Abstract of Paper Proposed for Presentation at the

43rd AIAA Aerospace Sciences Meeting and Exhibit

Reno, NV

10–13 January, 2005

A Reduced-Frequency Approach for Calculating Dynamic Derivatives

Scott M. Murman*
ELORET
MS T27B
Moffett Field, CA 94035
smurman@nas.nasa.gov

1 Introduction

Computational Fluid Dynamics (CFD) is increasingly being used to both augment and create an aerodynamic performance database for aircraft configurations. This aerodynamic database contains the response of the aircraft to varying flight conditions and control surface deflections. CFD currently provides an accurate and efficient estimate of the static stability derivatives, as these involve a steady-state simulation about a fixed geometry. The calculation of higher-order dynamic stability derivatives for general configurations and flow conditions is more costly however, requiring the simulation of an unsteady flow with moving geometry. For this reason the calculation of dynamic stability derivatives using CFD has been limited to either estimating the values at a handful of (hopefully) critical points and extrapolating to cover the range of interest, or using restrictive approximate methods. The need for more efficient, general CFD methods is especially acute as predicting dynamic derivatives with traditional methods, such as wind tunnel testing, is expensive and can be error prone. As aircraft designs continue to evolve towards highly-maneuverable unmanned systems, high-fidelity aerodynamic databases including dynamic derivatives are required to accurately predict performance and develop stability and control laws. CFD can provide a key technology for modeling the

*Senior Research Scientist, Member AIAA

dynamic performance of these advanced systems, with their extreme rate changes and flight conditions.

The current work presents a novel method for calculating dynamic stability derivatives which reduces the computational cost over traditional unsteady CFD approaches by an order of magnitude, while still being applicable to arbitrarily complex geometries over a wide range of flow regimes. Previous approaches can be broadly categorized as general methods which simulate an unsteady motion of the geometry (e.g. a forced oscillation)[1–5], or those which reduce the problem complexity in some manner with an attendant loss of generality. The former methods provide accurate results for arbitrary geometries and flow conditions, however they require a time-dependent moving-body flow simulation, which uses roughly an order of magnitude greater computational time than a static, steady-state simulation. Even predicting one of the roll, pitch, or yaw damping coefficients over the full flight regime is prohibitively expensive. Methods which compute the pitch damping using a lunar coning motion[6–8] can reduce the unsteady, moving-geometry problem to a static, steady-state computation, albeit in a non-inertial reference frame. While these methods provide computational efficiency, they are only applicable to longitudinal damping and require approximating (or ignoring) other damping coefficients. Weinacht[9] extended this approach to predict pitch and yaw damping, however it is only valid for axisymmetric bodies. Weinacht and Sturek[10] demonstrate roll damping calculations in a non-inertial frame for finned projectiles, which also reduces the problem to a steady-state flow solution, however this approach is only valid at $\alpha = 0.0°$. Linearized methods[11, 12] likewise greatly reduce the required computational cost, however with a loss of accuracy and a reduced range of applicable flow conditions and/or geometric complexity.

A time-dependent simulation supports a continuum of frequencies up to the limits of the spatial and temporal resolution. The primary thesis of this work is that the response to a forced motion can often be represented with a small, predictable number of frequency components without loss of accuracy. By resolving only those frequencies of interest, the computational effort is significantly reduced so that the routine calculation of dynamic derivatives becomes practical. Such "reduced-frequency methods" have recently been extended to retain the non-linearity of the original governing equations by Hall et al.[13, 14] for application to 2-D turbomachinery cascades. McMullen et al.[15, 16] followed this work, also focusing on 2-D turbomachinery flows. The current implementation uses this same non-linear, frequency-domain approach and extends the application to the 3-D Euler equations.

The current work uses a Cartesian, embedded-boundary method[17] to automate the generation of dynamic stability derivatives. The Cartesian method provides an efficient and robust mesh generation capability which can handle an arbitrarily-complex geometry description. Recently, a method to generate the water-tight surface triangulation required for Cartesian mesh generation directly from a CAD representation of the geometry has been developed[18]. This, combined with the Cartesian embedded-boundary method provides a robust and automatic mesh generation infrastructure which can be utilized through the design process. This meshing scheme has recently been combined with a parallel, multi-level scheme for solving time-dependent, moving-geometry problems, including a generalized rigid-domain motion capability[19]. This Arbitrary Langrangian-Eulerian (ALE) rigid-domain motion scheme provides the foundation upon which the current reduced-frequency method is implemented. The Cartesian methodology has been demonstrated as an efficient, robust method for automatically generating static stability derivatives[20, 21], and the current work extends this to include the prediction of dynamic derivatives.

