Effects of Gravity-Assist Timing on Outer-Planet Missions Using Solar-Electric Propulsion

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Missions to the outer planets for spacecraft with a solar-electric propulsion system (SEPS) and that utilize a single Venus gravity assist are investigated. The trajectories maximize the delivered mass to the target planet for a range of flight times. A comparison of the trajectory characteristics (delivered mass, launch energy and onboard propulsive energy) is made for various Venus gravity assist opportunities. Methods to estimate the delivered mass to the outer planets are developed.

Nomenclature

C	=	linear least square constant
c	=	magnitude of exhaust velocity
C_3	=	square of the earth-centered hyperbolic excess velocity
ΔE	=	total propulsive specific energy increment
ΔV_{SEPS}	=	velocity increment provided by onboard SEPS
g	=	gravitational acceleration at the surface of the Earth
I_{sp}	=	specific impulse
I_{sp}^*	=	average specific impulse of a SEPS trajectory
I_{sp}^m	=	median of I_{sp}^* range in all trajectories of this research
k	=	linear least square scale factor
m_0, m_F	=	initial injected mass and delivered mass
$\dot{m}$	=	mass flow rate
ξ, TOF	=	time of flight

I. Introduction

IN exploring the outer solar system planets, a planetary gravity-assist has commonly been used since one or more gravity-assists have the potential to save propellant, reduce time of flight (TOF), or both. Because of these advantages, many previous interplanetary missions (for example, Mariner 10, Voyager I, II, Galileo, Cassini and NEAR) exploited the gravity-assist.[1] However, for a ballistic interplanetary mission, there are very little chances that all flyby planets are in the desired positions at the desired time. Commonly, the launch may have to be postponed by a couple of years if a few-day launch opportunity is missed. Moreover, this short launch opportunity shrinks rapidly as the number of flyby planets increases. Solar Electric Propulsion Systems (SEPS) can provide longer launch opportunities than a ballistic mission. The trajectory shaping capability of SEPS as well as the longer launch opportunity increases mission design flexibility.

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The flexibility of SEPS missions also allows the gravity-assist to occur at different times even with the same TOF and the same launch year constraints. In a previous study,[2] we found that the different gravity-assist timing changes the mission performance in terms of delivered mass to target planets in outer-planet, single Venus gravity-assist missions. Revolution ratio (R-ratio) was introduced to represent the different Venus gravity-assist timing and was defined as the number of Venus revolutions for one revolution of a spacecraft around the Sun. For instance, a 3:1 R-ratio is one where roughly three Venus years occur during the period from launch to flyby of Venus by the spacecraft. For a SEPS mission, the R-ratio can be considered as the problem of trade-off between the launch energy and the onboard propulsive energy or similarly, the problem of allocating the flight time to before and after of the gravity-assist. For this study, we used SEPTOP (Solar Electric Propulsion Trajectory Optimization Program)[3] to generate SEPS, outer-planet optimal trajectories. In spite of the necessity of the R-ratio analysis, it can be difficult to obtain the different R-ratio optimal trajectories with SEPTOP. In this paper, the R-ratio problem is revisited to clarify the underlying fundamentals of the problem and methods to estimate the delivered mass of the different R-ratio trajectories are developed to support the R-ratio analysis.

II. Revolution Ratio

Figure 1 illustrates the trajectories in an Earth-Venus-Saturn (EVS) mission with three different R-ratios. The TOF of all trajectories in the figure is 6 years. It is clear that a spacecraft on the trajectory with the largest R-ratio spends the most time thrusting before the flyby occurs. Given the nature of SEPS performance – namely, that orbital energy addition is more efficient near the Sun due to greater power availability and control authority – it would seem that spending more time in close proximity to the Sun before heading outbound toward the destined target would be beneficial in delivering more mass. However, a larger R-ratio trajectory typically needs more launch energy, which in turn means a larger proportion of the total required energy being provided by an inefficient launch vehicle rather than the more efficient low-thrust engine. Trade-offs between the launch energy and the onboard propulsion result in an optimal R-ratio of 2:1 for this TOF 6 year mission. Here, the launch energy is provided by Delta IV M+(4,2) launch vehicle and High Thrust To Power 3900 s (HTTP 3900) thruster[4] is used as onboard propulsion system.

Figure 2 shows the delivered mass comparison for the EVS, Earth-Venus-Uranus (EVU), Earth-Venus-Neptune (EVN), and Earth-Venus-Pluto (EVP) missions. This comparison is a result of the launch energy and onboard propulsion trade-off. A more detailed comparison can be found in Ref. 2.

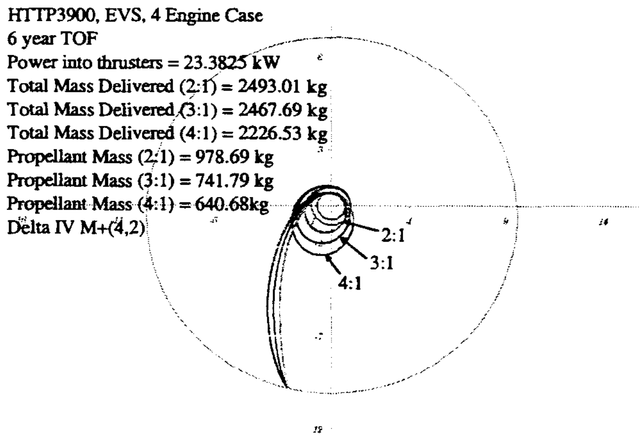

Figure 1. Trajectories with three different R-ratios for an EVS mission, Delta-IV M+ (4,2).

