SOUND TRANSMISSION THROUGH TWO CONCENTRIC CYLINDRICAL SANDWICH SHELLS

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ABSTRACT

This paper solves the problem of sound transmission through a system of two infinite concentric cylindrical sandwich shells. The shells are surrounded by external and internal fluid media and there is fluid (air) in the annular space between them. An oblique plane sound wave is incident upon the surface of the outer shell. A uniform flow is moving with a constant velocity in the external fluid medium. Classical thin shell theory is applied to the inner shell and first-order shear deformation theory is applied to the outer shell. A closed form for transmission loss is derived based on modal analysis. Investigations have been made for the impedance of both shells and the transmission loss through the shells from the exterior into the interior. Results are compared for double sandwich shells and single sandwich shells. This study shows that (1) the impedance of the inner shell is much smaller than that of the outer shell so that the transmission loss is almost the same in both the annular space and the interior cavity of the shells; (2) the two concentric sandwich shells can produce an appreciable increase of transmission loss over single sandwich shells especially in the high frequency range; and (3) design guidelines may be derived with respect to the noise reduction requirement and the pressure in the annular space at a mid-frequency range.

NOMENCLATURE

a ,b,	: thickness and width of intrinsic cell material of honeycomb core
c	: sound speed in fluid
D	: displacement component along radial direction
E, G, μ	: elastic constants of the face sheets and honeycomb core of sandwich shell
f	: frequency and mode dependent function
h, R	: thickness and radius of sandwich shells
H, J	: Hankel and Bessel functions
i	: $\sqrt{-1}$
k	: wave number
L	: differential operator
M	: Mach number
n	: mode number
p	: pressure in the external, annular, and internal cavities
r, z, θ	: cylindrical coordinates along radial, axial, and circumferential, directions
S	: cross area for annular or interior cavity
t	: time
TL	: transmission loss
u, v, w	: displacement components along axial, circumferential, and radial directions
V	: velocity of the external flow
W	: power flow per unit length
x, y, z	: Cartesian coordinates
Z	: modal impedance
γ	: incident angle of the plane sound wave
∇	: gradient
ε	: Neumann factor
ζ	: local coordinate along thickness direction of shell
ρ	: mass density of fluid and shell
ψ	: rotation of the shell
ω	: angular or rotational frequency

1. INTRODUCTION

Sound transmission through double walls separated by an airgap has been investigated by many researchers [1 to 5, 7, 10 to 14, 16, 18, 19, 22 to 24] since the construction will produce both noise attenuation and thermal insulation. These investigations have shown that double walls can increase transmission loss over a single wall. Among these studies, most of them assumed the double walls are flat with absorbent materials. Only a few focused on the study of double walls consisting of two concentric cylindrical shells. However, those investigations are mostly limited to the study of two isotropic shells and only thin shell theory is applied or to the use of the finite element method for composite double wall cylinders. Sandwich structures consisting of lightweight, flexible cores between relatively stiff skins can be used to increase the sound insulation [4, 6, 11, 20, 21]. The authors have studied sound transmission through single cylindrical sandwich shells with honeycomb core analytically. Results show that the sandwich shells can offer advantage over isotropic shells for noise reduction, especially at high frequencies [20]. In the aerospace and marine applications, cylindrical shells often occur as part of double shell constructions for the purpose of noise insulation, streamlining requirements, thermal shield, or interior finish requirements. The outer shell represents the exterior skin of an aircraft fuselage and the inner shell represents the trim panel in the aerospace application. In the marine application, the outer shell is thin to satisfy streamline requirements while the inner shell is thicker to withstand underwater pressures.

The objective of this paper is to study noise transmission through a system of two infinite concentric cylindrical sandwich shells excited by an incident oblique plane sound wave analytically. The shells are surrounded with fluids and there is air in the annular space between them. A uniform flow is moving with a constant velocity in the external fluid medium. Each of the sandwich shells is made of honeycomb core and face sheets that may be isotropic, orthotropic, or laminated fiber-reinforced composite materials. In this paper, we will focus on the aerospace application in which the outer shell is a thick shell while the inner is thin. The effect of the shear deformation and rotation can not be neglected for a thick shell [15, 17, 20]. Therefore, the first-order shear deformation theory is applied for the outer shell. For the inner shell, the classical thin shell theory is used. The concept under study is that a plane acoustic wave is incident and is reflected by the elastic shell in the exterior space, standing waves that consist of transmitted and reflected waves exist in the annular space, and transmitted waves exist in the interior cavity. To develop the solution, modal analysis is used to solve the coupled equations simultaneously including the convective wave equation for the external fluid, the wave equation for the annular and internal fluids, and the vibration equation for both outer and inner shells. A closed form expression for the transmission loss (TL) is derived. Calculations have been carried out for the impedances of both shells and the TL through the shells. Finally, comparisons of the TL between the double sandwich shells and single sandwich shells are made.

2. MATHEMATICAL ANALYSIS

2.1 Governing equations

Figure 1 shows a schematic of two infinite concentric cylindrical sandwich shells with radii R_1 and R_2 and wall thicknesses h_1 and h_2 for the outer and inner shells, respectively. The shells are surrounded by the external fluid and the internal fluid including that in the airgap (annular fluid) between them. The mass density and sound speed for the external, annular, and internal fluid media are $\{\rho_1, c_1\}$, $\{\rho_2, c_2\}$ and $\{\rho_3, c_3\}$. An oblique plane sound wave p^I is incident upon the system from the exterior of the shells with incident angle γ_1 (measured from the axial coordinate z). An airflow in the external fluid medium is moving with a constant velocity V along z direction. Without loss of generality, the angles of the incident wave with respect to the axes x and y are 0 and $\pi/2$, respectively.

In the exterior space, the pressure $p_1 = p^I + p_1^R$, where p^I is the incident wave and p_1^R is the reflected wave, satisfies the convected wave equation

$$c_1^2\nabla^2 (p^I + p_1^R) + \left(\frac{\partial}{\partial t} + V.\nabla\right)^2 (p^I + p_1^R) = 0 \qquad (1)$$

where ∇ the gradient and $\nabla^2 = \nabla.\nabla$ the Laplacian operator. The pressures here and in the following represent the perturbation pressures.