This abstract briefly covers methods for determining dynamic stability derivatives, and the reduced-frequency approach, including an 2-D oscillating airfoil example. Demonstration examples of calculating the roll damping for the basic finner missile are compared against experimental data. The final section discusses topics of future work that will be included in the full paper.

2 Dynamic Derivatives

The aerodynamic characteristics of an aircraft can be described by the force and moment coefficients about the body axes; the axial, normal, and lateral force coefficients (C_A, C_N, C_Y), and the roll, pitch, and yaw coefficients (C_l, C_m, C_n). In most cases it is sufficient to define these coefficients as functions solely of the flight conditions and aircraft configuration,

$$C_j = C_j\left(\alpha, \beta, M_\infty, h, \delta_i, p, q, r, \dot{\alpha}, \dot{\beta}\right) \tag{1}$$

where j represents each of the individual force and moment coefficients, h is the altitude, δ_i represents any configuration-dependent information such as control surface settings, and p,q, and r are the rotation rates about the body axes.* Each individual coefficient can be broken into two parts: a so-called static portion (subscript s) which depends only on the non-rotating parameters, and a dynamic portion (subscript d) which depends on both the rotational and non-rotating parameters.

$$C_j = C_{j_s}\left(\alpha, \beta, M_\infty, h, \delta_i\right) + C_{j_d}\left(\alpha, \beta, M_\infty, h, \delta_i, p, q, r, \dot{\alpha}, \dot{\beta}\right) \tag{2}$$

While the focus of the current work is the calculation of the dynamic portion of the force and moment coefficients, a complete evaluation of the static portion is of primary importance.

In general the aerodynamic coefficients are non-linear functions of all of the independent parameters, however in many cases this can be simplified so that a linear superposition of the individual effects of each parameter can be assumed, i.e.

$$C_{j_d} = C_{j_d}\left(\alpha, \beta, M_\infty, h, \delta_i, p\right) + C_{j_d}\left(\alpha, \beta, M_\infty, h, \delta_i, q\right) + \ldots \tag{3}$$

Further, each individual effect can be assumed to be due to a linear variation of that parameter, for example the roll variation is given by

$$C_{j_d}\left(\alpha, \beta, M_\infty, h, \delta_i, p\right) = C_{j_p}\Delta p = \frac{\partial C_{j_d}}{\partial p}\left(\alpha, \beta, M_\infty, h, \delta_i\right)\Delta p \tag{4}$$

*It is assumed that the rotation rates are suitably non-dimensionalized.

Notice that these dynamic derivatives are solely functions of the non-rotating parameters, similar to the static coefficients.

Experimentally, the two tools which are commonly used to provide dynamic data are the rotary-balance and forced-oscillation tests. While it is difficult to determine each of the individual dynamic derivatives in the general case, as the rotation about the body and wind axes are coupled, there is a large legacy of methodology for using data from these tests in linearized dynamic models such as described above (cf. Kalviste[22]). The initial focus of the current work is to simulate the rotary-balance and forced-oscillation tests using the reduced-frequency method, so that the results can be used directly within existing modeling procedures. This is seen as a necessary first step; before more complicated uses for CFD are entertained it must become an everyday tool for evaluating dynamic effects in the most common cases, similar to the manner it is currently being used to evaluate the static effects. A longer-term focus is to develop CFD methods which can compute the dynamic derivatives directly in the general case, and to extend the methods to provide efficient tools in non-linear flight regimes where the traditional methods begin to fail, and obtaining data is extremely difficult (cf. Refs. [23, 24]).

