MODELING AND DESIGN ANALYSIS METHODOLOGY FOR TAILORING OF AIRCRAFT STRUCTURES WITH COMPOSITES

By

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Introduction

Composite materials provide design flexibility in that fiber placement and orientation can be specified and a variety of material forms and manufacturing processes are available. It is possible, therefore, to "tailor" the structure to a high degree in order to meet specific design requirements in an optimum manner. Common industrial practices, however, have limited the choices designers make. One of the reasons for this is that there is a dearth of conceptual/preliminary design analysis tools specifically devoted to identifying structural concepts for composite airframe structures. Large scale finite element simulations are not suitable for such purposes.

The present project has been devoted to creating modeling and design analysis methodology for use in the tailoring process of aircraft structures. Emphasis has been given to creating bend-twist elastic coupling in high aspect ratio wings or other lifting surfaces. The direction of our work was in concert with the overall NASA effort Twenty-First Century Aircraft Technology (TCAT). A multi-disciplinary team was assembled by Dr. Damodar Ambur to work on wing technology, which included our project.

Summary of Accomplishments

Our current work has included the following items:

(1) Analysis to design elastically tailored wings with bend-twist coupling one cross section at a time. This work appears in Appendix I.

Tapered wings may be analyzed with the use of item (1) to discrete spanwise wing stations and with the loads known. Our grant monitor, Dr. Damodar Ambur, redirected us in 2003 to put a special emphasis on making the analysis of highly
tapered wings a significant priority and to make it efficient. The result is in items (2) and (3).

(2) Analysis of 3D wings with geometric taper, which includes variation of chord, airfoil thickness and cover wall thickness has been created and applied without bend twist coupling in the wing.

(3) Use of simplified design criteria which include the following steps:

(a) The wing cover thicknesses are defined by spanwise bending strain level. This is followed by a check for torsional divergence.

(b) “Rigid” wing loads are used to predict first order elastic deformations.

(c) Loading redistributions due to first order elastic deformations are evaluated. Effect on bending strain is assessed.

Use the results of (b) and (c) to set composite ply layups in the wing covers for “best” results. This will necessitate subdividing the wing planform into zones of constant ply layups. The zones influence the manufacturing process, and, therefore, require practical judgment.

Iteration may be required to define the cover preliminary thickness estimate.

Items (1) and (2) are complete. Item (3) has been conceived and applied to a fictitious Reno Air Racer with no elastic coupling in the wings, but not compared with other methodology such as indicated in item (1).

The differences among (1),(2), and (3) is that (1) requires only structures and materials technology. Item (2) and (3) require aerodynamic and mass information. Also, We have employed aerodynamic strip theory, which lacks 3D resolution, but which
should provide reliable preliminary design definition for the structural configuration. This work appears in Appendix II.

All of our work has been devoted to preliminary design analysis. The objective is to give the designer guidance in establishing the structural configuration. Once a viable configuration is identified, “fine tuning” to save weight and cost can proceed with the aid of large scale numerical simulation and perhaps, in addition, optimization. The key element is the definition of the ply layups.

**Concluding Remarks**

The effort that we directed to highly tapered wings at Dr. Ambur’s suggestion did not permit an evaluation of both taper and bend-twist elastic coupling. A request for additional supplemental funding and additional time was submitted to NASA Langley in order to reach all objectives. Unfortunately, the TCAT program was cancelled and no supplementary funds were provided.

Two Journal of Aircraft papers were written and presented at AIAA conferences. Both papers have been accepted for publication and are under revision. A complete record of the project accomplishments appears in Appendices I and II.
Appendix I

Rosita Cheung's
MS Thesis

2002
SOME STRATEGIES FOR CREATING BEND-TWIST COUPLING IN BOX BEAMS

By
ROSITA HIU-YEE CHEUNG

B.S. (University of California, Davis) 2001

THESIS

Submitted in partial satisfaction of the requirements for the degree of

MASTER OF SCIENCE

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of the

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DAVIS

Approved:

Committee in Charge

2002
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>Torsional stiffness matrix</td>
</tr>
<tr>
<td>a₁₂</td>
<td>Defined in eq. (14)</td>
</tr>
<tr>
<td>a₂₆</td>
<td>Defined in eq. (15)</td>
</tr>
<tr>
<td>Aᵢⱼ</td>
<td>Laminate membrane stiffnesses for wing box covers, or elements of matrix A</td>
</tr>
<tr>
<td>C₁₂</td>
<td>Effective Poisson's ratio, eq. (22)</td>
</tr>
<tr>
<td>C₂₆</td>
<td>Twist-camber coupling parameter, eq. (23)</td>
</tr>
<tr>
<td>Cᵢⱼ</td>
<td>Beam stiffness matrix</td>
</tr>
<tr>
<td>C₄₄</td>
<td>Torsional Stiffness, eq. (6)</td>
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<tr>
<td>C₄₅</td>
<td>Coupling Stiffness, eq. (7)</td>
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<tr>
<td>C₅₅</td>
<td>Bending Stiffness, eq. (8)</td>
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<td>Aerodynamic chord</td>
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<tr>
<td>cₛ</td>
<td>Chord of wing section structural box</td>
</tr>
<tr>
<td>E₁₁</td>
<td>Young's modulus of composite material in fiber direction</td>
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<tr>
<td>E₁₂</td>
<td>Young's modulus of composite material in transverse direction</td>
</tr>
<tr>
<td>G₁₂</td>
<td>In-plane shear modulus</td>
</tr>
<tr>
<td>H</td>
<td>Height of wing section structural box</td>
</tr>
<tr>
<td>h</td>
<td>Skin thickness of load bearing covers of the wing structural box</td>
</tr>
<tr>
<td>hₖ</td>
<td>Thickness of k-th ply of a laminate</td>
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<td>Kᵢⱼ</td>
<td>Structural box cover stiffnesses</td>
</tr>
<tr>
<td>K₁₁</td>
<td>Extensional stiffness, eq. (2)</td>
</tr>
<tr>
<td>K₁₂</td>
<td>Shear stiffness, eq. (3)</td>
</tr>
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K_{22}  
Extension-shear coupling stiffness, eq. (4)

k_{ij}  
Structural box cover stiffnesses per unit skin thickness

k_c  
Camber curvature kinematic matrix defined in eq. (21)

M_x  
Chordwise bending moment, eq. (17)

M_y  
Spanwise bending moment, eq. (18)

N_{xx}  
Axial running load due to bending in the wing box covers

N_{xs}  
Membrane shear flow or stress resultant

N_{ss}  
Circumferential (hoop) membrane stress resultant

\overline{Q}_{ij}  
Plane stress stiffnesses for each ply of a laminate

W_{xx}  
Bending curvature of wing box structural box, eq. (20)

x,y,z  
Cartesian coordinates

1,2  
Transformed coordinates

\beta^2  
Bend-twist coupling parameter, eq. (10)

\beta_1^2  
Bend-twist coupling parameter for covers only, eq. (11)

\epsilon  
Strain in fiber direction

\epsilon_{xx}, \epsilon_{ss}  
Components of extensional membrane strain in cell wall

\epsilon_{x\psi}, \epsilon_{y\psi}  
Strains in the x_\psi-y_\psi coordinate system

\gamma_{xs}  
Membrane shear strain in cell wall of wing section structural box

\mu\epsilon  
Microstrain, strain×10^6

\phi_x  
Rate of twist of wing box structural box, eq. (19)

\psi  
Laminate rotation angle with respect to the axis of the structure
\( \theta \) Dominant skin fiber orientation angle

**Superscripts**

- \( i,j,k \) Refers to indices assuming the values 1, 2, 6

**Superscripts**

- \( fw \) Refers to front spar web
- \( k \) Refers to identifying index
- \( l \) Refers to lower wing box cover
- \( rw \) Refers to rear spar web
- \( u \) Refers to upper wing box cover
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INTRODUCTION

Even many years after the invention of fiber-reinforced composite materials, many new uses are being found for them. They are used in many different applications because of their high strength-to-weight ratio, stiffness-to-weight ratio and increased design flexibility. The goal for designing is usually to reduce weight or cost while achieving high performance products. Although composite materials cost more than metal materials, they can produce cost competitive structures.

Elastic tailoring of composite materials utilizes their design flexibility. From Rehfield's definition\(^1\), an appropriate selection of structural concept, fiber orientation, ply stacking sequences and blending of materials can achieve a specific design goal. In past research, elastic tailoring has been used to influence the aerodynamics of the system\(^2\). This is defined as "aeroelastic tailoring."

From the definition of Shirk, Hertz and Weisshaar\(^2\), "Aeroelastic tailoring is the embodiment of directional stiffness into an aircraft structural design to control aeroelastic deformation, static or dynamic, in such a fashion as to affect the aerodynamic and structural performance of the aircraft in a beneficial way." The example of it is Grumman's X-29 technology demonstrator. The X-29's forward swept wing has high lift-to-drag ratio and increased resistance to torsional divergence.

The focus of the present work is mainly on bend-twist coupling in high aspect ratio wing boxes. This coupling can increase the stability of the aircraft system and resist the tip stall of subsonic wings. Bend-twist coupling is produced by the stiffness of the angle plies, which are the off-axis plies oriented at angles with respect to the bending axis.
of the structure. The laminated ply stiffness depends upon the material and fiber orientation. The coupling created by the stiffness can create the desired effect.

Two design strategies are used to understand the bend-twist coupling, which are angle ply rotation and laminate rotation. These two design strategies have been applied to two types of models. One is the ideal tailored box model from Rehfield’s work\(^3\), in which only the upper and lower covers are the load-bearing structural elements. The second is similar to the first one but with the additional sides or spar webs being modeled. In the latter model, all elements of the box are load-bearing. After extensive evaluation, the second model is chosen for our study. The first model tends to underestimate torsional stiffness.

An illustration for a large transport wing section is studied to help understand the two design strategies for producing bend-twist coupling.

**DESIGN STRATEGIES**

**Preliminary Remarks**

Bend-twist coupling is produced by creating stiffness which is not aligned with the bending axis of the primary structure. In wing covers made of laminated composites, some off-axis plies will serve to produce the desired effect. This is the context which is considered here.

**Angle Ply Rotation**

In this thesis, two design strategies are considered: Angle Ply Rotation and Laminate Rotation. The angle ply rotation uses axis-oriented plies and an unbalanced set of angle plies. If desired, transverse plies also can be included to increase damage
tolerance. In this method, bend-twist coupling is produced by the unbalanced layup. The angle ply orientation can be varied in order to enhance coupling. As shown in Figure 1, the axial plies stay in the same position but the angle of the unbalanced set of angle plies varies. The angle, \( \theta \), rotates with respect to the axial axis of the structure.

**Laminate Rotation**

The second design strategy is called "Laminate Rotation." This method has been applied to the swept forward wing of the X-29 and the analysis of bend-twist coupling in Ref. 3. This approach chooses an established layup configuration, which includes axial plies and angle plies. Transverse plies also can be included if desired. As shown in Figure 2, the entire laminate is rotated from the \( x-y \) coordinate system to the \( x_\psi-y_\psi \) coordinate system with respect to the axial axis of the structure. The coordinates natural to the solution of the structure problem are the structure coordinates \( x, y \), whereas the principal material coordinates are \( x_\psi, y_\psi \). The rotation angle, \( \psi \), is the angle measured from the \( x \)-axis to the \( x_\psi \)-axis. It is recommended to choose a balanced layup configuration in order to avoid or minimize warping due to processing. After the rotation, the plies are no longer aligned with the axis and bend-twist coupling is introduced.

The angle plies in the Angle Ply Rotation method and the laminate in the Laminate Rotation method are rotated until they produce the desired bend-twist coupling. It is efficient to have all angle plies in angle ply rotation to produce bend-twist coupling. However, there is a disadvantage that arises from the common manufacturing-related warping produced by processing thermoset composite materials for unbalanced
configurations. The advantages of angle ply rotation method over the laminate rotation method will inspire innovative utilization. For this method, warping effects due to unbalanced configurations will be neglected.