In the annular space, the pressure $p_2 = p_2^T + p_2^R$, where p_2^T is the transmitted wave and p_2^R is the reflected wave, satisfies the acoustic wave equation

$$c_2^2\nabla^2 (p_2^T + p_2^R) + \frac{\partial^2(p_2^T + p_2^R)}{\partial t^2} = 0 \qquad (2)$$

In the interior cavity, the pressure $p_3 = p_3^T$, where p_3^T is transmitted wave, satisfies the acoustic wave equation

$$c_3^2\nabla^2 p_3^T + \frac{\partial^2 p_3^T}{\partial t^2} = 0 \qquad (3)$$

It is assumed here that the interior cavity inside the shells is totally absorptive. This indicates there exists only inward-traveling wave.

For the shells, let $\{u_i^0, v_i^0, w_i^0\}$ be the displacement components at the middle surface of the shell in the axial, circumferential, radial directions, where the subscript i denotes the variables associated with the outer shell (i = 1) and the inner shell (i = 2). Let $\{\psi_z, \psi_\theta\}$ be the rotations of the normal to the undeformed midsurface of the outer shell, where θ is the angle in the circumferential direction. The governing shell equations for both the thin shell and the first-order shell theories have been shown in the authors' previous study [20]. Eliminating u_1^0, v_1^0, ψ_z, and ψ_θ for the outer shell and u_2^0 and v_2^0 for the inner shell, one can obtain differential equations in terms of displacement components in the radial direction w_1^0 and w_2^0

$$L_1(w_1^0) = p_{12} = p_2^T + p_2^R - (p_1^I + p_1^R) \qquad (4)$$

$$L_2(w_2^0) = p_{23} = p_3^T - (p_2^T + p_2^R) \qquad (5)$$

where L_1 and L_2 are the differential operators.

At the interfaces between the shells and fluids, the following equations must be satisfied

$$\left.\frac{\partial(p^I + p_1^R)}{\partial r}\right|_{r=R_1} = \rho_1 \left(\frac{\partial}{\partial t} + V.\nabla\right)^2 w_1 \qquad (6)$$

$$\left.\frac{\partial(p_2^T + p_2^R)}{\partial r}\right|_{r=R_1} = \rho_2 \frac{\partial^2 w_1}{\partial t^2} \qquad (7)$$

$$\left.\frac{\partial(p_2^T + p_2^R)}{\partial r}\right|_{r=R_2} = \rho_2 \frac{\partial^2 w_2}{\partial t^2} \qquad (8)$$

$$\left.\frac{\partial p_3^T}{\partial r}\right|_{r=R_2} = \rho_3 \frac{\partial^2 w_2}{\partial t^2} \qquad (9)$$

where r is the radial coordinate of the shells.

2.2 Transmission loss

For a harmonic incident wave, assume that p^I can be expanded as

$$p^I(r, z, \theta, t) = p_0.\sum_{n=0}^{\infty} \varepsilon_n (-\mathrm{i})^n J_n(k_{1r}r)^* \cos[n\theta] \exp[\mathrm{i}(\omega t - k_{1z}z)] \qquad (10)$$

with p_0 is the amplitude of the incident wave; n the number of the circumferential mode; $\varepsilon_0 = 1$ and $\varepsilon_n = 2$ otherwise; J_n Bessel function of the first kind of order n; ω annular or rotational frequency; and

$$k_{1r} = k_1 \sin(\gamma_1), \quad k_{1z} = k_1 \cos(\gamma_1) \qquad (11)$$

where $k_1 = (\omega/c_1)/[1+M \cos(\gamma_1)]$ and $M = V/c_1$.

Following the procedures presented in the authors' previous paper [20], one can expand the pressures p_1^R, p_2^T, p_2^R, and p_3^T which satisfy Eqs. (1) to (3) and displacements w_1^0 and w_2^0 in terms of $\cos[n\theta]\exp[\mathrm{i}(\omega t - k_{1z}z)]$. Then substitute these results into Eqs. (4) to (9) to solve for unknowns coefficients of p_1^R, p_2^T, p_2^R, p_3^T, w_1^0, and w_2^0. The derivation and the closed form solutions are given in the appendix.

In order to define the transmission loss, consider the transmitted power flow per unit length along the axial direction of the shells in the interior cavity inside both shells, W^T, which is given by the following

$$W^T = \frac{1}{2}\,\mathrm{Re}\left\{ \int_{S_2} (p_3^T\, \dot{w}_2^{0*})|_{r=R_2}\, dS \right\} \tag{12}$$

where $S_2 = 2\pi R_2$ and Re{.} and the superscript * represent real part and the complex conjugate of the argument, respectively. Substitution of Eqs. (A10) and (A12) for p_3^T and w_2^0 into above equation yields an expression for the components of W^T

$$W_n^T = \frac{2\,p_0^2}{\rho_3\,\varepsilon_n\,\omega}|f|^2 \tag{13}$$

Here, |.| is the absolute value of the argument and f is a frequency and modal dependent function,

$$f = \frac{\rho_3 k_{1r}}{\rho_1 k_{3r}} \times \frac{H_n^{1'}(k_{2r}R_2)}{H_n^{1'}(k_{2r}R_1)} \times \frac{J_n'(k_{1r}R_1)}{H_n^{1'}(k_{3r}R_2)} \times \frac{(Z_{11}^I+Z_{11}^R)(Z_{22}^T+Z_{22}^R)\omega^2}{(c_1^2\,k_1^2)\,\Delta} \tag{14}$$

$$\Delta = (Z_1^s+Z_{21}^T+Z_{11}^R)(Z_2^s+Z_{32}^T+Z_{22}^R)-(Z_1^s+Z_{11}^R-Z_{21}^R)(Z_2^s+Z_{32}^T-Z_{22}^T)\times H_n^{1'}(k_{2r}R_2)\,H_n^{2'}(k_{2r}R_1)/[H_n^{1'}(k_{2r}R_1)H_n^{2'}(k_{2r}R_2)] \tag{15}$$

