

Aeroelastic Deflection of NURBS Geometry

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Abstract

The purpose of this paper is to present an algorithm for using NonUniform Rational B-Spline (NURBS) representation in an aeroelastic loop. The algorithm is based on creating a least-squares NURBS surface representing the aeroelastic deflection. The resulting NURBS surfaces are used to update either the original Computer-Aided Design (CAD) model, Computational Structural Mechanics (CSM) grid or the Computational Fluid Dynamics (CFD) grid. Results are presented for a generic High-Speed Civil Transport (HSCT).

1 Introduction

A critical element in the application of Multidisciplinary Design Optimization (MDO) of an engineering system is the introduction of a consistent parametric geometric representation. Such a representation guarantees that the same geometry model is used to derive the computational models required for different disciplinary analyses, and the field data can be transferred among disciplines without loss of accuracy. With the introduction of CAD into an optimization process, it is possible to study complex configurations using higher fidelity CFD and CSM with consistent geometry representation.

Another critical issue for MDO applications is the strong interactions between CSM and CFD. Such interactions can prompt physically important phenomena such as those occurring in aircraft due to aeroelasticity.

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Correct modeling of these complex aeroelastic phenomena requires a direct coupling of CSM and CFD for flexible structure (e.g., wing). The interaction requires the manipulation of the original CAD geometry that is stored commonly as a set of NURBS. This author [1] has proposed earlier an approach to incorporate a CAD model in an MDO environment where multiple disciplines have access to a consistent parametric geometric representation.

Smith et al. [2] evaluated six methods for transferring information between CFD and CSM. They were Infinite-Plate Spline (IPS), Multiquadric-Biharmonic (MQ), NURBS, Thin-Plate Spline (TPS), Finite-Plate Spline (FPS), and Inverse Isoparametric Mapping (IIM) methods. Out of these six methods, they recommended to study IIM and NURBS further. They indicated that IIM showed great promise for two-dimensional applications, and it needs to be extended to three-dimensions. One key ingredient for this extension is the ability to map information known on the CFD model to the CSM model and vice-versa. A typical CSM model consists of triangle and quadrilateral elements. This author [3] has shown a process by which a set of three-dimensions points (e.g. CFD model) can be mapped to either a triangle, quadrilateral, or a NURBS surface. This procedure can be used to extend the IIM to three-dimensions.

Also Smith et al. [2] indicated that NURBS has excellent promise, but there were some problems that needed to be resolved before implementation. One problem was that the input data must be a structured (regular) grid. This requirement forces the data, at best, to be approximated. In most realistic cases, this step is either impossible or time consuming. In a similar study, this author [1] proposed and studied a method to use NURBS representation for data transfer among various disciplines. Since this method is based on a general three-dimensional least-squares representation, it does not require the input to be a structured grid. Another advantage of this approach is the control over the trade off between smoothness and accuracy. This paper extends and demonstrates this method to a generic HSCT.

2 NURBS

This section contains a brief overview of NURBS curves and surfaces, and readers should consult [4] for a detailed discussion. A NURBS curve, $\vec{R}(U)$, can be represented as

$$\vec{R}(U) = \frac{\sum_{i=1}^I B_{i,p}(U) W_i \vec{P}_i}{\sum_{i=1}^I B_{i,p}(U) W_i}, \quad (1)$$

where $\vec{P}_i$ are the control points (forming a control polygon), I is the number of control points, parameter U is bounded by $U_{min} \leq U \leq U_{max}$, and W_i are the weights. The $B_{i,p}$ are the p -th degree B-spline basis functions defined on the non-periodic and nonuniform knot vector (U). This completes the mapping between the one-dimensional parameter space, U , and the three-dimensional Euclidean space, $\vec{R}$. A NURBS surface is a parametric surface and is defined as a function of two parameters.

