NONLINEAR ELASTIC EFFECTS ON THE ENERGY FLUX DEVIATION OF ULTRASONIC WAVES IN GR/EP COMPOSITES

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INTRODUCTION

In isotropic materials, the direction of the energy flux (energy per unit time per unit area) of an ultrasonic plane wave is always along the same direction as the normal to the wave front. In anisotropic materials, however, this is true only along symmetry directions. Along other directions, the energy flux of the wave deviates from the intended direction of propagation. This phenomenon is known as energy flux deviation and is illustrated in Fig. 1.. The direction of the energy flux is dependent on the elastic coefficients of the material [1,2]. This effect has been demonstrated in many anisotropic crystalline materials. In transparent quartz crystals, Schlieren photographs have been obtained which allow visualization of the ultrasonic waves and the energy flux deviation [3].

The energy flux deviation in graphite/epoxy (gr/ep) composite materials can be quite large because of their high anisotropy. The flux deviation angle has been calculated for unidirectional gr/ep composites as a function of both fiber orientation and fiber volume content [4]. Experimental measurements have also been made in unidirectional composites [5]. It has been further demonstrated that changes in composite materials which alter the elastic properties such as moisture

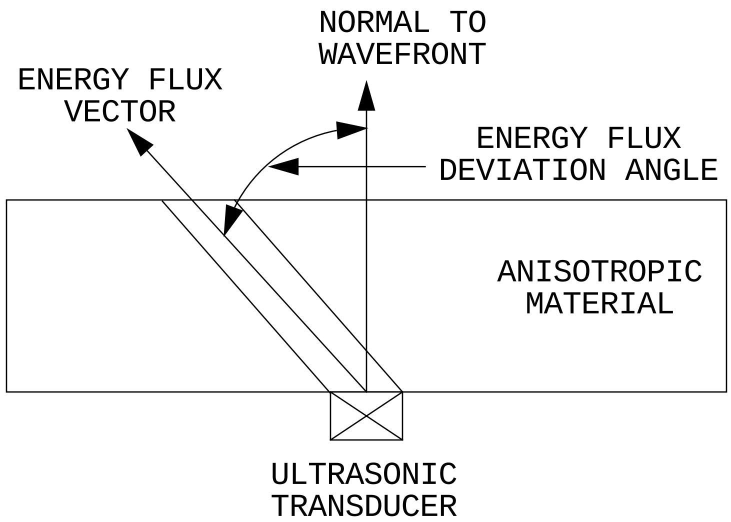

Fig. 1. Illustration of energy flux deviation of an ultrasonic wave in an anisotropic material.

absorption by the matrix or fiber degradation, can be detected nondestructively by measurements of the energy flux shift [6].

In this research, the effects of nonlinear elasticity on energy flux deviation in unidirectional gr/ep composites were studied. Because of elastic nonlinearity, the angle of the energy flux deviation was shown to be a function of applied stress. This shift in flux deviation was modeled using acoustoelastic theory and the previously measured second and third order elastic stiffness coefficients for T300/5208 gr/ep [7,8]. Two conditions of applied uniaxial stress were considered. In the first case, the direction of applied uniaxial stress was along the fiber axis (x_3) while in the second case it was perpendicular to the fiber axis along the laminate stacking direction (x_1).

THEORY

Assuming linear elasticity, the components of the energy flux vector (E_j) are a function of the linear elastic stiffness coefficients (c_{ijkl}) and the spatial and time derivatives of the displacement vector (u_i) [1,2]. The equation is given by

$$E_j = -c_{ijkl}\left(\frac{\partial u_k}{\partial x_l}\right)\left(\frac{\partial u_i}{\partial t}\right) \tag{1}$$

where the Einstein summation convention on repeated indices is assumed throughout this paper. The angle of energy flux deviation can then be calculated as the angle between the energy flux vector and the normal to the plane wave front.

To include nonlinear elastic effects on the energy flux deviation, acoustoelastic theory is used. This theory predicts an "effective" linear elastic stiffness tensor (c^*_{ijkl}) that is a function of the second and third order elastic coefficients and the applied stress (σ_{ij}) [9]. The expression for the "effective" stiffness tensor is given by

$$c^*_{nlij} = k_{nlij} + \sigma_{nj}\delta_{li} \tag{2}$$

where δ_{li} is the Kronecker delta and k_{nlij} is given by

$$\begin{aligned} k_{nlij} = {} & c_{nlij} + c_{rlij}\varepsilon_{nr} + c_{nsij}\varepsilon_{ls} + c_{nlpj}\varepsilon_{ip} + c_{nliq}\varepsilon_{jq} \\ & + c_{nlijuv}\varepsilon_{uv} + c_{rlijuv}\varepsilon_{uv}\varepsilon_{nr} + c_{nsijuv}\varepsilon_{uv}\varepsilon_{ls} \\ & + c_{nlpjuv}\varepsilon_{uv}\varepsilon_{ip} + c_{nliquv}\varepsilon_{uv}\varepsilon_{jq} \ . \end{aligned} \tag{3}$$