where primes represent the derivative, J_n are Bessel functions of the first kind of order n, and H_n^1 and H_n^2 are Hankel functions of the first and second kinds of order n, respectively,

$$k_2 = \omega/c_2\,,\quad k_{2z} = k_{1z}\,,\quad k_{2r} = \sqrt{k_2^2 - k_{2z}^2} \tag{16}$$

$$k_3 = \omega/c_3\,,\quad k_{3z} = k_{1z}\,,\quad k_{3r} = \sqrt{k_3^2 - k_{3z}^2} \tag{17}$$

$$Z_{11}^I(n,\omega) = \mathrm{i}\omega\frac{\rho_1}{k_{1r}}\left(\frac{k_1c_1}{\omega}\right)^2\frac{J_n(k_{1r}R_1)}{J_n'(k_{1r}R_1)} \tag{18}$$

$$Z_{11}^R(n,\omega) = -\mathrm{i}\omega\frac{\rho_1}{k_{1r}}\left(\frac{k_1c_1}{\omega}\right)^2\frac{H_n^2(k_{1r}R_1)}{H_n^{2'}(k_{1r}R_1)} \tag{19}$$

$$Z_{21}^T(n,\omega) = \mathrm{i}\omega\frac{\rho_2\,H_n^1(k_{2r}R_1)}{k_{2r}H_n^{1'}(k_{2r}R_1)},\quad Z_{21}^R(n,\omega) = -\mathrm{i}\omega\frac{\rho_2\,H_n^2(k_{2r}R_1)}{k_{2r}H_n^{2'}(k_{2r}R_1)} \tag{20}$$

$$Z_{22}^T(n,\omega) = \mathrm{i}\omega\frac{\rho_2\,H_n^1(k_{2r}R_2)}{k_{2r}H_n^{1'}(k_{2r}R_2)},\quad Z_{22}^R(n,\omega) = -\mathrm{i}\omega\frac{\rho_2\,H_n^2(k_{2r}R_2)}{k_{2r}H_n^{2'}(k_{2r}R_2)} \tag{21}$$

$$Z_{32}^T(n,\omega) = \mathrm{i}\omega\frac{\rho_3}{k_{3r}}\frac{H_n^1(k_{3r}R_2)}{H_n^{1'}(k_{3r}R_2)} \tag{22}$$

In above equations, Z_1^s and Z_2^s are the modal impedances of the outer and inner shells defined by

$$Z_1^s = p_{12}/\dot{w}_1^0\,,\quad Z_2^s = p_{23}/\dot{w}_2^0 \tag{23}$$

The transmission loss is, therefore, defined by

$$TL = -10\log_{10}\sum_{n=0}^{\infty}\frac{W_n^T}{W^I} \tag{24}$$

where W^I is the incident power flow per unit length along the axial direction of the shells

$$W^I = \frac{R_1\cos(\gamma_1)}{\rho_1\,c_1}p_0^2 \tag{25}$$

Then, a closed form for the transmission loss can be obtained by substituting Eqs. (13) and (25) into Eq. (24)

$$TL = -10\log_{10}\sum_{n=0}^{\infty}\frac{2\rho_1\,c_1}{\varepsilon_n\,\rho_3\,\omega\,R_1\cos(\gamma_1)}|f|^2 \tag{26}$$

When $R_1 = R_2$, the expression can be decomposed into the TL for single shell [20] with shell modal impedance $Z_1^s + Z_2^s$.

2.3 Equivalent constants of the honeycomb core

To calculate the equivalent mass density and elastic material constants of the honeycomb core, assume that the core is constructed from a regular hexagonal structure. The equivalent mass density is given by [8]

$$\rho = \rho^s\frac{2}{\sqrt{3}}\frac{a}{b} \tag{27}$$

where ρ^s is the intrinsic cell wall material mass density of the honeycomb core and a and b are its thickness and width, respectively, as shown in Fig. 1. The in-plane elastic constants of the honeycomb, E_z, E_6, $\mu_{z\zeta}$, and $\mu_{\theta\zeta}$ (ζ is the local coordinate along the radial direction of the considered shell) are also given by [8]

$$E_z = E_\theta = E_s\frac{4}{\sqrt{3}}\frac{a^3}{b^3}\,,\quad \mu_{z\zeta} = \mu_{\theta\zeta} = 1 \tag{28}$$

where E_s is the intrinsic Young's modulus.

Since the outer shell is a thick shell, the transverse mechanical properties need to be known. The paper [9] presented a formula for the transverse shear modula of a honeycomb core of a regular hexagonal structure

$$G_{z\zeta} = G_{\theta\zeta} = \sqrt{3}\,G_s\frac{a}{b} \tag{29}$$

where G_s is the intrinsic shear modulus. The third shear modulus in this study is defined by

$$G_{\xi\theta} = \frac{E_s}{\sqrt{3}}\frac{a^3}{b^3} \tag{30}$$

3. NUMERICAL ANALYSIS

3.1 Given constants

Numerical studies will illustrate the analysis presented here by considering a typical aircraft fuselage made from two concentric cylindrical sandwich shells. The outer shell consists of titanium face sheets and titanium honeycomb core and the inner shell consists of four layer laminated cross-ply graphite/epoxy face sheets and aluminum honeycomb core. The fiber orientation for the inner shell is {90^0, 0^0, 90^0, 0^0, honeycomb core, 0^0, 90^0, 0^0, 90^0} with axial direction measured from the exterior surface of the inner shell. The radius and wall thickness are R_1 = 1.88m and h_1 = 5.079cm for the outer shell and R_2 = 1.84m and h_2 = 0.635cm for the inner shell. The face sheets are made from the same material and with the same thickness for each shell. The thickness ratio of the core and the total for each shell is 0.84. The structural loss factor is η = 0.01. The material properties of the face sheet are given in Table. 1, where α is the fiber direction and β is the direction perpendicular to the fiber. The equivalent material properties of honeycomb core can be obtained by substitution of a/b = 0.1 and the material properties of the intrinsic core materials, titanium (given in Table. 1) and aluminum (ρ^s = 2750kg / m^3, E = 72GPa, μ = 0.3) into Eqs. (27) to (30), as given in Table. 2.

