



A SUPERIOR KIRCHHOFF METHOD FOR AEROACOUSTIC NOISE PREDICTION: The Ffowcs Williams–Hawkings equation

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Introduction

- **Prediction of aeroacoustic noise important**
 - all new aircraft must meet noise certification requirements
 - local noise standards can be even more stringent
 - NASA noise reduction goal: reduce perceived noise levels by a factor of two in 10 years
- **Several prediction methods available**
 - direct computation
 - CFD based methods
 - near field only
 - best coupled with integral method for far-field prediction
 - Acoustic Analogy (Ffowcs Williams–Hawkings Equation)
 - Kirchhoff formula
- **Confusion over relationship between methods exists**

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Comments on Integral Methods

■ Technology

- **acoustic formulations and algorithms mature**
 - widely used for rotating blade noise prediction
 - potentially useful for airframe noise, engine noise, etc.
- **flow field computation feasible in many cases**
 - required for input data
 - provided by CFD
- **high quality experiments aid validation**

■ This talk will demonstrate the superiority of the FW–H approach over the Kirchhoff method for aeroacoustics

- **analytically**
- **numerically**

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Advantages and Disadvantages

■ FW–H method

- + three source terms (thickness, loading, quadrupole) have physical meaning
- + three source terms are independent
- + mature and robust algorithms
- quadrupole source is a volume source (more computational resources needed when volume integration included)

■ Kirchhoff method

- + surface sources (only surface integration required)
- + applicable to problems described by the wave equation
- source terms not easily related to flow physics or design parameters
- not as much experience with algorithms for Kirchhoff problems

■ Analytical/Numerical comparison needed

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Analytical Comparison: FW–H Derivation Procedure

■ Embed exterior flow problem in unbounded space

- define generalized functions valid throughout entire space
- interpret derivatives as generalized differentiation

$$\begin{aligned}\tilde{\rho} &= \begin{cases} \rho & f > 0 \\ \rho_o & f < 0 \end{cases} \\ \rho \tilde{u}_i &= \begin{cases} \rho u_i & f > 0 \\ 0 & f < 0 \end{cases} \\ \tilde{P}_{ij} &= \begin{cases} P_{ij} & f > 0 \\ 0 & f < 0 \end{cases}\end{aligned}$$

■ Generalized conservation equations:

$$\frac{\partial \tilde{\rho}}{\partial t} + \frac{\partial \rho \tilde{u}_i}{\partial x_i} = (\rho' \frac{\partial f}{\partial t} + \rho u_i \frac{\partial f}{\partial x_i}) \delta(f) \quad \text{continuity}$$

$$\frac{\partial \rho \tilde{u}_i}{\partial t} + \frac{\partial \rho \tilde{u}_i \tilde{u}_j}{\partial x_j} + \frac{\partial \tilde{P}_{ij}}{\partial x_j} = (\rho u_i \frac{\partial f}{\partial t} + (\rho u_i u_j + P_{ij}) \frac{\partial f}{\partial x_i}) \delta(f) \quad \text{momentum}$$

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Analytical Comparison: FW–H Derivation Procedure

- Manipulate conservation laws into form of inhomogeneous wave equation

$$\begin{aligned} \square^2 p'(\vec{x}, t) = & \frac{\bar{\partial}^2}{\partial x_i \partial x_j} [T_{ij} H(f)] & \frac{\partial f}{\partial t} = -v_n \\ & - \frac{\partial}{\partial x_i} [(P_{ij} \hat{n}_j + \rho u_i (u_n - v_n)) \delta(f)] & \frac{\partial f}{\partial x_i} = \hat{n}_i \\ & + \frac{\partial}{\partial t} [(\rho_o v_n + \rho (u_n - v_n)) \delta(f)] \end{aligned}$$

- Don't assume integration surface $f=0$ is coincident with body
 - given in this form by Ffowcs Williams
 - demonstrated for rotors by di Francescantonio; Brentner & Farassat

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Analytical Comparison: Kirchhoff Derivation Procedure

■ Use embedding procedure on wave equation

- define generalized pressure perturbation:

$$\tilde{p}' = \begin{cases} p' & f > 0 \\ 0 & f < 0 \end{cases}$$

- use generalized derivatives
- generalized wave equation is Kirchhoff governing equation:

$$\square^2 p'(\vec{x}, t) = -\left(\frac{\partial p'}{\partial t} \frac{M_n}{c} + \frac{\partial p'}{\partial n}\right) \delta(f) - \frac{\partial}{\partial t} \left(p' \frac{M_n}{c} \delta(f) \right) - \frac{\partial}{\partial x_i} (p' \hat{n}_i \delta(f))$$
$$\equiv Q_{kir}$$

