# PROPAGATION OF FLEXURAL MODE AE SIGNALS IN
# GR/EP COMPOSITE PLATES

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## INTRODUCTION

It has been documented that AE signals propagate in thin plates as extensional and flexural plate modes. This was demonstrated using simulated AE sources (pencil lead breaks) by Gorman [1] on thin aluminum and gr/ep composite plates and by Gorman and Prosser [2] on thin aluminum plates. A typical signal from a pencil lead break source which identifies these two modes is shown in Figure 1. AE signals from transverse matrix cracking sources in gr/ep composite plates were also shown to propagate as plate modes by Gorman and Ziola [3]. Smith [4] showed that crack growth events in thin aluminum plates under spectrum fatigue loading produced signals that propagated as plate modes. Additionally, Prosser et al. [5] showed that AE signals propagated as plate modes in a thin walled composite tube.

This fact has important implications for the interpretation of AE data in thin plates, shells, and tubes. First, it has been demonstrated by Gorman and Prosser [2] that the source orientation can be determined by analysis of the plate mode amplitudes. Smith [4] pointed out how this could be used to discriminate real AE events from extraneous noise events. Second, Gorman [1] discussed how erroneous source location could be obtained using conventional first threshold crossing or peak arrival techniques because of the presence of plate modes which propagate with different and dispersive velocities. Such source location errors were substantiated by Ziola and Gorman [6] and an alternative method for source location based on cross-correlation of the flexural mode waves was demonstrated.

In order to apply the flexural mode cross-correlation source location technique, accurate knowledge of the dispersion of the flexural plate mode is needed. Furthermore, it has been pointed out by Mal et al. [7], Veidt and Sayir [8], and Dean [9] that measurements of flexural mode dispersion might be useful in determining the elastic constants of composite plates. In this research, measurements were made of the flexural mode dispersion on four different composite laminates. The ply layups for these laminates
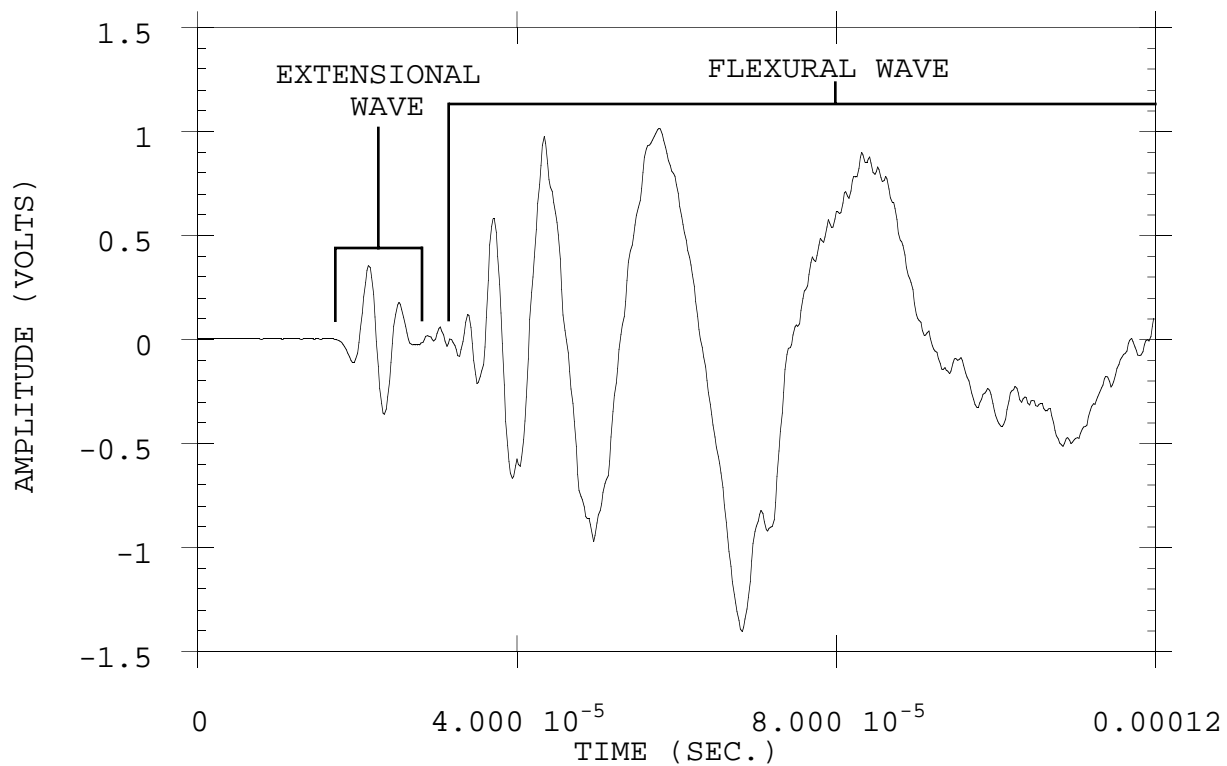
Figure 1    Typical AE signal in thin plate identifying extension-
al and flexural plate modes.