Table 1. Material properties of the face sheet of the sandwich shell.

titanium	
mass density: ρ_1^s (kg / m^3)	4510
elastic modulus: E (GPa)	120.02
Poison's ratio: μ	0.361
graphite / epoxy layer	
mass density: ρ_2^s (kg / m^3)	1580
elastic modulus: E_α (GPa)	181
elastic modulus: $E_\beta = E_z$ (GPa)	10.3
shear modulus: $G_{\alpha\beta} = G_{z\alpha}$ (GPa)	7.17
shear modulus: $G_{\beta z}$ (GPa)	2.87
Poison's ratio: $\mu_{\beta\alpha}$	0.33
Poison's ratio: $\mu_{z\alpha} = \mu_{z\beta}$	0.28

Table 2. Equivalent material properties of the honeycomb core.

titanium honeycomb core	
mass density: ρ_1^s (kg / m^3)	520.77
elastic modulus: $E_z = E_\theta$ (GPa)	0.277
elastic modulus: E_ζ (GPa)	120.02
shear modulus: $G_{z\zeta} = G_{\theta\zeta}$ (GPa)	2.545
shear modulus: $G_{z\theta}$ (GPa)	0.069
Poison's ratio: $\mu_{z\theta}$	1
Poison's ratio: $\mu_{z\zeta} = \mu_{\theta\zeta}$	0.361
aluminum honeycomb core	
mass density: ρ_2^s (kg / m^3)	317.54
elastic modulus: $E_z = E_\theta$ (GPa)	0.166
elastic modulus: E_ζ (GPa)	72
shear modulus: $G_{z\zeta} = G_{\theta\zeta}$ (GPa)	1.599
shear modulus: $G_{z\theta}$ (GPa)	0.042
Poison's ratio: $\mu_{z\theta}$	1
Poison's ratio: $\mu_{z\zeta} = \mu_{\theta\zeta}$	0.3

The properties of the fluids are given in the followings: The aircraft is in cruising flight at 25,000ft altitude (ρ_1 = 0.5489kg / m^3, c_1 = 309.966m / s) with interior pressurized to 10,000ft altitude (ρ_3 = 0.9041kg / m^3, c_3 = 328.558m / s). The mass density and sound speed of the fluid in the annular space between two shells are the same as that in the interior cavity. Three flight conditions encompassing subsonic, transonic, and supersonic conditions with Mach numbers M = 0.5, 1.0, and 2.0 are considered. The incident angle of the oblique plane sound wave is $\gamma_1 = 30^0$.

3.2 Results

Since the structural impedance of each shell Z_1^s and Z_2^s plays an important role in calculating the TL, figure 2 shows the modulus of the shell impedance versus frequency for M = 0.5, 1.0, and 2.0. For the outer shell, the resonances do not influence the impedance for M = 0.5. However, the resonances will result in peaks in the impedance for M = 2.0 at low frequencies, for instance, less than 1.5kHz. With an increase of the Mach numbers, the effect of the resonances on the impedance becomes significance. For the inner shell, the impedance is strongly affected by the resonances for both subsonic and supersonic Mach numbers at low frequencies. The minima in the impedance at the coincidence are shown in the inner shell and they are shifted upwards with increasing Mach number. The minima can not be observed in the outer shell for M = 0.5 and 1.0. Comparison of (a) and (b) reveals that the impedance is much larger in the outer shell and than in the inner shell. This illustrates that the outer shell transmits much less incident energy than the inner shell so that noise will be reduced largely after it transmits the outer shell.

Figure 3 shows the TL in the annular space and interior cavity of the shells. The formula for the TL in the cavity is given by Eq. (26) while in the space it is defined by

$$\widetilde{TL} = -10 \log_{10} \frac{\widetilde{W}^T}{W^I} \tag{31}$$

The transmitted power flow $\widetilde{W}^T$ is defined by

$$\widetilde{W}^T = \frac{1}{2} \mathrm{Re}\, \{\int_{S_1} [(p_2^T + p_2^R)\dot{w}_1^{0*}]|_{r=R_1}\, dS\} \tag{32}$$

in which $S_1 = 2\pi R_1$. The closed form solutions for p_2^T, p_2^R, and w_1^0 are given in the appendix. The power flow can be obtained by substituting the expressions of p_2^T, p_2^R, and w_1^0 into above equations.

Major minima in the TL corresponding to coincidence frequencies are shown in these figures. These minima are shifted upwards with increasing the Mach numbers. The effect of the shell resonances on the TL can be observed only for M = 1.0 and 2.0. The transmission loss is almost the same in the interior cavity and annular space at low frequencies. This is what we have expected because the impedance of the inner shell is much smaller than that of the outer shell, as shown in Fig. 2.

Figure 4 shows a comparison of the TL between the double sandwich shells and single sandwich shells. The single sandwich shell consists of either the outer shell or the inner shell. At low frequencies, not much difference exists for the TL between the double sandwich shells and single outer sandwich shell. However, with increasing frequencies, the advantage that the double sandwich shells can offer more noise reduction than single shells is revealed except near coincidence. Double sandwich shells can produce an appreciable increase of the TL over single sandwich shells at high frequencies.

The effect of the pressure of the fluid in the annular space between the shells on the TL is shown in Fig. 5. The space is pressurized to 10,000ft, 15,000ft (ρ_2 = 0.7708kg / m^3, c_2 = 322.463m / s), 20,000ft (ρ_2 = 0.6523kg / m^3, c_2 = 316.062m / s), or unpressurized at 25,000ft. The interior cavity is pressurized to 10,000ft in all cases. Results demonstrate that the variation of the pressures will lead to the difference in the TL within 2dB when the frequency is less about 160Hz. The difference will increase gradually to 12dB in the mid-frequency range, i. e., between 160Hz and 500Hz, so that a criterion can be made according to noise reduction requirement by selecting the pressure in the annular space at this range. When the frequency is greater than 500Hz, the difference of the TL is within 6dB except near coincidence. The minima in the TL are shifted upwards slightly with the decrease of the mass densities.