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Source Term Comparison

- Manipulate FW–H source terms into form of Kirchhoff source terms (inviscid fluid)

$$\begin{aligned}\square^2 p'(\vec{x}, t) = & Q_{kir} + \frac{\bar{\partial}^2}{\partial x_i \partial x_j} [T_{ij} H(f)] \\ & - \frac{\partial}{\partial x_j} [\rho u_i u_j] \hat{n}_i \delta(f) - \frac{\partial}{\partial x_j} [\rho u_i u_n \delta(f)] \\ & + \frac{\partial}{\partial t} [p' - c^2 \rho'] \frac{M_n}{c} \delta(f) + \frac{\partial}{\partial t} \left[(p' - c^2 \rho') \frac{M_n}{c} \delta(f) \right]\end{aligned}$$

- Extra source terms are 2nd order in perturbations quantities
- FW–H and Kirchhoff source terms
 - equivalent in linear region $(p' \approx c^2 \rho' \quad u_i \ll 1)$
 - NOT equivalent in nonlinear flow region

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Integral Formulation of FW–H equation

- New variables put FW–H equation into standard form

$$Q = \rho u_n - \rho' v_n; \quad L_i = P_{ij} + \rho u_i (u_n - v_n)$$

hence

$$\square^2 p'(\vec{x}, t) = \frac{\bar{\partial}^2}{\partial x_i \partial x_j} [T_{ij} H(f)] - \frac{\partial}{\partial x_i} [L_i \delta(f)] + \frac{\partial}{\partial t} [Q \delta(f)]$$

- Integral representation of solution (formulation 1A)

$$\begin{aligned} 4\pi p'(\vec{x}, t) = & \int_{f=0} \left[\frac{\dot{Q} + \dot{L}_r / c}{r(1 - M_r)^2} \right]_{ret} dS + \int_{f=0} \left[\frac{L_r - L_M}{r^2(1 - M_r)^2} \right]_{ret} dS \\ & + \int_{f=0} \left[\frac{(Q + L_r / c)(r\dot{M}_r + c(M_r - M^2))}{r^2(1 - M_r)^3} \right]_{ret} dS \end{aligned}$$

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Kirchhoff Formulation for Moving surfaces

■ Kirchhoff integral formulation

$$4\pi p'(\vec{x}, t) = \int_{f=0} \left[\frac{E_1}{r(1-M_r)} \right]_{ret} dS + \int_{f=0} \left[\frac{p'E_2}{r^2(1-M_r)} \right]_{ret} dS$$

where

$$E_1 = (M_n^2 - 1) \frac{\partial p'}{\partial n} + M_n \vec{M}_t \cdot \nabla_2 p' - \frac{M_n}{c} \dot{p}'$$
$$+ \frac{1}{c(1-M_r)} \left[(\dot{n}_r - \dot{M}_r - \dot{n}_M) p' + (\cos \theta - M_n) \dot{p}' \right] + \frac{1}{c(1-M_r)^2} \left[\dot{M}_r (\cos \theta - M_n) p' \right] ;$$
$$E_2 = \frac{(1-M^2)}{(1-M_r)^2} (\cos \theta - M_n)$$

