An Inverse Interpolation Method Utilizing In-Flight Strain Measurements for Determining Loads and Structural Response of Aerospace Vehicles

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ABSTRACT

An important and challenging technology aimed at the next generation of aerospace vehicles is that of structural health monitoring. The key problem is to determine accurately, reliably, and in real time the applied loads, stresses, and displacements experienced in flight, with such data establishing an information database for structural health monitoring.

The present effort is aimed at developing a finite element-based methodology involving an inverse formulation that employs measured surface strains to recover the applied loads, stresses, and displacements in an aerospace vehicle in real time. The computational procedure uses a standard finite element model (i.e., “direct analysis”) of a given airframe, with the subsequent application of the inverse interpolation approach. The inverse interpolation formulation is based on a parametric approximation of the loading and is further constructed through a least-squares minimization of calculated and measured strains. This procedure results in the governing system of linear algebraic equations, providing the unknown coefficients that accurately define the load approximation.

Numerical simulations are carried out for problems involving various levels of structural approximation. These include plate-loading examples and an aircraft wing box. Accuracy and computational efficiency of the proposed method are discussed in detail. The experimental validation of the methodology by way of structural testing of an aircraft wing is also discussed.

INTRODUCTION

Advanced structural health monitoring is generally regarded as a vital technology for the next generation of aeronautical and space systems [1]. This technology is aimed at preventing catastrophic structural failures and is comprised of three facets: (a) determination of stresses and deformations of structural components, (b) identification of external loads, and (c) detection of critical damage mechanisms such as cracking, delamination, and corrosion.

The development of the health-monitoring technology involves multidisciplinary research in the areas of computational mechanics, intelligent information systems, and sensor networks. Recent advances in the design of health-monitoring systems for aerospace applications are discussed in [1, 2].

The present effort is aimed at developing a finite element-based methodology involving an inverse formulation that employs measured surface strains to recover the applied loads, stresses, and displacements in an aerospace vehicle in real time. The determination of loads, stresses, and displacements using experimentally measured structural response (strains) is defined as an *inverse problem*. This type of problem may result in an ill-posed governing system of equations and, therefore, requires a special approach to obtain an accurate and stable solution. The mathematical fundamentals of inverse problems may be found in [3-5].

A review of recent literature indicates that inverse methods are used quite extensively in mechanics. Maniatty and co-workers proposed a finite element-based method for solving inverse elastic [6] and elastoviscoplastic [7] problems. Their method uses a regularization procedure involving two matrices that impose smoothness on the solution. Several one- and two-dimensional examples illustrate the application of the method. More recently, Maniatty [8] studied the regularization procedure utilizing a statistical approach by Tarantola [4]. This analysis provides an estimate of the errors in the solution of an inverse problem. The main drawback of this method is that it requires iterations. For complex three-dimensional structures, it may lead to convergence difficulties and high computational costs.

The application of artificial neural networks to load identification was proposed by Cao et al. [9]. In their approach, the load-strain relationships are established by using a learning algorithm for a multi-layer neural network. The convergence of this algorithm, however, strongly depends on the set of chosen learning parameters. The learning can even diverge if the combination of parameters is not appropriate.

Martin et al. [10] discussed a non-iterative method for the reconstruction of surface tractions using a boundary element method. The restriction of the method is that displacements and tractions have to be applied simultaneously on a portion of the surface structure. Okuma and Oho [11] studied the identification of dynamic characteristics using an inverse problem framework. The set of spatial matrices is determined by using experimentally measured frequency response functions. This method, however, requires additional constraint equations related to the damping matrix.

The present approach combines a computational mechanics methodology with experimentally measured strain data to determine the in-flight response characteristics in real time. In-flight internal and external loads, unlike other flight parameters, cannot be directly measured. Therefore, an inverse approach is required to construct the solution. A finite element-based method for experimental data analysis is developed and leads to the determination of stresses, displacements, and external loads. Numerical simulations are performed using a linearly elastic plate and wing structure to study the accuracy and computational effectiveness of the method.

INVERSE INTERPOLATION METHOD

Consider an airplane that performs a maneuver or is subjected to atmospheric turbulence. A change of the external and internal forces causes linear and angular displacements, strains, and stresses at each point of the structure. The sensors are embedded in the structure along specified patterns, allowing strain component measurements at certain locations. The strain sensors measure the changes in the strain components $\{\boldsymbol{\varepsilon}^*\}$ at n specified locations and appropriately store this information in an onboard computer. These strain data are used as input to the inverse analysis of the computational model based on a finite element formulation. Since the current approach is based on a finite element method, a finite element model of the airframe is first developed.