$$\vec{R}(\vec{U}) = \frac{\sum_{i=1}^I \sum_{j=1}^J B_{i,p}(U) B_{j,q}(V) W_{i,j} \vec{P}_{i,j}}{\sum_{i=1}^I \sum_{j=1}^J B_{i,p}(U) B_{j,q}(V) W_{i,j}}, \quad (2)$$

where the components of vector, $\vec{U} = (U, V)^T$, are the surface parameters and have no geometrical significance. However, as U increases for a constant V , the point $\vec{R}(\vec{U})$ moves always from one side of the surface to the other side. The $\vec{P}_{i,j}$ are control points (forming a control surface), $W_{i,j}$ are the weights, and $B_{i,p}$ and $B_{j,q}$ are the p -th and q -th degree B-spline basis functions defined on the non-periodic and nonuniform knot vector.

3 Formulation

The aeroelastic deflection, $\vec{d}_m$, is defined at each CSM grid points, $\vec{r}_{CSM} = \{x_m, y_m, z_m\}^T$. The goal is to modify the NURBS geometry definition, $\vec{R}(\vec{U})$, such that it reflects the deflection produced by CSM. The algorithm for deflection transfer has four steps:

1. Project the CSM grid points, $\vec{r}_m$, onto the original NURBS surface.
2. Create a NURBS surface based on the deflection, $\vec{D}(U, V)$, which has the same degree as the original NURBS surface.
3. Add/remove knots from the new surface to make it compatible with the original NURBS surface.
4. Add the control points to the original NURBS surface to form the new deflected NURBS surface.

The first step can be solved by mapping the CSM grid to the original NURBS surfaces, hence, reducing the dimension from four, $(\vec{d}(x_m, y_m, z_m))$, to three, $(\vec{d}(u_m, v_m))$. The u_m and v_m are in terms of the parametric coordinates of the original NURBS surface. This information may be available from the CSM grid generation process. If not, the CSM grid points can be projected onto the original NURBS surface [3]. The process of projecting a point, $\vec{r}_m = \{x_m, y_m, z_m\}^T$, on a surface, $\vec{R}(\vec{U})$, can be performed by

finding a $\vec{w}_m = \{u_m, v_m\}^T$ such that the distance, l_m , between the $\vec{r}_m$ and $\vec{R}(\vec{w}_m)$ is minimal and $\vec{w}_m$ is constrained to $\in [(a, b), (c, d)]$. The distance, l_m , can be written in terms of parameters $\vec{w}_m$ as

$$l^2(\vec{w}_m) = f(\vec{w}_m) = |\vec{R}(\vec{w}_m) - \vec{r}_m| \cdot |\vec{R}(\vec{w}_m) - \vec{r}_m|. \quad (3)$$

An efficient algorithm has been presented by this author [3] to minimize l_m .

The second step is to fit a three-dimensional surface, $\vec{D} = \vec{D}(U, V)$, to the deflection. A NURBS surface can be fitted based on a least-squares approximation [4, 6, 7, 8] that minimizes the approximation error. A three-dimensional curve is used to illustrate the least-squares algorithm.

A set of three-dimensional deflections, $\vec{d}_m$, can be fitted by a B-Spline curve, $\vec{D}(U)$. The B-Spline equation can be expressed at each parameter value, u_m , as

$$\vec{d}_m = B_{1,p}(u_m)\vec{D}_1 + B_{2,p}(u_m)\vec{D}_2 + \dots + B_{I,p}(u_m)\vec{D}_I, \quad (4)$$

where I is the number of control points, and p is the degree, $\vec{d}_m$ is the CSM deflection, M ($m = 1, M$) is number of CSM points, and $\vec{D}_i$ is the i -th B-Spline control point. The above equation can be expressed in matrix form as $[\vec{d}] = [B][\vec{D}]$, where

$$[B] = \begin{bmatrix} B_{1,p}(u_1) & \dots & B_{I,p}(u_1) \\ \vdots & \ddots & \vdots \\ B_{1,p}(u_m) & \dots & B_{I,p}(u_m) \\ \vdots & \ddots & \vdots \\ B_{1,p}(u_M) & \dots & B_{I,p}(u_M) \end{bmatrix}. \quad (5)$$