In this expression, c_{ijkluv} are the third order elastic stiffness coefficients and ε_{ij} are the strains resulting from the applied stresses. If the applied stresses are within the linear elastic regime, the strains are given by

$$\varepsilon_{ij} = s_{ijkl}\sigma_{kl} \tag{4}$$

where s_{ijkl} are the linear elastic compliances which are the inverse of the stiffnesses.

Thus, if the linear elastic stiffnesses and compliances, the third order elastic stiffnesses, and the applied stresses are known, an "effective" stiffness tensor can be calculated. This can then be used to compute the changes in energy flux deviation as a function of applied stress which are a result of nonlinear elastic effects.

MODEL CALCULATIONS

The effect of stress on the energy flux deviation was modeled for unidirectional T300/5208 gr/ep which was assumed to be transversely isotropic. The fiber axis was designated to be the x_3 axis while the laminate stacking direction which is perpendicular to the fibers was chosen to be the x_1 axis. The values of the previously measured nonzero independent linear elastic stiffness and compliance coefficients for this material are listed in Table 1. Likewise, the values of the previously measured nine independent nonzero third order stiffness coefficients are given in Table 2. In both tables, the contracted subscript matrix notation is used.

c_{ij}	(GPa)	s_{ij}	$(GPa)^{-1}$
c_{11}	14.26	s_{11}	0.092
c_{12}	6.78	s_{12}	-0.042
c_{13}	6.5	s_{13}	-0.003
c_{33}	108.4	s_{33}	0.0096
c_{44}	5.27	s_{44}	0.190

Table 1. Linear elastic stiffness and compliance coefficients.

c_{ijk}	(GPa)	c_{ijk}	(GPa)
c_{111}	-196	c_{155}	-49.1
c_{112}	-89	c_{344}	-47
c_{113}	-4	c_{133}	-236
c_{123}	65	c_{333}	-829
c_{144}	-33.4		

Table 2. Third order (nonlinear) elastic stiffness coefficients.

Calculations were performed for elastic waves propagating in the x_1x_3 plane. As in any anisotropic bulk material, three elastic waves will propagate along any direction in this plane. Of the three waves propagating in this plane, one of them is always a pure mode transverse (PT) wave with its particle displacement polarized perpendicular to the x_1 and x_3 axes (i.e. along the x_2 axis). The other two modes are quasi- mode waves with components of particle displacements both along their direction of propagation and perpendicular to it. One is a quasi-transverse (QT) mode wave while the other is a quasi-longitudinal (QL) mode wave. All three modes suffer energy flux deviation except for propagation along the fiber axis (x_3) and the laminate stacking axis (x_1). These are symmetry directions, along which, all three modes are pure mode waves and none suffers energy flux deviation.

For fiber orientations of less than 60 degrees, the QL mode propagates with a faster velocity and its energy flux deviates toward the fiber direction. The energy flux of the QT mode deviates in the opposite direction toward the x_1 axis. Between 60 and 90 degrees, these modes transition with the QL mode becoming the QT mode and vice-versa. Because of the complexity of the wave behavior in this region, it was excluded from this study.

The energy flux deviation was first computed for the condition of no applied stress. Then, calculations were performed for two different states of uniaxial stress. The first was stress along the fiber direction (x_3) of a magnitude of 1 GPa. The second was along the laminate stacking (x_1) direction with a magnitude of 0.1 GPa. These values are near the reported ultimate strengths of this material along the respective directions. This allows an estimate of the maximum effects of stress on energy flux deviation. For both conditions, the change in the energy flux deviation angle from the condition of zero applied stress was computed over the range of propagation directions of 0 to 60 degrees from the fiber axis at two degree intervals.

RESULTS AND DISCUSSIONS

The flux deviation angles of the three modes as a function of fiber orientation at zero stress are plotted in Fig. 2. The fiber orientation angle is the angle between the fibers and the normal to the wave front or the intended direction of propagation. A positive flux deviation angle implies the energy deviates away from the fiber direction toward the x_1 axis while a negative deviation means that the energy deviates toward the fibers. Over this range of fiber orientation angles, the energy of the QL and PT mode waves deviates toward the fibers while that of the QT deviates away from the fibers.