III. SEPS Thruster Model

The free and continuous thrust profile of SEPS provides serious difficulties in analytical trajectory design. In this paper, a thruster model that represents the overall thrust profile is developed to be exploited in the delivered mass estimation of SEPS trajectories. The thruster model includes constant thrust, mass flow rate and I_{sp} or exhaust velocity c. These parameters are varying during a mission and the variation is mission specific. Because of the variable characteristics of a SEPS, the established rocket equation is not applicable for a SEPS mission. With constant parameters of a SEPS thruster, it would be possible to use the rocket equation to estimate the delivered mass of the mission. The SEPS thruster model is a function of the input power so the modeling of the constant thruster parameters is

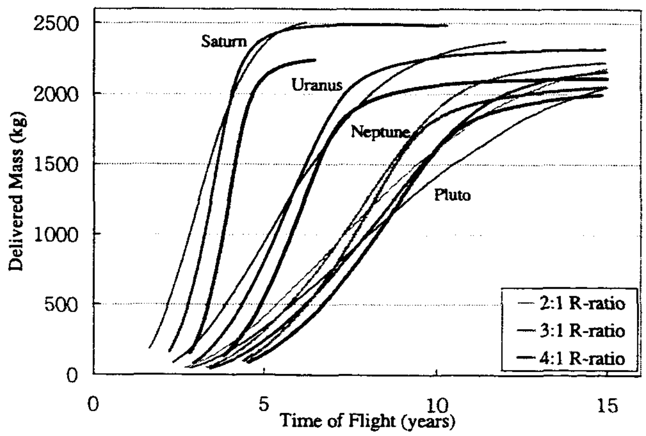

Figure 2. Delivered mass of three different R-ratios: EVS, EVU, EVN, and EVP missions.

equivalent to the selection process of the effective input power (P_{eff}) within the operating power range. Once the P_{eff} is determined, the thrust and mass flow rate are calculated from the original thruster model.[4] In case the duty cycle of the SEPS is not 100%, then the calculated thrust and mass flow rate are reduced by multiplying by the duty cycle percentage. For multiple thrusters, the number of thrusters is multiplied to the thrust and mass flow rate to find the total thrust and mass flow rate for a mission. With these results, the mission-specific overall I_{sp} (or magnitude of exhaust velocity c) is calculated by Eq. (1). The $\dot{m}$ is mass flow rate and g is gravitational acceleration at the surface of the Earth.

$$I_{sp} = {thrust}/{\dot{m}g} = {c}/{g} \tag{1}$$

If we define I_{sp}^* as an average mission-specific I_{sp}, the I_{sp}^* is calculated by the initial mass that is inserted to orbit by a launch vehicle m_0, the delivered mass m_F and the velocity increment provided by SEPS during the flight ΔV_{SEPS} from SEPTOP by Eq. (2) (Ref. 5).

$$I_{sp}^* = \frac{-\Delta V_{SEPS}}{g \cdot \ln(m_F / m_0)} \tag{2}$$

The I_{sp}^* represents the averaged effect of the thrusting in a trajectory. For all outer-planet trajectories of this research, the I_{sp}^* varies between 3166.357 s to 3628.537 s. The median of the I_{sp}^* variation I_{sp}^m is 3397.4 s and it is 90% of the maximum I_{sp} of the HTTP 3900 thruster. The P_{eff} that generate I_{sp}^m of 3397.4 s is calculated from the HTTP 3900 thruster characteristics and it results P_{eff} of 4.32 kWe. It is approximately 75% of the maximum operating power of the thruster. The I_{sp}^m (or c) is the representative value for a range of actual I_{sp}'s so there exists differences between the I_{sp}^m and the actual I_{sp}'s. The difference can be as large as 230 sec in this research so the validity of this modeling strategy should be reviewed before using the I_{sp}^m. The validity of the I_{sp}^m as a representative value for a SEPS thruster can be shown by examining the sensitivity of the delivered mass (m_F) to the variation of I_{sp}. In order to see this, consider the solution to the rocket equation in Eq. (3) (Ref. 5).

$$\exp\left[{-\Delta V_{SEPS}}/{c}\right] = \exp\left[{-\Delta V_{SEPS}}/{gI_{sp}}\right] = {m_F}/{m_0} \tag{3}$$

Examining first-order variations in I_{sp} and m_F.

$$\exp\left[\frac{-\Delta V_{SEPS}}{gI_{sp}}\right] \cdot \left(\frac{-\Delta V_{SEPS}}{g} \frac{-dI_{sp}}{I_{sp}^2}\right) = \frac{\Delta V_{SEPS}}{gI_{sp}^2} \cdot \exp\left[\frac{-\Delta V_{SEPS}}{gI_{sp}}\right] \cdot dI_{sp} = \frac{dm_F}{m_0} \tag{4}$$

which can be expressed using Eq. (2) as

$$\frac{\Delta V_{SEPS}}{gI_{sp}} \frac{dI_{sp}}{I_{sp}} = \frac{dm_F}{m_F} \tag{5}$$

In Eq. (5), $\Delta V_{SEPS}/gI_{sp}$ is typically a number less than one. For a typical EVS mission, $\Delta V_{SEPS}/gI_{sp}$ is about 3×10^{-1}. For sizable delivered masses of $m_F \approx 2000$ kg, this makes dm_F/m_F relatively insensitive to dI_{sp}/I_{sp} in a SEPS mission. On the other hand, for a chemical engine mission with the similar ΔV, the dm_F/m_F is more sensitive to dI_{sp} than for a SEPS mission because of the smaller I_{sp} range of 200 to 410 s (Ref. 6).