4. CONCLUDING REMARKS

This paper develops a mathematical model for prediction of sound transmission through two infinite concentric cylindrical sandwich shells excited by an incoming oblique plane sound wave. The shells are surrounded by fluid media and there is air in the annular space between them. Each sandwich shell is made of the honeycomb core and face sheets which can be isotropic, orthotropic, or laminated fiber-reinforce composite materials. The first-order shear deformation theory is applied for the outer shell and the classical thin shell theory is applied for the inner shell. A closed form for the transmission loss is derived including the effect of the external flow based the modal analysis. The following conclusions can be drawn:

(i). The transmission loss is almost the same in both the annular space and the interior cavity of the shells except near the coincidence since the impedance of the inner shell is much smaller than that of the outer shell.

(ii). The two concentric sandwich shells can offer appreciable advantage over the single outer sandwich shell and the single inner sandwich shell for noise reductions especially at high frequencies.

(iii). The transmission loss is not sensitive to the change of the pressure in the annular space in the low frequency range. However, in the mid-frequency range, an enhancement of the TL is achieved by selecting the pressure in the annular space.

5.. ACKNOWLEDGMENTS

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6. REFERENCE

[1] Beranek, L. L., *Sound transmission through multiple structures containing flexible blankets*, J. Acoust. Soc. Am., Vol. 21, No. 4, pp. 419-428, 1949.

[2] Beranek, L. L. (ed), *Noise and vibration control*, McGraw-Hill, 1988.

[3] Chen, S. S. and Rosenberg, G. S., *Dynamics of a coupled shell-fluid system*, Nucl. Eng. Des., Vol. 32, pp. 304-310, 1974.

[4] Coats, T. J., Silcox, R. J., and Lester, H. C., *Numerical study of active structural acoustic control in a double-walled cylinder*, Second Conference on Recent Advances in Active Control of Sound and Vibration, Blacksburg, Virginia, April 28-30, s11-s27, 1993.

[5] Desmet, W. and Sas, P., *Sound transmission of finite double-panel partitions with sound absorbing material and panel stiffeners*, The CEAS/AIAA 1st Joint Aeroacoustic Conference, Munich, Germany, Jun. 12-15, pp. 1049-1053, 1995.

[6] Dym, C. L. and Lang, M. A., *Transmission of sound through sandwich panels*, J. Acoust. Soc. Am., Vol. 56, No. 5, pp. 1523-1532, 1974.

[7] Ford, R. D., Lord, P., and Willams, P. C., *The influence of absorbent linings on the transmission loss of double-leaf partitions*, J. Sound Vib., Vol. 5, No. 1, pp. 22-28, 1967.

[8] Gibson, L. J., Ashby, M. F., Shajer, G. S., and Robertson, C. I., *The mechanics of two-dimensional cellular materials*, Pro. R. Soc. Lond. , A 382, pp. 25-42, 1982.

[9] Grediac, M., *A finite element study of the transverse shear in honeycomb cores*, Int. J. Solids Structures, Vol. 13, No. 13, pp. 1777-1788, 1993.

[10] Grooteman, F. P., Boer, D., and Schippers, H., *Vibro-acoustic analysis of double wall structures*, The CEAS/AIAA 1st Joint Aeroacoustic Conference, Munich, Germany, Jun. 12-15, pp. 1049-1053, 1995.

[11] Grosveld, F. W., Coats, T. J., Lester, H. C., and Silcox, R. J., *A numerical study of active structural acoustic control in a stiffened, double wall cylinder*, Noise-Con 94, Ft. Lauderdale, Florida, May 01-04, 1994.

[12] Krajcinovic, D., *Vibrations of two coaxial cylindrical shells containing fluid*, Nucl. Eng. Des., Vol 32, pp. 242-248, 1974.

[13] Levin, L. and Milan, D.,*Coupled breathing vibrations of two thin cylindrical coaxial shells in fluids*, Proc. Vib. Probl. Ind., Paper No. 616, Keswick, England, 1973.

[14] London, A., *Transmission of reverberate sound through double walls*, J. Acoust. Soc. Am., Vol. 22, No. 2, pp. 270-279, 1950.

[15] Mindlin, R. D., *Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates*, J. Mech. Vol. 18 (1), 1951, pp. 31-38.

[16] Mulholland, K. A. and Parbrook, H. D., and Cummings, A., *The transmission loss of double panels*, J. Sound Vib., Vol. 6, No. 3, pp. 324-334, 1967.

[17] Reissner, E., *The effect of transverse shear deformation on the bending of elastic plate*, J. Appl. Mech. Vol. 12, 1945, A69-A77.

[18] Skelton, E. A., *Acoustic scattering by a disk or annulus linking two concentric cylindrical shells, Part I: theory and results for heavy exterior fluid loading*, J. Sound Vib., Vol. 154, No. 2, pp. 205-220, 1992.

[19] Skelton, E. A., *Acoustic scattering by a disk or annulus linking two concentric cylindrical shells, Part II: results for heavy exterior fluid loading on both shells*, J. Sound Vib., Vol. 154, No. 2, pp. 221-248, 1992.

[20] Tang, Y. Y., Robinson, J. H., and Silcox, R. J., *Sound transmission through a cylindrical sandwich shell*, accepted by 34th AIAA Aerospace Science Meeting and Exhibit, Reno, NV, 1996.

[21] Vaicaitis, R. and Bofilios, D. A., *Noise transmission of double wall composite shells*, AIAA 10th Aeroacoustics Conference, Paper No. 85-0786-CP, Seattle, WA, July 9-11, 1986.

[22] Ver, I. L. and Holmer, C. I., *Interaction of sound waves with solid structures*, Noise and vibration control, ed. by Leo L. Beranek, pp. 270-361, 1988.

[23] White, P. H. and Powell, A., *Transmission of random sound and vibration through a rectangular double wall*, J. Acoust. Soc. Am., Vol. 44, No. 4, pp. 821-832, 1966.