If $M = I$, the matrix $[B]$ is a square matrix and the control points can be calculated directly by matrix inversion,

$$[\vec{D}] = [B]^{-1}[\vec{d}]. \quad (6)$$

In this case, the resulting B-Spline curve passes through each data point. However, if the number of data points, M , is greater than the number of control points, I , the problem is over-specified. A least-square method can solve the problem as,

$$[\vec{D}] = [[B]^T[B]]^{-1}[B]^T[\vec{d}]. \quad (7)$$

The smoothness of the least-square representation can be controlled by selecting degree (p) and the knot vectors. The least-squares approximation for surfaces is very similar to the least-squares approximation for the curves. The minimization error can be written as

$$Error = \sum_{m=1}^M \left[\vec{d}(u_m, v_m) - \frac{\sum_{i=1}^I \sum_{j=1}^J B_{i,p}(u_m) B_{j,q}(v_m) W_{i,j} \vec{D}_{i,j}}{\sum_{i=1}^I \sum_{j=1}^J B_{i,p}(u_m) B_{j,q}(v_m) W_{i,j}} \right]^2 \quad (8)$$

where $\vec{D}_{i,j}$ are control points for the NURBS surface representing the CSM deflection, $W_{i,j}$ are the weights, $B_{i,p}$ and $B_{j,q}$ are the p-th and q-th degree B-spline basis functions defined on the non-periodic and nonuniform knot vector, and M is the number of CSM grid points. This forms a system of linear equations that can be solved for control points of a NURBS surface representing the CSM deflection. Generally this NURBS surface has a different set of knot vectors than the original NURBS surface.

The third step is to add/remove knots from the new surface to make it compatible with the original NURBS surface. The details of this step can be found in [4]. The last step is to add the resulting control points from the previous step to the original NURBS surface to form a new deflected NURBS surface. This algorithm has following properties:

1. As $\vec{d}_m$ approaches zero, the method will reproduce the original NURBS surface, $\vec{R}(\vec{U})$ as shown in Eqs. 6 and 7.
2. Smoothness can be controlled on the resulting NURBS surface by selecting the degree of NURBS, the number and positions of the knot vectors.
3. The resulting surface is a NURBS surface with the same degree as the original NURBS surface.
4. It is possible to maintain the same knot vector as the original NURBS surface.

4 Results and Discussions

The algorithm has been tested for a generic HSCT geometry with two different sets of deflections. This geometry is made of three surfaces: fuselage, inboard, and outboard wings. Figure 1 shows the original NURBS surfaces, the deflected CSM grid, and the deflected NURBS surfaces. The goal of the first test case is to examine the capability of the deflection-transfer algorithm. For this case the CSM grid has a large and unrealistic deflection. Figure 2 shows an oblique view of the same test case. The resulting NURBS surfaces have an RMS error of less than one percent of mid-span

maximum thickness. For this test case, even though the inboard and outboard wings are fitted separately, it is interesting to note that no gaps or overlaps are observed. But this could be a potential problem.

This method has been simplified by applying the NURBS fitted deflection to the CFD grid directly. This direct coupling between CSM and CFD is based on a cubic NURBS surface using the least-squares method. For the last test case, a marching Euler CFD solver [9] is used to obtain the aerodynamics load. Then GENESIS [10] structural optimization code is used to calculate the structural deflection. The CSM model has a weak upper surface which has resulted in a noisy deflection. The CFD solvers are very sensitive to this artificial noise on the surface, and they must be smoothed. The IIM and NURBS methods are used to interpolate the deflection onto the CFD model. The results for IIM and NURBS methods are visually similar. Figure 3 shows the CSM and CFD model before and after deflection. The deflected CFD model shown in figure 3 is based on the IIM method. The average difference between IIM and NURBS results is relatively small (0.1% of mid-span maximum thickness).