3 Reduced-Frequency Method

The reduced-frequency method is derived from a general time-dependent scheme. An ALE rigid-domain motion approach is used in the current application to simulate a forced motion. The details of this time-dependent ALE scheme with a Cartesian embedded-boundary method are provided in [19], and a brief overview is given here. The time-dependent equations are

$$\frac{\partial \mathbf{Q}}{\partial t} + \mathbf{R}(\mathbf{Q}) = 0 \tag{5}$$

where t is the physical time, $\mathbf{Q}$ is the vector of conserved variables, and $\mathbf{R}(\mathbf{Q})$ is an appropriate numerical quadrature of the flux divergence, $\frac{1}{V}\oint_S \mathbf{f}\cdot\mathbf{n}dS$. This work uses an inviscid flux vector

$$\mathbf{f}\cdot\mathbf{n} = \left\{ \begin{array}{c} \rho u_n \\ \rho u_n \mathbf{u} + p\mathbf{n} \\ \rho u_n e + p\mathbf{u}\cdot\mathbf{n} \end{array} \right\} \tag{6}$$

where

$$u_n = (\mathbf{u} - \mathbf{u}_\Omega)\cdot\mathbf{n}$$

is the velocity relative to the moving boundary, and $\mathbf{u}_\Omega$ is the velocity of the moving domain.

Following Hall et al. [13, 14], both the conservative variables and $\mathbf{R}(\mathbf{Q})$ are assumed to be periodic functions of time (with frequency ω), and approximated with a finite Fourier series

$$\mathbf{Q}(\mathbf{x}, n\Delta t) \approx \sum_{k=-(N-1)/2}^{(N-1)/2} \hat{\mathbf{Q}}_k(\mathbf{x})\, e^{ik\omega n\Delta t}$$

$$\mathbf{R}(\mathbf{Q}, n\Delta t) \approx \sum_{k=-(N-1)/2}^{(N-1)/2} \hat{\mathbf{R}}_k(\mathbf{Q})\, e^{ik\omega n\Delta t}$$

where $\hat{\mathbf{Q}}_k$ and $\hat{\mathbf{R}}_k$ are complex Fourier coefficients, and $i = \sqrt{-1}$. As a result of this approximation $\mathbf{Q}$ can now only support a reduced set of frequencies, namely ω and the harmonics of ω. Since $\mathbf{R}$ is a non-linear function of $\mathbf{Q}$, the Fourier coefficient $\hat{\mathbf{R}}$ remains a function of $\mathbf{Q}$. Also, since $\mathbf{Q}$ and $\mathbf{R}$ are real, the Fourier coefficients of the negative wavenumbers are complex conjugates of their corresponding positive wavenumber.

The sampling rate Δt is chosen so that the functions are periodic over N samples, i.e. $\omega = \frac{2\pi}{N\Delta t}$ where $T = N\Delta t$ is the period. The Fourier coefficients can thus be evaluated using standard Fast Fourier Transform (FFT) algorithms. Substitution of the Fourier expansions for $\mathbf{Q}$ and $\mathbf{R}$ into the time-dependent equation, Eqn. 5, gives

$$ik\omega\hat{\mathbf{Q}}_k + \hat{\mathbf{R}}_k(\mathbf{Q}) = 0 \tag{7}$$

which form a set of N independent equations due to the orthogonality of the Fourier modes.* The solution procedure involves first performing an inverse Fourier transform to construct the N samples of $\mathbf{Q}$ from $\hat{\mathbf{Q}}_k$. These samples are used to construct N samples of $\mathbf{R}(\mathbf{Q})$, which are then transformed into the Fourier coefficients $\hat{\mathbf{R}}_k$. Equation 7 is then iterated to convergence.

The reduced-frequency approach outlined above has several convenient features. First, it can be applied to any set of time-dependent equations - inviscid, viscous, Reynolds-averaged turbulence model, etc. - without requiring any special procedures other than a discrete Fourier transform. The same non-linear operator $\mathbf{R}(\mathbf{Q})$ from the time-dependent scheme is computed. Secondly, the same convergence acceleration procedures that are common with static, steady-state solvers, such as local timestepping, multigrid, etc., can be utilized to solve Eqn. 7. In the current work all of the existing infrastructure from the parallel, multi-level Cartesian solver developed for steady-state[25], and unsteady dual-time schemes[19] has been re-used in the frequency-domain solver with only minor modifications.