Consider external loads applied to the airplane incrementally. Any change of forces acting on the aircraft is assumed to be small. At each increment or step of loading, the governing equations of the linear finite element approximation have the form

$$\left[\mathbf{K}\right]\ \left\{\mathbf{U}\right\} = \left\{\mathbf{P}\right\} \tag{1}$$

where $[\mathbf{K}]$ is a stiffness matrix, $\{\mathbf{U}\}$ is a nodal displacements vector, and $\{\mathbf{P}\}$ is a vector of equivalent nodal forces at the current load increment.

The nodal strains can be expressed in terms of nodal displacements using the strain-displacement matrix $[\mathbf{B}]$ as

$$\{\boldsymbol{\varepsilon}\} = [\mathbf{B}]\{\mathbf{U}\} \tag{2}$$

Given a surface load, $p(s)$, nodal forces are readily expressed as

$$\left\{\mathbf{P}\right\} = \int_s \left[\mathbf{N}\right]^T\ p(s)ds \tag{3}$$

where the integration is performed over the surface of the structure s.

The inverse problem is formulated as follows. First, one needs to establish a set of possible load cases, m. Each load case is characterized by the specifics of the aerodynamic load distribution and inertia forces. For the i^{th} load case, the load approximation is expressed in parametric form as

$$F_i(s) = \sum_{j=1}^{l} a_{ij} R_{ij}(s) \tag{4}$$

where $R_{ij}(s)$ are chosen spatial distribution functions and a_{ij} are unknown approximation parameters. These unknown parameters represent flight performance characteristics such as load factor, angle of attack, and speed. There can also be certain inequity constraints imposed on these parameters

$$\varphi\left(a_{ij}\right) \geq 0 \tag{5}$$

The approach proceeds with a direct finite element analysis performed for each i^{th} load case. The corresponding displacements, $\{\mathbf{U_{ij}}\}$, and strains, $\{\boldsymbol{\varepsilon}_{\mathbf{ij}}\}$, are determined from Eqs. (1) and (2) as

$$\left\{\mathbf{U_{ij}}\right\} = \left[\mathbf{K}\right]^{-1} \int_s \left[\mathbf{N}\right]^T R_{ij}(s)ds \tag{6}$$

$$\left\{\boldsymbol{\varepsilon}_{\mathbf{ij}}\right\} = \left[\mathbf{B}\right] \left[\mathbf{K}\right]^{-1} \int_s \left[\mathbf{N}\right]^T R_{ij}(s)ds \tag{7}$$

The stresses are then computed from Hooke's constitutive relations

$$\left\{\boldsymbol{\sigma}_{\mathbf{ij}}\right\} = \left[\mathbf{D}\right]\left\{\boldsymbol{\varepsilon}_{\mathbf{ij}}\right\} \tag{8}$$

where $[\mathbf{D}]$ denotes the elasticity matrix. The displacements, strains, and stresses corresponding to the i^{th} load are computed from the relations

$$\left\{\mathbf{U_i}\right\} = \sum_{j=1}^{m} a_{ij} \left\{\mathbf{U_{ij}}\right\} \tag{9}$$

$$\left\{\boldsymbol{\varepsilon}_{\mathbf{i}}\right\} = \sum_{j=1}^{m} a_{ij} \left\{\boldsymbol{\varepsilon}_{\mathbf{ij}}\right\} \tag{10}$$

$$\left\{\boldsymbol{\sigma}_{\mathbf{i}}\right\} = \sum_{j=1}^{m} a_{ij} \left\{\boldsymbol{\sigma}_{\mathbf{ij}}\right\} \tag{11}$$

