Reply by the Authors to C. K. W. Tam

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The prediction of noise generation and radiation by turbulence has been the subject of continuous research for over fifty years. The essential problem is how to model the noise sources when one's knowledge of the detailed space-time properties of the turbulence is limited. In Ref. [1] we attempted to provide a comparison of models based on acoustic analogies and recent alternative models. Our goal was to demonstrate that the predictive capabilities of any model are based on the choice of the turbulence property that is modeled as a source of noise. Our general definition of an acoustic analogy is a rearrangement of the equations of motion into the form $\mathcal{L}(u) = Q$, where $\mathcal{L}$ is a linear operator that reduces to an acoustic propagation operator outside a region $\mathcal{V}$; u is a variable that reduces to acoustic

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pressure (or a related linear acoustic variable) outside $\mathcal{V}$; and Q is a source term that can be meaningfully estimated without knowing u and tends to zero outside $\mathcal{V}$. There should be no dispute that if the details of the turbulence were known in sufficient detail, then an acoustic analogy, or any other method, could be used to predict the radiated noise: see Colonius et al. [2]. It should also be noted that models based on the acoustic analogy provide excellent predictions of rotorcraft and propeller noise [3], [4], as well as broadband airfoil noise [5]. In addition, the acoustic analogy yields very good predictions of statistical properties, such as the two-point cross correlation of the noise radiated by jets, outside the region where refraction effects are important [6]. What is at issue here is whether an acoustic analogy, in whatever form, is capable of describing the noise radiated by turbulence, when the details of the turbulence are not known completely.

In Ref. [7], addressing our paper, as well as in Ref. [8], it is argued that an acoustic analogy is unable to provide a description of the physical sources of aerodynamic noise: but, the Tam and Auriault [9] theory can. However, Lighthill's acoustic analogy was formulated on the basis that "we never know a fluctuating fluid flow very accurately. [10]" An acoustic analogy, as its name indicates, formulates the aerodynamic noise generation problem in terms of equivalent sources that give "the effect of a fluctuating external force field, known if the flow is known, acting on the said uniform acoustic medium at rest, and hence radiating sound in it according to the ordinary laws of acoustics. [10]"

The concept that the gradients of the instantaneous Reynolds stress, or a vortex force [1], provide an unsteady force on the fluid that results in noise generation and radiation seems to us to be one very viable picture of how turbulence generates noise. The force associated with the Reynolds stress gradient is an important feature of other fluid dynamic problems and models. These include acoustic streaming [11] and, of course, turbulence modeling in the Reynolds averaged Navier-Stokes (RANS) equations. The key question is how to model this effective force.

The issues raised in Ref. [7] go far beyond anything contained in our original paper. So, our response will only try to address a few specific issues: however, we hope that this response will highlight open issues that should be the subject of continued constructive debate.

Reference [8] provides three examples of the application of the acoustic analogy to problems with either exact or numerically exact solutions. The first example is the initial value problem of sound initiated by a pressure pulse with a Gaussian spatial distribution and the second is a boundary value problem of sound radiation from a sphere whose surface temperature oscillates in time. Both of these cases are ones in which an acoustic analogy is not needed, as the problems (equations and initial and boundary conditions) are defined exactly. However, whether the problems are solved directly, or by a solution of a formulation based on Lighthill's Acoustic Analogy, the correct answer for the radiated noise is obtained. On the basis of these examples, in Ref. [8] the question is posed "whether the Acoustic Analogy is a reliable way to identify the true sources of noise in real practical aeroacoustics problems, especially in turbulent flows?" Since the Acoustic Analogy was never formulated to identify the "true sources" of noise, a more pertinent question would be "whether the Acoustic Analogy is a useful way to identify the effective sources of noise in real practical aeroacoustics problems where the details of the flow are not known precisely, especially in turbulent flows?"

