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ABSTRACT

An approach is presented to treat computational aerodynamics as a process, subject to the fundamental quality assurance principles of process control and process improvement. We consider several aspects affecting uncertainty for the computational aerodynamic process and present a set of stages to determine the level of management required to meet risk assumptions desired by the customer of the predictions.

NOMENCLATURE

CA	Computational Aerodynamics
$C_{l,max}$	maximum section lift coefficient
r/c	leading-edge-radius to chord ratio
R_c	chord Reynolds number, $U_{oo}c/\nu$
SPC	Statistical Process Control
V&V	Verification and Validation

INTRODUCTION

Computational aerodynamics has a rich and long history as methods have evolved hand-in-glove with the evolution of high-speed computing. These methods are anchored in fluid mechanical and aerodynamic theory, and a hierarchy of techniques has been developed over many decades that differ primarily in fidelity, flow physics representation, and computational efficiency. Although most current work is focused on Computational Fluid Dynamics (CFD), Computational Aerodynamics (CA) embraces a broader scope than CFD. Computational Aerodynamics has as an end objective the development of full-scale vehicle capability as well as the a-priori prediction of the full-scale vehicle properties. As such, this entails the use of one or more CFD methods, reduced physics methods, prediction or extrapolation techniques, and calibration information that can come from experimentation. The particular suite of methods used for CA is driven by a combination of technical issues (such as method capability) and business factors (such as schedule, resource limitations, historical approach, etc.)

Two contrasting views toward computational aerodynamics are presented in Figure 1 [Zang 2002, and Wahls 2002] in the context of computational fluid dynamics. In this figure, CFD utility is illustrated for a notional full-envelope range of vehicle operating conditions. Figure 1a illustrates the current or traditional approach to CFD that is characteristic of deterministic CFD. Here, CFD is illustrated to provide trusted predictions (acceptable accuracy) for both low-speed and cruise attached-flow aerodynamics. This has resulted from sustained modeling efforts of these flows. However, separated and unsteady flow effects can be important for a significant fraction of the overall flight envelope. Modeling for separated and unsteady flows has proven to be a daunting task (as have experimental studies), and as such CFD has not demonstrated sufficient accuracy for these flows.

An alternate view is presented in Figure 1b and is referred to as uncertainty-based CFD. This view is the focus of the present paper. Emphasis with this approach is placed upon quantifying the uncertainty of computational aerodynamic methods throughout the flight domain. This means quantifying not only the value of some prediction (e.g., lift), but also the error bounds and confidence level associated with such a prediction, and doing so in the context of a well-defined and repeatable process. Because accuracy requirements are not necessarily as stringent in the whole envelope as they are at cruise conditions, the uncertainty-based CFD results could provide the confidence needed to address off-cruise-condition requirements. Such information can also then be related to risk issues for a decision maker in the context of vehicle design, development, or modification processes.

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The overall goal for computational aerodynamics uncertainty therefore is to establish a process to produce credible, quantified, unambiguous, and enduring statements of computational uncertainty for aerodynamic methods. We also anticipate that, with such a process, the quality of aerodynamic predictions could be assured the first time the process is performed, such that the current practice of adjusting computations to match experimental findings could be greatly reduced or even eliminated. Under these circumstances, computational aerodynamics could be more heavily relied upon for aerodynamic predictions.

To achieve confidence in aerodynamic predictions will require confidence in error bounds. We need to quantify the error bounds as they presently exist and also understand the sources and mechanisms of the error. The impact of these uncertainties is a separate consideration and more the concern of the customer of the computational predictions. Mission priorities then dictate which error bounds warrant further research and reduction.

Here we are drawing a clear distinction between the Voice of Customer and the Voice of the Process as enunciated by Wheeler and Chambers [1992]. The Voice of the Customer is the specifications required, including tolerances. The Voice of the Process for an overall computational aerodynamics result would be what the computational process produces, including uncertainties, as long as the process continues to operate stably. These two Voices must be considered and dealt with separately if the notion of uncertainty-based CA is to be useful. It should be noted that the Voice of the Process has no meaning if the process is not stable [Wheeler and Chambers 1992].