The parameters of the load interpolation, Eq. (4), are computed using a least squares procedure minimization, i.e., for the i^{th} load case, we have

$$S_i = \left\{\boldsymbol{\varepsilon}_{\mathbf{i}} - \boldsymbol{\varepsilon}_{\mathbf{i}}^{*}\right\}^T \left\{\boldsymbol{\varepsilon}_{\mathbf{i}} - \boldsymbol{\varepsilon}_{\mathbf{i}}^{*}\right\} = \left\{\sum_{j=1}^{m} a_{ij} \left\{\boldsymbol{\varepsilon}_{\mathbf{ij}}\right\} - \varepsilon_i^{*}\right\}^T \left\{\sum_{j=1}^{m} a_{ij} \left\{\boldsymbol{\varepsilon}_{\mathbf{ij}}\right\} - \varepsilon_i^{*}\right\} \tag{12}$$

The minimization of Eq. (12) with respect to the parameters a_{ij}, while accounting for the constraints (5), results in a governing system of linear algebraic equations that is readily solved for the a_{ij} coefficients. In this manner, a set of the a_{ij} solutions for m load cases can be obtained.

It is clear that a suitable criterion must be established in order to select the most appropriate load case. One such criterion employs a quality function, $Q(S_i, m, n)$, in which the calculated S_i values are used. A particular form of this function is constructed based on computer simulations and experimental statistics. For example, if Chebishev's polynomials are chosen to represent the $R_{ij}(s)$ functions, the quality function can be represented as [5]

$$Q\left(S_i, m, n\right) = \frac{S_i}{1 - \sqrt{\frac{1}{n}\left\{\left(m+1\right)\left[\ln\left(\frac{n}{m+1}\right)+1\right] - \ln H\right\}}} \tag{13}$$

where $1-H$ is the probability at which this estimate is valid. A minimum of the quality function $Q(S_i, m, n)$ corresponds to the k^{th} load case, providing the best solution to the overall inverse problem. For the known a_{kj} $(j=1,l)$ parameters, the resulting forces acting on the structure are calculated from Eq. (4), followed by the displacement, strain, and stress computations using Eqs. (6-11).

NUMERICAL SIMULATIONS

The present approach is validated using a two-step procedure. A finite element model is first developed for a given linearly elastic structure under the applied load $p(s)$. Surface strains $\{\boldsymbol{\varepsilon}^*\}$ are then computed at n specified locations, with these quantities representing the "measured" experimental strains. In the second step of the analysis, an inverse method is employed that uses these measured strains, $\{\boldsymbol{\varepsilon}^*\}$, and the parametric approximation of the load, $F_i(s)$, as given in Eq. (4). The strains are computed from the loads represented by the $R_{ij}(s)$ functions. The parametric approximation of the load is obtained via a least-squares minimization of Eq. (12). Then, the applied loads and structural response are recovered.

Two numerical examples are presented. The first is a cantilever plate under transverse loading, whereas the second is a simplified model of an aircraft wing box. The material properties used are those for a typical aluminum alloy with Young's modulus E = 72 GPa and Poisson's ratio ν=0.3. The finite element analysis is performed using a commercial finite element code, ANSYS [12], with both models employing "SHELL 63" shell elements.

Plate Analysis

Consider a cantilevered rectangular plate under transverse loading $p(x,y) = b_1 x^2 + b_2 y^2 + b_3 y + b_4$ applied normal to the top of the plate as shown in Figure 1. Two loading cases are considered: (a) a dominant bending load given by the parameters $b_1 = -0.25$, $b_2 = -4$, $b_3 = 0$, and $b_4 = 1.5$; and (b) a dominant torsion load with $b_1 = -0.25$, $b_2 = -4$, $b_3 = 15$, and $b_4 = 0.67$. Normal strains in the x-direction, $\{\boldsymbol{\varepsilon}_x^*\}$, and in the y-direction, $\{\boldsymbol{\varepsilon}_y^*\}$, simulating experimental measurements, are first computed from the finite element analysis (first step) at 17 strain-gage locations $\left(n=34\right)$, uniformly spaced along the line $y=-0.125$ on the upper surface of the plate, $z=0$. The plate is discretized using a mesh of 4 × 16 shell elements.