**MODEL AND ANALYSIS**

**Model**

A simple model is employed to analyze the wing box using the two design strategies. This model is based upon a refinement of the one in Ref. 3, which considers load bearing covers and spar webs or sides which are not modeled. The model shown in Fig. 3 is used for this study. It factors in the effects of the webs being modeled. As a result, this model has contributions from every portion of the box. This approach is more realistic. Based upon a consensus of Refs. 4-6, the webs are assumed to be made of balanced angle plies. The preferred choice is $[\pm45]$ plies$^{4,6}$. In addition, the recommended choice for the web wall thickness is less than half of the covers.

The coordinate system of the box model is shown in Fig. 4, where $x$ and $s$ are indicated. Also, the figure shows the ply orientation for the covers. The upper and lower covers are tailored identically and asymmetrically. They are mirror images of each other, which is the “best” way to create bend-twist coupling. The angle $\theta$ is the angle between the dominant off-axis and the ply orientation. The laminates for the upper and lower covers can be made of axial plies, angle plies, and transverse plies. Since the $s$ coordinate surrounds the box model, the signs for the angles are different for the covers. As shown in Fig. 5, the angle for the upper cover is positive, but negative for the lower cover.
Analysis of the Model

The analysis methodology is based upon the Bernoulli-Euler bending assumption applied to Rehfield’s theory of thin-walled composite beams. The warping effects and transverse shear deformations are neglected here. The walls of the model carry loads in plane stress, and the stress resultants for the walls are denoted \( N_{xx}, N_{xs} \) and \( N_{ss} \). The axial running load due to bending is \( N_{xx} \), and \( N_{xs} \) is the running load due to torsion (shear flow). The circumferential stress resultant, \( N_{ss} \) is assumed to be negligible. This implies no internal pressure. Consequently,

\[
\begin{bmatrix}
N_{xx} \\
N_{xs}
\end{bmatrix} = 
\begin{bmatrix}
K_{11} & K_{12} \\
K_{12} & K_{22}
\end{bmatrix} 
\begin{bmatrix}
\varepsilon_{xx} \\
\gamma_{xs}
\end{bmatrix}
\]

(1)

\( K_{11}, K_{12} \) and \( K_{22} \) corresponds to uniaxial extension, shear, and coupling stiffnesses, respectively. They are defined as

\[
K_{11} = A_{11} - \frac{(A_{12})^2}{A_{22}}
\]

(2)

\[
K_{12} = A_{16} - \frac{A_{16}A_{26}}{A_{22}}
\]

(3)

\[
K_{22} = A_{66} - \frac{A_{26}^2}{A_{22}}
\]

(4)

The membrane stiffnesses, \( A_{ij} \), are independent of the stacking sequence, which can be determined by simply adding the plane stress stiffnesses, \( \overline{Q_{ij}} \), for each ply. For a laminate of \( N \) plies,

\[
A_{ij} = \sum_{k=1}^{N} \overline{Q_{ij}} h_k \quad (i, j = 1, 2, 6)
\]

(5)

where \( h_k \) is the thickness of \( k^{th} \) ply.
Since the webs have contributions of stiffnesses for the present model, the expressions of the global stiffnesses are different from the previous model in Ref. 7. The global stiffnesses are

\[ C_{44} = \frac{C_s^2H^2}{(C_s + H)^2} \left[ \left( K_{22}^u + K_{22}^l \right) C_s + h \left( f_i \bar{Q}_{66}^{fw} + f_r \bar{Q}_{66}^{rw} \right) H \right] \quad (6) \]

\[ C_{45} = \frac{C_s^2H^2}{2(C_s + H)} \left( K_{12}^u - K_{12}^l \right) \quad (7) \]

\[ C_{55} = \frac{C_sH^2}{4} \left( K_{11}^u + K_{11}^l \right) \quad (8) \]

where \( C_{44} \), \( C_{45} \) and \( C_{55} \) are the torsional, coupling, and bending stiffnesses, respectively. In eqs. (6)-(8), the superscripts \( u \) and \( l \) denote the upper and lower covers, respectively, and \( fw \) and \( rw \) denote the front and rear spar webs, respectively. The web wall thickness is a fraction of the cover wall thickness, \( h \), which \( f_i \) is the fraction for front spar web and \( f_r \) is the fraction for the rear spar web.

Since a balanced configuration is chosen for the webs, the coupling stiffness is not affected. Furthermore, the bending effects from the webs are neglected; the only stiffness that is affected is \( C_{44} \), the torsional stiffness, which is increased.

The stiffnesses require the determination of wall thickness by using layup coefficients for a unit thickness in the strain limit equation. It is convenient to define the membrane stiffness in terms of thickness

\[ K_{ij} = h k_{ij} \quad (i, j = 1, 2) \quad (9) \]

and the bend-twist coupling parameters that are convenient to define

\[ \beta^2 = \frac{(C_{45})^2}{C_{44}C_{55}} \quad (10) \]
Design Analysis Methodology for Angle Ply Rotation

The design criterion used is to limit the axial strain in the fiber direction of a ply. Using the limit of fiber strain as a criterion is in use at Northrop Grumman. The wall thickness corresponds to one of the ply types reaching the strain limit. Before comparing the strains in the fiber direction, the strains in the x-s coordinate system have to be determined first. The inversion of Eq. (1) shows the expressions for them.

\[
\begin{bmatrix}
\varepsilon_{xx} \\
\gamma_{xs}
\end{bmatrix}
= \frac{1}{1 - \beta_1^2}
\begin{bmatrix}
\frac{1}{K_{11}} & -\frac{K_{12}}{K_{11}K_{22}} \\
\frac{K_{12}}{K_{11}K_{22}} & \frac{1}{K_{22}}
\end{bmatrix}
\begin{bmatrix}
N_{xx} \\
N_{xs}
\end{bmatrix}
\]

Then the strain transformation equations are used to determine fiber direction strains for all ply types. The strain for each ply type with a rotation angle \( \theta \) with respect to the x-s coordinate system is

\[
\varepsilon = \frac{1}{h(1 - \beta_1^2)} \left[ \left( \frac{a_{12}}{k_{11}} - \frac{a_{26} \beta_1^2}{k_{12}} \right) N_{xx} + \left( \frac{a_{26}}{k_{22}} - \frac{a_{12} \beta_1^2}{k_{12}} \right) N_{xs} \right]
\]

where \( a_{12} \) and \( a_{26} \) are defined as follows

\[
a_{12} = \cos^2 \theta - \sin^2 \theta \left( \frac{A_{12}}{A_{22}} \right)
\]

\[
a_{26} = \sin \theta \cos \theta - \sin^2 \theta \left( \frac{A_{26}}{A_{22}} \right)
\]

The cover wall thickness is determined by comparing the strains for all the ply types in the configuration and determine when the limit is reached first. The steps in this analysis are indicated in Fig. 6.
Design Analysis Methodology for Laminate Rotation

The stress resultants in the $x$-$s$ coordinate system are transformed to the $x_w$-$y_w$ coordinate system with an angle $\psi$ in order to get the strain-stress resultant relationship.

\[
\begin{bmatrix}
\varepsilon_{x_w} \\
\varepsilon_{y_w} \\
\gamma_{12}
\end{bmatrix} = [A]^{-1}
\begin{bmatrix}
N_{x_w} \\
N_{y_w} \\
N_{xy_w}
\end{bmatrix}
\]

where $[A]^{-1}$ is the inverse of the $A$ matrix.

Using the same design criterion as Angle Ply Rotation, the wall thickness also corresponds to one of the ply types reaching the strain limit. The strain in the fiber directions can be found by using strain transformation equations. They are known by transforming the strains in the $x_w$-$y_w$ coordinate system to $0^\circ$ for axial plies, $0^\circ$ for balanced angle plies and $90^\circ$ for transverse plies. The steps in this analysis are shown in Fig. 7.

Deformation Parameters

After the wall thickness has been found for the appropriate design strategy, the behavior of the structure can be determined. For only torsion and spanwise bending, the governing equations are

\[
M_x = C_{44}\phi_x + C_{45}(-W_{xx})
\]

(17)

\[
M_y = C_{45}\phi_x + C_{55}(-W_{xx})
\]

(18)

Eqs. (17) and (18) relate the moments to the rate of twist, $\phi_x$, and bending curvature, $W_{xx}$, of the structure. The moments in the $x$ and $y$ directions are $M_x=2CsHN_{xs}$, nose up moment, and $M_y=-N_{x}CsH$, spanwise lift (Fig. 3). The rate of twist and bending curvature of the structure can be determined from Eqs. (17) and (18) by inversion.
Camber curvature is estimated by the following equations:

\[ \phi_{x} = \frac{1}{C_{44}C_{55}(1 - \beta^2)}(C_{55}M_{x} - C_{45}M_{y}) \]  \hspace{1cm} (19) 

\[ W_{xx} = \frac{1}{C_{44}C_{55}(1 - \beta^2)}(C_{45}M_{x} - C_{44}M_{y}) \]  \hspace{1cm} (20)

where \( \epsilon \) characterizes the coupling between twist and camber.

\[ \epsilon \equiv \frac{2}{h} \left( -\bar{C}_{12} \epsilon + \bar{C}_{26} \frac{N_{ss}}{K_{22}} \right) \]  \hspace{1cm} (21)

where \( \bar{C}_{12} \) is an effective Poisson’s ratio

\[ \bar{C}_{12} = \frac{A_{12}}{A_{22}} \frac{A_{26}K_{12}}{A_{26}K_{22}} \]  \hspace{1cm} (22)

and \( \bar{C}_{26} \) characterizes the coupling between twist and camber

\[ \bar{C}_{26} = \frac{A_{26}}{A_{22}} \]  \hspace{1cm} (23)

This completes the analysis for each configuration. Note that no special effort has been made to tailor for camber response. The focus has been only on bend-twist coupling.

TRANSPORT ILLUSTRATION

A specific transport illustration is created for analysis using the two design strategies. The model shown in Figure 3 with the dimensions of the C-130 aircraft center wing box (Fig. 8) is used. The material system for the simple box model is AS4/3501-6 carbon-epoxy. Its elastic properties are shown in Table 1. The stress resultants in the covers are assumed to be 25,000 pounds per inch for \( N_{xx} \), and 5,000 pounds per inch for \( N_{ss} \). The limit for the axial strain in the fiber direction of a ply is set to be 4500
microstrain. The configuration for the Angle Ply Rotation design strategy is 50% of axial plies and 50% of angle plies; the configuration for the Laminate Rotation design strategy is 50% of axial plies and 50% of $[\pm 45]$. These 50-50 configurations are simplified approximations to the wings of the F-18 and the AV-8B aircraft. The transverse plies are neglected for both design strategies for simplicity. The webs are fixed at $[\pm 45]$, which is based on the unanimous recommendations of Refs. 4-6. For simplicity, the thicknesses for the front spar web and the rear spar web are chosen to be the same. The web wall thickness is chosen to be forty percent of the cover thickness. The analysis is based on this specific wing section and loads. Results are presented for designs using Laminate Rotation in Figs. 9-13 and Angle Ply Rotation in Figs. 14-18.

RESULTS AND DISCUSSION

Preliminary Remarks

The illustration corresponds to a transport aircraft wing section, which is that of the center wing of the C-130. Thus, the results presented strictly correspond to this configuration. This geometry, the chosen material system, design criterion adopted, loading considered and strategies used, all contribute to the analytical results and the discussion of them. The intention has been to provide an overall, unique assessment of wing sections with bend-twist coupling. Additional parametric studies and further research are needed to determine if the trends are general.

Laminate Rotation

The angle $\psi$ corresponds to the rotation of the laminate with respect to the axis of the structure.
The following observations are based upon the results in Figs. 9-13. The observations are:

1. The thickness curves have the same geometric shape as the bending and torsional stiffness curves with identically placed cusps (Fig. 9 and 10).