For example, the AIAA Applied Aero Technical Committee conducted a Drag Prediction Workshop in 2001 to evaluate CFD prediction of transonic cruise drag predictions for transport configurations. Workshop participants calculated forces and moments on the DLR-F4 transport model at the nominal cruise condition. This configuration was chosen because there were data available from three different wind tunnels. A total of 35 flow solutions were contributed from 14 different CFD codes ranging from research codes to commercially available codes. Hemsch [2002] performed a statistical analysis of the results and determined that the CFD results exhibit a 2-sigma confidence interval of roughly +/- 40 counts of drag (1 count of drag corresponds to a C_D change of 0.0001) at the cruise condition and the experimental data exhibits a 2-sigma confidence interval of roughly +/- 8 counts. These results represent the Voice of the Process. Aircraft manufacturers report that they require drag predictions within +/- 1 count of drag. This represents the Voice of the Customer. Clearly, neither the CFD predictions nor the wind tunnel data are capable of meeting this requirement.

In almost all cases, process improvement requires two steps: first, elimination of assignable causes in order to create a stable, predictable process and second, modification of the process, if necessary, to reduce the residual variation. The DPW workshop participants identified that the grids used for the predictions were inadequate. For this particular application, lack of grid convergence led to an unpredictable process (14% of the solutions were outside the statistical limit boundaries of the core solutions) and to process variation considerably larger than required (+/- 40 counts versus +/- 1 count respectively.) This issue is being addressed in the second AIAA Drag Prediction Workshop scheduled for the summer of 2003. The results of the first workshop clearly indicate the need for more rigorous processes to quantify and manage uncertainty for computational aerodynamics.

Computational uncertainty assessments embrace a variety of technologies and issues. In the next section, we review some critical CA uncertainty process considerations. In the following section, we discuss fundamental management issues for implementing the CA uncertainty process in different risk assumption environments.

PROCESS CONSIDERATIONS

Some considerations of prediction will first be reviewed. This is followed by a discussion of inferences that may be drawn from the computational simulation. Finally, a list of verification, validation and uncertainty elements is presented.

Prediction to Unvalidated Conditions

Prediction has at least three different usages in terms of computational aerodynamics. First, there is a computational prediction for a point-wise match to data from an independent source. Here, we often see an a-posteriori comparison of a computation (usually CFD) against experiment. Second, prediction is used in the sense of interpolation to conditions that fall within those already anchored by a comparison between the computation and benchmark information. Third, prediction is used in the sense of an extrapolation to conditions that fall outside the domain for which the computation is anchored by independent data. This last form of prediction is the most desired and, unfortunately, usually comes with the highest uncertainty.

This extrapolation form of prediction (e.g., from ground-based to full-scale Reynolds number) entails many other considerations separate from traditional verification and validation issues. As such there can be separate and distinct sources of uncertainty associated with the prediction process itself. These include issues that could range from ground-based test technique concerns (e.g., wind-tunnel wall interference, etc.) to full-scale vehicle medium uncertainty, such as for planetary entry.

Trucano [1998] brought a noteworthy perspective toward prediction by emphasizing the consequences associated with prediction uncertainty. This helps establish how much confidence is needed in the error estimate. Consequences in the prediction (or full-scale) domain need to be considered heavily in setting priorities for which problems get worked, and how they are worked. There may be high consequence issues, with relatively simple physics, that would be more important to address than lower consequence issues that would none-the-less require sophisticated computational technology. For example, late in the Shuttle development program there were lingering concerns for tile loads, including those due to the flow in the gaps between the tiles. Dwoyer et al [1982] demonstrated that this flow could be modeled with the Stokes flow approximation and obtained sufficiently accurate simulations in time to contribute to the resolution of this program concern.