were $[0_{16}]$, $[0_4,90_4]_s$, $[0,90]_{4s}$, and $[0,45,-45,90]_{2s}$.  A Fourier phase
technique was used to measure the dispersion up to a frequency of 160 kHz.
on signals generated by pencil lead breaks (Hsu-Neilsen sources).

     For future applications of this source location technique, it would
be more convenient to analytically predict the dispersion behavior for a
given laminate with known material properties rather than make experimen-
tal measurements.  The ability of existing plate theories for predicting
flexural mode dispersion was investigated.  The experimentally measured
dispersion curves were compared with theoretical predictions based on
classical plate theory (CPT) using elastic moduli calculated from lami-
nated plate theory.  The lack of agreement between theory and experiment
at the higher frequencies demonstrated the limitations of CPT for compos-
ite materials. These are caused by neglecting the effects of shear defor-
mation and rotatory inertia.  A higher order plate theory (HOPT) which
includes these effects was also used to predict the dispersion behavior
of this mode.  The predictions based on the HOPT were in much better agree-
ment with the experimental measurements.

<div align="center">THEORY</div>

     In this research, two theoretical approaches were used for predicting
the dispersion of flexural mode waves in gr/ep composite laminates.  The
first theoretical predictions were based on classical plate theory (CPT).
This is a widely used approximate theory for describing motion in thin
plates where the wavelength ($\lambda$) is much larger than the plate thickness
(h).  For flexural waves, the plate is assumed to be under a state of pure

bending in which plane sections of the plate remain plane and perpendicular to the midplane of the plate. Thus, shear deformation is not included in this theory. A state of plane stress is also assumed and the effects of rotatory inertia are neglected. A number of authors have presented CPT in detail including Graff [10] who derives the equation of motion for isotropic materials and Whitney [11] who includes the effects of anisotropy. The CPT equation of motion for an orthotropic composite laminate in the absence of body forces is

$$D_{11}\frac{\partial^4 w}{\partial x^4} + 4D_{16}\frac{\partial^4 w}{\partial x^3 \partial y} + 2\left(D_{12} + 2D_{66}\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + 4D_{26}\frac{\partial^4 w}{\partial x \partial y^3} + D_{22}\frac{\partial^4 w}{\partial y^4} + \rho h \frac{\partial^2 w}{\partial t^2} = 0 \qquad \text{Eq. 1}$$

where the $D_{ij}$'s are the anisotropic bending stiffness coefficients obtained from laminated plate theory as described by Whitney [11] or Tsai and Hahn [12]. In the previous equation, w is the displacement along the z axis or normal to the plane of the plate, x and y are orthogonal axes in the plane of the plate, and $\rho$ is the density.

The dispersion behavior for the flexural mode using CPT is obtained by substituting the displacement for a plane wave propagating in an arbitrary direction into the equation of motion. This displacement is of the form

$$w = A_0 e^{i\left(\omega t - kl_x x - kl_y y\right)} \qquad \text{Eq. 2}$$

where $A_0$ is the amplitude, $\omega$ is the angular frequency, $l_x$ and $l_y$ are the direction cosines of the direction of propagation, and k is the wavenumber. After substitution and reduction of terms, the resulting CPT dispersion relation is found to be

$$c_f = \sqrt[4]{\frac{S}{\rho h}}\sqrt{\omega} \qquad \text{Eq. 3}$$

where $c_f$ is the velocity of the flexural mode and

$$S = D_{11}l_x^4 + 4D_{16}l_x^3 l_y + 2\left(D_{12} + 2D_{66}\right)l_x^2 l_y^2 + 4D_{26}l_x l_y^3 + D_{22}l_y^4 \qquad . \qquad \text{Eq. 4}$$

Thus, in CPT, the velocity is dependent on the direction of propagation and increases as the square root of the frequency without limit.

