Refinements to the Mixed-Mode Bending Test for Delamination Toughness

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ABSTRACT

The mixed-mode bending (MMB) test for delamination toughness was first introduced in 1988. This simple test is a combination of the standard Mode I (opening) test and a Mode II (sliding) test. This MMB test has become widely used in the United States and around the world for mixed-mode toughness measurements. Because of the widespread use of this test method, it is being considered for standardization by ASTM Committee D30. This paper discusses several improvements to the original test method.

The improvements to the MMB test procedure include an improved method for calculating toughness from the measured test quantities, a more accurate way of setting the mixed-mode ratio to be tested, and the inclusion of a new alignment criterion for improved consistency in measured values.

INTRODUCTION

The mixed-mode bending (MMB) test shown in Figure 1 combines the double cantilever beam (DCB) test which is the standard test for Mode I toughness, with the end notched flexure (ENF) test for Mode II toughness. Although originally designed to test the static delamination toughness of composites, researchers have extended its use to fatigue loading and to toughness testing of adhesives. Because of the number of laboratories using this test, it is important that standardization of the test protocol occur so that future tests will be performed in a uniform manner making the resulting data directly comparable.

The proportion of Mode I and Mode II loading in a MMB test is controlled by setting the lever loading position, c, shown in Figure 1. The MMB test has several advantages over other mixed-mode delamination tests, including the use of a single test specimen configuration to test over almost the entire range of Mode I to Mode II ratios, a closed form model that is used to calculate the toughness values from measured quantities, and a mixed-mode ratio that remains essentially constant during delamination growth [1]. The original MMB fixture showed a nonlinearity in the loading curve due to the rotation of the lever. This nonlinearity caused

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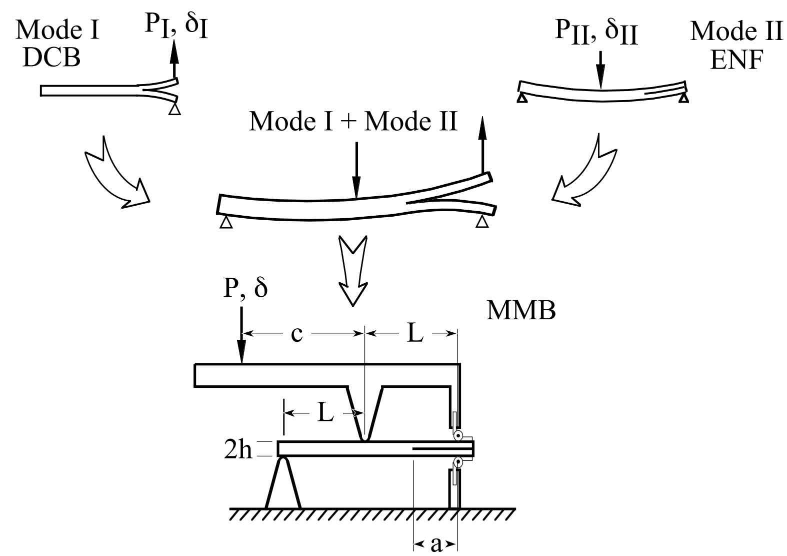


Figure 1. The mixed-mode bending test.

significant errors in the calculated toughness values and changed the mixed-mode ratio. It was found that this problem could be largely avoided by changing the way in which the lever was loaded and the modified apparatus is shown in Figure 2[2].

During the MMB test, load and load-point displacement are recorded as shown in Figure 3. The MMB apparatus is loaded in displacement control until the delamination begins to grow. Several different loads can be taken from the load displacement curve to be associated with the delamination initiation. The loads include: the point where the loading curve deviates from linearity, $P_{NL;}$ the point where delamination is visually observed to grow, P_{VIS}; and the load associated with a 5% increase in compliance, $P_{5\%}$; the maximum load point, P_{MAX}. The P_{NL} has been shown to provide repeatable conservative values of toughness in the Mode I test[3] and is recommended for MMB testing as well. Depending on the mixed-mode ratio and the material being tested, the delamination may grow in a stable or an unstable manner.