The extrapolation form of prediction from computational aerodynamic methods naturally leads to inference space considerations. The nature of the inference space, as a context to the computational predictions, can significantly affect the underlying uncertainty of these predictions.

Inference Space

Aerodynamic flow domains provide a useful context to perform aerodynamic inferences. This leads to the more general concept of a physics-based inference space.

A simple example of flow domain considerations is taken from Polhamus [1996] for airfoil stall characteristics, Figure 2. In this work Polhamus retained the stall characterization developed by Gault [1957]. Figure 2a shows three of these distinctions as being: *i*) thin-airfoil stall, *ii*) leading-edge stall, *iii*) trailing-edge/leading-edge stall, and not shown is *iv)* trailing-edge stall. As the flow sketches indicate, the flow physics of each of these classifications is quite different, and the consequences on the lift properties near $C_{l,max}$ are also quite different. The ability of a computational method to predict one of these classes of stall would not necessarily imply that it could predict the other classes since the underlying flow physics are different in each case.

Polhamus modeled the airfoil stall flow types in terms of two parameters, airfoil leading-edge radius and chord Reynolds number as shown in Figure 2b. The data for this figure are from the NACA 6-series of airfoils, and, subject to the available data, Polhamus estimated boundaries between the various stall classifications. It is hypothesized that one can have increased confidence extrapolating a validated (or calibrated) computational method in the parameter-space variables (r/c and R_c in this example) to conditions beyond the validation conditions *so long as a domain boundary is not crossed*. The domains, as shown in Figure 2b, are examples of what constitutes a physics-based inference space. It is interesting that with this view toward a physics-based inference space, a direct link between basic flow physics (e.g., turbulent reseparation) and aggregate aerodynamic properties, like $C_{l,max}$, are readily established. Having the critical physics linked to prediction metrics (like $C_{l,max}$) is not only crucial to method validation, but also should greatly reduce the uncertainty in these predictions.

The fact that there are flow domains is nothing new. Examples can be found for many flow considerations, such as the work of Stanbrook and Squire [1965] and later Miller and Wood [1985] who established domains of supersonic leading-edge vortex separation and subsequent classes of shock-vortex structures. What is new is the approach of exploiting these domains, and building new domain knowledge, in the context of computational uncertainty and inference of prediction uncertainty. These domains, or physics-based inference spaces, generally exist in a similarity space within which we hypothesize that extrapolation becomes tractable with greatly reduced uncertainty as compared to other means.

Transformation from primitive to similarity variables generally results in reduced dimensionality for both independent and dependent variables. Although a simple example of this reduced dimensionality is given by transonic similarity, examples for more complex flows have also been developed. For example, Stahara [1981] used the concept of strained coordinates [Lighthill 1949] to demonstrate the utility of similarity-based methodology for predicting transonic airfoil aerodynamics with or without flow discontinuities. However, the flows must be topologically similar.

The method of strained coordinate used by Stahara [1981] can be useful in determining the uncertainty of a solution where no validation data exists, provided that the flows are topologically similar and the interpolation remains in a physics-based inference space. First, determine the uncertainty at two points within the physics based inference space where validation data exists using error propagation or N-version testing. Then apply the method of strained coordinates to interpolate this uncertainty to the new solution point.

Crucial knowledge for a physics-based inference space to be useful comes down to *i*) knowledge of the underlying flow physics to the aerodynamic topic of interest (which could be anchored in theoretical or experimental considerations) and *ii*) knowledge of the boundary region for said physics. This emphasis on physics-based boundary regions between aerodynamic flow states could present a new view toward aerodynamic testing for the purposes of code calibration and validation. A simple example is the establishment of the boundary between attached-flow cruise aerodynamics and separated flow for a given vehicle class. Data for the onset of separation would be useful for enhancing prediction confidence within the attached flow domain. The data could also guide the development of boundary prediction methodology to further reduce the uncertainty associated with state-change flow physics.