If the change of I_{sp} is so large that the first-order analysis is not accurate, one must compare the nonlinear values directly. Starting from Eq. (2), let

$$\frac{dm_{F1}}{m_0} = \exp\left[\frac{-\Delta V_{SEPS}}{gI_{sp1}}\right] \tag{6}$$

$$\frac{dm_{F2}}{m_0} = \exp\left[\frac{-\Delta V_{SEPS}}{gI_{sp2}}\right] \tag{7}$$

and define

$$\begin{aligned} \Delta m_F &= m_{F2} - m_{F1} \\ \Delta I_{sp} &= I_{sp2} - I_{sp1} \end{aligned} \tag{8}$$

then

$$\frac{\Delta m_F}{m_0} = \frac{m_{F2}}{m_0} - \frac{m_{F1}}{m_0} = \exp\left[\frac{-\Delta V_{SEPS}}{gI_{sp2}}\right] - \exp\left[\frac{-\Delta V_{SEPS}}{gI_{sp1}}\right] \tag{9}$$

and finally,

$$\begin{aligned} \frac{\Delta m_F}{m_{F1}} &= \frac{\exp\left[\frac{-\Delta V_{SEPS}}{gI_{sp2}}\right] - \exp\left[\frac{-\Delta V_{SEPS}}{gI_{sp1}}\right]}{\exp\left[\frac{-\Delta V_{SEPS}}{gI_{sp1}}\right]} = \exp\left[\frac{-\Delta V_{SEPS}}{gI_{sp2}} + \frac{\Delta V_{SEPS}}{gI_{sp1}}\right] - 1 \\ &= \exp\left[\frac{-\Delta V_{SEPS}}{g}\left(\frac{I_{sp1} - I_{sp2}}{I_{sp1} I_{sp2}}\right)\right] - 1 = \exp\left[\frac{\Delta V_{SEPS}}{g} \cdot \frac{\Delta I_{sp}}{I_{sp1} I_{sp2}}\right] - 1 \end{aligned} \tag{10}$$

The representative I_{sp}^{m} value for the HTTP 3900 thruster is 3397.447 s and the variation of I_{sp}^{*} in all trajectories with various TOF is $I_{sp}^{m} \pm 230$ s. The maximum $\Delta m_F/m_{F1}$ deviation for the range of I_{sp}^{m} is less than 5 % for ΔV_{SEPS} = 20 km/s case. In Fig. 3 the variation of m_F is shown for the variation of I_{sp}^{*} near the representative I_{sp}^{m} value. In the figure, the maximum m_F estimation error caused by the I_{sp}^{m} modeling inaccuracy is ~ 4.8 % of the m_F which is acceptable error for preliminary mission planning. In summary, the delivered mass is not highly sensitive to the specific impulse modeling inaccuracy for the SEPS outer-planet missions and therefore, a constant specific impulse can be used for several mission planning purposes.

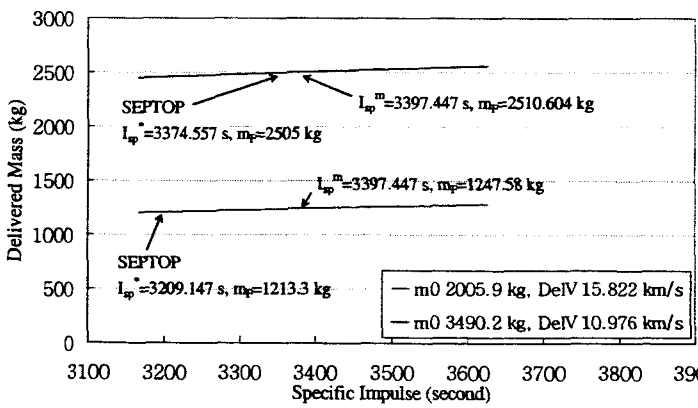

Figure 3. Delivered mass sensitivity to the specific impulse variation.

IV. Estimation of Delivered Mass of Different R-ratio Trajectories

A. Estimation of Delivered Mass with Linear Least Square Method

The bases of the delivered mass estimation method are the regularities of the relation between the delivered mass and the total specific propulsive energy increment. The total propulsive specific energy increment is defined as

$$\Delta E = \Delta V_{SEPS}{}^{2} + C_3 \tag{11}$$

where ΔE is the total propulsive specific energy increment (excluding an energy increment by a gravity assist) and the launch energy C_3 is the square of the earth-centered hyperbolic excess velocity of a spacecraft when it has just separated from the launch vehicle. Figure 4 shows the ΔE of three R-ratio trajectories for EVS to EVP missions. In Fig. 4, the ΔE curves for a target planet have similar shape regardless of the different R-ratio. In other words, ΔE curves with different R-ratios are related by a shift along with the TOF axis and the ΔE axis. The amount of the horizontal shifts (along with the TOF axis) is investigated to determine one Venus year (224.701 days) and the amount of the vertical shifts (along with the ΔE axis) is found to be 30 km^2/s^2 between 3:1 and 2:1 R-ratios and 15 km^2/s^2 between 4:1 and 3:1 R-ratios in Fig. 4. Figure 5 compares the 2:1 and 4:1 R-ratio ΔE curves from SEPTOP with the shifted 3:1 R-ratio ΔE curve for EVS mission. Overall, the shifted curve matches 2:1 and 4:1 R-ratio ΔE curves accurately. This result means that the ΔE curves with different R-ratio to a target planet have similar profiles in spite of their different delivered mass profile. There are many potential applications of this result; for example, it is possible to predict the delivered mass of a mission with different R-ratio. Also, the search space for optimization can be dramatically reduced by previewing the delivered mass profiles of the different R-ratio trajectories and selecting the most promising R-ratio before starting the optimization. The comparison results for EVU, EVN and EVP missions are not included in this paper for conciseness but their results are similar with Fig. 5.