Toughness can be calculated in many ways. Because the MMB test is a combination of the pure mode tests, one way to calculate strain energy release rate is to separate the loading into Mode I and Mode II components. These pure mode loads can then be used in equations for toughness developed for the pure mode tests. The pure mode components have been shown to be given by [1]

$$P_I = P\left(\frac{3c - L}{4L}\right) \quad \text{and} \quad P_{II} = P\left(\frac{c + L}{L}\right) \tag{1}$$

In a similar way the pure mode displacement components can be shown to combine to form the load point displacement from the MMB test [4].

$$\delta = \left(\frac{3c - L}{4L}\right)\delta_I + \left(\frac{c + L}{L}\right)\delta_{II} \tag{2}$$

Simple beam theory analyses of the pure mode tests ignore shear deformations and bending deformations that occur about the crack tip. Kinloch et al. [5, 6] suggested correcting for these deformations by adding a correction term to the delamination length as shown in the following equations:

$$G_I = \frac{12 P_I^2 (a + \chi h)^2}{b^2 h^3 E} \quad \text{and} \quad G_{II} = \frac{9 P_{II}^2 (a + 0.42 \chi h)^2}{16 b^2 h^3 E} \tag{3}$$

$$C_I = \frac{\delta_I}{P_I} = \frac{8(a + \chi h)^3}{b h^3 E} \quad \text{and} \quad C_{II} = \frac{\delta_{II}}{P_{II}} = \frac{2L^3 + 3(a + 0.42 \chi h)^3}{8 b^2 h^3 E} \tag{4}$$

where the correction term $\chi = \sqrt{\frac{E_{11}}{11 G_{13}} \left\{ 3 - 2 \left(\frac{\Gamma}{1 + \Gamma} \right)^2 \right\}}$, $\Gamma = 1.18 \frac{\sqrt{E_{11} E_{22}}}{G_{13}}$ (5)

b is the specimen width and E is the material modulus that controls the bending of the test specimen. The appropriate choice of modulus, E, in Eq. 3 and 4 will be discussed later. Finite element analyses have demonstrated that these correction factors model the specimen response accurately[7].

By measuring a critical value of load at which the delamination extends, Eq. 1 can be used to calculate critical values of Mode I and Mode II loadings P_I and P_{II}. These loads can then be substituted into Eq. 3 to determine the critical values of G_I and G_{II}. Combining the individual modes as shown in Eq. 6, total toughness and the mixed-mode ratio can be determined. Note that the mixed-mode ratio is normally expressed as the fraction of Mode II to the total fracture toughness.

$$G = G_I + G_{II} \quad \text{and} \quad {G_{II}}/{G} = \frac{G_{II}}{G_I + G_{II}} \tag{6}$$

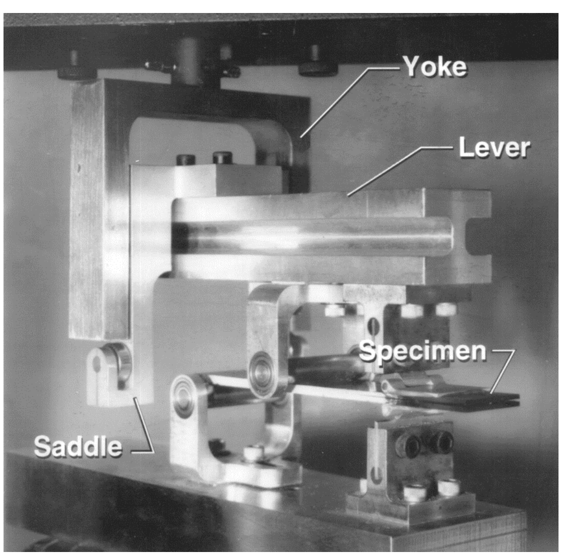


Figure 2. Modified MMB apparatus.