If a physics-based inference space is bounded, that is, if the boundary is established (say, experimentally) then it may be possible to validate the CA code within that domain. However, in an unbounded inference space a CA code can at best be validated at discrete flow conditions. Predictive capability within the bounded physics-based inference space should be possible with much higher confidence. These ideas are abstracted in Figure 3. Computational uncertainty near domain boundaries will certainly be higher than in the interior, not only because of the more complex state-change physics, but also because of the inherent uncertainty of the conditions causing this change of state (i.e., the parameter space range of the boundary). Physics to model and eventually predict these boundaries will in most cases be the hardest capability to be developed and thus may depend the most heavily on experimental technology or more advanced numerical simulation.

The boundary may be analogous to the intermediate solution in an asymptotic expansion. It provides a neighborhood through which the solutions match (consider, for example, the overlap region in the log-law/law of the wall from turbulent boundary layer theory). The scale/extent of the boundary will be different for different flows, and this scale itself will be a metric of interest. The boundaries of the inference spaces more than likely manifest higher-order flow physics with corresponding implications on computational uncertainty – the closer a solution is to a boundary the higher the likely uncertainty, especially if the location of the boundary cannot be predicted exactly.

The approach of validating computational aerodynamic techniques in the context of a physics-based inference space could have significant consequences for inferences that must be drawn in extrapolating from experimental test conditions to full-scale application conditions. Figure 4 illustrates that testing generally includes sub-scale conditions from which inferences must be drawn for the full-scale vehicle application. (Easterling [2001] has published a more sophisticated treatment of this topic). As one practical example, consider extrapolating in Reynolds number from wind tunnel to flight conditions. As illustrated in Figure 4, an extrapolation in the sense of parameter-based inference space can essentially become an interpolation in the sense of a physics-based inference space.

The nature of ground-based testing would most likely be altered to support the physics-based inference space approach to extrapolation. For the Reynolds number scaling example, wind tunnel data would need to be obtained at sufficiently high values to assure that the full-scale flow physics were fully established, albeit at subscale values. Computational techniques, validated with these data, and in the context of the physics-based inference space (e.g., fully turbulent transonic attached flow) could then be used as shown in Figure 4b. It is anticipated that this approach could greatly reduce the uncertainty associated with full-scale aerodynamic predictions.

Two facets of the physics-based inference approach to computational aerodynamic uncertainty could contribute to reduce ground-based testing requirements. First, the coupling of the physics-based inference space approach and aerodynamic similarity principles could be highly desirable as a means to reduce test conditions. Second, Modern Design of

Experiments principles could also be highly effective within the physics-based inference space to further reduce test condition requirements [DeLoach 1998].

Verification and Validation Elements

The AIAA definitions for verification and validation are given as follows [AIAA Guide]:

> *Verification*: The process of determining that a model implementation accurately represents the developer's conceptual model and the solution to the model.
>
> *Validation*: The process of determining the degree to which a model is an accurate representation of the real world from the perspective of the intended uses of the model.

The quantification of uncertainty in a numerical prediction can be broken down into several sources. We use the following four sources: method V&V, process control, parameter uncertainty, and model form uncertainty (Figure 5). Here we use method V&V to distinguish sub-process V&V (such as for a CFD code) from the overall V&V process as defined in the AIAA guide. Method V&V and process control are primarily concerned with the quantification and control of numerical errors and uncertainty. Parameter and model form uncertainty are concerned with quantification of uncertainty in the physical process being modeled.

Method V&V. Code verification and validation are necessary first steps in quantifying uncertainty. Roache [1997] distinguishes between verification and validation of a code and verification of a solution. Roache [1998] states "Verification is *completed* (at least in principle, first for the code, then for a particular calculation) whereas Validation is ongoing (as experiments are improved and/or parameter ranges are extended)."