2. The cusps correspond to distinct transitions between ply types reaching the design strain limit. If the design limit were raised for the same material system, the thickness curve would be lowered, but would maintain the same shape.

3. If the ratio between $N_{xx}$ and $N_{xz}$ is nearly constant, but with absolute values reduced, the thickness curve would be lowered as above.

4. Changing the mix of plies would likely alter the cusp points, but not the general shape of the curves.

5. Due to cusp placement, the curve of $C_{45}$ is not exactly skew symmetric (Fig. 10).

6. The rate of twist and bending curvature curves have the same cusp points but opposite curvature for the thickness curve. As a higher thickness corresponds to greater stiffness, these trends make physical sense (Fig. 11 and 12).

7. A different trend can be noticed at approximately −60 and −30 degrees in Figs. 11 and 12, respectively. In the neighborhood of −60 degrees, the rate of twist is extremely low. In contrast, the bending curvature reaches a minimum near −30 degrees.

8. If it is desired to stabilize aeroelastically produced lift due to tailoring, the −60 degrees is a good choice for Laminate Rotation, since it tends to reduce section stall and increase resistance to torsional divergence.
9. The camber curvature is small, and it does not follow similar trends to the other parameters. Recall that no effort has been made to design for camber (Fig. 13).

10. More bend-twist coupling does not necessarily correspond to the coupled global stiffnesses, as shown in Figs. 11-13.

**Angle Ply Rotation**

The following observations are based upon the results in Figs. 14-20. Due to different configurations of ply layups, the results for Angle Ply Rotation cannot be directly compared to the results for Laminate Rotation. The angle $\theta$ is the angle ply orientation with respect to a fixed $[0]$ axis. The following observations are made:

1. There is one less cusp to the curves for thickness, stiffness, rate of twist and bending curvature. There is a cusp at $\theta=0^\circ$, which could be evaluated by a separate analysis. The way the current program is written as shown in Appendix A, $\theta=0^\circ$ corresponds to dividing by zero, which cannot be determined.

2. Even though the two design strategies cannot be compared directly, there are still some noticeable differences between them. From Fig. 19, the thickness for Angle Ply Rotation that is required to satisfy the design requirements is less than thickness for Laminate Rotation.

3. There is less bend-twist coupling and greater twisting stiffness for negative ply rotation (Fig. 15). The bending stiffness is greater for negative angle ply rotation, $\theta$, than for negative laminate rotation, $\psi$. The curves are affected by the cusps.

4. Compared to Laminate Rotation, the coupled bending stiffness for Angle Ply Rotation is increased and the coupled torsional stiffness is decreased. Based on
the changes in stiffnesses, the value of the rate of twist for Angle Ply Rotation is greater than Laminate Rotation.

5. Not only the rate of twist has been affected; the global compliance of bending curvature for Angle Ply Rotation is less than Laminate Rotation.

6. The camber curvature is greater for Angle Ply Rotation than Laminate Rotation for positive rotation, but the reverse is true for negative rotations (Fig. 20).

Concluding Remarks

An aircraft illustration of the two design strategies has been presented. The trends are likely to be affected by selected loading magnitudes and their respective ratios, which may be altered by control surfaces and internal features of the structure such as ribs.

SUMMARY

In this study, two design strategies specifically chosen to produce spanwise bend-twist coupling have been presented and illustrated. The illustration that has been chosen is the center wing section of a transport wing. Even though the results of the two design strategies show some similarities, the wing cover ply layups are so different that they cannot be directly compared. It does appear, however, that the new angle ply rotation strategy may produce a lighter weight structure.

This study is only a beginning. In order to understand these two design strategies well, much future work is needed.
REFERENCES


<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiber direction, tension modulus, $E_1$</td>
<td>$20.0 \times 10^6$ psi</td>
</tr>
<tr>
<td>Transverse direction, tension modulus, $E_{22}$</td>
<td>$1.7 \times 10^6$ psi</td>
</tr>
<tr>
<td>In-plane shear modulus, $G_{12}$</td>
<td>$0.85 \times 10^6$ psi</td>
</tr>
<tr>
<td>Poisson's ratio, $\nu_{12}$</td>
<td>0.30</td>
</tr>
</tbody>
</table>

Table 1. Nominal unidirectional mechanical properties for AS4/3501-6 graphite-epoxy (room temperature, dry).
FIGURE 1 - ANGLE PLY ROTATION
FIGURE 2 - LAMINATE ROTATION
FIGURE 3 - BOX MODEL WITH WEBS MODELED

- NONSTRUCTURAL ELEMENTS
- LOAD BEARING COVERS

AERODYNAMIC CENTER

BENDING AXIS

WEB

M_x

M_y

y_{AC}

y

V_\infty

C_a

C_s

H
FIGURE 4 - SIGN CONVENTION FOR PLY ORIENTATION AND LAYUP
FIGURE 5 - PLY ORIENTATION FOR THE COVERS

Upper Cover

Lower Cover
Rotate the angle plies from the x-s coordinate system to the 1-2 coordinate system with an angle, \( \theta \), where \( \theta \) is the angle from the x-axis to the 1-axis.

Express the strain-stress resultant relationships in the 1-2 coordinate system.

Determine the expressions for the strains in the [0] and [\( \theta \)] plies.

Limit the axial strain in the fiber direction of a ply to 4500 \( \mu \varepsilon \). Compare the strains in the [0] and [\( \theta \)] plies. Determine which one will reach the limit strain level first.

The cover thickness is controlled by the ply that reaches the limit strain level first.

Global stiffnesses and camber curvature can then be determined.
Rotate the laminate from the \( x-s \) coordinate system to the \( x_{\psi}-y_{\psi} \) coordinate system with an angle, \( \psi \), where \( \psi \) is the angle from the \( x \)-axis to the \( x_{\psi} \)-axis.

Transform the fixed stress resultants, \( N_{xx} \) and \( N_{xs} \), from the \( x-s \) coordinate system to the \( x_{\psi}-y_{\psi} \) coordinate system.

Express the strain-stress resultant relationships in the \( x_{\psi}-y_{\psi} \) coordinate system.

Determine the expressions for the strains in the \([0]\), \([\pm \theta]\) and \([90]\) plies.

Limit the axial strain in the fiber direction of a ply to 4500 \( \mu \varepsilon \). Compare the strains in the \([0]\), \([\pm \theta]\) and \([90]\) plies. Determine which one will reach the limit strain level first.

The cover thickness is controlled by the ply that reaches the limit strain level first.

Global stiffnesses and camber curvature can then be determined.
FIGURE 8 - C-130 BOX DIMENSIONS

NOTE: ALL UNITS ARE IN INCHES
FIGURE 9 THICKNESS VS. LAMINATE ROTATION for (50% [0], 50% [±45], 0.4 web's thickness)
FIGURE 10 STIFFNESSES VS. LAMINATE ROTATION
for (50% [0], 50% [±45], 0.4 web's thickness)
FIGURE 11 RATE OF TWIST VS. LAMINATE ROTATION
for (50% [0], 50% [±45], 0.4 web's thickness)
FIGURE 12 BENDING CURVATURE VS. LAMINATE ROTATION
for (50% [0], 50% [±45], 0.4 web's thickness)
FIGURE 13 CAMBER CURVATURE VS. LAMINATE ROTATION
for (50%[0], 50% [±45], 0.4 web's thickness)
FIGURE 14 THICKNESS VS. ANGLE PLY LAYUP ORIENTATION
for (50% [0], 50% [θ], 0.4 web's thickness)
FIGURE 15 BENDING, TWISTING and COUPLING STIFFNESSES VS. ANGLE PLY LAYUP ORIENTATION
for (50% [0], 50% [θ], 0.4 web's thickness)
FIGURE 16 RATE OF TWIST VS. ANGLE PLY LAYUP ORIENTATION
for (50% [0], 50% [θ], 0.4 web’s thickness)
FIGURE 17 BENDING CURVATURE VS. ANGLE PLY LAYUP ORIENTATION
for (50% [0], 50% [θ], 0.4 web's thickness)
FIGURE 18 CAMBER CURVATURE VS. ANGLE PLY LAYUP ORIENTATION
for (50% [0], 50% [θ], 0.4 web's thickness)
FIGURE 19 COMPARATIVE THICKNESS
for (50% [0], 50% [θ], 0.4 web's thickness)
FIGURE 20 COMPARATIVE CAMBER CURVATURE
for (50% [0], 50% [θ], 0.4 web's thickness)

Camber Curvature ($\text{in}^{-1} \times 10^{-4}$)

Angle Ply Rotation Angle ($\theta$) or Laminate Rotation Angle ($\psi$), (deg)
APPENDIX A

PROGRAM FOR ANGLE PLY ROTATION

Variable and Parameter Definitions

Appendix A is the Fortran 90 program that is used to analyze Angle Ply Rotation.

The following inputs are required for the program.

Inputs:

- Width of the box model, Cs
- Height of the box model, H
- Elastic properties of the materials used for the box model, $E_{11}$, $E_{22}$ and $G_{12}$
- Limit strain for the design criterion, $\varepsilon$
- Axial loading due to bending, $N_{xx}$, and due to torsion, $N_{xs}$
- Configuration of the ply layup – fractions of axial plies, angle plies and transverse plies, which are $f_0$, $f_9$, $f_{90}$
- Orientation for the ply layup of the webs, $w$
- Fraction of the wall thickness for the front spar web, $f_f$
- Fraction of the wall thickness for the rear spar web, $f_r$

After inputting the values for the above parameters, the program generates the following outputs.

Outputs:

- Bend-twist coupling parameter, $f^2$
- Thickness of the covers, $h$
- Rate of twist, $\phi_x$
- Bending curvature, $W_{xx}$
- Camber curvature, $k_c$
- Global stiffnesses, $C_{44}$, $C_{55}$, and $C_{45}$
- Relative weight, which is compared to a balanced benchmark reference of [0] and [45] plies.

Notes: All of the inputs are in degrees and English units.

Program Listing

```
c==================================
c PROGRAM for Angle Ply Rotation
c==================================

Program coupling
real*8 E11,E22,G12,v12,v21,Q11,Q12,Q22,Q66,C44,C45,C55
```
real*8 Qbar11u,Qbar12u,Qbar22u,Qbar16u,Qbar26u,Qbar66u,eps
real*8 Qbar11l,Qbar12l,Qbar22l,Qbar16l,Qbar26l,Qbar66l
real*8 Qbar11w,Qbar12w,Qbar22w,Qbar16w,Qbar26w,Qbar66w
real*8 Qbar11b,Qbar12b,Qbar22b,Qbar66b
real*8 Qb110,Qb120,Qb220,Qb160,Qb260,Qb660
real*8 Qb11u,Qb12u,Qb22u,Qb16u,Qb26u,Qb66u,Q(4)
real*8 Qb119,Qb129,Qb229,Qb169,Qb269,Qb669
real*8 Qb11l,Qb12l,Qb22l,Qb16l,Qb26l,Qb66l
real*8 Qb11p,Qb12p,Qb22p,Qb16p,Qb26p,Qb66p
real*8 Qb11n,Qb12n,Qb22n,Qb16n,Qb26n,Qb66n
real*8 Qb115,Qb125,Qb225,Qb165,Qb265,Qb665,d,c,betamax
real*8 k11u,k12u,k22u,theta,betasq,Cs,He,sumf,f0,frheta,f90
real*8 k11l,k12l,k22l,c12u,c26u,kcu,thick,thickb,wgain,thetamax
real*8 k11w,k12w,k22w,k11b,k22b,betal,hbb,htb,Mx,My,dphi
real*8 theta1,pi,a,c,Nxx,Nxs,a12,a26,a12b,a26b,bb,ht,w,ffw,frw,Wxx
integer i