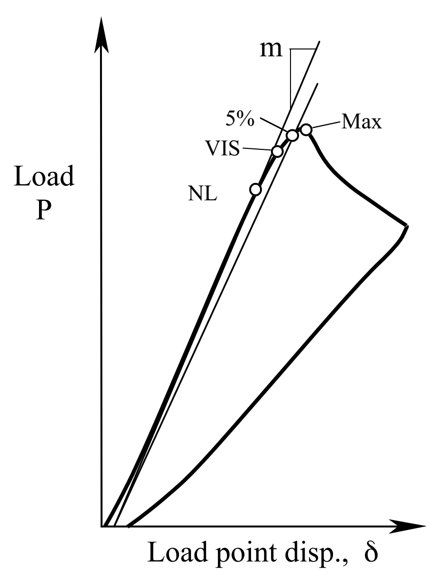


Figure 3. Load-displ. plot from MMB test.

CHOICE OF MODULUS FOR TOUGHNESS MEASUREMENTS

The choice of material modulus to be used in the G calculation is not obvious for the MMB test. The extensional modulus, $E_{11,}$ in the longitudinal direction of the test specimen would control deformation of a homogenous material. However, it has been shown that the measured stiffness of this type of composite bending specimen is significantly less than that predicted by a simple bending model using a modulus value measured in tension[8]. In other words, the effective modulus for these specimens in bending is normally less than the value of modulus measured in tension. It is believed that this difference is due to resin rich layers on the surface of the composite significantly reducing the bending rigidity. To correct for this, a bending modulus can be measured by loading the specimen in bending. Often a 3-point bending specimen is used, along with a simple beam theory analysis, to calculate modulus from the slope of the loading curve. The approach presented here is to use the slope from the MMB loading curve to back calculate the effective modulus to be used in the calculation of toughness. This has several advantages. First, the moduli of these types of bending specimens vary significantly from specimen to specimen. For this reason, compliance calibration methods are suggested for both the DCB Mode I test[9] and the ENF Mode II test[8]. The type of compliance calibration used with the DCB specimen does not work with the MMB specimen because the delamination growth is not always stable. The type of compliance calibration used with the ENF specimen does not work with the MMB test because the bonded hinges keep the specimen from easily being shifted in the apparatus to measure compliance for several different delamination lengths. Several researchers [4, 7, 10] have attempted compliance calibration methods for the MMB test, but in general, the approaches complicated the running of the test by requiring extra displacement measurements and have not resulted in reducing the experimental scatter.

In order to incorporate the compliance of the test specimen into the toughness calculation it is suggested that the slope of the load-displacement curve be used to determine the effective modulus of the specimen. The slope of the loading curve is given by m=P/δ, so combining Eqs. 1, 2 and 4 results in:

$$\begin{aligned}\frac{1}{m} = \frac{\delta}{P} &= \frac{\left(\frac{3c-L}{4L}\right)\delta_I + \left(\frac{c+L}{L}\right)\delta_{II}}{P} = \left(\frac{3c-L}{4L}\right)\frac{\delta_I}{P} + \left(\frac{c+L}{L}\right)\frac{\delta_{II}}{P} \\ &= \left(\frac{3c-L}{4L}\right)^2\frac{\delta_I}{P_I} + \left(\frac{c+L}{L}\right)^2\frac{\delta_{II}}{P_{II}} = \left(\frac{3c-L}{4L}\right)^2 C_I + \left(\frac{c+L}{L}\right)^2 C_{II} \qquad (7) \\ &= \left(\frac{3c-L}{4L}\right)^2\left(\frac{8(a+\chi h)^3}{bh^3E}\right) + \left(\frac{c+L}{L}\right)^2\left(\frac{2L^3+3(a+.42\chi h)^3}{8bh^3E}\right)\end{aligned}$$

Often the displacement measurement for the MMB test is taken from the stroke of the test machine. However, the compliance of the load cell and load frame can cause significant errors in the displacement measurements. It is, therefore, important to determine and correct for the compliance of the loading system.