Code verification is a process to ensure that the model equations are solved correctly. The Method of Manufactured Solutions [Roache 1998] is a powerful technique for code verification. A non-trivial solution is specified and forcing functions added to the governing PDEs to satisfy this solution. A grid convergence study then verifies the order of accuracy of the code [Salari and Knupp 2000].

However, the use of a verified code is insufficient. A grid convergence study is required to estimate the error on new calculations. Additionally, a grid convergence study should be used to check that the order of convergence on the new prediction matches the advertised accuracy of the code. A reduced order of accuracy is an indication of an error in the calculation. Roache [1997] gives several examples of calculations where an esoteric error in a new code application resulted in a reduced order of accuracy.

Grid convergence studies (extending into the asymptotic range) using verified codes are the most common and direct method to quantify numerical uncertainty.

Process Uncertainty. CFD code application remains a labor-intensive activity. The process requires specification of (usually) complex geometry and discretization of the flow volume. The practitioner must make appropriate decisions about the flow physics (incompressible or compressible, laminar, transitional or turbulent, etc.) and choose appropriate physical models. Budgets and deadlines limit the resources available for the analysis and prevent the analyst from exploring alternate models. Simple errors in input decks can invalidate the results.

Process control, in the sense of modern quality assurance, not only minimizes the blunders due to misapplication of codes and simple user errors, but it also enables the creation and management of a stable, predictable process with known variation – the target of Figure 1b. We believe that generally available best practices (e.g., ERCOFTAC [2000], Chen [2002]) will provide a foundation for developing and establishing such processes.

Parameter Uncertainty. The conceptual model that is implemented in a computer code is an idealization of the physical world that it is used to model. For example, manufactured vehicles vary from the design specifications with each vehicle being slightly different, aircraft deformations under load are difficult to accurately measure or predict, physical properties vary or are difficult to estimate. The aeroelastic deformation of a wing in flight can have a large effect on the resulting flowfield. It is important that the actual aeroelastic shape of the aircraft be incorporated in the numerical model to accurately predict the flow. The uncertainty in this deformation can be accommodated using error propagation techniques. Additionally, there is uncertainty in the value of physical constants (e.g. thermal conductivity, viscosity, specific heats, etc.) that can be important for some classes of problems. Several techniques exist to propagate the effects of these variations on system response; sensitivity analysis, Monte-Carlo, and polynomial chaos among them [Hills and Trucano 1999, Walters 2003]. For example, Pelletier et al. [2002] showed the effect of geometric uncertainty and physical constant uncertainty on the validation study of a laminar free-convection problem with variable fluid properties.

Model Form Uncertainty. Validation is confirmatory evidence that the conceptual model is an adequate representation of the physical process for the intended use of the model. Model form uncertainty is a quantification of a model's predictive accuracy.

Current modeling efforts focus on unit problems. Models are typically developed from theory, validated on unit problems and then tested on more complex configurations. The resulting model becomes widely accepted when a sufficiently large number of unit problems and configurations have been computed with results equivalent or better than the previous best model. This ad hoc procedure for model acceptance does not provide an estimate of uncertainty for model predictions. A framework needs to be developed to provide quantification for model form uncertainty. Very little work has been done in this area [Hills and Trucano 1999, Luis and McLaughlin 1992].

There is some ambiguity whether certain sources of uncertainty should be classified as parameter uncertainty or model form uncertainty. We have chosen to classify physical constants under parameter uncertainty. However, we feel that model coefficients, such as used in turbulence models, are part of the model and must be considered as part of model form uncertainty.

Interrelationships among Uncertainty Sources. Although the uncertainty sources of Figure 5 each have unique aspects, it is important to acknowledge that they also interact and affect one another. Our current view of these interactions is shown in Figure 6. Also included in this figure are a number of additional key topics (not shown in Figure 5) that contribute to aerodynamic prediction uncertainty.