Figure 4 shows a close-up view of the original deflection contours and the NURBS fitted deflection contours. The average difference in deflection for the CSM model is under 0.5% of mid-span maximum thickness. As seen in this figure the noise in the deflection has been smoothed out by using NURBS representation. Also this figure reveals that the aeroelastic deflection modifies the wing camber as well. Figure 5 shows the interpolated deflection based on IIM on the left hand-side. Using algorithm described in this paper, a NURBS surface is fitted through the CSM deflection and then evaluated on the CFD grid. The resulting CFD deflection is shown on the right hand-side of figure 5. As shown in figures 4 and 5, the original CSM and IIM interpolated deflection contours are not very smooth. Also the IIM interpolation sags within each CSM element. This is due to incompatible CFD and CSM models. The unsmoothed CSM deflection and the sagging from IIM method exacerbate the problem of data transfer. As demonstrated the NURBS representation can smooth out the noise in the CSM deflection so that an accurate CFD solution can be obtained.

5 Summary

An algorithm is presented for using NonUniform Rational B-Spline (NURBS) representation in an aeroelastic loop. The results indicate that it is possible to update either the CAD model, CSM grid or the CFD grid. This permits the use of CAD geometry within an aeroelastic loop with strong interdisciplinary interactions. It is also possible to transfer smooth and accurate

deflections to a CFD model.

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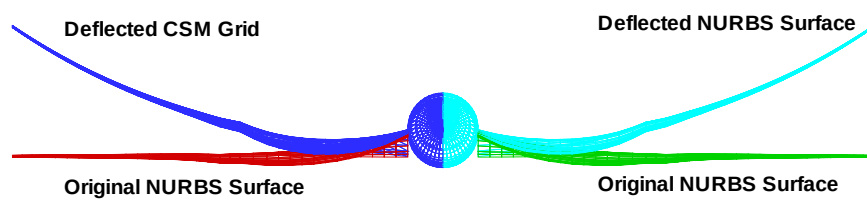