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Numerical Comparison

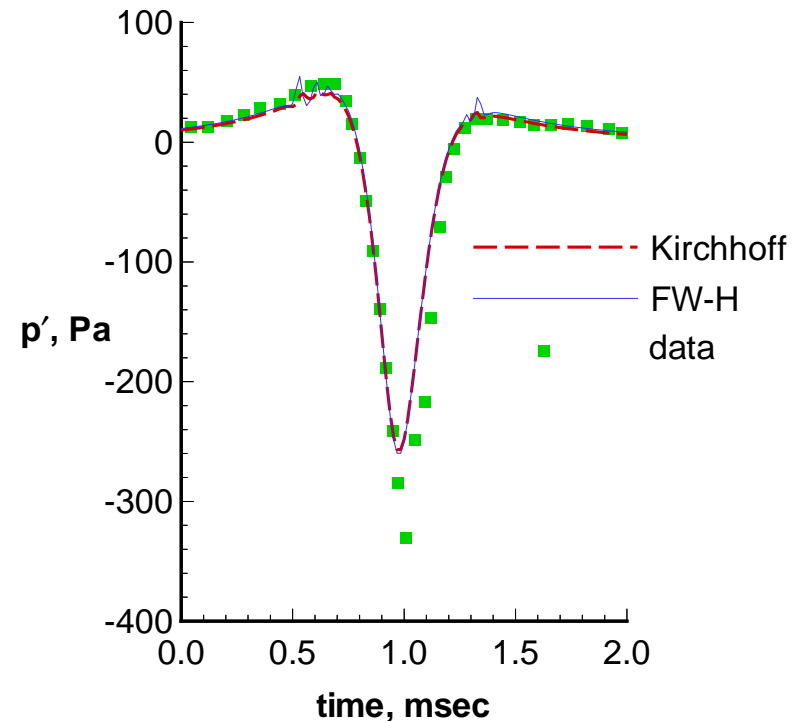
- **Kirchhoff code (RKIR)**
 - numerical implementation of Kirchhoff integration
 - code developed for helicopter rotors (Purdue/Sikorsky/NASA LaRC)
- **Prototype code developed (FW-H/RKIR)**
 - based on RKIR (Rotating Kirchhoff code - rotor noise prediction)
 - utilizes Farassat's formulation 1A
 - quadrupole source neglected; could be included
- **Cases for comparison**
 - hovering rotor
 - rotor in forward flight
 - viscous flow over a circular cylinder

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Numerical Comparison: UH-1H hovering rotor

- **UH-1H rotor**
 - 1/7th scale model
 - untwisted blade
- **Test setup (Purcell)**
 - Hover, $M_H = 0.88$
 - inplane microphone, 3.09 R from hub
- **Flow-field computation**
 - full potential flow solver used (FPRBVI)
 - 80 x 36 x 24 grid (somewhat coarse)

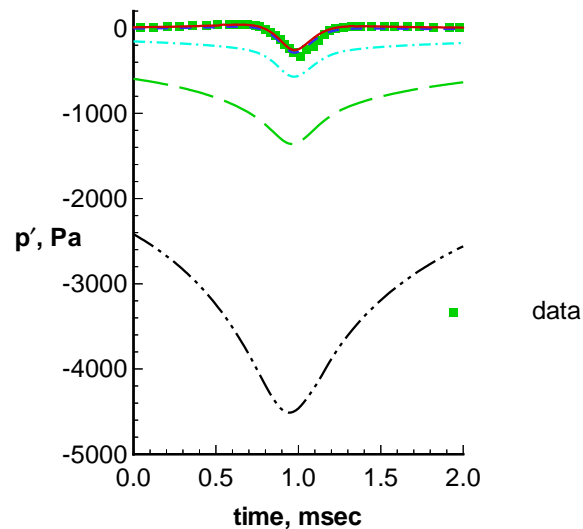


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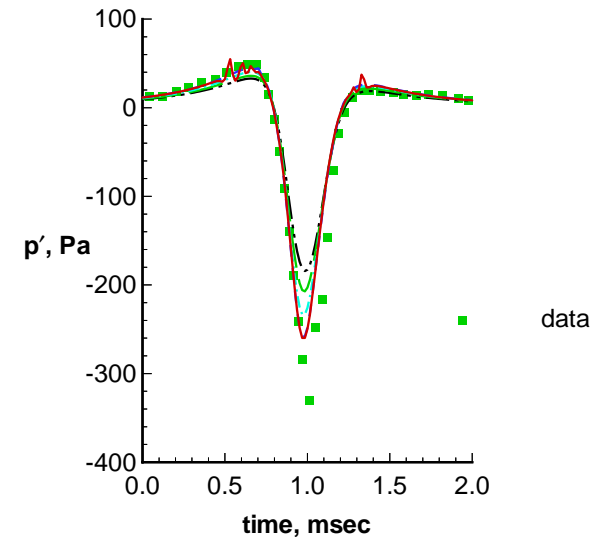
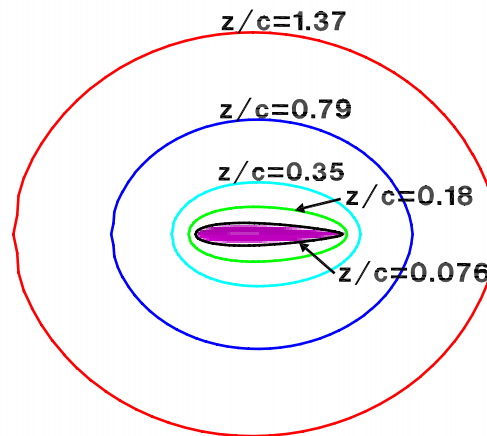