The system compliance should be determined for each mixed-mode ratio to be tested. This can be done by simply measuring the stiffness of a bar of known stiffness such as one made of steel. The stiffness of the bar can be derived from Eq. 7 by neglecting all crack length terms including crack correction terms and substituting the modulus of the bar for the modulus of the test specimen.

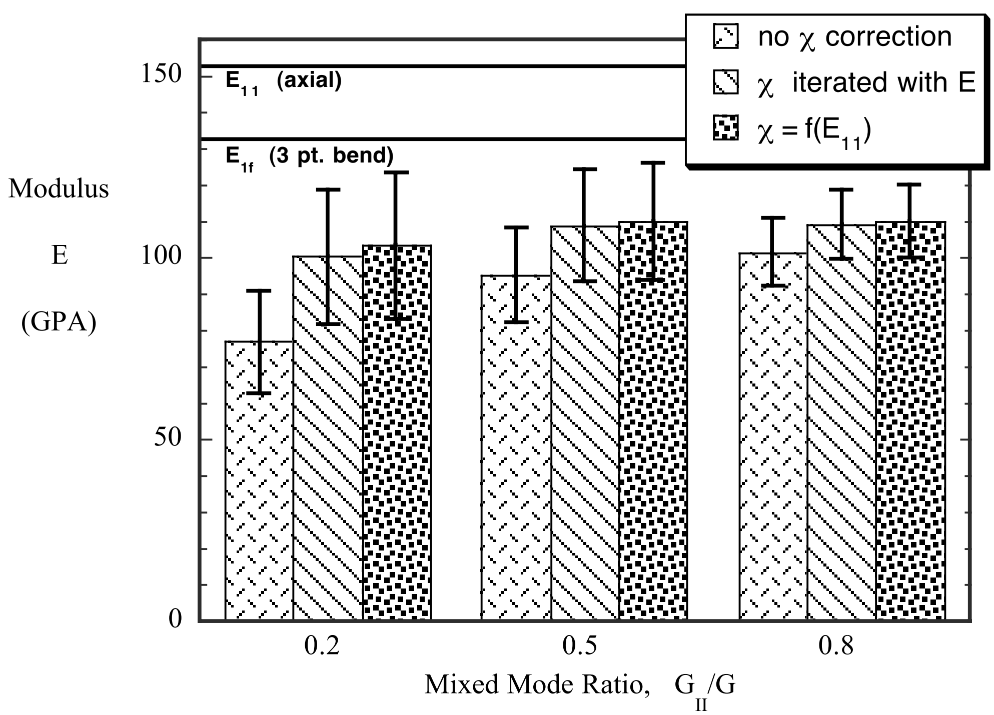
$$C_{bar} = \frac{2L\,(c+L)^2}{E_{bar}\,b_{bar}\,t^3} \quad \text{and} \quad C_{sys} = \frac{1}{m_{bar}} - C_{bar} \tag{8}$$

t is the total thickness of the bar (t=2h). It is advisable to choose a bar that is much stiffer than the delamination test specimen. This will cause the measured compliance to be dominated by the compliance of the loading system and errors caused by uncertainties in the compliance of the bar will, therefore, be small.

In the first attempt to incorporate the specimen stiffness, the deformation terms involving χ were neglected. This resulted in the calculated modulus varying with mixed-mode ratio, as shown in Figure 4, and indicating a problem with the calculation method. By rearranging Eq. 7 and subtracting the system compliance the following equation for specimen modulus incorporating the additional deformation terms was derived:

$$E = \frac{8(3c-L)^2(a+\chi h)^3 + (c+L)^2\left[4L^3 + 6(a+0.42\chi h)^3\right]}{16L^2bh^3\left(\frac{1}{m} - C_{sys}\right)} \tag{9}$$

The average at the 0.2 mixed-mode ratio is still slightly lower for the results shown



. **Figure 4. Calculated modulus values.**

but the average is well within the experimental scatter bands so this difference may not be significant. The back calculated E is less than even E_{1f} value measured in flexure. The reason for this is not well understood, but it may relate to the difference in bending of the full thickness beam measured by the 3-point bend test and the bending of the half thickness beam that dominates the deformation of the MMB test. The lower effective modulus will result in higher calculated toughness values than those determined using the higher extensional modulus.