In Fig. 3. the change in the predicted energy flux deviation due to the application of stress along the fibers is plotted as a function of fiber orientation angle. The energy of the QT mode wave suffers the largest shift in flux deviation reaching a maximum of three degrees at a propagation direction of approximately 20 degrees with respect to the fiber direction. That of the PT mode changes by a smaller amount in the

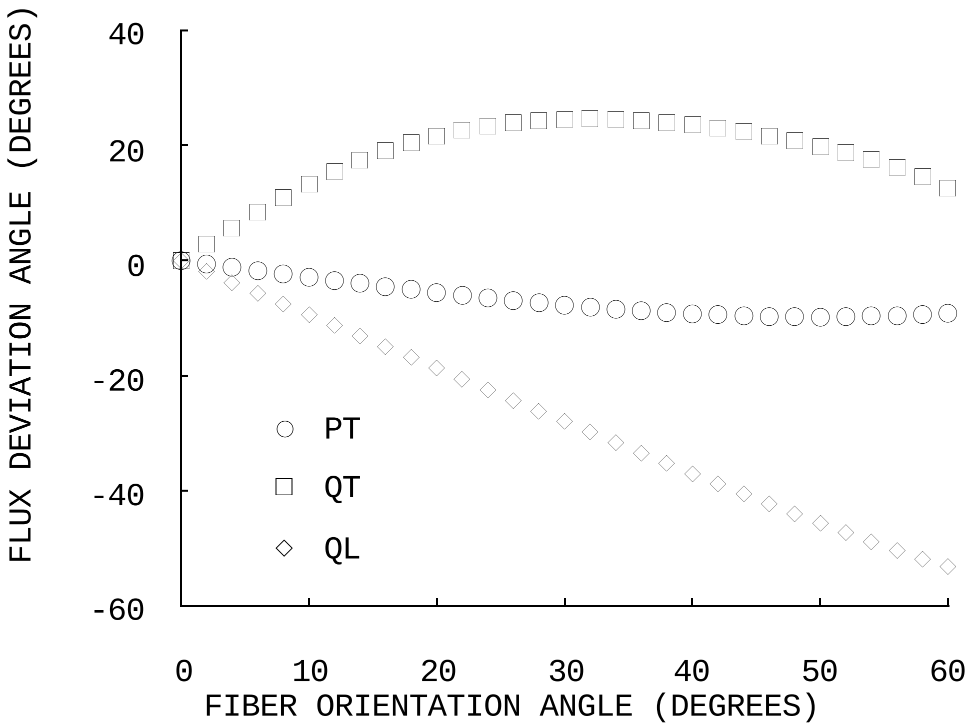

Fig. 2. Energy flux deviation as a function of fiber orientation angle for case of no applied stress.

opposite direction while the QL mode wave suffers a negligible shift.

The relative magnitudes of the flux deviation shifts of the different modes can be explained qualitatively by considering the ratios of the magnitude of the nonlinear elastic coefficients to the linear coefficients. The primary elastic coefficients affecting the propagation of the PT and QT modes are those dominated by the matrix properties. These are c_{11}, c_{12}, c_{44}, c_{111}, and c_{112}. The magnitudes of the nonlinear coefficients are over an order of magnitude larger than the linear coefficients in this case. However, the ratios of the nonlinear to linear coefficients which dominate the propagation of the QL wave (c_{33}, c_{133}, and c_{333}) are much smaller even though the magnitudes of the individual coefficients are larger. Therefore, the effect of nonlinear elasticity on the energy flux deviation should be much smaller for the QL mode wave. The previous measurements of the effect of matrix degradation on energy flux deviation also showed a larger change in the flux deviation of the QT mode wave with almost no change in the QL mode wave [6].