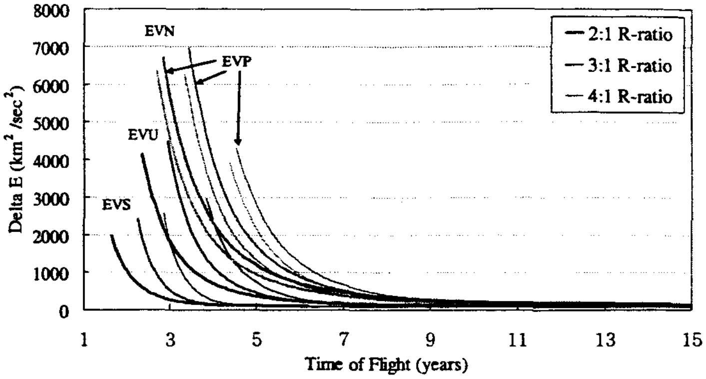

Figure 4. Total propulsive energy increment versus Time of flight.

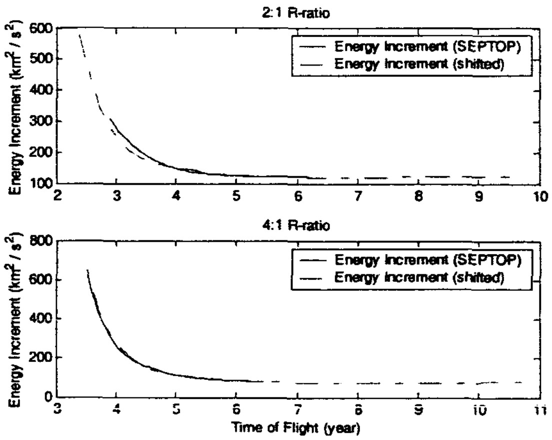

Figure 5. Total propulsive energy increment versus Time of flight.

Another result that should be mentioned is that the horizontal distance between the curves is constantly one Venus year in all missions. Together with the definition of R-ratio, it is an interesting result that explains the physical differences between the different R-ratio trajectories. The difference in TOF from the launch date to the flyby date between the different R-ratio trajectories is roughly one Venus year by the definition of R-ratio so, in the point of ΔE, the effect of the different R-ratio is that a very similar ΔE curve is repeated with the TOF difference of one Venus year. This result provides a method to generate a delivered mass profile of a different R-ratio trajectory without actually calculating it by SEPTOP reducing time to provide preliminary mission planning results. In generating a delivered mass profile from a ΔE curve, the relation between the ΔE curve and the delivered mass profile needs to be established.

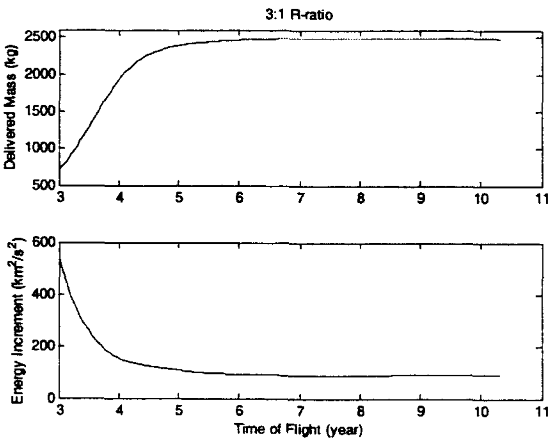

Figure 6. Delivered mass and ΔE for 3:1 R-ratio, EVS mission.

Figure 6 shows the delivered mass and ΔE for 3:1 R-ratio, EVS mission. The delivered mass generation method is based on the symmetry between the ΔE and the delivered mass relative to a horizontal line in Fig. 6. Because of their symmetry, the delivered mass profile can be represented by ΔE with a scale factor and a constant. Let $f_1(\xi)$ be the delivered mass function and $f_2(\xi)$ be the $\Delta E(\xi)$ function where ξ is the time of flight. Then,

Table 1. *k* and *C* for EVS to EVP missions

Target	R-ratio	k (kg/km^2/s^2)	C (kg)
Saturn	2:1	8.501	3374.258
	3:1	5.154	2928.289
	4:1	3.579	2481.234
Uranus	2:1	4.908	2800.503
	3:1	3.866	2616.894
	4:1	2.898	2329.991
Neptune	2:1	4.163	2592.702
	3:1	3.594	2506.627
	4:1	2.592	2269.293
Pluto	2:1	3.964	2584.694
	3:1	3.414	2492.664
	4:1	2.511	2233.285

$$f_1(\xi) + k \cdot f_2(\xi) = C \tag{12}$$

where k is a scale factor between the delivered mass and ΔE, and C is the representing constant for the trajectories to a target planet with the R-ratio. Equation (12) is rearranged to be used in linear least square fitting in the below equation.