Numerical Comparison: Sensitivity to Surface Placement

- A principal advantage of the FW-H approach is insensitivity to surface placement



Kirchhoff



FW-H

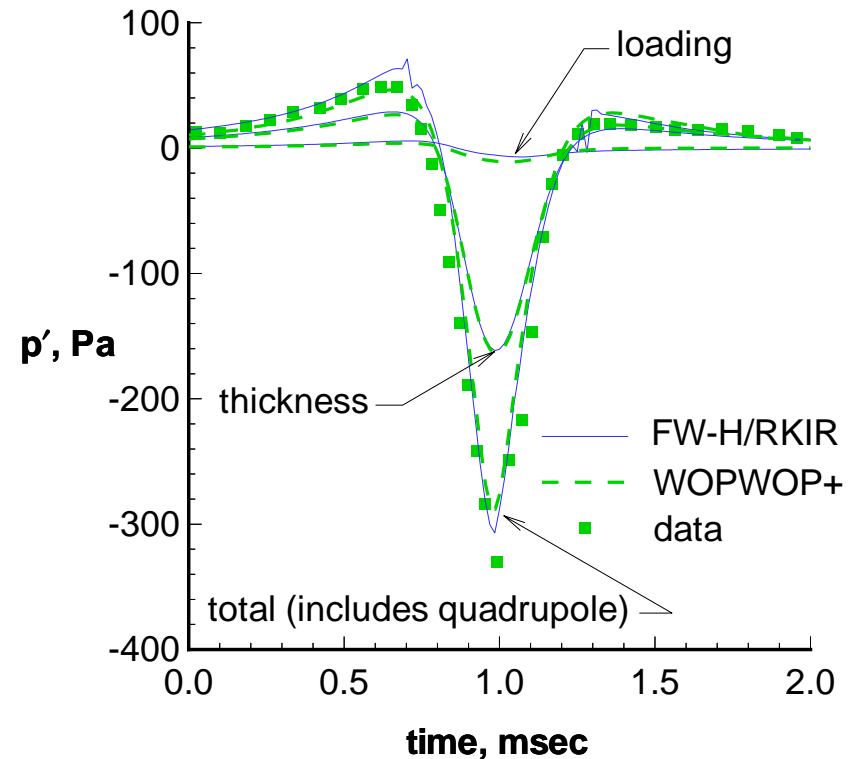
(Note difference in pressure scales)

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Identification of Noise Components

- Compare components from off FW-H/RKIR with WOPWOP+
 - UH-1H rotor in hover
 - Hover solution from TURNS (Baeder)
- Two predictions necessary with FW-H/RKIR
 - thickness and loading from surface coincident with rotor blade
 - total signal from a surface approximately 1.5 chords away from blade.
- New application of FW-H equation retains advantage of predicting noise components