[24] Yeh, T. T. and Chen, S. S., *Dynamics of a cylindrical shell system coupled by viscous fluid*, J. Acoust. Soc. Am., Vol. 62, No. 2, pp. 262-270, 1977.

APPENDIX: DERIVATION OF THE SOLUTIONS

To develop the solutions, assume the pressures p_1^R, p_2^T, p_2^R, and p_3^T which satisfy Eqs. (1) to (3) as

$$p_1^R(r, z, \theta, t) = \sum_{n=0}^{\infty} p_{1n}^R H_n^2(k_{1r}r) \cos[n\theta] \exp[\mathrm{i}(\omega t - k_{1z}z)] \quad \text{(A1)}$$

$$p_2^T(r, z, \theta, t) = \sum_{n=0}^{\infty} p_{2n}^T H_n^1(k_{2r}r) \cos[n\theta] \exp[\mathrm{i}(\omega t - k_{1z}z)] \quad \text{(A2)}$$

$$p_2^R(r, z, \theta, t) = \sum_{n=0}^{\infty} p_{2n}^T H_n^2(k_{2r}r) \cos[n\theta] \exp[\mathrm{i}(\omega t - k_{1z}z)] \quad \text{(A3)}$$

$$p_3^T(r, z, \theta, t) = \sum_{n=0}^{\infty} p_{3n}^T H_n^1(k_{3r}r) \cos[n\theta] \exp[\mathrm{i}(\omega t - k_{1z}z)] \quad \text{(A4)}$$

where p_{1n}^R, p_{2n}^R, p_{2n}^T, and p_{3n}^T are yet-to-be-determined complex amplitude factors.

The displacement components w_1^0, and w_2^0 are assumed as

$$w_1^0(z, \theta, t) = \sum_{n=0}^{\infty} w_{1n}^0 \cos[n\theta] \exp[\mathrm{i}(\omega t - k_{1z}z)] \quad \text{(A5)}$$

$$w_2^0(z, \theta, t) = \sum_{n=0}^{\infty} w_{2n}^0 \cos[n\theta] \exp[\mathrm{i}(\omega t - k_{1z}z)] \quad \text{(A6)}$$

where w_{1n}^0 and w_{2n}^0 are unknown complex amplitude factors. Substitution of Eqs. (A1) to (A6) into Eqs. (4) to (9) yields the solutions for p_{1n}^R, p_{2n}^R, p_{2n}^T, p_{3n}^T, w_{1n}^0 and w_{2n}^0

$$p_{1n}^R = -\frac{J_n(k_{1r}R_1)}{H_n^2(k_{1r}R_1)} \times \frac{\Delta_1}{\Delta} p_0 \varepsilon_n (\text{-i})^n \quad \text{(A7)}$$

$$p_{2n}^T = -\frac{\mathrm{i}\rho_2\omega}{k_{2r}} \times \frac{J_n'(k_{1r}R_1)}{H_n^{1'}(k_{2r}R_1)} \times \frac{H_n^2(k_{1r}R_1)}{H_n^{2'}(k_{1r}R_1)} \times \frac{\Delta_2}{\Delta} p_0 \varepsilon_n (\text{-i})^n \quad \text{(A8)}$$

$$p_{2n}^R = \frac{\mathrm{i}\rho_2\omega}{k_{2r}} \times \frac{J_n'(k_{1r}R_1)}{H_n^{1'}(k_{2r}R_1)} \times \frac{H_n^2(k_{1r}R_1)}{H_n^{2'}(k_{1r}R_1)} \times \frac{H_n^{1'}(k_{2r}R_2)}{H_n^{2'}(k_{2r}R_2)} \times \frac{\Delta_3}{\Delta} \times p_0 \varepsilon_n (\text{-i})^n \quad \text{(A9)}$$

$$p_{3n}^T = \frac{\rho_3 k_{1r}}{\rho_1 k_{3r}} \times \frac{\omega^2}{k_1^2 c_1^2} \times \frac{J_n'(k_{1r}R_1)}{H_n^{1'}(k_{3r}R_2)} \times \frac{H_n^{1'}(k_{2r}R_2)}{H_n^{1'}(k_{2r}R_1)} \times \frac{\Delta_4}{\Delta} p_0 \varepsilon_n (\text{-i})^n \quad \text{(A10)}$$

$$w_{1n}^0 = -\frac{\mathrm{i}}{\omega} \times \frac{J_n'(k_{1r}R_1)\, H_n^{1'}(k_{1r}R_1)}{H_n^{2'}(k_{1r}R_1)} \times \frac{\Delta_5}{\Delta} p_0 \varepsilon_n (\text{-i})^n \quad \text{(A11)}$$

$$w_{2n}^0 = \frac{k_{1r}}{\rho_1\omega^2} \times \frac{\omega^2}{k_1^2 c_1^2} \times \frac{J_n'(k_{1r}R_1)\, H_n^{1'}(k_{2r}R_2)}{H_n^{1'}(k_{2r}R_1)} \times \frac{\Delta_6}{\Delta} p_0 \varepsilon_n (\text{-i})^n \quad \text{(A12)}$$

where Δ are given in the text and

$$\Delta_1 = (Z_1^s + Z_{21}^T - Z_{11}^I)(Z_2^s + Z_{32}^T + Z_{22}^R) - (Z_1^s - Z_{11}^I - Z_{21}^R)(Z_2^s + Z_{32}^T - Z_{22}^T) \times H_n^{1'}(k_{2r} R_2)\, H_n^{2'}(k_{2r} R_1) / [H_n^{1'}(k_{2r} R_1) H_n^{2'}(k_{2r} R_2)] \quad \text{(A13)}$$

$$\Delta_2 = (1 + Z_{11}^I / Z_{11}^R)(Z_2^s + Z_{32}^T + Z_{22}^R) \quad \text{(A14)}$$

$$\Delta_3 = (1 + Z_{11}^I / Z_{11}^R)(Z_2^s + Z_{32}^T - Z_{22}^T) \quad \text{(A15)}$$

$$\Delta_4 = (1 + Z_{11}^I / Z_{11}^R)(Z_{22}^T + Z_{22}^R) Z_{11}^R \quad \text{(A16)}$$

$$\Delta_5 = (1 + Z_{11}^I / Z_{11}^R)\{(Z_2^s + Z_{32}^T + Z_{22}^R) - (Z_2^s + Z_{32}^T - Z_{22}^T) \times H_n^{1'}(k_{2r} R_2)\, H_n^{2'}(k_{2r} R_1) / [H_n^{1'}(k_{2r} R_1) H_n^{2'}(k_{2r} R_2)]\} \quad \text{(A17)}$$

$$\Delta_6 = (1 + Z_{11}^I / Z_{11}^R)(Z_{22}^T + Z_{22}^R) Z_{11}^R \quad \text{(A18)}$$