The cost of the reduced-frequency approach scales as roughly N times the cost of a static, steady-state solution, as each iteration requires N evaluations of $\mathbf{R}(\mathbf{Q})$. Further, it is required to store N copies of each variable in the scheme, which can be prohibitive in 3-D, especially using commodity desktop systems. Thus if it requires more than one or two Fourier modes to characterize the unsteady behavior, the reduced-frequency approach rapidly loses favor relative to solving the time-dependent equations, which require roughly an order of magnitude greater effort than a steady-state simulation, but can support a continuum of modes.

The current approach involves simulating the response to a prescribed periodic motion using an invsicid scheme. The simulations of the base state for these flowfields (steady-state simulations without a prescribed motion) usually result in a time-invariant flowfield, so that all of the unsteadiness in a forced oscillation simulation is due to the prescribed motion. The first mode of $\mathbf{Q}$ is thus identical to the forcing frequency. Further, in an inviscid simulation no

*N independent equations for the real and imaginary parts of the positive wavenumbers. The equations for the complex conjugate are redundant.

physical mechanism exists to transfer energy between modes, so that all of the energy remains in the primary mode.* Thus, the response to a forced oscillation computed using just a single mode with the reduced-frequency method is equivalent to a full time-dependent simulation for an inviscid scheme. This can be seen in Fig. 1, which shows the results of simulating the forced oscillation of a transonic NACA 0012 airfoil using three methods: a time-dependent simulation, and reduced-frequency simulations retaining one and two modes. After the initial transient of the time-dependent simulation, all three simulations are nearly identical, and all are in good agreement with the experimental data and capture the hysteresis in the normal force variation. This indicates that $\mathbf{Q}$ contains just a single mode at the forcing frequency, and that including higher harmonics provides no additional information. A comparison of numerical timings will be presented for 3-D simulations in the final paper.

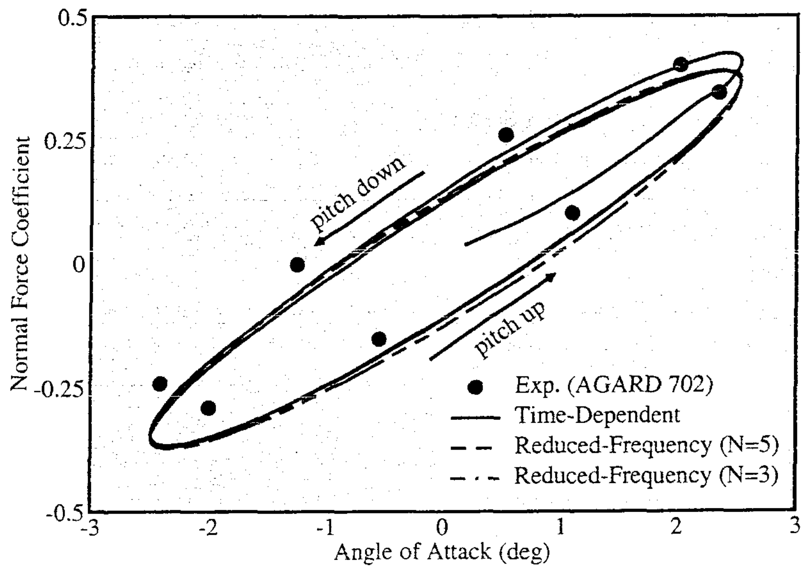

Figure 1: Variation of normal force coefficient with angle of attack for oscillating NACA 0012. ($M_\infty = 0.755$, $\alpha(t) = 0.016 + 2.51 \sin(2\pi 62.5t)$). The time-dependent simulation includes the initial transient portion of the calculation. Experimental data from [26].