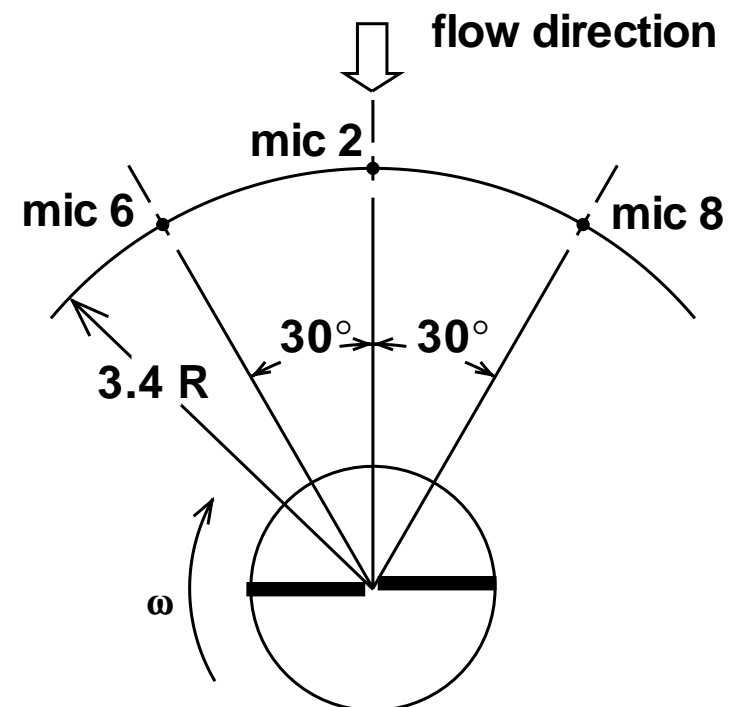


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Numerical Comparison: Forward Flight Case

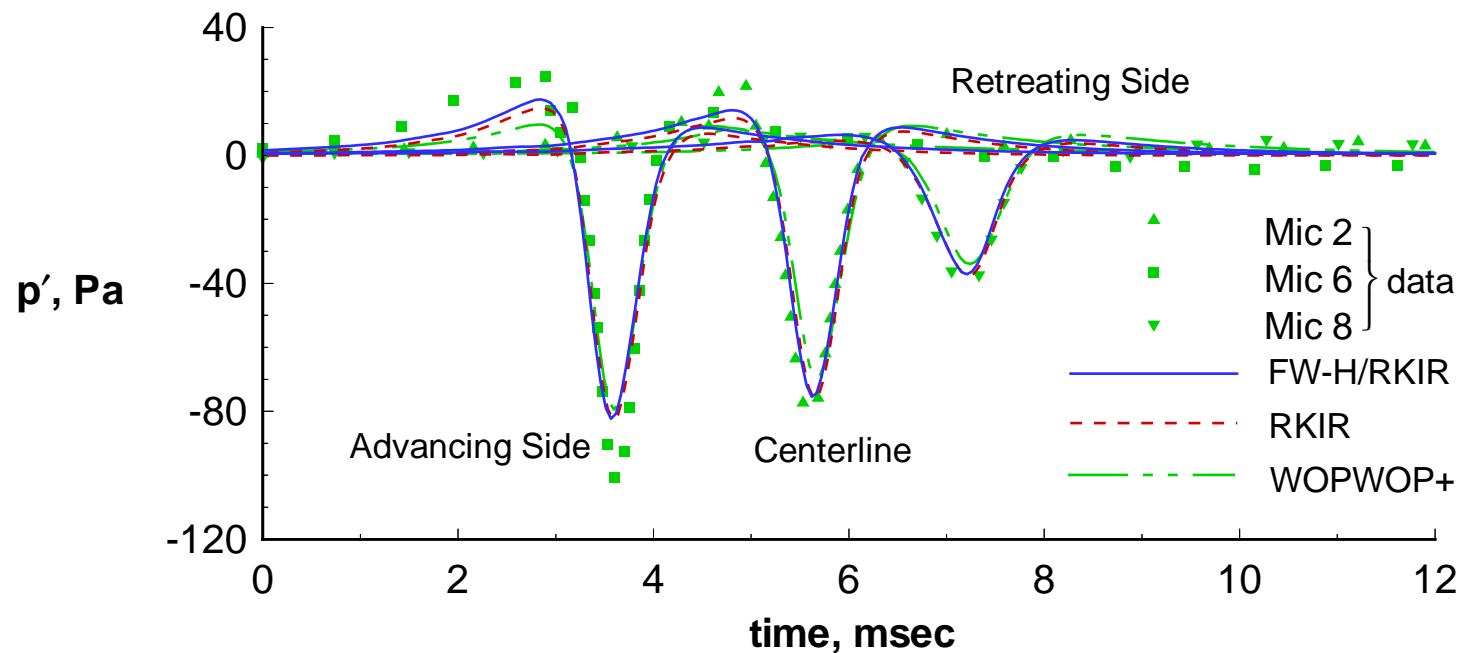
- **Test setup** (Schmitz et al.)
 - **Operational Loads Survey (OLS)**
1/7 scale model rotor
 - **3 inplane microphone used for comparison**
- **Operating conditions**
 - $M_{AT} = 0.84$
 - $\mu = 0.27$
- **Flow-field computation**
 - **flow solver: full potential code for rotors (FPRBVI)**
 - **80 x 36x 24 grid**



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Numerical Comparison: Forward Flight Case