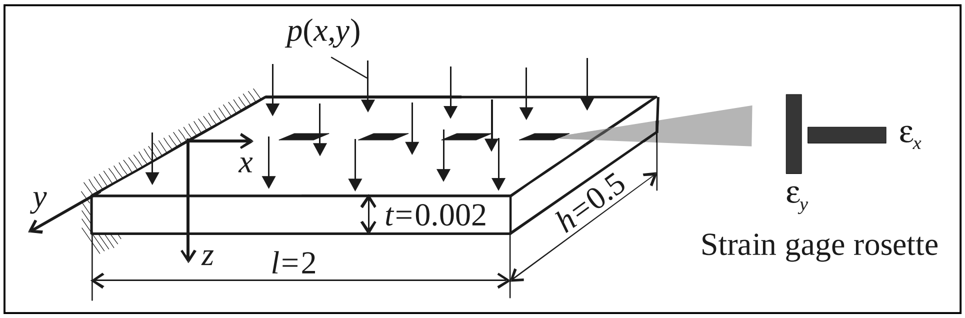

Figure 1. The cantilevered plate problem.

Assuming that the load approximation is represented by $F(x,y)=a_1x^2+a_2y^2$ $+a_3y+a_4$, the present inverse approach gives rise to the values of the a_j $(j=1,4)$ coefficients summarized in Table I. Their favorable comparison with the corresponding values of the b_j $(j=1,4)$ coefficients for the actual load case clearly demonstrates that the proposed method defines the load accurately.

TABLE I. COEFFICIENTS OF LOAD APPROXIMATION

Load Case	a_1	a_2	a_3	a_4
a	-0.245	-3.99	-0.004	1.499
b	-0.250	-3.99	14.99	0.670

Aircraft Wing Box

The second example concerns the wing box depicted in Figure 2. The upper and lower panels are stiffened with integral stringers. As shown in the figure, three ribs are attached spanwise on the inside of the wing. The load $p(x,y)=b_1\sqrt{l^2-x^2}$ $(b_1=0.5)$ is applied normal to the upper surface of the wing. There are 11 strain-gage locations for $\{\boldsymbol{\varepsilon}_x^*\}$ $(n=11)$, uniformly spaced along the line $y=-21.4$ on the upper surface of the wing.

Here, we select the load approximation function as $F(x,y)=a_1\sqrt{l^2-x^2}$. In order to study the effect of measurement errors on the accuracy of the method, the measured strains are assumed in the form $\{\boldsymbol{\varepsilon}_{\mathrm{X}}^*\}\{\boldsymbol{\Delta}^*\}$. The components of the vector of relative errors $\{\boldsymbol{\Delta}^*\}$ are given as $\Delta_i=1+0.05\,\delta_i$, where δ_i is a random variable having a standard normal distribution. The recovered a_1 coefficient is in the range of 0.494-0.502. These values agree well with the actual load parameter $b_1=0.5$.

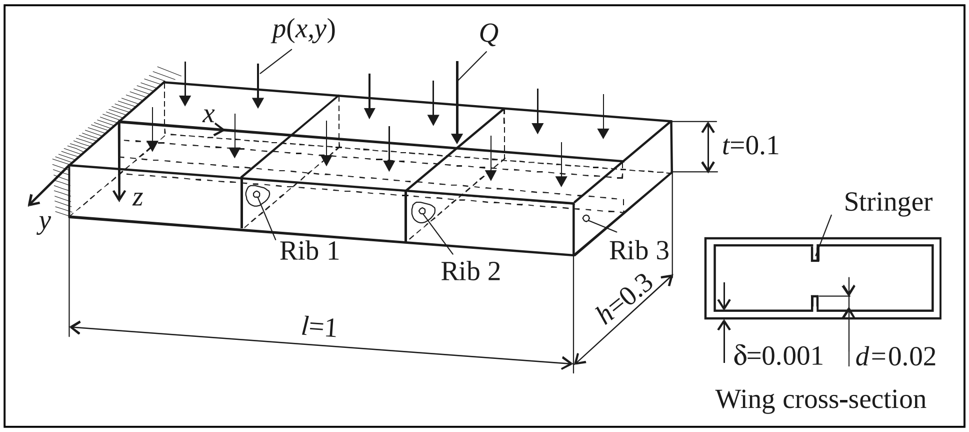

Figure 2. Aircraft wing box and loading.

FUTURE DEVELOPMENTS

Additional numerical studies based on different loadings will be performed. In order to correlate and quantify the results of the inverse approach, an experimental validation of the methodology is underway. Structural testing of an aircraft wing will be performed in an experimental setup comprised of a wing box, three-dimensional frame, and loading and measurement systems. These tests will allow: (a) the determination of the accuracy of the proposed method, (b) a study of the errors associated with the application of strain gages, and (c) development of recommendations for the number and locations of strain gages.

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