The second theory used to predict the dispersion was a higher order plate theory (HOPT) which includes the effects of shear deformation and rotatory inertia. This theory was put forth by Tang et al. [13] following earlier work by Yang et al [14] which was an extension of work by Mindlin [15]. The dispersion behavior for a symmetric orthotropic laminate predicted by this theory is obtained when the determinant of the following matrix of coefficients is set equal to zero

$$\begin{vmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{vmatrix}$$ 

Eq. 5

where

$$M_{11} = D_{11}k^2l_x^2 + 2D_{16}k^2l_xl_y + D_{66}k^2l_y^2 + A_{55} - I\omega^2$$

Eq. 6

$$M_{12} = D_{16}k^2 + (D_{12} + D_{66})\,k^2l_xl_y$$

Eq. 7

$$M_{13} = iA_{55}kl_x$$

Eq. 8

$$M_{21} = D_{16}k^2 + (D_{12} + D_{66})\,k^2l_xl_y$$

Eq. 9

$$M_{22} = D_{66}k^2l_x^2 + 2D_{16}k^2l_xl_y + D_{22}k^2l_y^2 + A_{44} - I\omega^2$$

Eq. 10

$$M_{23} = iA_{44}kl_y$$

Eq. 11

$$M_{31} = -iA_{55}kl_x$$

Eq. 12

$$M_{32} = -iA_{44}kl_y$$

Eq. 13

and

$$M_{33} = A_{55}k^2l_x^2 + A_{44}k^2l_y^2 - \rho^*\omega^2 \quad .$$

Eq. 14

In the previous equations,

$$(\rho^*,I) = \int_{z=-\frac{h}{2}}^{z=\frac{h}{2}} \rho\,(1,z^2)\,dz \quad ,$$

Eq. 15

and

$$A_{ij} = \kappa_i\kappa_j \int_{z=-\frac{h}{2}}^{z=\frac{h}{2}} (Q_{ij})_1\,dz \quad \text{for i,j = 4,5.}$$

Eq. 16

In Eq. 16, the $\kappa_i$ are shear correction factors which were determined to yield the best agreement with three dimensional elasticity theory when $\kappa_i^2 = 5/6$. The subscript l refers to the l'th layer of the laminate and the

$Q_{ij}$ are the stiffnesses for the l'th layer. Solving the determinant for the wavenumber as a function of $\omega$ yields a cubic in $k^2$. Only the root which approaches zero as the frequency approaches zero is the correct root. Once k as a function of $\omega$ is known, the phase velocity is determined as a function of frequency using the relation

$$c_f = \frac{\omega}{k} \quad .$$
Eq. 17

## EXPERIMENT

The composite laminates used in this study were made of AS4/3502 graphite/epoxy. All four laminates consisted of sixteen plys and had a nominal thickness of 2.26 mm. The dimensions were 0.508 m. along the x direction (0 degree ply direction) and 0.381 m. along the y direction. Measurements were made along the 0 degree (x direction), 45, and 90 degree directions for all four laminates. The nominal lamina properties for this material as obtained from the manufacturer are given in Table 1. These values were used in the laminated plate theory calculations to obtain the bending stiffness coefficients needed for the CPT and HOPT dispersion calculations.

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Lamina thickness = $1.413 \times 10^{-4}$ m.

Density = 1550 kg/m$^3$      Fiber volume = 60%

$Q_{xx}$ = 145.5 GPa.    $Q_{xy}$ = 2.91 GPa    $Q_{yy}$ = 9.69 GPa    $Q_{ss}$ = 5.97 GPa

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Table 1     Lamina properties of AS4/3502 graphite epoxy.

