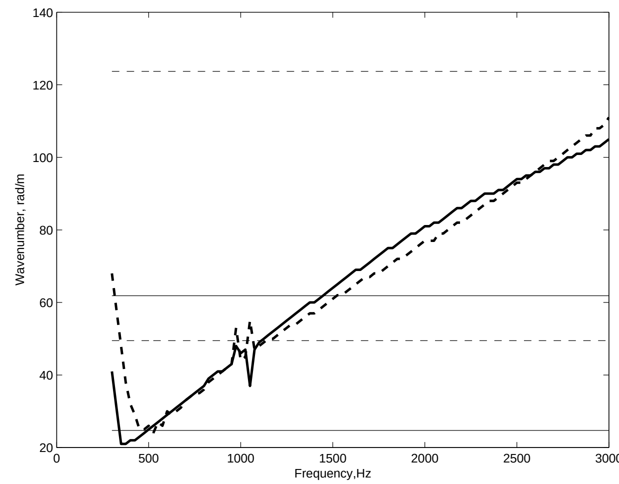

Figure 12. Example 3, Wave number data from 0.5 and 1 in. samples, dashed, and, 1 and 2 in. samples, solid.

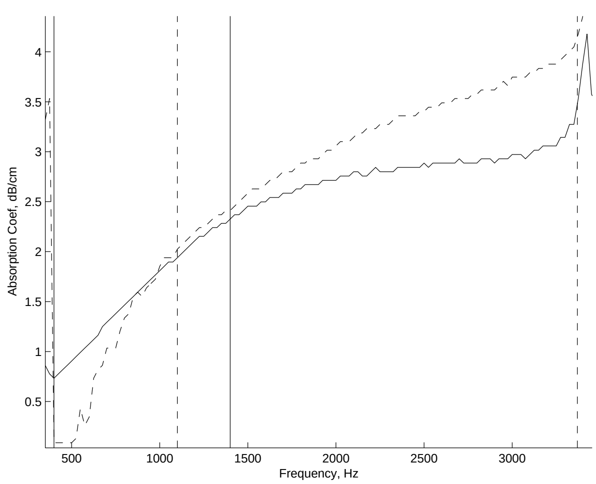

Figure 13. Absorption Coef from 0.5 and 1 in. samples, dashed, and, 1 and 2 in. samples, solid.

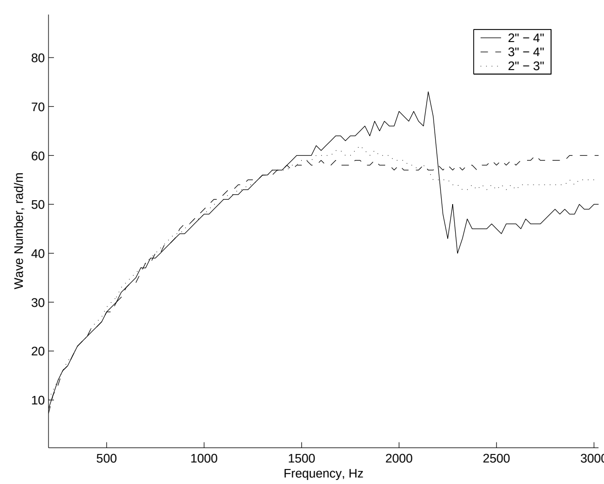

Figure 14. Wave number data derived from various material thicknesses.