$$k \cdot f_2(\xi) - C = -f_1(\xi) \tag{13}$$

For the given TOF ξ_i, $i = 1, \dots, n$, Eq. (13) can be used to construct below linear least square equation.

$$\begin{bmatrix} f_2(\xi_1) & -1 \\ \vdots & \vdots \\ f_2(\xi_n) & -1 \end{bmatrix} \begin{pmatrix} k \\ C \end{pmatrix} = -\begin{pmatrix} f_1(\xi_1) \\ \vdots \\ f_1(\xi_n) \end{pmatrix} \tag{14}$$

The Q-R factorization[7] is used in solving Eq. (14) to calculate the k and C. From ΔE's of three R-ratios for EVS to EVP missions, the k's and C's are calculated and shown in Table 1.

The thruster used is HTTP 3900 and the launch vehicle is Delta IV M+(4,2). The k and C depend on the thruster and the launch vehicle because the relation between the ΔE and the delivered mass is determined by them. The k represents the propulsion efficiency of ΔE to the delivered mass. Large k means more efficient thrusting (launch vehicle + SEPS) for a given delivered mass. The unit of k is kg $\cdot$ s^2 / km^2. The constant C can be interpreted as a limit of the maximum delivered mass of the R-ratio trajectory class to a target planet. According to Table 1, k's are larger for closer targets and that can be interpreted to mean that the thrusting is more efficient than for a farther targets. At the same time, C's for farther targets are smaller than C's for closer targets so the

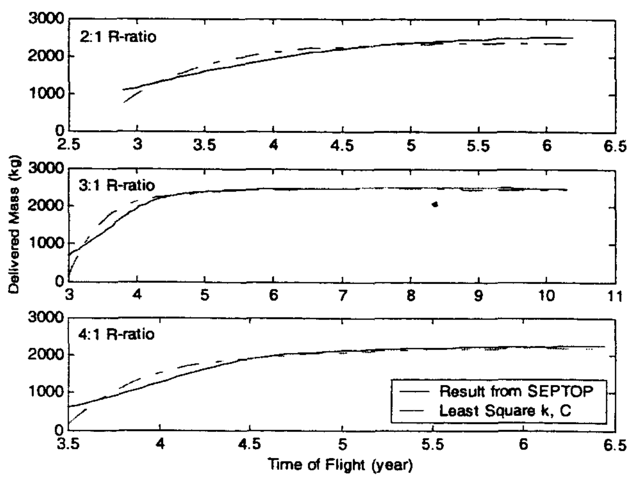

Figure 7. Delivered mass comparison between SEPTOP and *k*, *C* fitting for EVS mission.

limit of the delivered masses for farther targets are smaller. In Eq. (12), the k and C are regulating each other in the sense of increasing or decreasing the delivered mass to a target planet. The delivered mass is determined by the interplay between the k and C and the given profile of ΔE as a function of TOF.

In order to check the validity of k and C, the delivered mass profiles are regenerated using Eq. (13) and k and C of Table 1. Figure 7 shows the comparison of the delivered masses from SEPTOP with the delivered mass from k, C and ΔE from SEPTOP for an EVS mission. The overall predictive performance is observed in the figure. These results mean that if the k, C and ΔE were known for a mission, the delivered mass profile for the mission could be estimated without using SEPTOP. As previously explained, the ΔE curves for a target planet can be approximated by a planar shift of a known ΔE curve. At present, a ΔE curve to a target planet is required to be calculated by SEPTOP to approximate the ΔE of the different R-ratio trajectories to the target planet. On the other hand, the k and C for a mission can be closely approximated once the k and C to a different target planet but with the same R-ratio are known from the SEPTOP result. In other words, k and C of a mission to a target planet with an R-ratio have a relation with the k and C of a mission to a different target planet with the same R-ratio. The relation between k's and C's with the different R-ratio to a target planet needs more research to be clarified. The relations between k's and C's of different missions are shown in Figs. 8 and 9 as functions of inverse of the semimajor axis of target planets. In the figures, the k's and C's are shown as linear functions of the inverse of semimajor axis. It is also possible to calculate k's and C's for other target planets if one use the linear functions and then possible to estimate the delivered mass to the target planets. In summary, Fig. 10 illustrates a procedure of linear least square method.

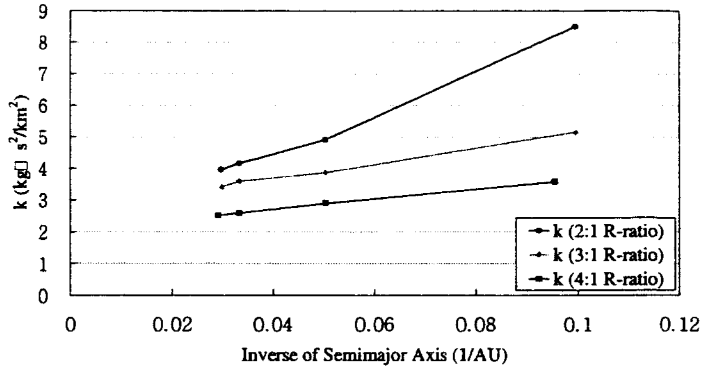

Figure 8. k variation as a function of inverse of semimajor axes.