A Fourier phase technique was used for the measurement of the flexural mode dispersion. This technique has been described by a number of authors including Sachse and Pao [16], Pao and Sachse [17], Veidt and Sayir [8], Dean [9], and Alleyne and Cawley [18]. In this technique, the elastic wave is detected at two different distances away from the source of the wave along the direction of propagation of interest. The phase ($\varphi$) of the wave at each position at a given frequency (f) is determined by performing a Fourier Transform on the signals and computing the phase. The phase must be unwrapped to remove the $2n\pi$ uncertainty. The phase difference ($\Delta\varphi$) over the distance between the two transducers ($\Delta x$) is then computed for each frequency. The wave number and velocity are then calculated at each frequency by

$$k(f) = \frac{\Delta\varphi(f)}{\Delta x}$$
Eq. 18

and

$$c\,(f)\ =\ \frac{2\pi f}{k\,(f)}\ =\ \frac{2\pi f \Delta x}{\Delta \phi\,(f)} \qquad .$$

The experimental setup used for these experiments is shown in Figure 2. The two receiving sensors were Panametrics 3.5 MHz broad band ultra-
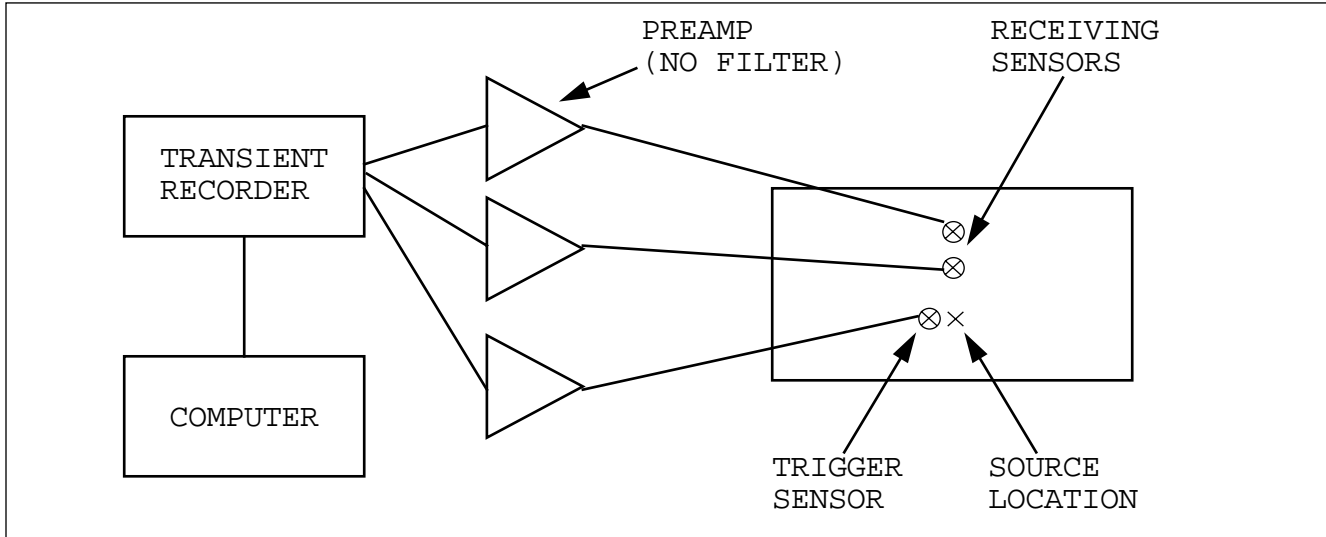


Figure 2      Experimental setup for flexural velocity measurements
            in composite plates.

sonic transducers.  These sensors have been shown by Prosser [19] to pro-
vide flat frequency, displacement sensitivity response to these low
frequency plate waves.  The trigger sensor was a Physical Acoustics Cor-
poration (PAC) model R15.  The preamplifiers (PAC model 1220A) were set
at 40 dB amplification with no filtering. The source was a pencil lead
break repeated with the transducers at separations of 1.91, 2.54, 3.18,
3.81, and 4.45 cm.  An average velocity and standard deviation for the
five different measurements was computed.  The source and receivers were
kept as nearly in the center of the plate as possible to minimize reflec-
tions.  The waveforms were digitized at a sampling frequency of 1 MHz with
a LeCroy 6810 transient recorder and then transferred to a personal com-
puter for processing.