Figure 1: Front View of HSCT with Large Deflection

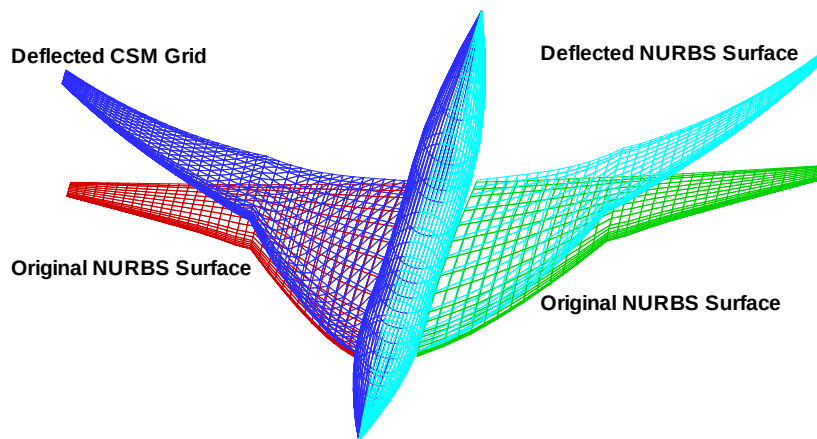


Figure 2: Oblique View of HSCT with Large Deflection

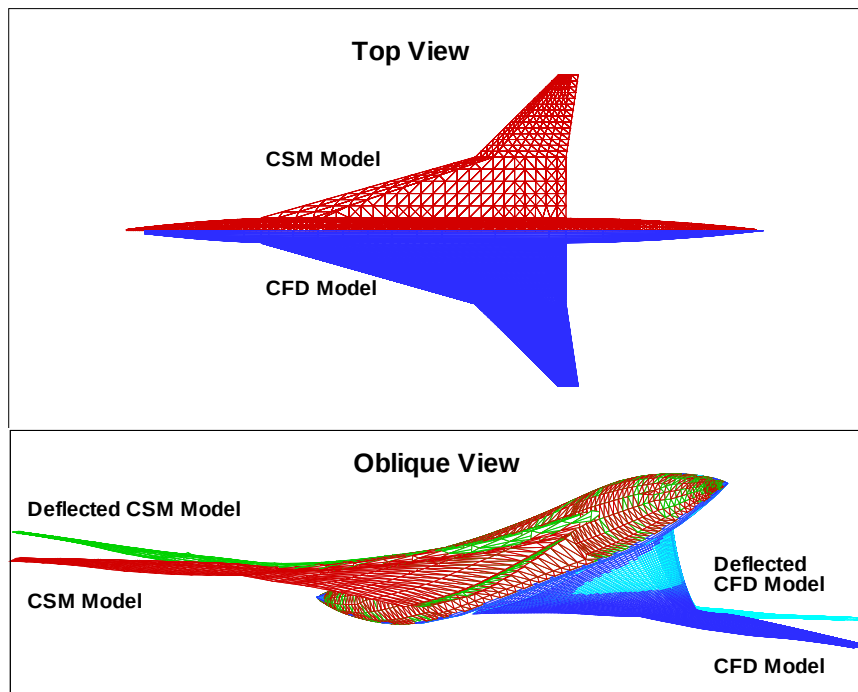


Figure 3: Detailed CSM and CFD Models of HSCT

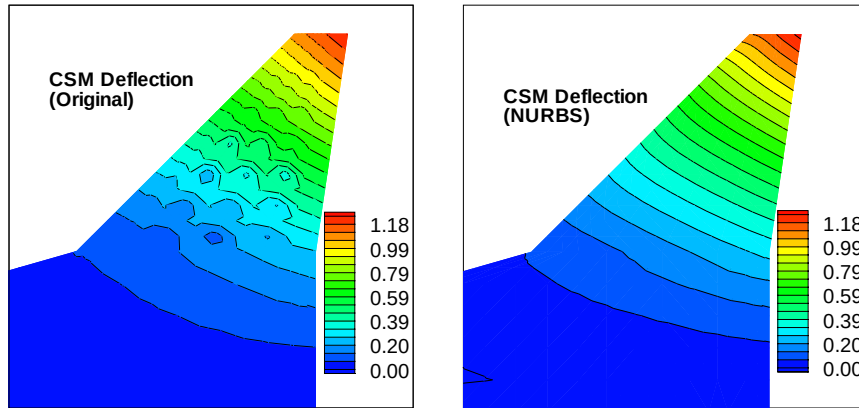


Figure 4: Original and NURBS Fitted Deflection for CSM Model

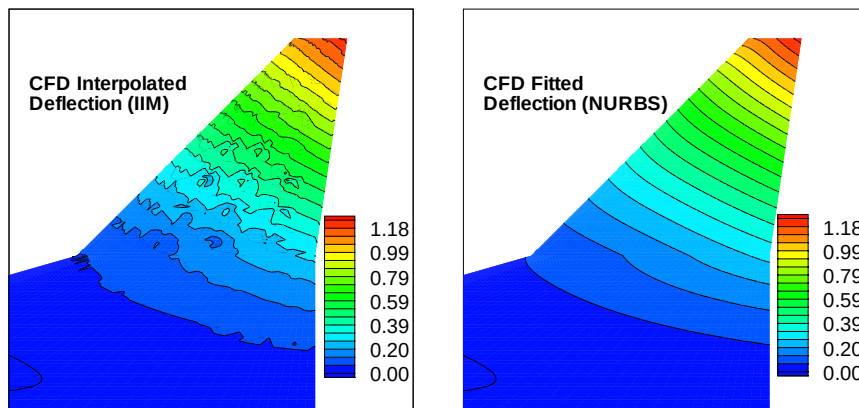


Figure 5: Interpolated (IIM) and NURBS Fitted Deflection for CFD Model