In a third example in Ref. [8], it is argued that the Acoustic Analogy is unable to obtain the weak solution to the nonlinear Euler equations. The example given is the propagation of a normal shock into a stationary gas in one-dimension. The gas conditions behind the shock are known to be given by the Rankine-Hugoniot relations. For example (using the notation of Ref. [8]),

$$\rho_2 = \rho_1 \left[\frac{(\gamma + 1)\, M_s^2}{(\gamma - 1)\, M_s^2 + 2} \right] \tag{1}$$

If this problem is formulated in the form of the Lighthill Acoustic Analogy the quadrupole source term in Lighthill's equation should be written correctly as,

$$\begin{aligned} Q(x,t) &= \frac{\partial^2}{\partial x^2}\left[\rho u^2 + p - a_o^2\rho\right] \\ &\quad +\Delta\left[\rho u^2 + p - a_o^2\rho\right]\delta'(x - c_s t) \end{aligned} \tag{2}$$

where $\Delta[\,]$ denotes the jump across the shock and $\delta'(\,)$ denotes the derivative of the Dirac delta function. The second term on the right hand side of Eqn. (2) arises since, for this problem, the spatial derivatives should be treated as generalized derivatives. However, in the one-dimensional case, this term makes no contribution to the solution. (This would not be the case for problems in two or three dimensions.) Thus the expression derived in Ref. [8] is correct:

$$\begin{aligned} \rho_2 &= \rho_1 + \frac{1}{a_o^2 - c_s^2}\left[-\frac{2\gamma p_1\left(M_s^2-1\right)}{\gamma+1} + \frac{2a_o^2\rho_1\left(M_s^2-1\right)}{(\gamma-1)M_s^2+2}\right. \\ &\quad \left. - \frac{4\rho_1\left(M_s^2-1\right)^2 c_s^2}{(\gamma-1)M_s^2+2}\frac{1}{(\gamma+1)M_s^2}\right] \end{aligned} \tag{3}$$

However, this result does not show that "the Acoustic Analogy is unable to reproduce a correct weak solution of the Euler equations. [8]" The following substitutions can be made,

$$a_o^2 - c_s^2 = -a_o^2\left(M_s^2 - 1\right) \tag{4}$$

$$p_1 = \rho_1 R T_1 = \rho_1 a_o^2/\gamma \tag{5}$$

$$c_s^2 = M_s^2 a_o^2 \tag{6}$$

Then, Eqn. (3) can be manipulated algebraically to give Eqn. (1), the Rankine-Hugoniot relation, **exactly**. Thus, the Acoustic Analogy is able to reproduce a weak solution of the Euler equations. However, this is a case in which the Acoustic Analogy is being used as a flow solver, which, as its name indicates, was never its intended use. The question of the effectiveness of an acoustic analogy to describe the sources of aerodynamic noise when the flow conditions are not known exactly is addressed next.

In Ref. [7] it is argued that the true noise source is the convective derivative of the kinetic energy of the turbulence per unit mass. To contrast this assertion with alternatives, we choose to cast the problem in a simplified form. First, we will neglect the effects of the mean flow and assume that the mean temperature, and hence the speed of sound, are constant. This makes the algebra less cumbersome and is a good approximation for sound radiation at 90^o to the jet axis where the comparisons of Ref. [1] were made. Secondly, we choose to write the Euler equations in terms of the logarithm of the pressure. This form is the basis for the development of the Phillips' and Lilley's equations and is more convenient for some subsequent developments below. The equations of motion can then be written,

$$\frac{\partial \pi}{\partial t} + \frac{\partial u_i}{\partial x_i} = 0 \tag{7}$$

$$\frac{\partial u_i}{\partial t} + c^2 \frac{\partial \pi}{\partial x_i} = 0 \tag{8}$$

where, $\pi = \gamma^{-1} \ln(p/p_o)$, p_o is the mean static pressure, u_i is the flow velocity, and c is the speed of sound. Following the formulation of Ref. [9], but using the present form of the Euler equations, the full set of governing equations for the generation of sound by fine-scale turbulence is,

$$\frac{\partial \pi'}{\partial t} + \frac{\partial u_i'}{\partial x_i} = 0 \tag{9}$$

$$\frac{\partial u_i'}{\partial t} + c_o^2 \frac{\partial \pi'}{\partial x_i} = -\frac{\partial q_s}{\partial x_i} \tag{10}$$

where q_s is now the kinetic energy of the turbulence per unit mass, primes denote linear acoustic fluctuations, c_o is the constant mean speed of sound, and $\pi' \simeq p'/(\gamma p_o)$. It should be noted that at this stage of the analysis, the "source" appears as the gradient of q_s. It is straightforward to show that in the far field,