    Prior to computing the FFT to determine the phase, the higher fre-
quency extensional mode and the reflections arriving later in the flexural
mode were zeroed out in the computer.  Previous Fourier analysis of the
flexural mode signals when digitized at much higher sampling frequencies
(100 MHz) showed that the maximum frequency component in the flexural mode
was about 200 kHz.  Thus, aliasing was not a concern even at the low sam-
pling frequency of 1 MHz.

## RESULTS AND DISCUSSION

    The average measured velocities for the 0, 45, and 90 degree direc-
tions in the $[0_{16}]$ graphite epoxy plate are plotted in Figure 3 to Figure
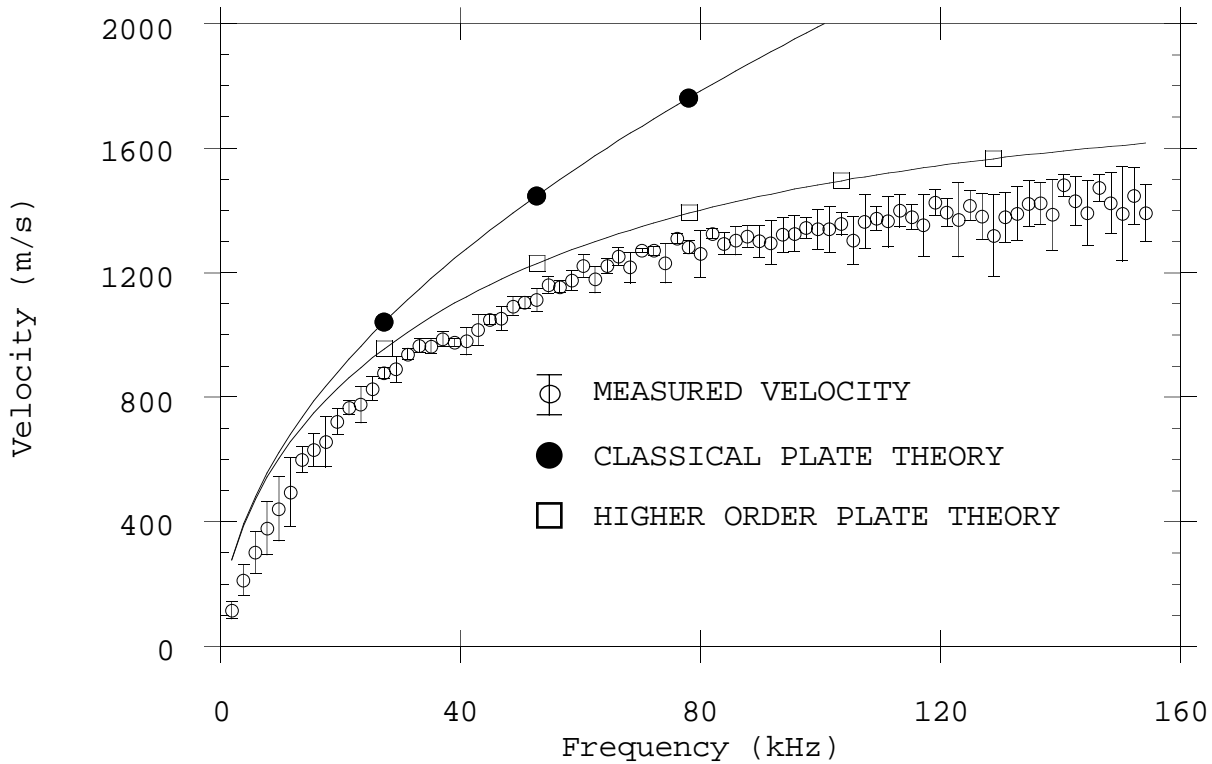5 with the standard deviation of the measured values indicated by error

Figure 3    Measured and theoretical flexural dispersion for 0 degree propagation in $[0_{16}]$ graphite/epoxy plate.

bars. The predicted velocity dispersion curves for CPT and HOPT are also shown in these plots. The agreement between measurement and HOPT is excellent for the 90 degree propagation direction. For propagation at 45 and 0 degrees, the measured values are consistently less than those predicted by HOPT.  The discrepancy in the HOPT predicted and measured velocities at 0 and 45 degrees was believed to be caused by differences in actual material properties from the nominal ones used in the calculations.  This is common for these materials and is due to fiber volume variations, cure processing variations, and variations in resin chemistry.  These discrepancies were consistent with differences in predicted and measured extensional velocities for these same plates reported by Prosser [19].