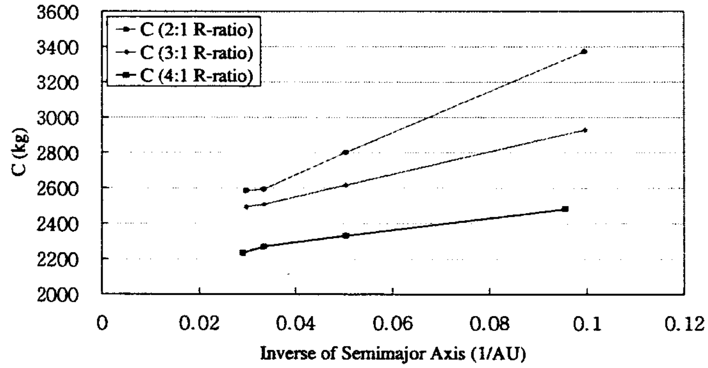

Figure 9. C variation as a function of inverse of semimajor axes.

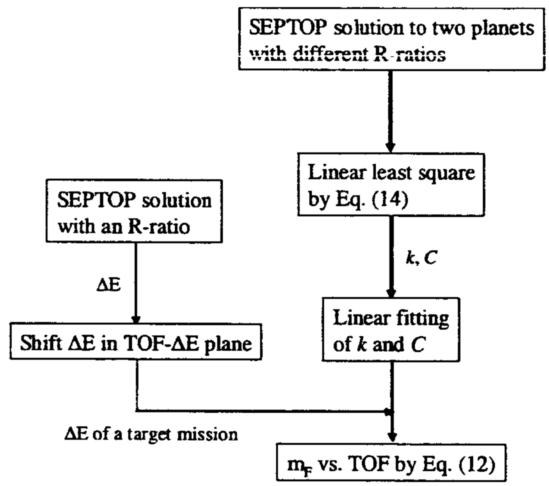

Figure 10. A procedure of linear least square estimation method.

Figure 11 and Fig. 12 shows the comparisons between the estimated delivered mass and the delivered mass from SEPTOP for EVS and EVP missions. The estimated delivered mass is calculated with k and C and planar shifted ΔE curves. EVU and EVN missions show similar results so they are excluded for the conciseness of the paper.

In Figs. 11 and 12, there are some differences between the regenerated delivered mass and the SEPTOP result in spite of the overall coincidences. The differences in short TOF range are caused by the incomplete symmetry between the delivered mass profile and the ΔE profile. The incompleteness is more outstanding in all of the 2:1 R-ratio cases than the other cases and this is the reason of the relatively larger differences in short TOF, 2:1 R-ratio cases. The source of the differences in intermediate to long TOF range is the linear least square error. The linear least square algorithm tries to minimize the total error in all of the data range so a small range of large error usually degrade the total accuracy and the error spread out over all the data range. Therefore, there are relatively larger errors in long TOF, 2:1 R-ratio cases than in other R-ratio cases. The maximum difference between the regenerated delivered mass and the SEPTOP result is about 500 kg in short TOF. This estimated delivered mass can be used for some mission planning purposes, because the estimated delivered mass curve is a good approximation for middle to long TOF range missions, however, a more precise method to predict the delivered mass is required to determine which R-ratio trajectory is the best candidate for a given TOF range of interest without spending lots of effort in SEPTOP.

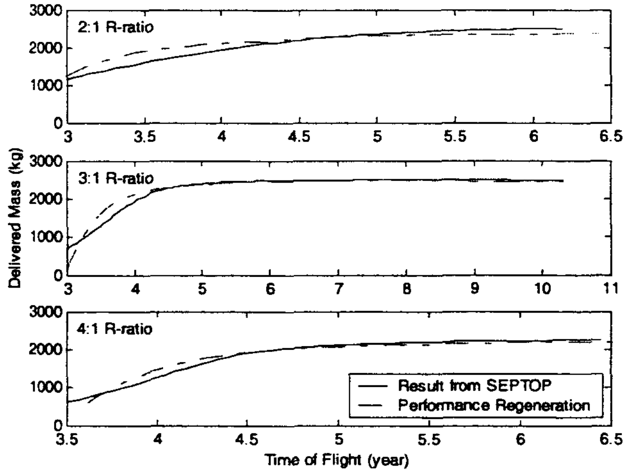

Figure 11. Delivered mass comparison, the estimated versus SEPTOP, EVS mission.

B. Estimation of Delivered Mass with Hybrid Method

The linear least square estimation method has some amount of error especially in its short TOF ranges. The error is caused by the incomplete symmetry between the delivered mass profile and the ΔE profile. In spite of the incomplete symmetry, however, the ΔE profile has significant advantage because the similarity in ΔE profiles among the different R-ratio cases provides a basis to predict the delivered masses of the different R-ratio cases. A more precise estimation method, a hybrid method, also relies on the similarity of ΔE profiles as the linear least square method but it also employs a method that uses the thruster model and rocket equation.

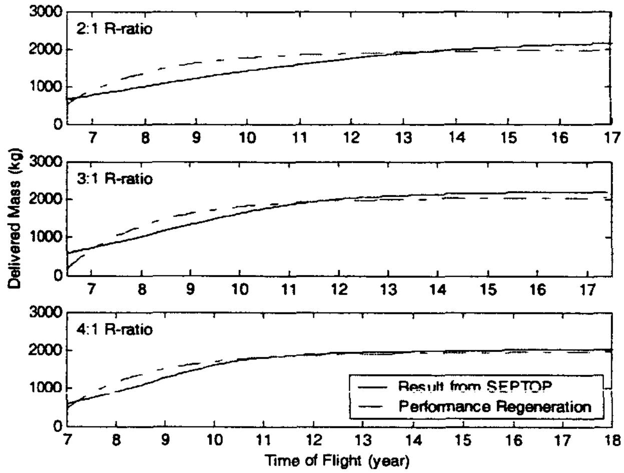

Figure 12. Delivered mass comparison, the estimated versus SEPTOP, EVP mission.

