AIAA Paper No. 75-453

Thickness Noise of Helicopter Rotors at High Tip Speeds

F. Farassat
Joint Institute for Acoustics and Flight Sciences
Hampton, VA

R. J. Pegg and D. A. Hilton
NASA Langley Research Center
Hampton, Virginia

AIAA 2nd Aero-Acoustics Conference
March 24-26, 1975
Hampton, Virginia
THICKNESS NOISE OF HELICOPTER ROTORS AT HIGH TIP SPEEDS

F. Farassat*, R. J. Pegg†, and D. A. Hilton‡†
NASA Langley Research Center
Hampton, Virginia

Abstract

A new formulation of helicopter rotor thickness, noise for hover and forward flight, is discussed. The parameters required for this formulation are rotor motion, planform and airfoil thickness distribution. A computer program has been developed to calculate the pressure signature due to blade thickness for a helicopter in arbitrary motion. Comparison with high-speed helicopter tests shows good agreement with calculations when the observer is in or near the horizontal plane in which the rotor disc lies. Characteristics of thickness noise are illustrated by numerical examples indicating strongly that the high-speed blade slap may be due primarily to the thickness effect. The methods of Deming and Arnoldi are discussed as the special cases of this technique.

I. Introduction

An unexplained phenomenon of rotor noise generation is the high-speed blade slap. This noise which appears at high advancing tip speeds has several distinctive characteristics. It is impulsive, that is, its signature has a high crest factor, and is directional towards the forward region of the flight. The peak pressure is in the rotor plane. The sound intensity appears to increase substantially as the relative tip speed approaches sonic speed in the medium. This noise is sometimes referred to as the compressibility noise for the obvious reason that the compressibility effects become apparent at high tip speeds.

It is the purpose of this paper to suggest the thickness noise as the possible mechanism of high-speed blade slap. The theory developed by Farassat (7) has been used to study the characteristics of the thickness noise. Some comparison with experimental measurements is made and reported here. The agreement with experiment and the characteristic features of the thickness noise suggest this mechanism as the source of the blade slap.

II. Thickness Noise Theory

The present theory is based on the solution of the wave equation:

\[
\frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} - \nabla^2 p = \frac{\delta}{\delta t} [\rho v_n \nabla \delta (r)]
\]

where \( p \) is the acoustic pressure, \( \tau \) is the source time, and \( v_n \) is the normal velocity of the blade surface which is defined by the equation \( f(y, \tau) = 0 \). The density of the undisturbed medium is \( \rho_0 \) and the speed of sound is denoted by \( c \). In Equation (1), \( \delta (r) \) is the Dirac delta function. This equation has several forms of solution which were originally studied by Frowes Williams and Hawkins (2). One form based on a collapsing sphere which was also proposed by Frowes Williams and Hawkins and developed by Farassat (3), seems to be appropriate for numerical calculations. The derivation of this solution is given in (1). The solution of Equation (1) used here is

\[
4\pi p(x, t) = \frac{3}{\delta t} \int \frac{\rho \omega v_n}{r \sin \theta} d \Gamma \, d \tau
\]

(2)

where \( x \) and \( t \) are the observer positions and time, respectively, \( v_n \) is the normal velocity of the blade surface and \( r = |\vec{x} - \vec{y}| \) where \( \vec{y} \) is the source location on the blade. In Equation (2), \( \Gamma \) is the curve of intersection of the blade surface and the surface \( g = t - t + \frac{x}{c} = 0 \) which for a fixed \( \vec{y} \) and \( t \) is a sphere whose radius collapses at speed of sound \( c \). The angle \( \theta \) is between the outward normal to the body and the direction \( \vec{T} = \vec{x} - \vec{y} \). Figure 1 illustrates the definition of some of the parameters.

![Figure 1](image)

Figure 1. Illustration of the Blade System Intersecting the Collapsing Sphere \( g = 0 \) Forming \( \Gamma \)-curve.