The effect of shear and rotatory inertia is clear when the CPT and the HOPT are compared in these plots. CPT and HOPT are in agreement at very low frequencies in all cases where the approximations of CPT are valid. The discrepancy between the two increases with increasing frequency as the velocity predicted by CPT increases without bound.

It is also apparent that the difference between HOPT and CPT is much greater for the 0 and 45 degree directions than for the 90 degree direction.  This is expected since the shear modulus is much smaller in comparison to the Young's modulus in those directions.  CPT is based on the assumption of no shear deformation which implies an infinite shear modulus.  Thus, better agreement is provided by CPT when the ratio of the shear modulus to the Young's modulus is larger.
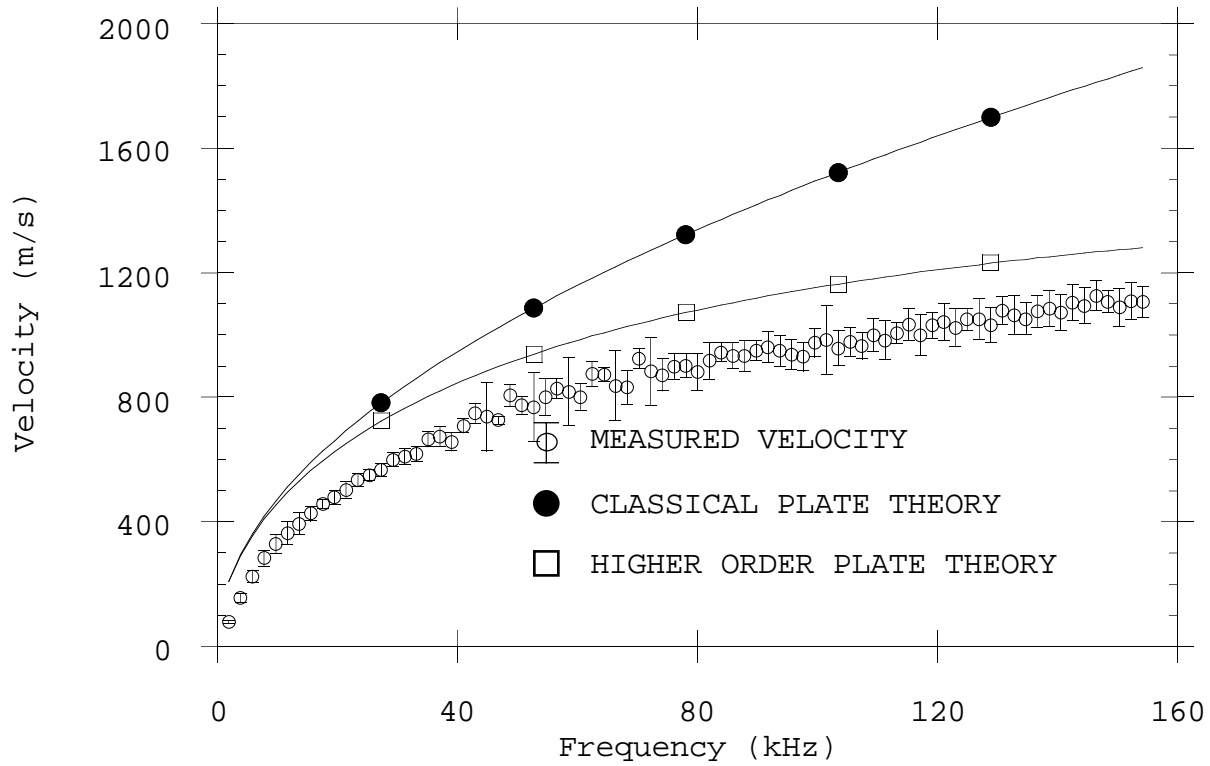
Figure 4    Measured and theoretical flexural dispersion for 45 de-
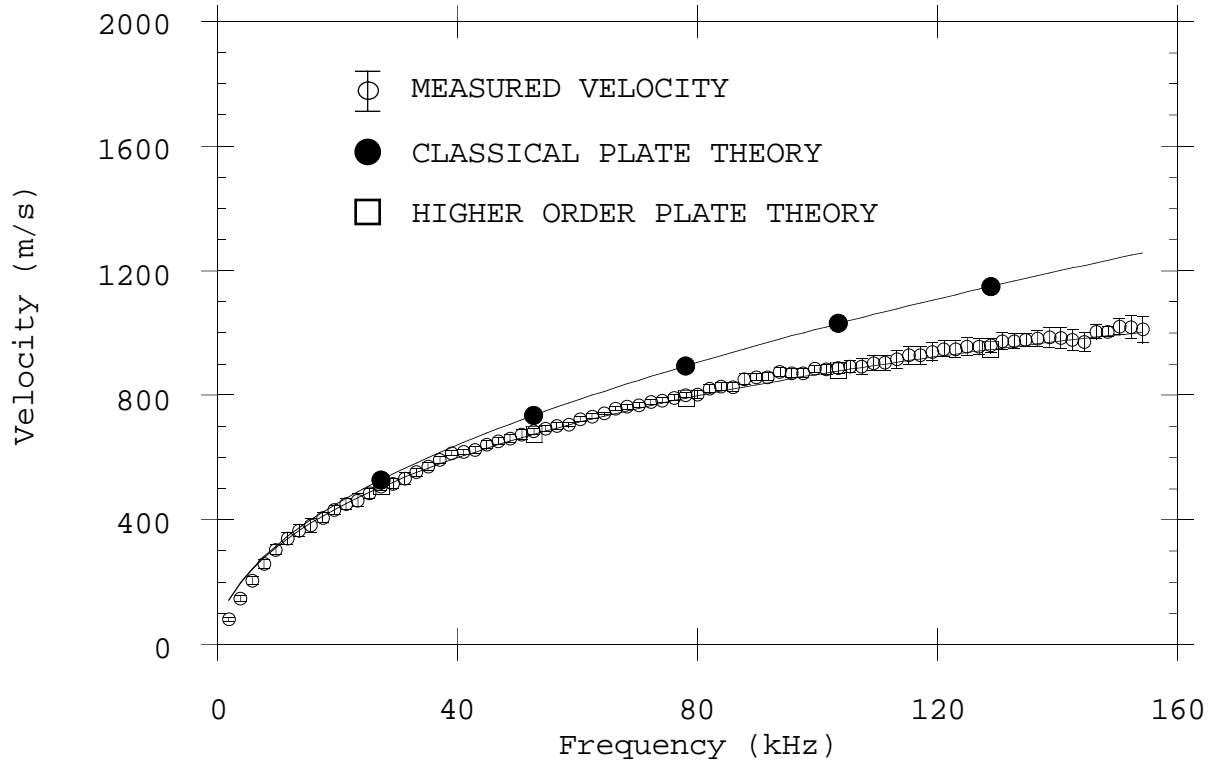gree propagation in [0₁₆] graphite/epoxy plate.



Figure 5    Measured and theoretical flexural dispersion for 90 de-
gree propagation in [0₁₆] graphite/epoxy plate.