$$p'(\mathbf{x},t) = \frac{\rho_o}{4\pi c_o^2 x} \iiint\limits_{-\infty}^{\infty} \frac{\partial^2 q_s}{\partial t^2}\left(\mathbf{x}_1, t - \frac{|\mathbf{x}-\mathbf{x}_1|}{c_o}\right) d\mathbf{x}_1 \tag{11}$$

where, $x = |\mathbf{x}| \simeq |\mathbf{x}-\mathbf{x}_1|$. An expression for the spectral density of the pressure is then given by,

$$\begin{aligned} S(\mathbf{x},\omega) &= \frac{\rho_o^2\omega^4}{32\pi^3 c_o^4 x^2} \iiint\limits_{-\infty}^{\infty} \iiint\limits_{-\infty}^{\infty} \int\limits_{-\infty}^{\infty} R_{q_s}(\mathbf{x}_1, \boldsymbol{\eta}, \tau) \\ &\quad \times \exp\left[i\omega\left(\tau - \frac{\mathbf{x}\cdot\boldsymbol{\eta}}{x c_o}\right)\right] d\tau d\boldsymbol{\eta} d\mathbf{x}_1 \end{aligned} \tag{12}$$

where, $\boldsymbol{\eta} = \mathbf{x}_2 - \mathbf{x}_1 = (\xi,\ \eta,\ \zeta)$, and

$$R_{q_s}(\mathbf{x}_1, \boldsymbol{\eta}, \tau) = \langle q_s(\mathbf{x}_1, t)\, q_s(\mathbf{x}_2, t+\tau)\rangle \tag{13}$$

If it is assumed that

$$R_{q_s}(\mathbf{x}_1, \boldsymbol{\eta}, \tau) = A^2 u_s^4 E(\boldsymbol{\eta}, \tau) \tag{14}$$

with,

$$E(\boldsymbol{\eta},\tau) = \exp\left\{-\frac{|\xi|}{\bar{u}\tau_s} - \frac{1}{\ell_s^2}\left[(\xi - \bar{u}\tau)^2 + \eta^2 + \zeta^2\right]\right\} \tag{15}$$

the far field spectral density at 90^o to the jet axis is given by,

$$S(\mathbf{x},\omega) = \frac{\sqrt{\pi}}{16\pi^2}\frac{\rho_o^2}{c_o^4 x^2} \iiint\limits_{-\infty}^{\infty} \frac{A^2 u_s^4 \ell_s^3}{\tau_s^3} \frac{\omega^4 \tau_s^4}{(1+\omega^2\tau_s^2)} \exp\left[-\frac{\omega^2\ell_s^2}{4\bar{u}^2}\right] d\mathbf{x}_1 \tag{16}$$

This is the result given by Model 2 based on the acoustic analogy in Ref. [1]. In Ref. [7] it is argued that this is not the correct result for the spectral density as it is the convective derivative of q_s that is asserted to be the true source in the Tam and Auriault model. It is very important to note at this point that the convective derivative of q_s only appears in the Tam and Auriault theory following an integration by parts to move the convective derivative from the adjoint Green's function onto q_s. There appears to be no reason, based on the physical arguments introduced by Tam and Auriault, for this assertion: other than the quality of the resulting predictions of the spectral density at or near to 90^o to the jet axis. We can easily reproduce the Tam and Auriault formula if, at the appropriate stage of the analysis, we use the following result, based on the assumption of stationary turbulent statistics,

$$\left\langle \frac{\partial^2 q_s}{\partial t^2}(\mathbf{x}_1, t)\frac{\partial^2 q_s}{\partial t^2}(\mathbf{x}_2, t+\tau)\right\rangle = -\frac{\partial^2}{\partial \tau^2}\left\langle \frac{\partial q_s}{\partial t}(\mathbf{x}_1, t)\frac{\partial q_s}{\partial t}(\mathbf{x}_2, t+\tau)\right\rangle \tag{17}$$

$$= -\frac{\partial^2 R_{q_s'}}{\partial \tau^2}(\mathbf{x}_1, \boldsymbol{\eta}, \tau) \tag{18}$$