The normal velocity \( v_n \) of the airfoil can be written in the terms of helicopter speed and rotor angular velocity \( \Omega \). Consider a rotating system of axes \( \vec{n} \) such that \( \vec{n}_1 \)-axis is parallel to the chord of a blade, the positive direction being from the leading edge to the trailing edge. The rotor...
system is in the $n_1n_2$-plane with the origin at the center of rotation. Then

$$y_n = \frac{F'(n_1)}{\sqrt{1 + F^2(n_1)}} (n_2 \Omega - V_1) + V_3 n_3$$  \hspace{1cm} (3)

where $F(n_1)$ is the thickness function of the airfoil and $V_1$ and $V_3$ are the helicoidal speed along $n_1$- and $n_2$-axes, respectively. Here $n$ is the unit outward normal to the blade surface.

One can approximate the $\Gamma$-curves with the intersection of the sphere $g = 0$ and the mean airfoil surface. The upper and lower surfaces of the airfoil contribute equally to the thickness noise so that one obtains, from Equations (2) and (3), the following:

$$f(x, t) = \frac{\beta c}{2\pi} \sum_{\text{blades}} \int \frac{d\Gamma}{\Gamma} \frac{F'(n_1)(n_2 \Omega - V_1)}{\sqrt{1 + F^2(n_1)}} \sin \theta$$

The summation notation is used here to denote the integration over all the blades of the rotor system for which the intersection with $g = 0$ occurs. Note that the axial velocity $V_3$ cancels out in the integrand in Equation (4) when combining the integrals over the upper and lower surfaces of the blades.

The condition $\theta = 0$ occurs when the surface of the blade and the sphere $g = 0$ are tangent. It may be shown that $f(x, t)$ Equation (2) can be written in the following form:

$$f(x, t) = \frac{1}{2\pi} \int \frac{\partial \phi(x, t)}{\partial \Gamma} \frac{\rho \sqrt{\rho c}}{\rho c} \frac{d\Gamma}{\Gamma}$$

where $\lambda = [1 + M_n^2 - 2M_n \cos \theta]^{1/2} - M_n$, $\rho$ is $\gamma/c$, and $\Gamma$ is the surface that the $\Gamma$-curves form in the space for fixed $x$ and $t$. The $\Gamma$-surface is given analytically by $f(y, t - \Gamma) = 0$. It is the locus of the points on the blades whose signal arrive simultaneously to the observer at the time $t$. From the relations

$$\lambda^2 = (1 - M_n^2)^2 + 2M_n (1 - \cos \theta),$$

$$1 + M_n^2 - 2M_n \cos \theta > 0,$$

one can see that the only possible singularity of Equation (5) is when $M_n = 1$ and $\theta = 0$ simultaneously. This condition requires more careful consideration\(^{(4)}\). In case $M_n \neq 1$, we see that the condition is a removable singularity of Equations (2) and (4) so that a very small portion of $\Gamma$-surface near such a point can be neglected. This can be incorporated in the numerical scheme and, thus, Equation (4) can be used for calculation of thickness noise. For the case $M_n = 1$ and $\theta = 0$ simultaneously at a point on the $\Gamma$-surface, either smaller mesh must be taken during numerical calculation or the contribution from the region near this point must be found analytically and be included in the numerical scheme. The differentiation with respect to the observer time can be performed numerically.

The extent of the $\Gamma$-surface or more precisely the extent of that region of the $\Gamma$-surface with large source strength, the observer position and the observer time interval of interest as well as the source time scale determine the compactness of a sound generator.

Apart from the neglect of the differences in the retarded time, the compactness character of the sound source makes it possible to relate the acoustic pressure to the global parameters of the source such as the net force on the body or the net rate of mass injection by the motion of the body surface. Thus, the surface structure of the source does not enter the acoustic calculations directly, but the structure indirectly influences these calculations by affecting the global parameters. For a compact source the $\Gamma$-surface and the body surface should be nearly identical and the differences between the retarded time from the observer should be much less than the typical time scale of the source. For non-compact sources, the surface structure does enter the acoustic calculations (3), as is also evident from Equations (2), (4), and (5). From this discussion, one can conclude that even for moderate tip Mach numbers, with the observer in or close to the plane where the rotor disc lies, the compactness assumption is not justified and one should use Equation (4).

III. Previous Work

The earliest published work on the thickness noise is by Earnesthausen\(^{(5)}\). He measured the near field sound radiation from a high-speed model propeller with symmetrical airfoil section and zero pitch angle. He explained the shape of the acoustic pressure signature by sweeping the pressure pattern around the airfoil past the observer. He discovered several features of the thickness noise such as the increase of the sound intensity and appearance of higher harmonics at higher tip speeds.