A plot of the measured velocities and the HOPT predictions for the [0,90]$_{4s}$ plate is shown in Figure 6. In order to better view this com-
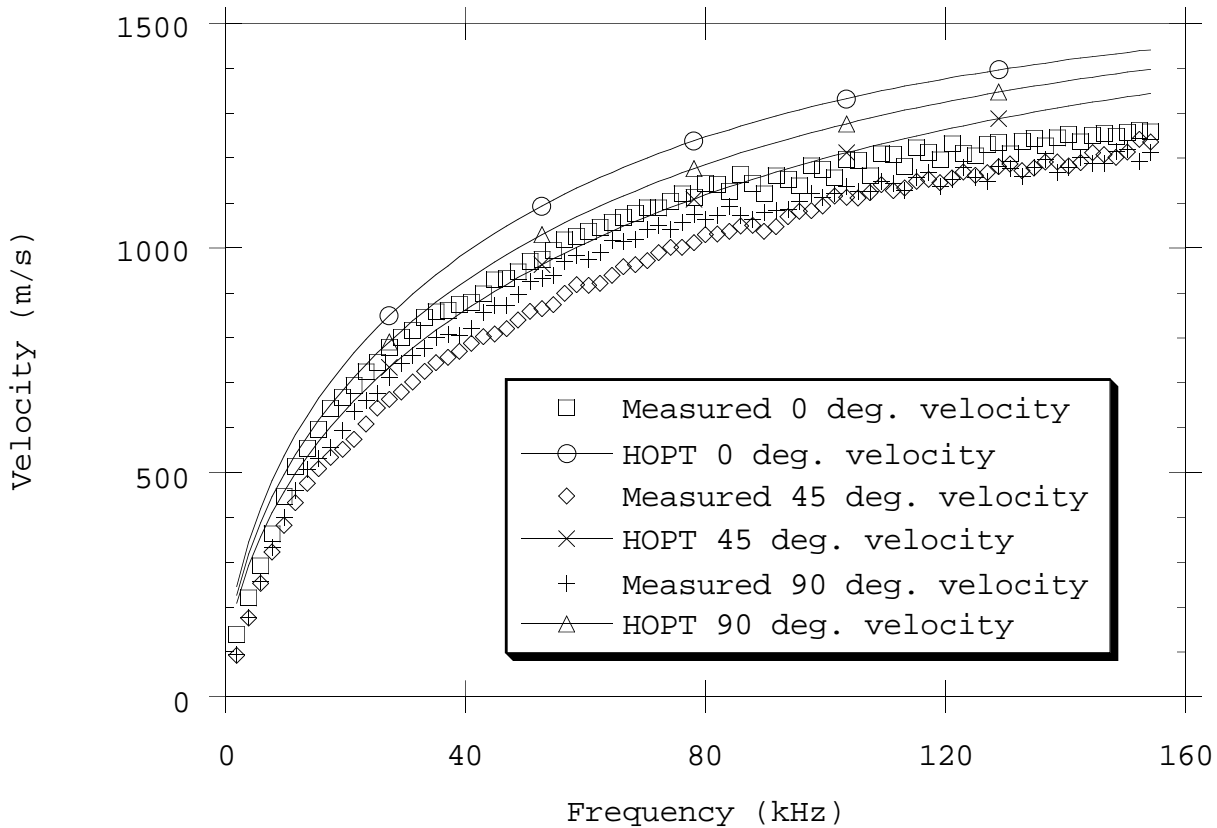


Figure 6   Measured and HOPT theoretical flexural dispersion for 0, 45, and 90 degree propagation directions in [0,90]$_{4s}$ graphite/epoxy plate.

plicated graph, the CPT predictions and the experimental uncertainties are not shown in the plot. In this plate, the measured velocities were less than theoretical predictions for all three directions of propagation. However, the measured and theoretical velocities occurred in the same order with the 0 degree velocity being the largest, followed next by the 90 degree velocity, and with the 45 degree velocity the smallest. The results for the other two laminates were similar with the measured veloc-ities consistently less than predicted by HOPT. This again seems to indi-cate that the actual material properties are less than the nominal properties used in the theoretical calculations.

In summary, a Fourier phase technique was used to measure the disper-sion of the flexural plate mode in four composite laminates. CPT was shown to be of limited value in predicting the dispersion of this mode because it assumes that the effects of shear deformation and rotatory inertia are negligible. HOPT, which includes these effects, gave much better agree-ment with the measured values. However, there was a consistent discrep-ancy between theory and experiment believed to be due to variations in actual material properties from those used in the calculations. Thus, HOPT looks promising for predicting the dispersion behavior of the flex-ural mode for use in the cross-correlation source location technique.

However, accurate material properties are needed.  If these are not known, an experimental technique for measuring this dispersion has been presented.

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