- Advancing-side acoustic pressure underpredicted
- Agreement with data is good
- All three codes agree with each other

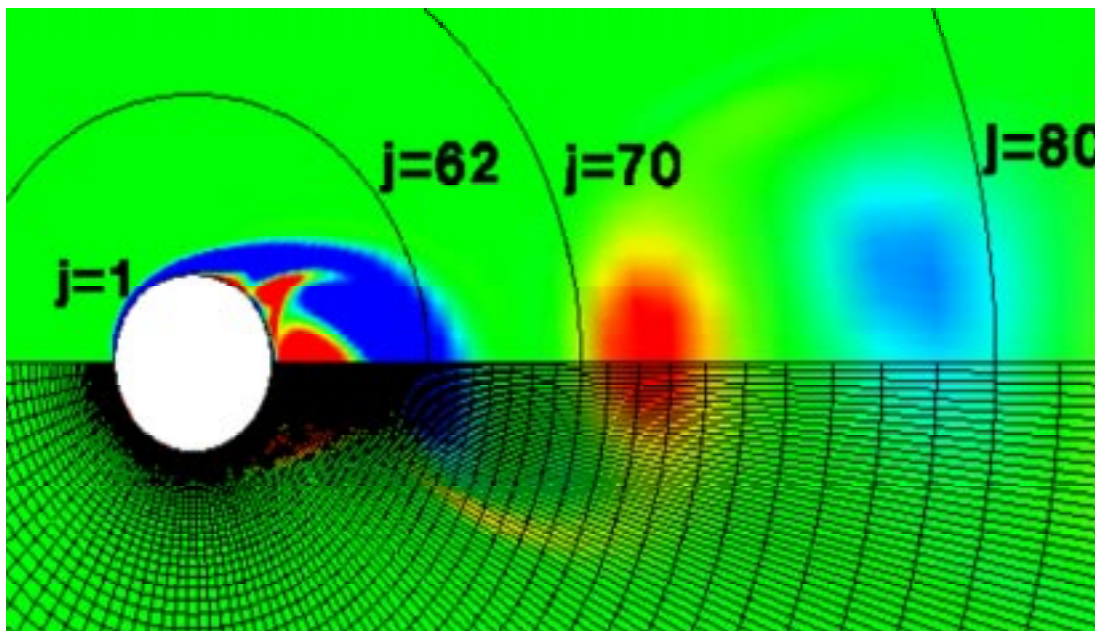
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Numerical Comparison: Circular Cylinder Flow

■ Problem:

- Viscous flow over a circular cylinder
- 2D, unsteady laminar CFD computation, $Re = 1000$.
- Acoustic calculation 3D, cylinder 40 dia long



Vorticity field
from N-S computation

CFD grid 193x97

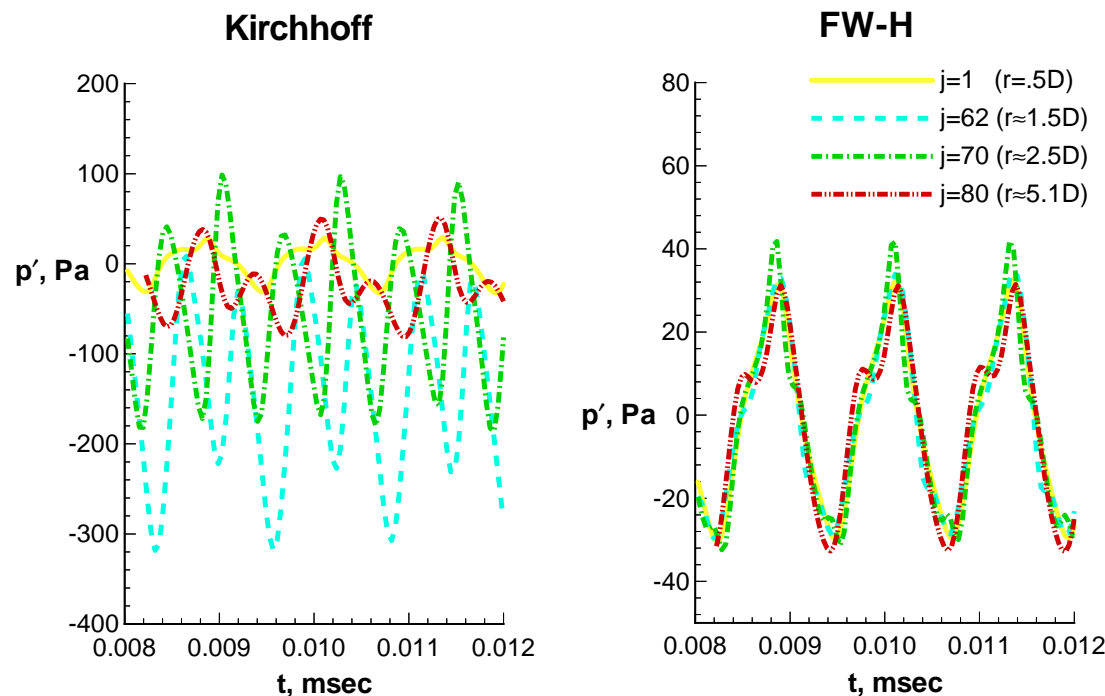
grid extends out 20 dia.