!--------------------------------------
!
! create the data files for the results
!
!--------------------------------------

open(unit=11, file='result.dat',status='unknown')
open(unit=12, file='result1.dat',status='unknown')
open(unit=13, file='result2.dat',status='unknown')
open(unit=14, file='result3.dat',status='unknown')
open(unit=15, file='result4.dat',status='unknown')
open(unit=16, file='result5.dat',status='unknown')

pi=3.14159
betamax=0.0
write(*,*) 'Enter the width for the single beam box in inches'
read(*,*) Cs
write(*,*) 'Enter its height in inches'
read(*,*) He
write(*,*) 'Enter the properties of the material'
write(*,*) 'E11?'
read(*,*) E11
write(*,*) 'E22?'
read(*,*) E22
write(*,*) 'G12?'
read(*,*) G12
write(*,*) 'v12?'
read(*,*) v12
write(*,*) 'strain?'
read(*,*) eps
write(*,*) 'Enter the axial load Nxx'
read(*,*) Nxx
write(*,*) 'Enter the shear flow Nxs'
read(*,*) Nxs
write(*,*) 'Enter the degree for the angle plies of the webs'
read(*,*) w
write(*,*) 'Enter the fraction for the front spar web'
read(*,*) ff
write(*,*) 'Enter the fraction for the rear spar web'
read(*,*) fr
sumf=0

write(*,*) 'Enter the fraction for zero degree plies'
read(*,*) f0
write(*,*) 'Enter the fraction for theta plies'
read(*,*) ftheta
write(*,*) 'Enter the fraction for 90 degrees plies'
read(*,*) f90
sumf=f0+ftheta+f90

Do while (sumf.NE.1.0)
write(*,*) 'The sum of the fraction for all plies has to be 1'
write(*,*) 'Enter the fraction for zero degree plies'
read(*,*) f0
write(*,*) 'Enter the fraction for theta plies'
read(*,*) ftheta
write(*,*) 'Enter the fraction for 90 degrees plies'
read(*,*) f90
sumf=f0+ftheta+f90
End Do

c determine the reduced stiffnesses at [0]

v21=v12*E22/E11
Q11=E11/(1-v12*v21)
Q12=v12*E22/(1-v12*v21)
Q22=E22/(1-v12*v21)
Q66=G12
Q(1)=Q11
Q(2)=Q12
Q(3)=Q22
Q(4)=Q66

c transform the reduced stiffnesses to different angles

da=0.0
call Qmat(a,Q,Qb110,Qb120,Qb220,Qb160,Qb260,Qb660)
c=90.0
call Qmat(c,Q,Qb119,Qb129,Qb229,Qb169,Qb269,Qb669)
d=45.0
call Qmat(d,Q,Qb115,Qb125,Qb225,Qb165,Qb265,Qb665)
e=d*pi/180
Qbar11b=f0*Qb110+ftheta*Qb115+f90*Qb119
Qbar12b=f0*Qb120+ftheta*Qb125+f90*Qb129
Qbar22b=f0*Qb220+ftheta*Qb225+f90*Qb229
Qbar66b=f0*Qb660+ftheta*Qb665+f90*Qb669
k11b = Qbar11b - Qbar12b**2/Qbar22b
k22b = Qbar66b
a12b = (cos(e))*2 - Qbar12b/Qbar22b*(sin(e))*2
a26b = sin(e)*cos(e)
hbb = (Nxx/k11b)/eps
htb = (a12b*Nxx/k11b+a26b*Nxs/k22b)/eps
if (hbb.LE.htb) then
  thickb = htb
else
  thickb = hbb
end if

C--------------------------------------
c determine the properties for the webs
C--------------------------------------
call Qmat(w,Q,Qb11p,Qb12p,Qb22p,Qb16p,Qb26p,Qb66p)
call Qmat(-w,Q,Qb11n,Qb12n,Qb22n,Qb16n,Qb26n,Qb66n)
Qbar11w = (Qb11p+Qb11n)/2
Qbar12w = (Qb12p+Qb12n)/2
Qbar16w = (Qb16p+Qb16n)/2
Qbar22w = (Qb22p+Qb22n)/2
Qbar26w = (Qb26p+Qb26n)/2
Qbar66w = (Qb66p+Qb66n)/2
k11w = Qbar11w - Qbar12w**2/Qbar22w
k12w = Qbar16w - Qbar12w*Qbar26w/Qbar22w
k22w = Qbar66w - Qbar26w**2/Qbar22w

theta = 1.0
betamax = 0.0
Do i = 1,90
  theta1 = theta*pi/180
  call Qmat(theta,Q,Qb11u,Qb12u,Qb22u,Qb16u,Qb26u,Qb66u)
call Qmat(-theta,Q,Qb11u,Qb12u,Qb22u,Qb16u,Qb26u,Qb66u)
  Qbar11u = f0*Qb11u+ftheta*Qb11u+f90*Qb11n
  Qbar12u = f0*Qb12u+ftheta*Qb12u+f90*Qb12n
  Qbar16u = f0*Qb16u+ftheta*Qb16u+f90*Qb16n
  Qbar22u = f0*Qb22u+ftheta*Qb22u+f90*Qb22n
  Qbar26u = f0*Qb26u+ftheta*Qb26u+f90*Qb26n
  Qbar66u = f0*Qb66u+ftheta*Qb66u+f90*Qb66n
  Qbar11l = f0*Qb11l+ftheta*Qb11l+f90*Qb11n
  Qbar12l = f0*Qb12l+ftheta*Qb12l+f90*Qb12n
  Qbar16l = f0*Qb16l+ftheta*Qb16l+f90*Qb16n
  Qbar22l = f0*Qb22l+ftheta*Qb22l+f90*Qb22n
  Qbar26l = f0*Qb26l+ftheta*Qb26l+f90*Qb26n
  Qbar66l = f0*Qb66l+ftheta*Qb66l+f90*Qb66n
\[ k_{11u} = \text{Qbar11u} - \text{Qbar12u}^2/\text{Qbar22u} \]
\[ k_{12u} = \text{Qbar16u} - \text{Qbar12u} \times \text{Qbar26u}/\text{Qbar22u} \]
\[ k_{22u} = \text{Qbar66u} - \text{Qbar26u}^2/\text{Qbar22u} \]
\[ k_{11l} = \text{Qbar11l} - \text{Qbar12l}^2/\text{Qbar22l} \]
\[ k_{12l} = \text{Qbar16l} - \text{Qbar12l} \times \text{Qbar26l}/\text{Qbar22l} \]
\[ k_{22l} = \text{Qbar66l} - \text{Qbar26l}^2/\text{Qbar22l} \]
\[ \beta_1 = k_{12u}^2/(k_{11u} \times k_{22u}) \]

c transform the strains from the xy axis to the angle plies

\[ a_{12} = (\cos(\theta_1))^2 \times \text{Qbar12u}/\text{Qbar22u} \times \text{Qbar22u}^*(\sin(\theta_1))^2 \]
\[ a_{26} = \sin(\theta_1) \times \cos(\theta_1) \times \text{Qbar26u}/\text{Qbar22u} \times (\sin(\theta_1))^2 \]

c determine the thickness

\[ h_b = (N_{xx} \times k_{11u} - N_{xs} \times \beta_1 \times k_{12u})/(\epsilon_s \times (1 - \beta_1)) \]
\[ h_t = ((a_{12} \times k_{11u} - a_{26} \times \beta_1 \times k_{12u}) \times N_{xx} + (a_{26} \times k_{22u} - a_{12} \times \beta_1 \times k_{12u}) \times N_{xs})/(\epsilon_s \times (1 - \beta_1)) \]

c determine the ply that controls the cover thickness

\[ \text{if (} h_b . \geq . h_t \text{) then} \]
\[ \text{thick} = h_b \]
\[ \text{else} \]
\[ \text{thick} = h_t \]
\[ \text{end if} \]

c determine the behavior of the structure

\[ M_{x} = 2 \times C_s \times H_e \times N_{xs} \]
\[ M_{y} = -N_{xx} \times C_s \times H_e \]
\[ c_{12u} = Q_{bar12u}/Q_{bar22u} - (Q_{bar26u} \times k_{12u})/(Q_{bar22u} \times k_{22u}) \]
\[ c_{26u} = Q_{bar26u}/Q_{bar22u} \]
\[ k_{cu} = (2/H_e) \times (c_{12u} \times \epsilon_s + c_{26u} \times N_{xs}/(\text{thick} \times k_{22u})) \]
\[ C_{44} = \text{thick} \times ((k_{22u} + k_{22l}) \times C_s + k_{22w} \times (f_f + f_r) \times H_e) \]
\[ & \times (C_s \times H_e/(C_s + H_e))^2) \]
\[ C_{45} = \text{thick} \times (0.5 \times (k_{12u} - k_{12l}) \times (C_s \times H_e)^2)/(C_s + H_e) \]
\[ C_{55} = \text{thick} \times (0.25 \times (k_{11u} + k_{11l}) \times C_s \times H_e^2) \]
\[ \beta_{s q} = C_{45} \times (C_{44} \times C_{55}) \]
\[ \text{if (} \beta_{max} . \leq . \beta_{s q} \text{) then} \]
\[ \beta_{max} = \beta_{s q} \]
\[ \theta_{max} = \theta \]
\[ \text{end if} \]
\[ \phi = (C_{55} \times M_{x} - C_{45} \times M_{y})/(C_{44} \times C_{55} \times (1 - \beta_{s q})) \]
\[ W_{xx} = \frac{(C_{45} \cdot M_x - C_{44} \cdot M_y)}{(C_{44} \cdot C_{55} \cdot (1 - \beta_{sq}))} \]
\[ w_{gain} = \frac{(2 \cdot \text{thick} + (f_{fw} + f_{frw}) \cdot \text{thick})}{(2 \cdot \text{thick}_b + (f_{fw} + f_{frw}) \cdot \text{thick})} - 1 \]

```
c write the results to the data files
c
write(11,*) theta, beta_sq, beta1
write(12,*) thick, hb, ht
write(13,*) dphi, kcu, W_{xx}
write(14,*) theta, hbb, htb
write(15,*) thick_b, wgain
write(16,*) C_{44}, C_{45}, C_{55}
theta = theta + 1.0
End Do
write(*,*) 'Maximum of beta squared at'
write(*,*) 'theta=', theta_max
write(*,*) 'max. beta squared=', beta_max
End
```

```
c SUBROUTINE for transforming reduced stiffness matrix
c
subroutine Qmat(deg, Qb11, Qb12, Qb22, Qb16, Qb26, Qb66)
imPLICIT none
real*8 deg, Qb11, Qb12, Qb22, Qb16, Qb26, Qb66
real*8 Q11, Q12, Q22, Q66, deg1
intent(in):: deg, Q
intent(out):: Qb11, Qb12, Qb22, Qb16, Qb26, Qb66
Q11 = Q(1)
Q12 = Q(2)
Q22 = Q(3)
Q66 = Q(4)
deg1 = deg * 3.14159 / 180
Qb11 = Q11 * (cos(deg1))**4 + 2 * (Q12 + 2 * Q66) * (sin(deg1))**2
& * (cos(deg1))**2 + Q22 * (sin(deg1))**4
Qb12 = (Q11 + Q22 - 4 * Q66) * (sin(deg1))**2 + 2 * (cos(deg1))**2
& + Q12 * (sin(deg1))**4 + (cos(deg1))**4
Qb22 = Q11 * (sin(deg1))**4 + 2 * (Q12 + 2 * Q66) * (sin(deg1))**2
& * (cos(deg1))**2 + Q22 * (cos(deg1))**4
Qb16 = (Q11 - Q12 - 2 * Q66) * sin(deg1) * (cos(deg1))**3 + (Q12 - Q22 + 2 * Q66)
& * (sin(deg1))**3 * (cos(deg1))
Qb26 = (Q11 - Q12 - 2 * Q66) * (sin(deg1))**3 * cos(deg1) + (Q12 - Q22 + 2 * Q66)
& * (sin(deg1))**3 * (cos(deg1))
Qb66 = (Q11 + Q22 - 2 * Q12 - 2 * Q66) * (sin(deg1))**2 + 2 * (cos(deg1))**2
& + Q66 * ((sin(deg1))**4 + (cos(deg1))**4)
return
end subroutine Qmat
```
APPENDIX B

PROGRAM FOR LAMINATE ROTATION

Variable and Parameter Definitions

Appendix B is the Fortran 90 program that is used to analyze Laminate Rotation.