Then, if it assumed that,

$$R_{q_s'}(\mathbf{x}_1, \boldsymbol{\eta}, \tau) = \frac{A^2 u_s^4}{\tau_s^2} E(\boldsymbol{\eta}, \tau) \tag{19}$$

it is readily shown that,

$$S(\mathbf{x}, \omega) = \frac{\sqrt{\pi}}{16\pi^2}\frac{\rho_o^2}{c_o^4 x^2}\iiint\limits_{-\infty}^{\infty} \frac{A^2 u_s^4 \ell_s^3}{\tau_s^3}\frac{\omega^2 \tau_s^2}{(1+\omega^2\tau_s^2)}\exp\left[-\frac{\omega^2 \ell_s^2}{4\bar{u}^2}\right] d\mathbf{x}_1 \tag{20}$$

This is the prediction formula derived by Tam and Auriault that results in very good predictions for the spectral density at 90^o to the jet axis if the velocity, length, and time scales, u_s, ℓ_s, and τ_s, are extracted from a RANS calculation.

Even though acoustic analogies have been developed on the basis of effective noise sources, one can ask whether an acoustic analogy can provide guidance concerning the true noise sources. If the acoustic analogy is written in the form given in Ref. [1], with only the second order unsteady force included, the equations for the acoustic analogy are,

$$\frac{\partial \pi'}{\partial t} + \frac{\partial u_i'}{\partial x_i} = 0 \tag{21}$$

$$\frac{\partial u_i'}{\partial t} + c_o^2 \frac{\partial \pi'}{\partial x_i} = -f_i'' \tag{22}$$

Then the far field spectral density is given exactly by Eqn. (20). This time, in modeling the appropriate turbulence statistics, it is assumed that

$$\left\langle f_x''\left(\mathbf{x}_1, t\right) f_x''\left(\mathbf{x}_2, t+\tau\right)\right\rangle = A^2 \frac{m^2 u_s^4}{\ell_s^2} E\left(\boldsymbol{\eta}, \tau\right) \tag{23}$$

where f_x'' is the fluctuating force in the direction of the observer and $m = u_s/c_o$. Thus, as we proposed in Ref. [1], an acoustic analogy can yield the same prediction formula as the Tam and Auriault theory if the appropriate statistical properties of the turbulence are modeled.

Finally, we consider whether the Tam and Auriault theory is fundamentally different from the original dilatation theory proposed by Ribner [12]. He imagined that the turbulent eddies, he termed "jetlets," would collide and generate regions of compression followed by expansion. He argued that "this unsteady *dilatation* of fluid elements, driven by inertial (momentum) effects, generates sound.[13]" He also noted that "a turbulent flow contains an apparently random pattern of momentum, vorticity and pressure. These variables are interconnected by the governing dynamic and continuity equations. Any one of the three can serve as the vehicle for predicting noise. [13]" They can indeed. An examination of Eqn. (7) shows that the dilatation rate is related exactly to the convective derivative of the pressure (in the non-zero mean flow case). So one could either treat the noise sources in terms of the convective derivative of the pressure or the dilatation rate. They will have an identical effect in terms of noise generation and radiation. If a dilatation rate source $\theta\left(\mathbf{x}, t\right)$, is placed on the right hand side of Eqn. (21), and the source is removed from the momentum equation, the resulting far field pressure is again given exactly by Eqn. (20). In this instance, the appropriate description of the source statistics is,

$$\langle \theta(\mathbf{x}_1, t)\, \theta(\mathbf{x}_2, t+\tau) \rangle = A^2 \frac{m^4 u_s^2}{\ell_s^2} E(\boldsymbol{\eta}, \tau) \tag{24}$$

It is important to note that in these last two cases it is the term on the right hand side of the linearized equations that can be treated as the source and their statistics, not those of some derived property, are the ones that determine the radiated noise.

In conclusion, we have shown that Lighthill's Acoustic Analogy is able to provide correct solutions to model problems where the initial or boundary conditions are given exactly. Though, its use is neither necessary nor appropriate for such problems as it should be recalled that the Acoustic Analogy represents aerodynamic noise generation and propagation in terms of equivalent sources. It has also been shown that the far field noise prediction formula depends, not so much on the formulation of the noise generation model, but on the choice of model for the turbulent source statistics. One final caveat is necessary. All of the statements made in Ref. [1], as well as in this response, are based on comparisons with the far field spectral density at 90^0 to the jet axis. Similar conclusions are not yet possible based on comparisons at other angles, where other issues, such as convective amplification or the contributions of a different source mechanism, remain to be resolved.

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