4 Sample Results

The reduced-frequency method outlined above is used to calculate the roll damping of the basic-finner missile configuration (Fig. 2) at $M_\infty = 2.5$ and angles of attack $\alpha = 0.0°$, $10.0°$, and $20.0°$. The results are compared to a general time-dependent method, and the experimental results of Jenke[27]. The time-dependent method computes the rolling moment

*Numerical dissipation does provide a numerical mechanism to transfer energy, however the numerical dissipation is orders of magnitude lower than the physical kinematic viscosity, and hence is not effective. This does not imply that a viscous simulation automatically would show a wide energy band, or that these higher modes must be resolved to provide an effective estimate of the response to a forced oscillation.

on the missile at constant roll rates p, and then differences the results to determine the roll-damping derivative C_{l_p}. One approach for applying the reduced-frequency method to this configuration is to oscillate the body at a fixed amplitude and frequency. Following the modeling of dynamic derivatives in Sec. 2, the variation of rolling moment with roll rate is assumed to be linear, and hence the roll-damping can be determined from a forced-oscillation by a linear regression. The summary of these results are presented in Table 1. The reduced-frequency method accurately predicts the roll-damping at all angles of attack at a fraction of the cost of the general time-dependent method.

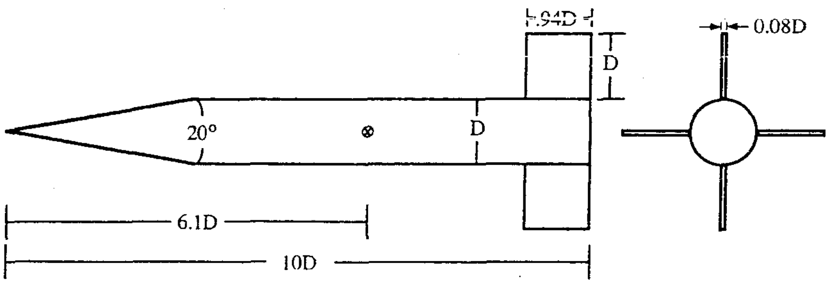

Figure 2: Basic-finner geometry is a cone-cylinder fuselage with square fins in the + configuration. The cone section has a 10° half-angle, and the center of mass is located 6.1 diameters from the nose along the longitudinal axis of the body.

α	Experiment[27]	Time-Dependent	Reduced-Frequency
0.0°	-17.4	-17.7	-17.3
10.0°	-19.6	-19.2	-18.6
20.0°	-21.3	-21.2	-19.9

Table 1: Computed roll-damping C_{l_p} for the basic-finner configuration, $M_\infty = 2.5$.

5 Future Work

The current abstract contains an overview of the methodology for computing dynamic derivatives using a reduced-frequency method. The proposed paper will include a thorough validation of the scheme against experimental data for dynamic configurations of interest, namely rotary-balance and forced-oscillation test results. These validation cases will include transonic and supersonic results for both complete aircraft and missile configurations. The final paper will also include a summary of the computational cost for the reduced-frequency approach compared against the general time-dependent method.

References

[1] Pechier, M.., Guillen, P., and Cayzac, R., "Magnus Effect over Finned Projectiles," *Journal*

of Spacecraft and Rockets, 38(4):542–549, July 2001.

[2] Oktay, E. and Akay, H. U., "CFD Predicitions of Dynamic Derivatives for Missiles," AIAA Paper 2002-0276, Jan. 2002.

[3] Murman, S.M., Aftosmis, M.J., and Berger, M.J., "Numerical Simulation of Rolling-Airframes Using a Multi-Level Cartesian Method," AIAA Paper 2002-2798, June 2002.

[4] Cormier, J., Champagneux, S., Moreuxt, V., and Collercandy, R., "Prediction of Quasi-Stationary Aerodynamic Coefficients Using the ALE Method," in *ECCOMAS 2000*, (Barcelona, Spain), Sept. 2000.

[5] Park, S.H., Kim, Y., and Kwon, J.H., "Predicition of Dynamic Damping Coefficients Using the Unsteady Euler Equations," *Journal of Spacecraft and Rockets*, 40(3):356–362, 2003. Also AIAA Paper 2002-0715.

[6] Schiff, L.B., "Nonlinear Aerodynamics of Bodies in Coning Motion," *AIAA Journal*, 10 (11):1517–1522, 1972.

[7] Park, S.H. and Kwon, J.H., "Navier-Stokes Computations of Stability Derivatives for Symmetric Projectiles," AIAA Paper 2004-0014, Jan. 2004.

[8] Weinacht, P., Sturek, W.B., and Schiff, L.B., "Navier-Stokes Predictions of Pitch-Damping for Axisymmetric Shell Using Steady Coning Motion," AIAA Paper 91-2855, Aug. 1991.