Deming developed a theory for static propellers\(^{(6)}\). In this case, because of the periodicity of the signal, the spectrum of the sound can be obtained easily. His equation and the boundary condition is equivalent to the time Fourier transform of the single Equation (1). His result is, therefore, equivalent to the Fourier transform of Equation (4) for the case of a static propeller or a hovering rotor. Because of unavailability of high-speed computers, Deming was obliged to make further simplifying assumptions to get analytic expressions for his computations.

Recently, Hawking and Lowson\(^{(7)}\) published a work which includes a thickness term identical to Deming's using a slightly different approach.

Using the work of Billing\(^{(8)}\), Arnoldi obtained a theory for moving and static propellers\(^{(9)}\). His theory is too detailed to be discussed here, but the bulk of manipulations is due to the complex helical pattern of the source motion. The basic idea behind the theory was explained by Hawking and Lowson\(^{(10)}\) using the modern mathematical tool of generalized functions. Arnoldi's
solution is identical to the solution of the equation
\[
\frac{1}{c^2} \frac{\partial^2 P}{\partial \tau^2} - \nabla_p^2 P = - \frac{\partial^2}{\partial y \partial y} \left\{ \rho_0 v_0 \delta (y-y_H(\tau)) \right\}
\]
where \( \dot{u}_i \) is the source speed which is at the position \( y_H(\tau) \) and has the volume \( V_0 \). In other words, Arnoldi's work is restricted to compact sources only. Also, his analysis is restricted to uniform axial motion of the propeller which cannot be applied to helicopter rotors. In time domain, one can easily solve the above equation numerically.

Figure 2. Theoretical variation of acoustic pressure signature due to thickness with blade tip speed for a hovering helicopter. No. of blades = 2, Rotor diameter = 10 m, Hub diameter = 1.4 m, Blade aspect ratio = 10.75, Blade chord = 0.4 m, airfoil type-biconvex parabolic, observer in the plane of the rotor 50 meters from the center of rotation, \( M_t \) = tip Mach number (tip velocity/c). Thickness ratio = 10 percent.
IV. Numerical Calculations and Comparison With Experiment

Some of the characteristic features of the thickness noise are discussed here by numerical examples. Also, the results of two helicopter tests are compared with calculations. The computer program with some discussion of the theory will be published at a later date. The basic features of the present method is that all the calculations are performed in the time domain by constructing the T-curves for each source time t and performing the integral in Equation 4 by finite difference method.

For Figures 2 and 3, the rotor system is 10 meters in diameter with two rectangular blades. The airfoil section is a biconvex parabolic arc. The computer program is not restricted to airfoil sections with sharp leading edge as Figures 5 and 6 show. The chord length is 0.4 meter and the airfoil thickness ratio is 10 percent. The aspect ratio of each blade is 10.75.

Figure 2 shows the effect of changing tip speed from tip Mach number $M_t$ (tip velocity/c) of 0.4 to 1.1 for a hovering helicopter. The acoustic pressure signatures at a point 50 meters from the center of the rotation and in the plane of the disc are given for one period of the signal. One should note the considerable increase in amplitude with increase of the tip speed. Starting with approximately equal positive and negative peaks, the negative peak becomes dominant from about $M_t = 0.6$. For sonic and low supersonic Mach numbers, there are two large positive and one large negative peak. The negative peak for $M_t$ in the range 0.4 to 1.1 follows closely a variation given by $\exp (8.94 M_t^2)$. The appearance of higher harmonics is also evident in Figure 2. For higher tip Mach numbers, one obtains a signal which is concentrated in a shorter portion of the period with larger peaks.

Figure 3 indicates the change in the pressure signature with observer on a sphere 50 meters in radius and changing elevation from the plane of the disc up to 45° elevation in 15° increments. The tip Mach number is 0.9 and the helicopter is in hovering position. The signature becomes less impulsive with the increase in elevation and the amplitude decreases considerably indicating strong directionality in the plane of the rotation.