The shift in energy flux deviation due to applied stress

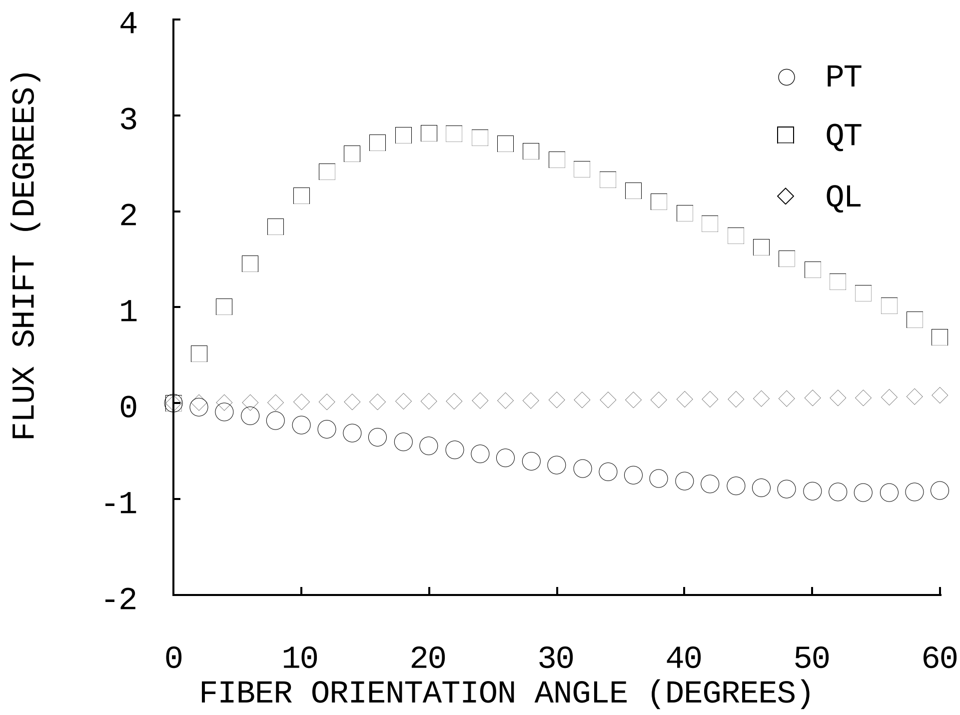

Fig. 3. Change in energy flux deviation due to a 1 GPa stress applied along the x_3 or fiber direction.

along the x_1 axis is shown in Fig. 4. Again the QT mode wave suffers the largest change in flux deviation angle while the QL mode is almost unchanged. It is interesting to note that the direction of the change in energy flux is in the opposite direction from the case of applied stress along the fiber direction.

These calculations demonstrate the effect of nonlinear elasticity on the energy flux deviation of ultrasonic waves in gr/ep composite materials. The modes indicate the angles of fiber orientation and wave modes that suffer the maximum shift in flux deviation for the cases of applied stress considered. This will aid in future experimental measurements of this effect. Although the models presented were for bulk waves propagating through a thick composite material, the same effect is expected for plane plate waves propagating in thin plates. The longer propagation paths possible along plates would make the effect more measurable and thus could improve the stress resolution possible. This effect could be used to develop a new nondestructive method for monitoring stress in composite materials or as a new method for measuring their nonlinearity.

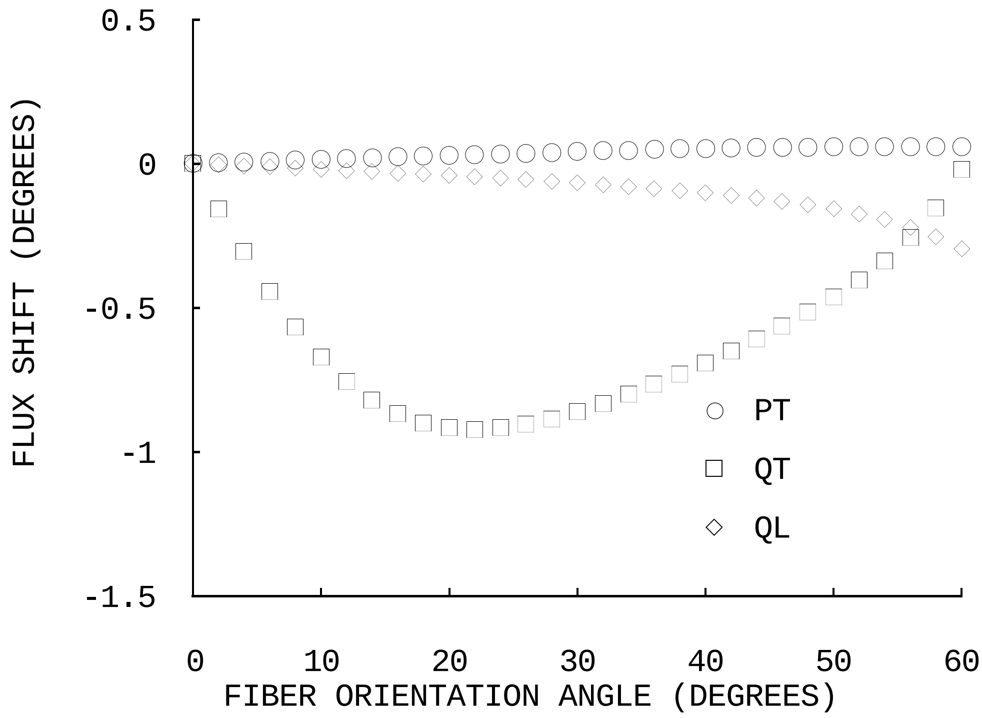

Fig. 4. Change in energy flux deviation due to a 0.1 GPa stress applied along the x_1 or laminate stacking direction.

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