The following inputs are required for the program.

Inputs:

- Width of the box model, $C_s$
- Height of the box model, $H$
- Elastic properties of the materials used for the box model, $E_{11}$, $E_{22}$ and $G_{12}$
- Limit strain for the design criterion, $\varepsilon$
- Axial loading due to bending, $N_{xx}$
- Load due to torsion, $N_{ts}$
- Configuration of the ply layup – fractions of axial plies, balanced angle plies and transverse plies. Also, the angle for the balanced angle plies, $\theta$.
- Orientation for the ply layup of the webs, $w$
- Fraction of the wall thickness for the front spar web, $f_f$
- Fraction of the wall thickness for the rear spar web, $f_r$

After inputting the values for the above parameters, the program generates the following outputs.

Outputs:

- Bend-twist coupling parameter, $\beta$
- Thickness of the covers, $h$
- Rate of twist, $\phi_{xx}$
- Bending curvature, $W_{xx}$
- Camber curvature, $k_c$
- Global stiffnesses, $C_{44}$, $C_{55}$, and $C_{45}$
- Relative weight, which is compared to a balanced benchmark reference of [0] and [45] plies.

Notes: All of the inputs are in degrees and English units.

Program Listing

c=====================================
c PROGRAM for Laminate Rotation
c=====================================
Program coupling
real*8 Cs,He,E11,E22,G12,v12,theta,beta1max
real*8 sumf,f0,ftheta,f90,v21,111,122,222,66,deg1max
real*8 a,bn,bp,c,c,beta1,C44,C55,C45,k11u,k12u,k22u,Q(4)
real*8 Qb110,Qb120,Qb220,Qb160,Qb260,Qb660
real*8 Qb119,Qb129,Qb229,Qb169,Qb269,Qb669
real*8 Qb11p,Qb12p,Qb22p,Qb16p,Qb26p,Qb66p
real*8 Qb11n,Qb12n,Qb22n,Qb16n,Qb26n,Qb66n
real*8 Qb111,Qb121,Qb221,Qb161,Qb261,Qb661
real*8 Qb112,Qb122,Qb222,Qb162,Qb262,Qb662
real*8 Qbar11w,Qbar12w,Qbar16w,Qbar22w,Qbar66w,ffw,frw
real*8 w,k11w,k12w,k22w,k11l,k12l,k22l,betasq,betasqmax,degsqmax
real*8 Qfu(3,3),Qfl(3,3),Qbar(3,3),Nxx,Nxs,eps,theta1,thick,h1
real*8 Mx,My,dphi,c12,c26,cu,kc,N11,N22,N12,thickb,wgain
real*8 h2,h3,theta2,theta3,s11,s12,s45p,s45n,detA,Al(3,3)
integer i

C--------------------------------------
c create the data files for the results
C--------------------------------------
open (unit=11,file='result.dat',status='unknown')
open (unit=12,file='result1.dat',status='unknown')
open (unit=13,file='result2.dat',status='unknown')
open (unit=14,file='result3.dat',status='unknown')
open (unit=15,file='result4.dat',status='unknown')
p=3.14159

write(*,*)'Enter the width for the single beam box in inches'
read(*,*) Cs
write(*,*)'Enter its height in inches'
read(*,*) He
write(*,*)'Enter the properties'
write(*,*)'E11?'
read(*,*) E11
write(*,*)'E22?'
read(*,*) E22
write(*,*)'G12?'
read(*,*) G12
write(*,*)'v12?'
read(*,*) v12
write(*,*)'strain?'
read(*,*) eps
write(*,*)'Enter the axial load, Nxx'
read(*,*) Nxx
write(*,*)'Enter the shear flow, Nxs'
read(*,*) Nxs
sumf=0
betalmax=0.0
betasqmax=0.0
c state the fractions for [0] or [theta] or [90] plies in the laminates
c---------------------------------------------------------------

Do while (sumf.NE.1.0)
   write(*,*) 'Enter the fraction of zero degree plies'
   read(*,*) f0
   write(*,*) 'Enter the degree for the theta plies'
   read(*,*) theta
   write(*,*) 'Enter the fraction of theta plies'
   read(*,*) ftheta
   write(*,*) 'Enter the fraction of 90 deg plies'
   read(*,*) f90
   write(*,*) 'Enter the angle of the webs'
   read(*,*) w
   write(*,*) 'Enter the fraction for the front spar web'
   read(*,*) ff
   write(*,*) 'Enter the fraction for the rear spar web'
   read(*,*) fr
   sumf=f0+ftheta+f90
End Do

c transform the reduced stiffnesses to different angles
C------------------------------------------------------
c=90.0
bp=theta
bn=-theta
Qmat(bp,Q,Qb11p,Qb12p,Qb22p,Qb16p,Qb26p,Qb66p)
Qmat.bn(Q,Qb11n,Qb12n,Qb22n,Qb16n,Qb26n,Qb66n)
Qbar(1,1)=f0*Qb110+ftheta/2*(Qb11p+Qb11n)+f90*Qb119
Qbar(1,2)=f0*Qb120+ftheta/2*(Qb12p+Qb12n)+f90*Qb129
Qbar(1,3)=f0*Qb160+ftheta/2*(Qb16p+Qb16n)+f90*Qb169
Qbar(2,1)=Qbar(1,2)
Qbar(2,2)=f0*Qb220+ftheta/2*(Qb22p+Qb22n)+f90*Qb229
Qbar(2,3)=f0*Qb260+ftheta/2*(Qb26p+Qb26n)+f90*Qb269
Qbar(3,1)=Qbar(1,3)
Qbar(3,2)=Qbar(2,3)
Qbar(3,3)=f0*Qb660+ftheta/2*(Qb66p+Qb66n)+f90*Qb669

c-determine the properties for the webs

call Qmat(w,Q,Qbl11,Qb111,Qb112,Qb211,Qb212,Qb611,Qb612)
call Qmat(-w,Q,Qbl12,Qb122,Qb121,Qb222,Qb221,Qb621,Qb622)
Qbar11w=(Qbl11+Qb111)/2
Qbar12w=(Qbl12+Qb112)/2
Qbar22w=(Qbl22+Qb212)/2
Qbar16w=(Qbl16+Qb122)/2
Qbar26w=(Qbl26+Qb222)/2
Qbar66w=(Qbl66+Qb612)/2
k11w=(Qbar11w-Qbar12w)**2/Qbar22w
k12w=(Qbar16w-Qbar12w)*Qbar26w/Qbar22w
k22w=(Qbar66w-Qbar26w)**2/Qbar22w

Do i=0,90
  e=i*3.14159/180
  theta1=theta*3.14159/180
  theta2=-theta*3.14159/180

c-transform the stiffnesses into a certain rotation angle

call Tran(e,Qbar,Qfl)
call Tran(-e,Qbar,Qfu)
k11u=Qfu(1,1)-Qfu(1,2)**2/Qfu(2,2)
k12u=Qfu(1,3)-Qfu(1,2)*Qfu(2,3)/Qfu(2,2)
k22u=Qfu(3,3)-(Qfu(2,3))**2/Qfu(2,2)
k11f=Qfl(1,1)-Qfl(1,2)**2/Qfl(2,2)
k12f=Qfl(1,3)-Qfl(1,2)*Qfl(2,3)/Qfl(2,2)
k22f=Qfl(3,3)-(Qfl(2,3))**2/Qfl(2,2)


c-determine the coupling parameter

beta1=k12u**2/(k11u*k22u)
if (beta1max.LT.beta1)
  beta1max=beta1
deg1max=e
\[ \det A = Q_{11}(1,1)Q_{11}(2,2)Q_{11}(3,3) + 2Q_{11}(1,2)Q_{11}(1,3) \]
\[ & \quad -Q_{11}(1,3)^2 + 2Q_{11}(2,2) - Q_{11}(2,3)^2 \]
\[ & \quad -Q_{11}(1,2)^2 \]
\[ AI(1,1) = Q_{11}(2,2)Q_{11}(3,3) - Q_{11}(2,3)^2 \]
\[ AI(1,2) = Q_{11}(1,3)Q_{11}(2,3) - Q_{11}(1,2)Q_{11}(3,3) \]
\[ AI(1,3) = Q_{11}(1,2)Q_{11}(2,3) - Q_{11}(1,3)^2 \]
\[ AI(2,1) = AI(1,2) \]
\[ AI(2,2) = Q_{11}(1,1)Q_{11}(3,3) - Q_{11}(1,3)^2 \]
\[ AI(2,3) = Q_{11}(1,2)Q_{11}(1,1)Q_{11}(2,3) \]
\[ AI(3,1) = AI(1,3) \]
\[ AI(3,2) = AI(2,3) \]
\[ AI(3,3) = Q_{11}(1,1)Q_{11}(2,2) - Q_{11}(1,2)^2 \]

\[ s_{11} = AI(1,1)N_{11} + AI(1,2)N_{22} + AI(1,3)N_{12} \]
\[ s_{22} = AI(2,1)N_{11} + AI(2,2)N_{22} + AI(2,3)N_{12} \]
\[ s_{12} = AI(3,1)N_{11} + AI(3,2)N_{22} + AI(3,3)N_{12} \]
\[ h_1 = s_{11}/\varepsilon \]
\[ s_{45p} = \cos(\theta_2)^2s_{11} + \sin(\theta_2)^2s_{22} + \]
\[ & \quad \sin(\theta_2)s_{12}^2 \]
\[ h_2 = s_{45p}/\varepsilon \]
\[ s_{45n} = \cos(\theta_3)^2s_{11} + \sin(\theta_3)^2s_{22} - \]
\[ & \quad \sin(\theta_3)s_{12}^2 \]
\[ h_3 = s_{45n}/\varepsilon \]

\[ \text{End If} \]

\[ \text{c transform the fixed stress resultants to the 1-2 coordinate system} \]
\[ \text{c} \]
\[ \begin{align*}
N_{11} &= \cos(\theta_1)^2N_{xx} + 2\sin(\theta_1)\cos(\theta_1)N_{xs} \\
N_{22} &= \sin(\theta_1)^2N_{xx} - 2\sin(\theta_1)\cos(\theta_1)N_{xs} \\
N_{12} &= -\sin(\theta_1)\cos(\theta_1)N_{xx} + (\cos(\theta_1)^2N_{xs} \\
& \quad -\sin(\theta_1)^2N_{xs} \\
\end{align*} \]
\[ \det A = Q_{11}(1,1)Q_{11}(2,2)Q_{11}(3,3) + 2Q_{11}(1,2)Q_{11}(1,3) \]
\[ & \quad -2Q_{11}(2,2) - Q_{11}(2,3)^2 \]
\[ AI(1,1) = (Q_{11}(1,1)Q_{11}(3,3) - Q_{11}(1,3)^2)/\det A \]
\[ AI(1,2) = (Q_{11}(1,3)Q_{11}(2,3) - Q_{11}(1,2)Q_{11}(3,3))/\det A \]
\[ AI(1,3) = (Q_{11}(1,2)Q_{11}(2,3) - Q_{11}(1,3)^2)/\det A \]
\[ AI(2,1) = AI(1,2) \]
\[ AI(2,2) = (Q_{11}(1,1)Q_{11}(3,3) - Q_{11}(1,3)^2)/\det A \]
\[ AI(2,3) = (Q_{11}(1,2)Q_{11}(1,1)Q_{11}(2,3))/\det A \]
\[ AI(3,1) = AI(1,3) \]
\[ AI(3,2) = AI(2,3) \]
\[ AI(3,3) = (Q_{11}(1,1)Q_{11}(2,2) - Q_{11}(1,2)^2)/\det A \]