The inclusion of the crack length correction term χ to calculate E raised another issue because χ is a function of the modulus of the material. One option would be to calculate the effective modulus iteratively. This results in the value of χ being consistent with the value of modulus. Relying on an iterative solution for a test protocol would be somewhat cumbersome, and as seen in the figure, the effect of calculating χ using E_{11} rather than the back-calculated E from the MMB test, makes a negligible difference. For this reason, it is suggested that extension modulus, E_{11}, be used to calculate the crack length correction term, χ, which is then used to calculate the effective modulus and the fracture toughness.

CALCULATION OF LEVER LENGTH

A second issue arose while developing the protocol for an ASTM standard test for the MMB specimen. Normally it is preferred to test at a given mixed-mode ratio. The original versions of the test protocol determined the lever length with an equation derived from simple beam theory. After the test, when the toughness values were calculated using the corrections for delamination length (Eq. 3), the mixed-mode ratio was found to be significantly different from the desired value. One solution to this was to invert the toughness equations including the delamination length corrections so that the lever length, c, was expressed as a

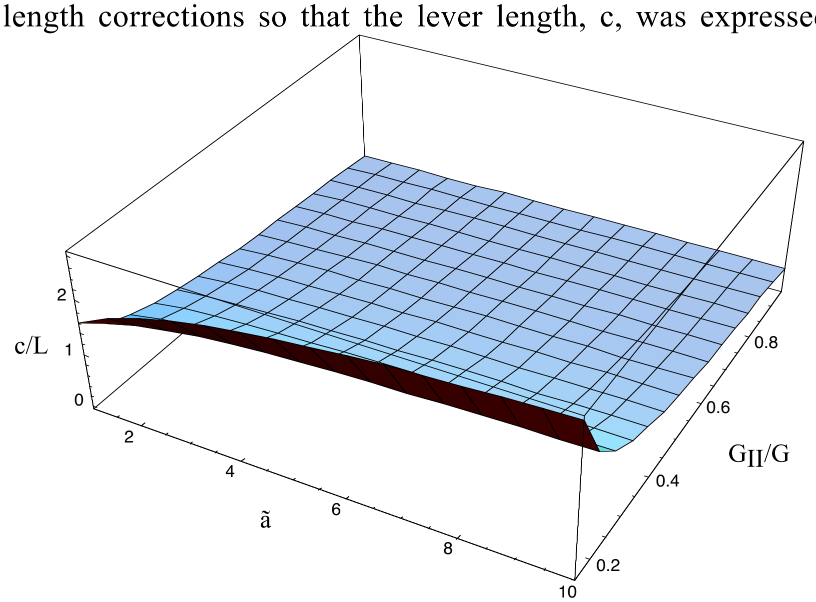


Figure 5. Required lever length for desired mixed-mode ratio.

function of the mixed-mode ratio. Unfortunately, this expression was quite complicated even after rearranging the expression so that it was expressed as a function of only two variables: the mixed-mode ratio G_{II}/G and a non-dimensional delamination length term $\tilde{a} = \frac{a}{h\chi}$. The inverted equation is plotted in Figure 5 but the equation is too long to include here and is also too complicated to be easily used to determine the lever length for a desired mixed-mode ratio. As seen in Figure 5, the desired function is fairly regular in shape. Because of this, it was hoped that it could be fit with a simpler equation without inducing too much error. Initial attempts at dropping less significant terms failed because they resulted in significant error over some portion of the plotted region. A second attempt at curve fitting was made by starting with a constant term and adding one term at a time to try to include any missing functionality observed from a plot of the error. This was a trial and error process, but with each functional form, a least-squares type optimization was performed to determine the optimal coefficient values. Eq. 10 presents a curve fit to the contour shown in Figure 5.