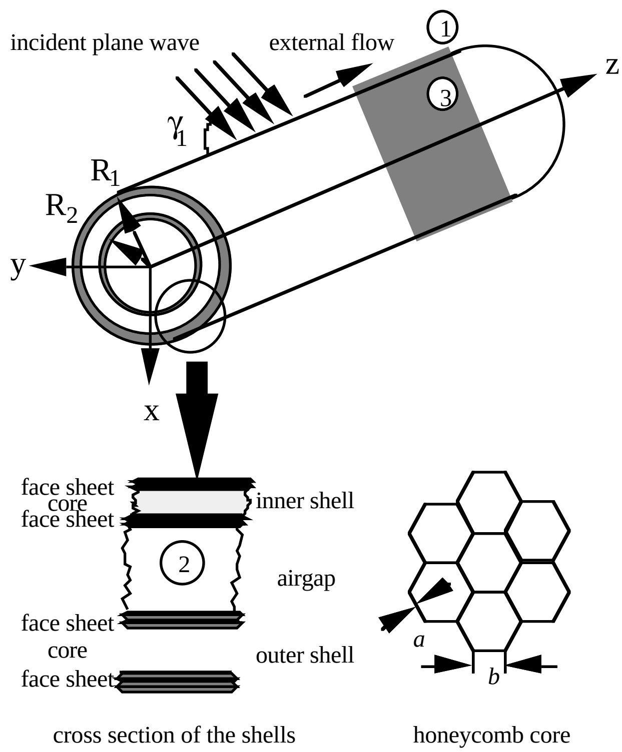

Figure 1. Schamatic of two concentric shells and their geometry construction.

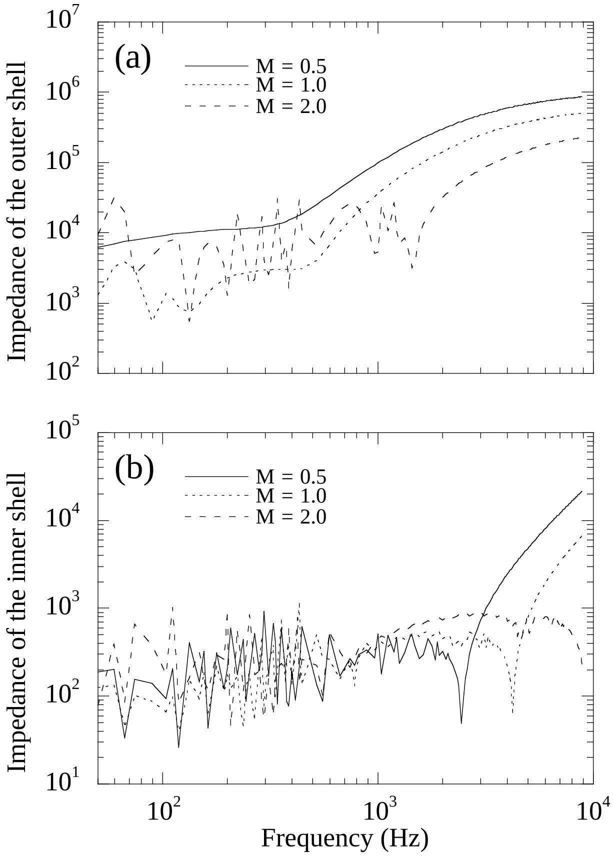

Figure 2. Modulus of impedance of the sandwich shells.