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Noise Generated by Flow Over Cylinder

- Location 128 dia from cylinder, 90 deg from freestream



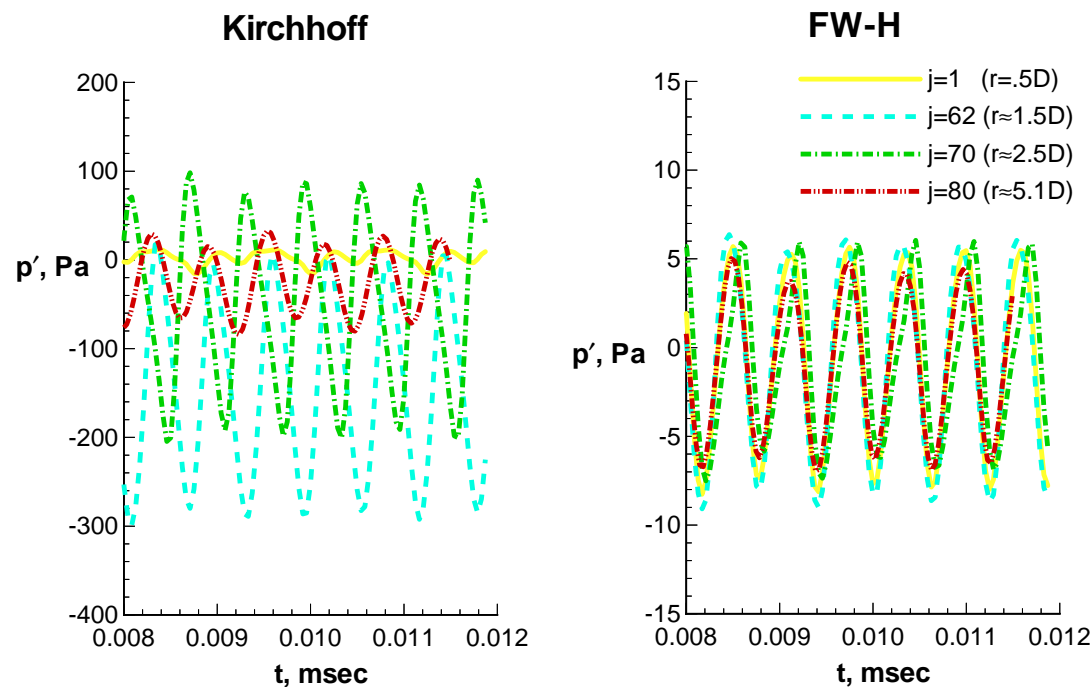
- FW-H predictions show small sensitivity to surface placement
- Kirchhoff predictions meaningless

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Noise Generated by Flow Over Cylinder

■ Location 128 dia from cylinder, downstream



■ differences in FW-H prediction due to:

- CFD inaccuracy
- Increased integration error (grid size)

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Conclusions

- **FW–H method of choice for aeroacoustic problems**
 - conservation of mass and momentum built in
 - unified theory with thickness, loading, and quadrupole source terms
 - insensitive to integration surface placement
- **FW–H approach the “better” than linear Kirchhoff because:**
 - valid in linear and nonlinear flow regions
 - surface terms include quadrupole contribution enclosed
 - physical noise components can be identified with two surfaces
- **The Kirchhoff approach**
 - valid only in the linear flow region (not known a priori)
 - input data must satisfy the wave equation
 - wakes and potential flow field can cause major problems
 - solution can be sensitive to placement of Kirchhoff surface

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