[9] Weinacht, P., "Navier-Stokes Predictions of the Individual Components of the Pitch-Damping Sum," *Journal of Spacecraft and Rockets*, 35(5):598–605, Oct. 1998.

[10] Weinacht, P. and Sturek, W., "Computations of the Roll Characteristics of a Finned Projectile," *Journal of Spacecraft and Rockets*, 33(6):769–775, Nov. 1996.

[11] Green, L., Spence, A.M., and Murphy, P.C., "Computational Methods for Dynamic Stability and Control Derivatives," AIAA Paper 2004-0015, Jan. 2004.

[12] Stalnaker, J., "Rapid Computation of Dynamic Stability Derivatives," AIAA Paper 2004-0210, Jan. 2004.

[13] Hall, K., Thomas, J., and Clark, W., "Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique," in *9th International Symposium on Unsteady Aerodyamics, Aeroacoustics, and Aeroelasticity of Turbomachines*, (Lyon, France), Sept. 2000.

[14] Hall, K., Thomas, J., and Clark, W., "Computation of Unsteady Nonlinear Flows in Cascades Using a Harmonic Balance Technique," *AIAA Journal*, 40(5):879–886, May 2002.

[15] McMullen, M., Jameson, A., and Alonso, J., "Acceleration of Convergence to a Periodic Steady State in Turbomachinery Flows," AIAA Paper 2001-0152, Jan. 2001.

[16] McMullen, M., Jameson, A., and Alonso, J., "Application of a Non-Linear Frequency Domain Solver to the Euler and Navier-Stokes Equations," AIAA Paper 2002-0120, Jan. 2002.

[17] Aftosmis, M.J., Berger, M.J., and Melton, J.E., "Robust and Efficient Cartesian Mesh Generation for Component-Based Geometry," *AIAA Journal*, 36(6):952–960, June 1998.

[18] Haimes, R. and Aftosmis, M.J., "On Generating High Quality "Water-tight" Triangulations Directly from CAD," in *Meeting of the International Society for Grid Generation (ISGG) 2002*, June 2002.

[19] Murman, S.M., Aftosmis, M.J., and Berger, M.J., "Implicit Approaches for Moving Boundaries in a 3-D Cartesian Method," AIAA Paper 2003-1119, Jan. 2003.

[20] Rogers, S. E., Aftosmis, M. J., Pandya, S. A., Chaderjian, N. M., Tejnil, E. T., and Ahmad, J. U., "Automated CFD Parameter Studies on Distributed Parallel Computers," AIAA Paper 2003-4229, June 2003.

[21] Murman, S.M., Aftosmis, M.J., and Nemec, M., "Automated Parameter Studies Using a Cartesian Method," accepted for publication, AIAA Paper 2004-5076, Aug. 2004.

[22] Kalviste, J., "Use of Rotary Balance and Forced Oscillation Test Data in Six Degrees of Freedom Simulation," AIAA Paper 82-1364, 1982.

[23] O'Connor, C.J., Ralston, J.N., Fitzgerald, T., "Evaluation of the NAWC/AD F/A-18 C/D Simulation Including Database Coverage and Dynamic Data Implementation Techniques," AIAA Paper 96-3365, 1996.

[24] Brandon, J.M. and Foster, J.V., "Recent Dynamic Measurements and Considerations for Aerodynamic Modeling of Fighter Airplane Configurations," AIAA Paper 98-4447, 1998.

[25] Aftosmis, M.J., Berger, M.J., and Adomavicius, G., "A Parallel Multilevel Method for Adaptively Refined Cartesian Grids with Embedded Boundaries," AIAA Paper 2000-0808, Jan. 2000.

[26] Landon, R.H., "Compendium of Unsteady Aerodynamic Measurements," AGARD Report No. 702, Aug. 1982.

[27] Jenke, L.M., "Experimental Roll-Damping, Magnus, and Static-Stability Characteristics of Two Slender Missile Configurations at High Angles of Attack (0 to 90 deg) and Mach Numbers 0.2 through 2.5," AEDC Technical Report AEDC-TR-76-56, July 1976.