\[ \text{c compare the strains in different fiber directions} \]
\[ \text{c} \]
\[ \begin{align*}
s_{11} &= AI(1,1)N_{11} + AI(1,2)N_{22} + AI(1,3)N_{12} \\
s_{22} &= AI(2,1)N_{11} + AI(2,2)N_{22} + AI(2,3)N_{12} \\
s_{12} &= AI(3,1)N_{11} + AI(3,2)N_{22} + AI(3,3)N_{12} \end{align*} \]
\[ h_1 = s_{11}/\varepsilon \]
\[ s_{45p} = \cos(\theta_2)^2s_{11} + \sin(\theta_2)^2s_{22} + \]
\[ & \quad \sin(\theta_2)s_{12}^2 \]
\[ h_2 = s_{45p}/\varepsilon \]
\[ s_{45n} = \cos(\theta_3)^2s_{11} + \sin(\theta_3)^2s_{22} - \]
\[ & \quad \sin(\theta_3)s_{12}^2 \]
\[ h_3 = s_{45n}/\varepsilon \]

\[ \text{c determine the ply that controls the cover thickness} \]
\[ \text{c} \]
\[ \begin{align*}
\text{if } (h_1 >= h_2 . \text{AND. } h_1 >= h_3) \text{ then} \\
\text{thick} &= h_1 \\
\text{else if } (h_2 >= h_3 . \text{AND. } h_2 >= h_1) \text{ then} \\
\text{thick} &= h_2 \\
\text{else} \\
\text{thick} &= h_3 \\
\text{end if} \\
\text{if } (e \text{EQ} 0) \text{ then} \\
\text{thick} &= \text{thick} \\
\end{align*} \]
end if

C determine the behavior of the structure

wgain=(2*thick+(ffw+frw)*thick)/(2*thickb+(ffw+frw)*thick)-1

C44=thick*((Cs*He/(Cs+He))**2)*((k22u+k22l)*Cs
& +((ff+fr)*k22w*He)

C45=0.5*thick*(Cs*He)**2/(Cs+He)*(k11u+k12l)
betasq=C45**2/(C44*C55)

Mx=2*Cs*He*Nxs
My=-Nxx*Cs*He
dphi=(C55*Mx-C45*My)/(C44*C55*(1-betasq))
cur=(C45*Mx-C44*My)/(C44*C55*(1-betasq))
c12=Qfu(1,2)/Qfu(2,2)-(Qfu(2,3)*k12u)/(Qfu(2,2)*k22u)
c26=Qfu(2,3)/Qfu(2,2)
kc=(2/He)*(c12*eps+c26*Nxs/(thick*k22u))
\( \text{if (betasqmax}\LT\text{betasq) then}
\)
\( \text{betasqmax=betaq}
\)
\( \text{degsqmax=e}
\)

end if

C write the results to the data files

write(11,*) e,beta1,betasq
write(12,*) h1,h2,h3
write(13,*) e,thick,wgain
write(14,*) dphi,cur,kc
write(15,*) C44,C45,C55

End Do

write(*,*) 'max. beta1'
write(*,*) deg1max,beta1max
write(*,*) 'max. betasq'
write(*,*) degsqmax,betasqmax

End

C SUBROUTINE for transforming reduced stiffness matrix

C------------------------------------------------------------------------

subroutine Qmat(deg,Q,Qb11,Qb12,Qb22,Qb16,Qb26,Qb66)

implicit none
real*8 deg,Qb11,Qb12,Qb22,Qb16,Qb26,Qb66,Q(4)
real*8 Q11,Q12,Q22,Q66,deg1
intent(in)::deg,Q
intent(out)::Qb11,Qb12,Qb22,Qb16,Qb26,Qb66
Q11=Q(1)

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\[ Q_{12} = Q(2) \]
\[ Q_{22} = Q(3) \]
\[ Q_{66} = Q(4) \]

deg1 = deg * 3.14159/180

\[ Q_{b11} = Q_{11} (\cos(\text{deg1}))*4 + 2*(Q_{12} + 2*Q_{66})*\sin(\text{deg1}))*2 \]
\& *\cos(\text{deg1}))*2 + Q_{22}*(\sin(\text{deg1}))*4

\[ Q_{b12} = (Q_{11} + Q_{22} - 4*Q_{66})*(\sin(\text{deg1}))*4 + 2*(Q_{12} + 2*Q_{66})*\sin(\text{deg1}))*2 \]
\& *\cos(\text{deg1}))*2 + Q_{22}*(\cos(\text{deg1}))*4

\[ Q_{b16} = (Q_{11} - Q_{12} - 2*Q_{66})*\sin(\text{deg1})*(\cos(\text{deg1}))*3 + (Q_{12} - Q_{22} + 2*Q_{66}) \]
\& *\sin(\text{deg1}))*3*\cos(\text{deg1})

\[ Q_{b26} = (Q_{11} - Q_{12} - 2*Q_{66})*(\sin(\text{deg1}))*4 + (Q_{12} - Q_{22} + 2*Q_{66}) \]
\& *\sin(\text{deg1})*(\cos(\text{deg1}))*4

\[ Q_{b66} = (Q_{11} + Q_{22} - 2*Q_{12} - 2*Q_{66})*\sin(\text{deg1})*(\cos(\text{deg1}))*2 \]
\& + Q_{66}*(\sin(\text{deg1}))*4 + (\cos(\text{deg1}))*4

return
end subroutine Qmat

c===============================================
c SUBROUTINE for transforming laminate into rotation angle
c=================================================================

subroutine Tran(d1,Qbar,TR2)
implicit none
real*8 d1,d,Qbar(3,3),TR1(3,3),TR2(3,3)
intent(in)::d1,Qbar
intent(out)::TR2

d=d1*3.14159/180

TR1(1,1)=Qbar(1,1)*(\cos(d))*2 + Qbar(1,2)*\sin(d))*2 + Qbar(1,3)*\sin(d))*2 + Qbar(1,1)*\sin(d))*2 + Qbar(1,2)*\cos(d))*2 + Qbar(1,3)*\cos(d))*2
\& - Qbar(1,1)*\sin(d))*2 + Qbar(1,2)*\cos(d))*2 + Qbar(1,3)*\cos(d))*2 - (\sin(d))*2

TR1(1,2)=Qbar(1,1)*\sin(d))*2 + Qbar(1,2)*\cos(d))*2 + Qbar(1,3)*\cos(d))*2 + Qbar(1,1)*\cos(d))*2 + Qbar(1,2)*\sin(d))*2 + Qbar(1,3)*\sin(d))*2
\& - Qbar(1,1)*\cos(d))*2 + Qbar(1,2)*\sin(d))*2 + Qbar(1,3)*\sin(d))*2 - (\cos(d))*2

TR1(1,3)=Qbar(1,1)*\sin(d)*\cos(d) - Qbar(1,2)*\sin(d)*\cos(d)
\& + Qbar(1,3)*(\cos(d))*2 - (\sin(d))*2

TR1(2,1)=Qbar(2,1)*\cos(d))*2 + Qbar(2,2)*\sin(d))*2 + Qbar(2,3)*\sin(d)*\cos(d)
\& - Qbar(2,1)*\sin(d))*2 + Qbar(2,2)*\cos(d))*2 + Qbar(2,3)*\cos(d))*2

TR1(2,2)=Qbar(2,1)*\sin(d))*2 + Qbar(2,2)*\cos(d))*2 + Qbar(2,3)*\sin(d)*\cos(d)
\& + Qbar(2,1)*\cos(d))*2 + Qbar(2,2)*\sin(d))*2 + Qbar(2,3)*\sin(d))*2

TR1(2,3)=Qbar(2,1)*\sin(d)*\cos(d) - Qbar(2,2)*\sin(d)*\cos(d)
\& + Qbar(2,3)*(\cos(d))*2 - (\sin(d))*2

TR1(3,1)=Qbar(3,1)*\cos(d))*2 + Qbar(3,2)*\sin(d))*2 + Qbar(3,3)*\sin(d)*\cos(d)
\& - Qbar(3,1)*\sin(d))*2 + Qbar(3,2)*\cos(d))*2 + Qbar(3,3)*\cos(d))*2

TR1(3,2)=Qbar(3,1)*\sin(d))*2 + Qbar(3,2)*\cos(d))*2 + Qbar(3,3)*\sin(d)*\cos(d)
\& + Qbar(3,1)*\cos(d))*2 + Qbar(3,2)*\sin(d))*2 + Qbar(3,3)*\sin(d))*2

TR1(3,3)=Qbar(3,1)*\sin(d)*\cos(d) - Qbar(3,2)*\sin(d)*\cos(d)
\& + Qbar(3,1)*\cos(d))*2 - (\sin(d))*2

end subroutine Tran
TR2(1,1)=TR1(1,1)*(cos(d))**2+TR1(2,1)*(sin(d))**2 &
       -TR1(3,1)*2*sin(d)*cos(d)
TR2(1,2)=TR1(1,2)*(cos(d))**2+TR1(2,2)*(sin(d))**2 &
       -TR1(3,2)*2*sin(d)*cos(d)
TR2(1,3)=TR1(1,3)*(cos(d))**2+TR1(2,3)*(sin(d))**2 &
       -TR1(3,3)*2*sin(d)*cos(d)
TR2(2,1)=TR1(1,1)*(sin(d))**2+TR1(2,1)*(cos(d))**2 &
       +TR1(3,1)*2*sin(d)*cos(d)
TR2(2,2)=TR1(1,2)*(sin(d))**2+TR1(2,2)*(cos(d))**2 &
       +TR1(3,2)*2*sin(d)*cos(d)
TR2(2,3)=TR1(1,3)*(sin(d))**2+TR1(2,3)*(cos(d))**2 &
       +TR1(3,3)*2*sin(d)*cos(d)
TR2(3,1)=TR1(1,1)*sin(d)*cos(d)-TR1(2,1)*sin(d)*cos(d) &
       +TR1(3,1)*((cos(d))**2-(sin(d))**2)
TR2(3,2)=TR1(1,2)*sin(d)*cos(d)-TR1(2,2)*sin(d)*cos(d) &
       +TR1(3,2)*((cos(d))**2-(sin(d))**2)
TR2(3,3)=TR1(1,3)*sin(d)*cos(d)-TR1(2,3)*sin(d)*cos(d) &
       +TR1(3,3)*((cos(d))**2-(sin(d))**2)
return
end subroutine Tran
Appendix II

Brett Sikola’s
MS Project

2004
Linear Modeling of Aeroelastic Twist of a High Aspect Ratio Wing with Significant Taper

Brett Sikola, Lawrence W. Rehfield

Abstract

The most significant aeroelastic deformation that occurs in optimized cruise flight is the elastic twist that occurs about the spanwise direction of the wing. By shifting the distribution of the spanwise aerodynamic loads, this aeroelastic twist can have a negative impact on induced drag and structural loading. Hence preliminary design tools that can characterize this twist, particularly in cases of significant spanwise taper such as presented in this paper, are important for beginning the design of efficient aircraft with such wings.