$$c = \left(0.167 + 0.000137\tilde{a}^2 - 0.108\sqrt{\ln(\tilde{a})}\left(G_{II}/G\right)^4 + \frac{-1400 + 0.725\tilde{a}^2 - 141\ln(\tilde{a}) - 302\ln(G_{II}/G)}{219 - 5000\,G_{II}/G + 55\ln(\tilde{a})} \right) L \qquad (10)$$

The error caused by using the curve fit equation instead of the full solution is plotted in Figure 6. The figure shows that the error in mixed-mode ratio is less than 1% over the range 15% < G_{II}/G < 95% and 3 < ã <15 which should include the vast majority of cases of interest. Although the curve fit equation is not simple, it is simple enough that one can use it to calculate a lever length for a desired mixed-mode ratio, and the resulting error in mixed-mode ratio should be negligible. Although this curve fit is not a unique solution, it does give a manageable way of setting the lever length.

APPARATUS ALLIGNMENT

Finally, through ASTM Round Robin testing, it was found that the alignment of the test fixture is extremely important. An exploratory Round Robin test program

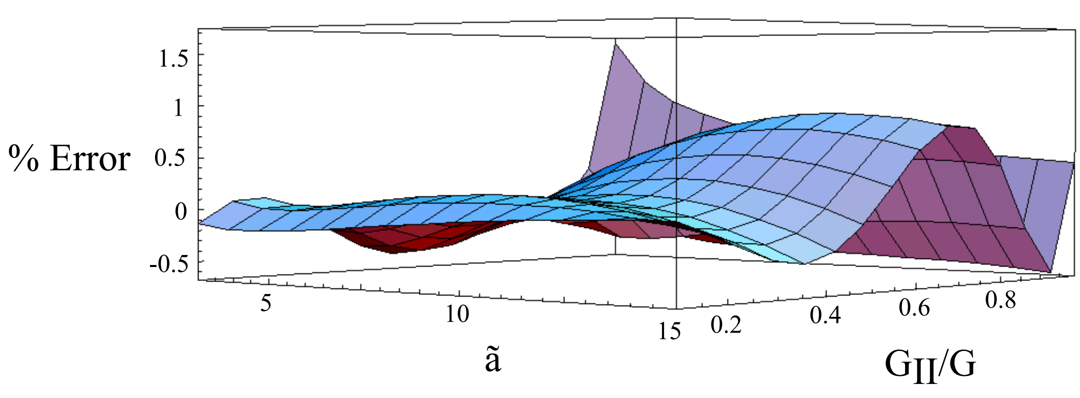


Figure 6. Error due to curve fit.

was performed in hopes of developing the statistical data for the test protocol. When the data was returned, some labs reported toughness values that were less than 50% of the results from other labs performing the same tests on specimens from the same batch of material. It was discovered that the data was significantly lower for labs that did not meticulously align the MMB apparatus for each test specimen so that the specimen was loaded evenly across its width. The fracture surfaces from these lower toughness specimens showed evidence that the delamination originated at a corner instead of uniformly across the specimen, which explained the lower toughness values. To prevent this problem, the apparatus should be aligned for each specimen so that contact between rollers and test specimen is made uniformly across the specimen width. To highlight this sensitivity, a new alignment criterion was added to the proposed test protocol.

CONCLUSIONS

Efforts to improve the MMB test protocol have resulted in: an improved method of calculating fracture toughness that accounts for the measured stiffness from each individual test specimen; a method to calculate the correct lever length for a desired mixed-mode ratio; and includes a fixture alignment criterion to reduce scatter in the test results. These advances to the test protocol of the mixed-mode bending test have improved the performance of the test method significantly.

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