Figure 4 corresponds to the condition of Figure 2(f), but with rotor hub diameter of 8.4 meters (84 percent of the rotor diameter) resulting in blades with aspect ratio of 2 instead of 10.75. This figure shows that it is the tip region of the blades which is responsible for most of the noise generation.

Figure 5 is for a two-bladed rotor system of a helicopter at forward speed of 87.5 m/sec (170 kts). The rotor diameter is 13.42 meters, the blade chord is 0.686 meter and the rotor RPM is 324. The airfoil thickness ratio is 9.3 percent. Figures 5(a), (b), and (c) show the theoretical pressure signatures for blades using a biconvex parabolic arc, an NACA four-digit and a supercritical airfoil, respectively. At emission time, the observer is 80 meters ahead of the helicopter which is flying at 15 m altitude. These signatures are calculated to show the effect of the change in thickness distribution of blade section. It is seen that the biconvex airfoil is the least noisy and the supercritical airfoil the most noisy of the three sections. It must be mentioned, however, that the use of supercritical airfoils may make it possible to reduce the thickness ratio near the tip and, thus, result in both improved performance and better noise characteristics.
Figure 4. The influence of blade tip region on the generation of thickness noise. Rotor hub diameter = 8.4 m, \( M_\infty = 0.9 \). Other parameters as in Figure 2. Compare this figure with Figure 2(f).

Figure 5. Effect of change of thickness distribution on the theoretical acoustic pressure signatures. No. of blades = 2, Rotor diameter 13.42 m Blade chord = 0.686 m, RPM = 324, Helicopter altitude = 15 m, observer location = 80 m (ahead) of helicopter at emission time, Thickness ratio = 9.3 percent.

Figure 6 is for the rotor system of Figure 5, but with an NACA four-digit airfoil section. The observer position is as for Figure 5. Figures 6(a) and (b) are for forward speed of 72.0 m/sec (140 kts) and 87.5 m/sec (170 kts), respectively. The corresponding experimental acoustic pressure signatures for a test helicopter with the same rotor parameters and operating conditions, obtained at NASA Langley Research Center, are superimposed on the figure. The experimental signatures also include the tail rotor noise. It is seen that the thickness noise theory can explain the observed experimental signature. The discrepancy may be due to the errors of numerical differentiation in using Equation (4). It may also be due to the neglect of the contribution from the pressure sources on the blades which is not considered in this paper (7). For the observer in or close to the plane where the rotor disc lies (near or far field), thickness noise appears to be of the correct order of magnitude for experimentally observed pressure levels of high-speed blade slap.

Figure 6. Comparison for theoretical and experimental acoustic pressure signatures for a test helicopter. Airfoil type = NACA four-digit. Other parameters as in Figure 5.

V. Conclusions

This paper presents a new formulation of the thickness noise theory which is not restricted to either compactness or far-field assumptions. Due to the complex geometry of the \( z \)-surfaces or the \( z \)-surfaces for a rotor system, it is difficult to draw general qualitative conclusions based on Equation (4), for the arbitrary motion of the helicopter. Numerical examples which are worked out indicate that:
1. The blade noise due to thickness at high tip speeds is impulsive in character and can have large amplitudes of the order observed for high-speed blade slap.

2. The thickness noise is directional with peak pressure in the rotation plane towards the forward region of flight.

3. The tip region of the blade contributes mainly to the generation of the thickness noise. The signature is directly proportional to the thickness ratio(1).

4. The variation in the thickness distribution of the blade section can have substantial influence on the noise radiation.

To achieve noise reduction, one may reduce the blade thickness ratio and change the thickness distribution near the tip region of the blades. The effect of the planform changes has not been studied here, but this offers another means of controlling the radiated noise.

Based on the fact that the present theory exhibits the observed features of high-speed blade slap of helicopters, thickness noise is strongly suspected to be the source of the blade slap at high relative tip speeds. Additional investigation is needed to verify this conclusion.

The nonlinear propagation effects which may be important if the propagation distance is large, has been studied by Hawkins and Lowson (7). The change in the wave form when included in the analysis, gives better agreement with experimental results according to these authors.

The first author acknowledges the support from NASA Grant NGR-09-010-085, entitled "Noise Reduction." The computer program was developed with the help of Mr. G. H. Mall of Computer Sciences Corporation.

References