The preliminary design tool presented here makes several simplifying assumptions. Airfoil sections are assumed to behave linearly in both an elastic sense and a sectional aerodynamic sense. As this is primarily a structural tool, aerodynamic behavior is further simplified to assume that individual airfoil sections are assumed to not affect each other in the spanwise direction (Trefftz plane). This is classical "strip theory aerodynamics" that has been used for high aspect ratio lifting surfaces for generations. Furthermore, with composite construction that is balanced with respect to a spanwise axis, the axis serves as an elastic axis (EA) which permits bending and torsion to be elastically uncoupled. Emphasis is placed here, therefore, on elastic twist. Simple, but rational and familiar, design criteria are utilized to size the wing.
Nomenclature

EA = Elastic Axis
\(c_a(r)\) = Root Aerodynamic Chord
\(\lambda\) = Taper Ratio
\(q\) = Dynamic Pressure
\(y_{ac}\) = Chordwise Separation Distance of the Center of Pressure and EA
\(c_l\) = 2-D Coefficient of Lift
\(a_0\) = 2-D Lift Slope of Airfoil
\(c_m\) = Constant Coefficient of Moment
\(c_{\text{turn}}\) = 2-D Coefficient of Lift Under Turning Conditions
\(f_s\) = Factor of Safety
\(k_{11}\) = Axial Reduced Stiffness per Unit Thickness
\(k_{22}\) = Torsional Reduced Stiffness per Unit Thickness

Introduction

Aeroelastic response is difficult to assess in preliminary design. Many of the tools used to predict aeroelastic response are difficult to use or require large run times (such as ANSYS multi-physics engine runs or a NASTRAN run) and hence are limited mostly to final analysis rather than true parametric optimization of an aeronautical structure.

It is the intent of this work to present an alternative means of simultaneous aerodynamic/structural analysis of high aspect ratio tapered aerodynamic structures loaded normal to their spanwise structural axis that can not only give a realistic first
scaling of the necessary thicknesses of structural box skinning material, but also of its subsequent deformation. This formulation is also applicable to swept wings, as only the component of the airflow normal to the wing need be considered as causing aerodynamic loading, as is commonly done in preliminary swept wing analysis. Further, and most importantly, the taper of the wings need not be constant or continuous in any sense, so long as a leading edge normal chord can be specified at any point along the span.

Methodology

The geometric parameters of the wing (or other lifting surface), and its structural composition, must first be described. Secondly, appropriate candidate structural concepts and material systems must be identified.

The material system later illustrated makes use of composite materials, but isotropic materials may easily be used. In the case of anisotropic materials, such as carbon-composite, reduced stiffnesses must be calculated for laid up plies in a manner consistent with sectional analysis. Also the design strain limit must be specified, a common design limit. Finally, the minimum manufacturable wall thickness of the material must be established based upon experience.

The subsequent example and discussion only describes the method for a linear taper, but as long as all the parameters can be given as either explicit functions or points that may be interpolated, the methodology can be applied. Currently, many wing dimensions are reduced dimensions or ratios taken from other dimensions. For the current example, the root chord, \( c_a(r) \), the taper ratio of tip chord divided by root aerodynamic
chord, $\lambda$, and the wing half-span. It is assumed that the internal structure scales equivalently with the external aerodynamic structure. Consequently, the dimensions of the wing structure box must be given at the root, including the thickness of the vertical webs. The webs are often sized as a fraction of the thickness of the upper and lower covers based upon experience and transverse shear flow. It is assumed that the wing structure box is more or less rectangular and a closed section. While the global structure is a tapered prism, the individual finite element sections are modeled as untapered. As the bending and twisting deformations are elastically uncoupled, aeroelastic twist can be estimated independently$^3$.

The structural material is a 50/50 mix of [0] degree spanwise plies and [±45] torsionally stiff plies, i.e. [0, ±45], composed of AS4/3501-6 composite material. This laminate mimics the F/A-18 and AV-8B without chordwise plies and gives a fairly good compromise between bending and twisting stiffness$^1$.

The dynamic pressure is the only dimensional aerodynamic loading parameter, $q$. The dimensionless aerodynamic loading parameters are the sectional aerodynamic parameters: the coefficient of lift at cruise (1-g) loads, $c_l$, the lift slope of the section, $a_0$, the perceived maximum multiple of 1g that will be experienced in turns, and the angle-of-attack independent coefficient of moment, $c_m$. The inclusion of the $c_m$ is a departure from the earlier work of Reference 4., in addition to several other extensions made here of that work.

The separation of the quarter chord from the elastic center of the structural box is given as $y_{ac}$. Finally the factor of safety, $f_s$, is used to multiply the maximum g-loading to provide for reasonable design margin.
As one last convergence parameter, the number of elements used in the analysis must be specified. A consideration of the convergence of the solution is given subsequently.

The analysis of the aerodynamic structure makes use of a series of aerodynamic independent sections that interact elastically along the EA. There is one degree of elastic freedom per element, twisting about the spanwise axis. Each element experiences a load at its outboard tip (node) and twists elastically according to this load. The outboard load and the applied aerodynamic load, based on both the initial $c_1$ the induced angle of attack times $a_0$, are applied at the inboard station (node). The amount of elastic twist experienced by each element under the load applied to it is based on its sectional torsional stiffness, itself dictated by the structural box and the stiffnesses of the material system. In order to remain consistent with this model, the torsional displacement of the inboard node (wing root) of the innermost element is fixed at zero ($c_1$ stays at its original value), while the load applied to the outboard node of the most outboard element is set to zero; hence this element sees no relative elastic displacement. The loadings just described can also be seen below in Fig.1.

A system of linear equations results from the equilibrium of the global structure. Note that only the twisting moment about the spanwise axis is considered in the displacement of the system; while the system does displace in bending parallel with the $c_1$ loading, the impact on the aerodynamic load distribution is minor, and, hence, it is only considered in the context of the bending design strain limit. Once the nodal displacements have been found, all of the resultant loadings can be found.
In the entire system implemented in the coming example, the loadings are actually found three times. In the first run, no displacements are allowed and the bending loads are found for the worst case scenario of the maximum g-loading multiplied times the factor of safety. These are then used in conjunction with the calculated bending stiffness to find the skin thickness necessary to reach the bending strain limit at the root; the minimum thickness is also enforced at this step in order to ensure manufacturability.

In the second run, the loads are calculated in a manner similar to the thickness determination loads, only now the $c_i$ is reflective of cruise conditions. These loads are used as a baseline for comparison with the loads that are found in the elastically displaced case. The final case, of course, finds the loads produced in the wing that has elastically deformed to its loads.

One final interesting calculation that is carried out is the divergence speed estimation. By holding constant the dynamic pressure used to calculate the skin thickness but varying the dynamic pressure used to calculate the elastic twist of the wing, the displacement of the outer tip of the wing can be found as a function of dynamic pressure and speed. Thus, the divergence speed can be found graphically by finding where the transverse displacement asymptotically approaches infinity. In the following example, the result of this calculation is compared to the typical 70-75% chord calculation.

Example Structure

The example considered in this paper is the wing of a Reno Unlimited Class Air Racer outlined in a preliminary design study and illustrated in Fig. 2. The Reno airport at the time of the Reno air races is generally at about nine thousand feet real altitude, and
competitive racers in the unlimited class generally race at about 644 to 724 km/h (400 to 450 M.P.H.), which sets the dynamic pressure. The rest of the pertinent data for the racer are listed Table 1. Note that the sign convention for the twist is positive nose up. Although the taper of this wing is constant, the methodology is still general enough to analyze a wing of arbitrary taper, as mentioned earlier.

**Results and Discussion: Reno Air Racer**

1. As is visible in Fig.3, the combination of the separation of the coefficient of lift from the aerodynamic and the negative coefficient moment results in the negative coefficient of moment ultimately pushes the induced angles of attack negative. Due to aeroelastic effects, the section lift coefficient has decreased by about 0.018, a decrease of about 15%. This seems unlikely to cause radical problems, but there is certainly a perturbation from any initially “rigid” optimized wing configuration. Further notice the two inflection points of the twist distribution of the wing. The inboard inflection point is mostly the result of the non-constant thickness of the section; otherwise the loading tends to scale proportionally to the stiffness with the current scaling. The second inflection point is a result of the requirement that the rate of change of the twist will reach zero at the outboard section, as this section is free of twisting loads.

2. The twisting load certainly seems affected by the aeroelastic deformation, reducing it by about twelve percent, visible in Fig.4. While the twist does not
directly affect performance, it is interesting to note its decrease and resulting impact on the balance of the aircraft.

3. The bending moment is less affected by the change in sectional angle of attack, with a reduction in overall lift in the wing less than 10%, as seen in Fig. 5. The qualitative curve exhibits no dramatic shift from its original shape. This same trend is seen in a relatively minor 1.6” shift in the spanwise center of pressure.

4. The minimum manufacturability criterion was triggered slightly before the half-span mark, visible in Fig. 6. From the sole perspective of rigid bending loads it would seem that very small thicknesses would be all that would be required to avoid the strain limit in the outboard sections of the wing; however, not including the manufacturability requirement produced unrealistic and undesirable outboard displacements. It is seen, therefore, that reducing the minimum manufacturable thickness would not necessarily save any appreciable weight.

5. As expected, the torsional and bending stiffness scale with both the span and the stiffness, as is evident from the kink at about half-span in Fig. 7. Notice that both never reach zero, as this would produce infinite tipwise displacements and loads, neither of which is realistic.

6. As the loading is initially only $2/15^{\text{th}}$s that of the turn conditions that set the thickness, it is not surprising that the strain remains far below the 4500 microstrain limit set for the material, visible in Fig. 8. Had the wing been held rigid, the strain would have been constant for the initial inboard section. Clearly the strains have been lessened due to the induced angles of attack of the sections.
Note that once the strain was no longer held constant it drops off rapidly; hence the strain limit is not reach in the outboard sections.

7. In a comparison of the current models estimate of the divergence speed with that of the 70-75% section model, both predict divergence at flight speeds over 100 M.P.H. (160 km/h) faster than that of the anticipated flight envelope, as seen in fig. 9. By both estimates then it would seem that the current structural configuration is a good initial estimate for preventing divergence.

Convergence Study

As is common with lower order elements, such as those with only two nodes, a large number of nodes are required to achieve satisfactory convergence; in this case of the order of 100. Additionally the outermost element is a fairly gross estimation when the elements are fairly large, and in fact one can almost draw a line connecting the kinks in the station twists that are created by the final rigid element. It would seem logical to conclude that holding the last element to a certain much smaller fraction the size of the other elements would increase accuracy for the same computational cost.

Conclusions

The preliminary design tool presented in this paper can clearly provide useful feedback in the preliminary design of elastic high aspect ratio wings of arbitrary taper. By this code alone it can be reasonably construed that many higher order models of the aerodynamic performance on "rigid" airframes, such as CFD and wind tunnel analysis,
might fail to fully predict the behavior of the structure, and may be unwarranted without including aeroelastic coupled effects.

From the specific case in question, several other conclusions may be drawn. Depending on the magnitude of the coefficient of moment and the coefficient of lift, the elastic response of the wing may induce positive or negative changes in the angle of attack. The traditional 70-75% span model for divergence prediction is more conservative for this illustration. Structural rigidity towards the wingtips is generally not as important at preventing failure of the material as keeping deformations and changes in load under control. It may also be noted here that by setting the sectional aerodynamic coefficients of the outer part of the wing corresponding to control surfaces and to values consistent with a control input, control reversal may also be modeled.

It is the intention of this preliminary design methodology to allow for analysis of more diverse planforms than that analyzed here, and, as designed it can be easily applied to a host of other lifting surfaces uncoupled elastic bend-twist response. Finally it runs in only a few seconds on almost any desktop computer running Matlab.

Final Remarks

What is new?