In the hybrid method, the TOF range is divided to two ranges, short TOF and remaining TOF range, and a method that uses the rocket equation is applied to that short TOF range whereas the linear least square method that uses k and C is applied to the remaining TOF range. First, a solution to the rocket equation is shown below.[5]

$$m_F = m_0 \exp\left(\frac{-\Delta V_{SEPS}}{c}\right) \tag{15}$$

The delivered mass can be calculated given m_0, ΔV_{SEPS} and c. The way to determine the m_0 and ΔV_{SEPS} for a given TOF is explained below and the c is modeled by a constant I_{sp} SEPS model.

In estimating the delivered mass, the given data are the ΔE and the delivered mass profile of an R-ratio case (3:1 R-ratio case in this research). By a planar shift, the ΔE profile of 2:1 and 4:1 R-ratio cases can be approximated. However, it is required to find the ΔV_{SEPS} out of ΔE because Eq. (15) needs ΔV_{SEPS} to calculate the m_F and ΔE is the sum of the C_3 and the square of ΔV_{SEPS}. Of course, the ΔV_{SEPS} and also C_3 are assumed to be known for an R-ratio case in order to calculate the ΔE profile that is to be shifted. Therefore, the $C_3/\Delta E$ profiles of all R-ratio cases are compared to find the relation between them. Figure 13 shows the variation of $C_3/\Delta E$ versus ΔE profile for the short TOF range, for all target missions. By comparing the ΔE values with Fig. 4, one can notice that the TOF gets shorter as ΔE value increases in Fig. 13. The $C_3/\Delta E$ profiles are generated with SEPTOP results and almost identical in all R-ratio missions. This fact provides a way to find the ΔV_{SEPS} out of the shifted ΔE profile. If we assume the $C_3/\Delta E$ profiles of other R-ratio cases are identical with the $C_3/\Delta E$ profile of the known R-ratio case, which is true for this research, the C_3 and ΔV_{SEPS} can be extracted from the shifted ΔE profile and the extracted ΔV_{SEPS} can be put into the rocket equation solution to predict the delivered mass of other R-ratio cases. The reason why the $C_3/\Delta E$ profiles are almost identical in all missions can be explained as follow. Because all missions use the same launch vehicle, Delta IV M+(4,2) and the same SEPS (HTTP 3900 thruster), and the shapes of trajectories are similar except its R-ratio, the portion of C_3 (or ΔV_{SEPS}) in ΔE profile, i.e. $C_3/\Delta E$ profile, are identical within an allowable error range for all target missions. Figure 14 illustrates the procedure of the hybrid estimation method for the short TOF range. Once we have a trajectory data for an R-ratio case from SEPTOP, the ΔE profile and the $C_3/\Delta E$ profile of the R-ratio case are available for the next step. By shifting the ΔE profile in TOF-ΔE plane, the ΔE profile of different R-ratio case can be approximated. The ΔV_{SEPS} is computed with the assumed $C_3/\Delta E$ profile and the definition of ΔE. Then with the constant exhaust velocity from the constant thruster model and the ΔV_{SEPS}, the delivered mass is computed for a different R-ratio case without using SEPTOP. If the trajectories in different missions significantly differ from each other, then this assumption in the similarity of $C_3/\Delta E$ profile is not valid. In this research, however, all trajectories have similar characteristics except the different R-ratios in order to compare the effect of R-ratio to the delivered mass performance of a trajectory.

Another important application of the $C_3/\Delta E$ profiles is its peak point. The peak point is interpreted as the maximum relative usage of C_3 and there is a change in thrusting strategy in the left and right parts of the peak point. The peak point is equivalent to the beginning of stationary state of ΔE profile in Fig. 4.

Because the $C_3/\Delta E$ profiles of other R-ratio cases are assumed to be identical with the known $C_3/\Delta E$ profile, the peak $C_3/\Delta E$ points of other R-ratio cases are also identical with the known peak point. The peak point is used as the border between the short TOF range and the remaining TOF range in the hybrid method. Therefore, the method that uses the rocket equation solution is applied for the TOF ≤ peak TOF (the TOF value for the peak point) and the previous method that uses k and C is applied for the TOF ≥ peak TOF.

The m_0 in Eq. (15) can be calculated by a

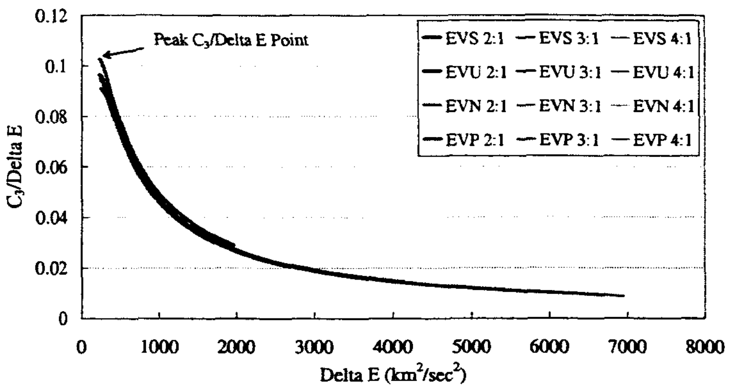

Figure 13. $C_3/\Delta E$ vs. ΔE, short TOF, all R-ratios, all target missions.