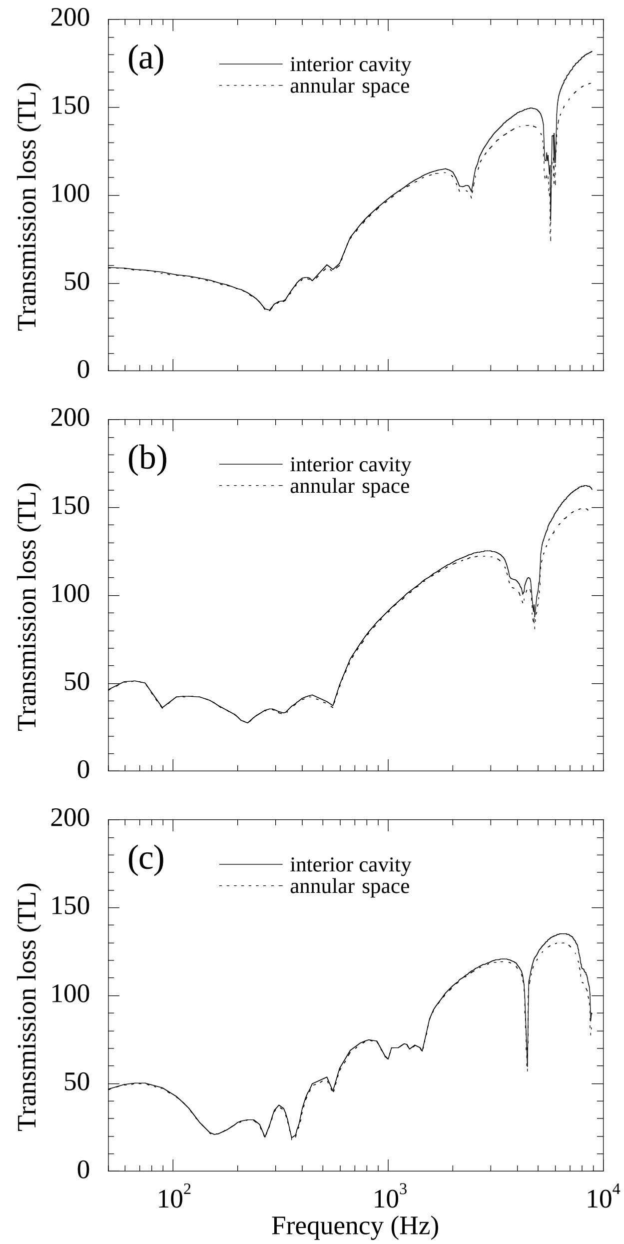

Figure 3. The TL at annular space and interior cavity:
(a) M = 0.5; (b) M = 1.0; (c) M = 2.0.

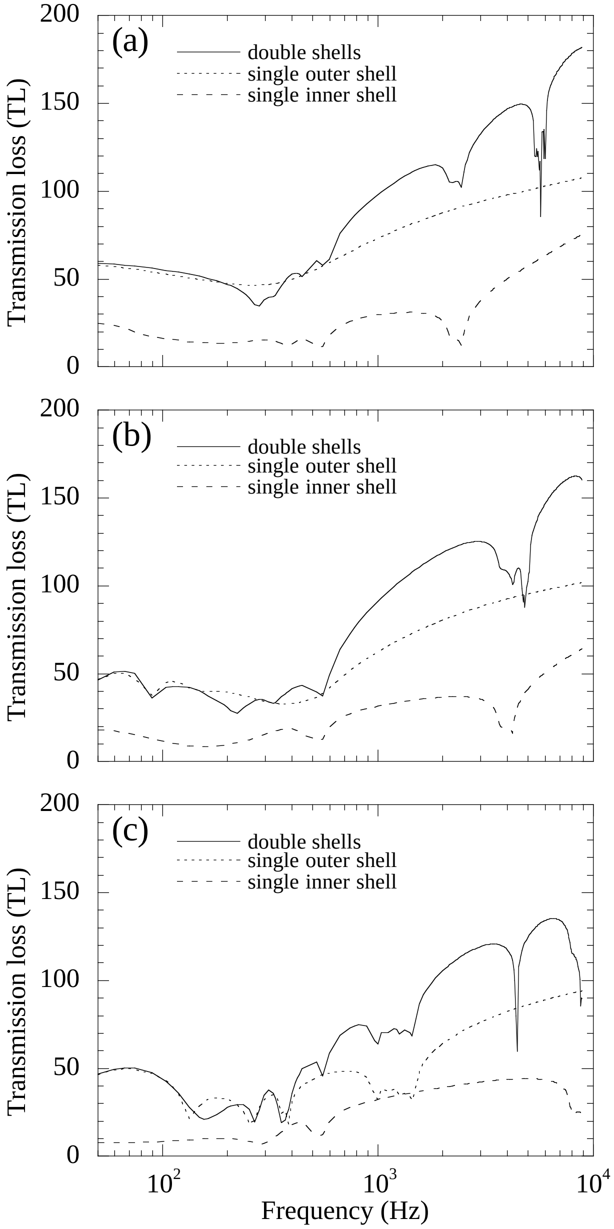

Figure 4 A comparison of the TL between double shells and single shells: (a) M = 0.5; (b) M = 1.0; (c) M = 2.0.

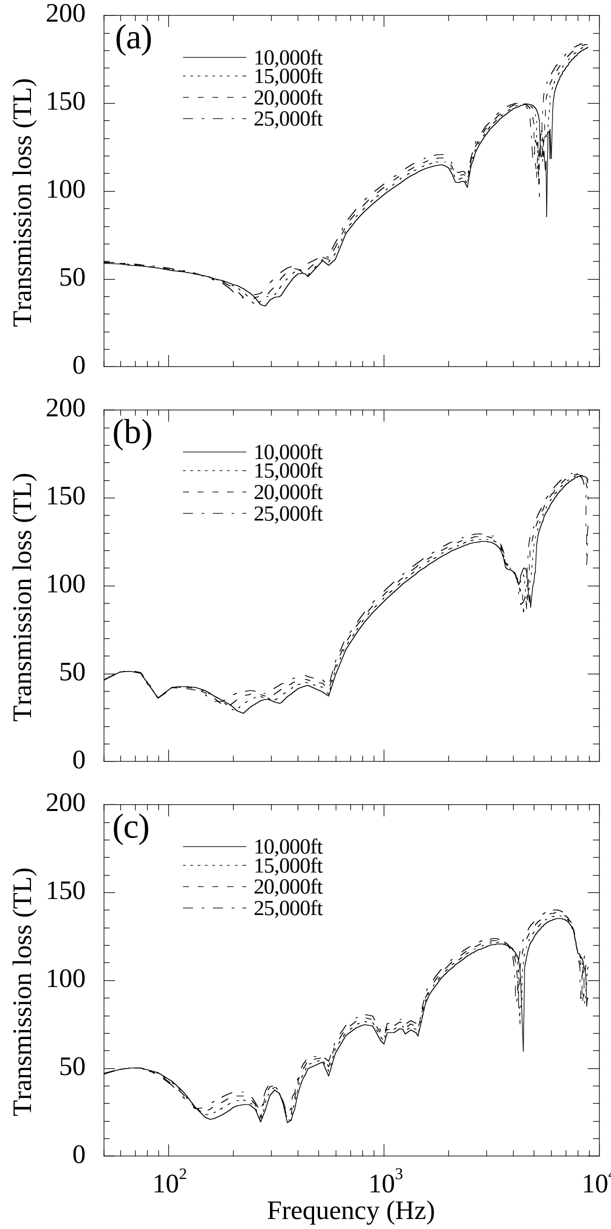

Figure 5. The effect of the annular pressure on the TL: (a) M = 0.5; (b) M = 1.0; (c) M = 2.0.