1. Analysis efficiency – suitable for preliminary design or conceptual structural design on desktop computers

2. Simple, but rational, established, and validated modeling approach.
3. Illustration – extreme taper of wing, realistic parameter values for aircraft type, minimum gauge drives design of outboard wing sections (helps reduce tip twist and outboard elastic twist effects)

4. Valid only for wing covers with balanced ply layups.

References


2. Rehfield, L.W., EAE-135, Aerospace Structures (Class Notes), University of California Davis, 2002


Appendix: Matlab Code

Source Matlab code for design methodology
<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>q</td>
<td>$19.6 \times 10^3(2.848)$</td>
<td>(Pa)lb/in$^2$</td>
</tr>
<tr>
<td>$c_{a0}$</td>
<td>1.122(44.19)</td>
<td>m(inches)</td>
</tr>
<tr>
<td>Wing box width at root</td>
<td>$0.4 \times c_a(r)$</td>
<td>m(inches)</td>
</tr>
<tr>
<td>Wing box height at root</td>
<td>$0.1 \times c_a(r)$</td>
<td>m(inches)</td>
</tr>
<tr>
<td>$y_{ac}$ at the root</td>
<td>$0.2 \times c_a(r)$</td>
<td>m(inches)</td>
</tr>
<tr>
<td>Wing half span</td>
<td>2.925(115.6)</td>
<td>m(inches)</td>
</tr>
<tr>
<td>$c_l$</td>
<td>0.12</td>
<td>-</td>
</tr>
<tr>
<td>$a_0$</td>
<td>5.7</td>
<td>-</td>
</tr>
<tr>
<td>$c_m$</td>
<td>$-3.22 \times 10^{-2}$</td>
<td>-</td>
</tr>
<tr>
<td>$c_{lum}$</td>
<td>$5 \times c_l$</td>
<td>-</td>
</tr>
<tr>
<td>$f_s$</td>
<td>1.5</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.39</td>
<td>-</td>
</tr>
<tr>
<td>$k_{11}$</td>
<td>$81.5 \times 10^9(11.1 \times 10^6)$</td>
<td>Pa(lbf/in$^2$)</td>
</tr>
<tr>
<td>$k_{22}$ box covers</td>
<td>$21.34 \times 10^6(3.095 \times 10^6)$</td>
<td>Pa(lbf/in$^2$)</td>
</tr>
<tr>
<td>$k_{22}$ vertical box members</td>
<td>$35.58 \times 10^6(5.16 \times 10^6)$</td>
<td>Pa(lbf/in$^2$)</td>
</tr>
<tr>
<td>vertical box members fraction</td>
<td>0.4</td>
<td>-</td>
</tr>
<tr>
<td>microstrain criteria</td>
<td>0.0045</td>
<td>m/m(in/in)</td>
</tr>
<tr>
<td>minimum thickness</td>
<td>$1.27 \times 10^{-3}(0.05)$</td>
<td>m(inches)</td>
</tr>
</tbody>
</table>

Table 1, Design Data for Example Reno Air Racer
Figures
Deformation producing loads

Aerodynamic
Loads

Outboard element

Figure 1, Twisting elastic forces on each element

Figure 2, Example Reno Air Racer Wing Planform
Figure 3, Station Twist
Figure 4, Twisting Moment
Figure 5. Bending Moment
Figure 6, Skin (Cover) Thickness
Figure 7, Stiffnesses
Figure 8, Spanwise Microstrain under Cruise Conditions
Figure 9, Torsional Divergence Prediction
Figure 10, Convergence Study
%NASA funded wing twist approximation
%initial debug matlab convergence study

clear all
close all
format short e
clc

taper_ratio=.39 ;
root_aero_chord=44.190 ;
root_cover_height=.1*root_aero_chord ;
root_cover_length=.4*root_aero_chord ;
root_y_ac=2.*root_aero_chord ;
wing_half_span=115.6250 ;

S=wing_half_span*(root_aero_chord+root_aero_chord*taper_ratio)/2 ; %in^2

g=2.848 ;
ao=5.7 ;

q=2.848 ;

aos=5.7 ;

ao=5.7 ;

fs = 1.5 ;

k11=11.82e6 ;
k22=3.095e6 ;
k22web=5.16e6 ;
webf=0.4 ;

h_min=.05 ;

n=100 ; %input('enter the number of elements, 2 or greater');

m=wing_half_span/n ; %individual segment length, constant

a=zeros(n,1);

b=a; c=a; d=a;

y_ac=a; ca=a; cs=a; H=a; C44=a; scale=a; mom=a;

scale(i)=1-(1-taper_ratio)*((i-1)/n + 1/2/n) ; %as all linear dimensions scale prop
optionally

\[ y_{ac}(i) = scale(i) \times root_{y_{ac}}; \]
\[ ca(i) = scale(i) \times root_{aero\_chord}; \]
\[ cs(i) = scale(i) \times root_{cover\_length}; \]
\[ H(i) = scale(i) \times root_{cover\_height}; \]
\end{end}

\non--------------------------%find h using minimum manufacturability and turn bending (rigid)%-----------------------------
\non--------------------------

\on lift\_turn(1)=q*ca(1)*l*( cl\_turn*fs ) ;
for i=2:n
    lift\_turn(i)=q*ca(i)*l*( cl\_turn*fs ) ;%rigid segmentary 1 ✓
lifting
end

shear\_turn(n)=lift\_turn(n);
for i=(n-1):-1:1
    shear\_turn(i)=lift\_turn(i)+shear\_turn(i+1); %rigid segment rootside shearing moment
end

bend\_turn(n)=l/2*l*lift\_turn(n);
for i=(n-1):-1:1
    bend\_turn(i)=bend\_turn(i+1)+l/2*l*lift\_turn(i)+l*shear\_turn(i+1); %rigid bending mom ✓
end

for i=1:n
    h(i)=bend\_turn(i)/(k11*H(i)*cs(i)*microstrain); %needed bending stiffness calculation
    if h(i) < h_min
        h(i)=h_min; %enforce minimum thickness
    end
end

%vector of thicknesses complete%

%fill out 'd' matrix, torque forces that are constant regardless of aoa
for i=1:n
    d(i)=(cl*y_{ac}(i)+cm*ca(i))*ca(i)*q*1;
end

%rigid cruise case %

mom\_rigid(n)=d(i) ;
for i=(n-1):-1:2
    mom\_rigid(i)=d(i) + mom\_rigid(i+1); % rigid twisting moment
end
mom\_rigid(1)=d(1)+mom\_rigid(2) ;

lift\_rigid(1)=q*ca(1)*l*cl ;
for i=2:n
    lift\_rigid(i)=q*ca(i)*l*cl; %rigid segmentary lifting
end
total_lift_rigid=sum(lift_rigid);

shear_rigid(n)=lift_rigid(n);
for i=(n-1):-1:1
    shear_rigid(i)=lift_rigid(i)+shear_rigid(i+1);
end

bend_rigid(n)=1/2*l*lift_rigid(n);
for i=(n-1):-1:1
    bend_rigid(i)=bend_rigid(i+1)+1/2*l*lift_rigid(i)+l*shear_rigid(i+1);
end

cp_rigid=bend_rigid(1)/shear_rigid(1);

% find sectional stiffnesses %
for i=1:n
    C44(i)=c(i)^2*H(i)^2/(cs(i)+H(i))^2*(2*h(i)*k22*cs(i) + 2*h(i)*webf*k22web*H(i)) ;
    C55(i)=2*h(i)*k11*cs(i)*H(i)^2/4 ;
end

% fill out a,b,c vectors %
for i=3:n
    a(i)=-C44(i-1)/l; %a(1) and a(2) are zero
end

b(1)=1;
for i=2:(n-1)
    b(i)=( C44(i-1)+C44(i) )/l - ao*ca(i)*q*y_ac(i)*1; %not special end conditions
end
b(n)=C44(n-1)/l - ao*ca(n)*q*y_ac(n)*1;
for i=1:(n-1)
    c(i)=-C44(i)/l; %c(n)=0 as its original value was retained
end

%solve a,b,c,d tri-diagonal matrix (i.e. strictly 3 banded sparse matrix)%
torque_alphas=tridiag(a,b,c,d);

% Reduce data from the elastic simultaneous case %

mom(n)=d(n) + ao*ca(n)*q*y_ac(n)*1*torque_alphas(n) ;
for i=(n-1):-1:2
    mom(i)=d(i) + ao*ca(i)*q*y_ac(i)*1*torque_alphas(i) + mom(i+1); % induced twisting moment
end
mom(1)=d(1)+mom(2);
lift(1)=cl*q*l*ca(1) ;
for i=2:n
    lift(i)=q*ca(i)*l*(cl+ao*torque_alphas(i)) ; % induced segmentary lifting
end
total_lift=sum(lift);

shear(n)=lift(n);
for i=(n-1):-1:1
    shear(i)=lift(i)+shear(i+1); % induced segment rootside shearing moment
end

bend(n)=1/2*l*lift(n);
for i=(n-1):-1:1
    bend(i)=bend(i+1)+1/2*l*lift(i)+l*shear(i+1) ; % induced bending moment
end

cp=bend(1)/shear(1); % induced center of pressure

% failure criteria, microstrain and divergence 

strain(i)=bend(i)/(kll*H(i)*cs(i)*h(i)); % needed bending stiffness calculation

x_alphas=[0 1/n:1/n:(1-1/n) 1];
x=1/2/n : 1/n : (1-1/2/n);
q_td=qtd(x,n,C44,y_ac,1,S,ao)

%PLOT OUTPUT%

figure(1) %alphas
 torque_degrees=torque_alphas*180/pi ;
 torque_degrees(n)
y=[0 ;torque_degrees(2:n); torque_degrees(n)];
plot(x_alphas,y);
xlabel('dimensionless span');, ylabel('station twist (degrees)');, title('Station twist');

figure(2) %twisting moment
 plot(x,mom,x,mom_rigid,'-.') ;
 legend('flexible moment','rigid moment') ;
title('Twisting Moment')
xlabel('dimensionless span');
ylabel('twisting moment (lbf-in)');

figure(3) %shear
 plot(x,shear,x,shear_rigid,'-.') ;
 legend('flexible shear','rigid shear') ;
title('Shear')
xlabel('dimensionless span');
ylabel('shear (lbf)');

figure(4) %bending moment
plot(x.bend, x.bend_rigid, '-.');
legend('flexible bending moment', 'rigid bending moment');
title('Bending moment')
xlabel('dimensionless span');
ylabel('bending moment (lbf-in)');

figure(5) % h, thickness
plot(x,h);
title('Skin thickness')
xlabel('dimensionless span');
ylabel('skin thickness (inches)');

figure(6) % C44, C55
plot(x,C44,x,C55,-'');
legend('C44', 'C55');
title('stiffnesses')
xlabel('dimensionless span');
ylabel('stiffness (lbf-in^2)');

figure(7) % microstrain
plot(x,strain)
title('spanwise microstrain, flexible case')
xlabel('dimensionless span')
ylabel('strain (in/in)')

%%%%%%%  
%- end -% 
%%%%%%%
function [result]=tridiag(a,b,c,d)

% gauss-jordan tri-diag solver

% declare an intermediate matrix to which solutions will begin
% being written. Declare size of matrix to speed execution
m=length(d);
bb=zeros(m,1);
cc=bb;
dd=bb;
result=bb;
% the first row is unaffected by the row ops that eliminate the
% front row in the first loop, so write it in
bb(1)=b(1);
cc(1)=c(1);
dd(1)=d(1);

% the first loop eliminates the front row by taking the row above it
% and multiplying it by a value that would make it the same as the leading
% entry in the next row, and then subtracting with this modified row
for j=2:m
    bb(j)=b(j)-(a(j)/bb(j-1))*cc(j-1);
    cc(j)=c(j);
    dd(j)=d(j)-(a(j)/bb(j-1))*dd(j-1);
end

% this leaves one equation in one unknown in the last row, which is solved
% explicitly
result(m)=dd(m)/bb(m);
% the row above it then becomes defined, and this is carried through the
% remainder of the equations, solving the tri-diagonal system
for j=m-1:-1:1
    result(j)=(dd(j)-cc(j)*result(j+1))/bb(j);
end
function [result]=qtd(x,n,C44,y_ac,l,S,ao)

%find station at closest to .75 dimensionless span

found=0;

for i=1:n
    test=x(i);
    if test > 0.70
        if found == 0
            answeri=i ;
            found = 1 ;
        end
    end
end

Kinverse=l/C44(1) ;
for i=2:answeri
    Kinverse=l/C44(i)+Kinverse ;
end
K=1/Kinverse ;

result=K/S/y_ac(answeri)/ao ;