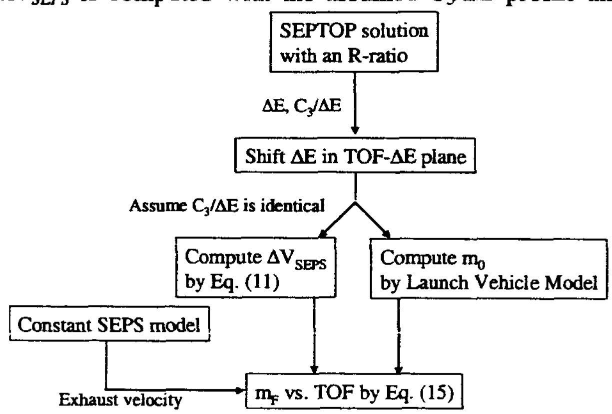

Figure 14. Procedure of hybrid method for short TOF range estimation.

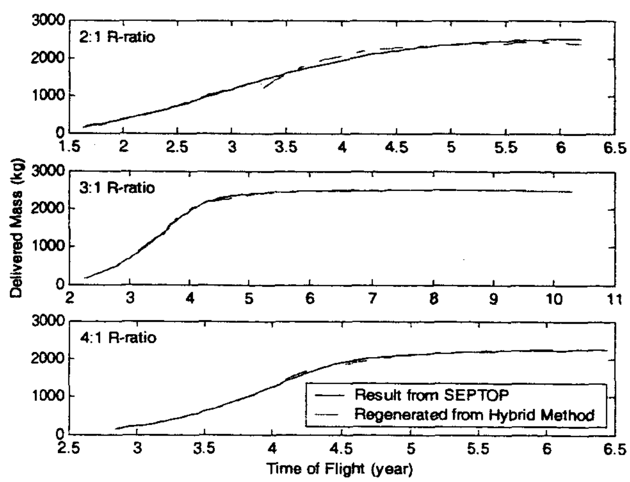

Figure 15. Hybrid Method Delivered mass comparison, the estimated versus SEPTOP, EVS mission.

given launch vehicle model if C_3 is known. The C_3 and ΔV_{SEPS} can be extracted from the assumed $C_3/\Delta E$ profile and the shifted ΔE profile as previously explained. The exhaust velocity c is not a constant for a SEPS but it is modeled as a constant with sufficient accuracy. Now, one can use the rocket equation solution to predict the delivered mass for the short TOF missions. The ΔE of 3:1 R-ratio case is chosen to be shifted and its $C_3/\Delta E$ is used for all other R-ratio cases. Figure 15 and Fig. 16 show the comparison between the estimated delivered mass curves and the delivered mass from SEPTOP for all R-ratio cases for EVS and EVP missions. The estimated delivered mass curves consist of two parts and their border is the peak TOF point of $C_3/\Delta E$ profile. It is clear that there is significant enhancement of the accuracy of the delivered mass estimation compared with Figs. 10 and 11 of the linear least square method. The maximum error in the short TOF range is less than 70 kg and the maximum error in the long TOF range is less than 100 kg for the hybrid method. For a comparison, the maximum error in the linear least square method was about 500 kg. The reasons of this accuracy enhancement are first, the similarity of the $C_3/\Delta E$ profiles in short TOF range of the different R-ratio cases and second, the better linear least square fitting with smaller TOF range for the long TOF range. On the other hand, the error of the hybrid method comes from the assumptions about the ΔE profile, $C_3/\Delta E$ profile and the modeling error of the constant exhaust velocity of SEPS for the short TOF range. The incomplete symmetry between the delivered mass profile and the ΔE profile and the linear least square error are the sources of the errors in the longer TOF range.

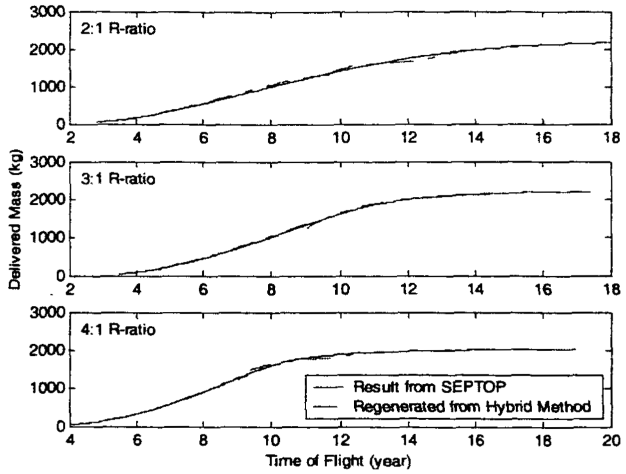

Figure 16. Hybrid Method Delivered mass comparison, the estimated versus SEPTOP, EVP mission.

V. Conclusion

Various R-ratio trajectories to outer planets are investigated in this paper. The trade-off between the launch energy and the onboard propulsive energy of SEPS for Venus gravity-assist missions is explained. An estimation method is presented for predicting mission performance for different R-ratio trajectories and can successfully regenerate the delivered mass profile only from one converged set of mission data and a given thruster model. Extensions of this method are currently being investigated.

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