PROCESS MANAGEMENT

The two sections below provide an approach for moving toward uncertainty-based computational aerodynamics. The first section regards relationships between computational output and Statistical Process Control. The second section addresses management and risk considerations.

The Three Aspects of any CA Output and Their Relationship to Statistical Process Control

In the CA uncertainty process, there are fundamentally two aspects to every number of importance that is generated as part of the code output: (1) the generated value itself and (2) the difference between the so-called true value and the generated value, i.e. the error. However, in all practical cases of interest, the true value is not known and, hence, the error is also unknown. To deal with this issue statistically, we would define a computational process. But now there are necessarily *three* aspects to every process output value: (1) the average of the sample realizations of the process, which is the estimate of the population mean, (2) the sample standard deviation, which is the estimate of the population standard deviation, and (3) the number of realizations, which reflects how well the population parameters are known.

The CA uncertainty process is concerned with aspects (2) and (3) whether they are known qualitatively or quantitatively. The risk for a decision maker is not associated with the value itself nor even the size of the standard deviation since modern probabilistic methods, at least in principle, can propagate those effects, but rather how well the second value is known.

For the last four decades, methods for building confidence in the second and third aspects have been developed for precision metrology by the National Institute for Standards and Technology (NIST) and its predecessor, the National Bureau of Standards (NBS) [Eisenhart 1969]. Those methods, which are based on the notions of statistical process control (SPC) used in manufacturing [Wheeler and Chambers 1992], have been recently adapted to force and moment testing in wind tunnels [Hemsch et al. 2000]. We suggest that the CFD community consider such methods for improving the CA uncertainty process. A qualitative definition of SPC for measurement processes is given below followed by a more mathematical version [Belanger 1984]:

A measurement process is in a state of statistical control if the amount of scatter in the data from repeated measurements of the same item over a period of time does not change with time and if there are no sudden shifts or drift in the data. (Qualitative)

A measurement process is in a state of statistical control if the resulting observations from the process, when collected under any fixed experimental conditions within the scope of the a priori well-

defined conditions of the measurement process, behave like random drawings from some fixed distribution with fixed location (mean) and fixed scale (standard deviation) parameters. (Quantitative)

For the purposes of CA uncertainty, we would consider the sources of error to be different grids, observers, codes, turbulence models, etc.

Management of the CA Uncertainty Process for Decision Makers and for Resource Allocation

It has been our experience that effective response of stakeholders (funders, managers and workers) to CA uncertainty issues is largely a function of their understanding of the present state of their CA uncertainty process either for an individual software development project or for a vehicle design effort or for the organization as a whole. It may be less well known that it is also a function of their knowledge of potential CA uncertainty stages. We propose in this section a particularly simple way to consider and display those stages to stakeholders. Knowing where the organization or project is relative to those stages and the actions required for each one makes it possible to allocate resources effectively, particularly if the risk state that the customer is willing to assume is clear.

It is convenient to think of the decision maker as the customer of the CA uncertainty process and their desired risk assumption level as the Voice of the Customer, a term from the Statistical Quality Control literature [Wheeler and Chambers 1992]. In manufacturing, the Voice of the Customer might be a specified tolerance interval defined by a design department. Similarly, we could call the actual performance of the CA uncertainty effort, the Voice of the Process. Again, in manufacturing, the Voice of the Process might represent the actual variation in the output from a series of manufacturing steps [Wheeler and Chambers 1992].

Below we offer five representative CA uncertainty stages and correlate them with decision-maker risk assumption level. Readers who are familiar with the Software Capability Maturity Model [Paulk et al 1993] or the Process Maturity Model [Gardner 2001] will recognize that the CA uncertainty stages share some, but not all, of their features. Consider the five possible stages shown in Figure 7. They are:

1. **No systematic uncertainty effort**. Activity in this stage is best described as ad hoc. Success depends upon the experience of particular individuals acting alone or as a team. It would be reasonable to expect to find research efforts at this level, particularly when entirely new code functionality is being attempted. For this level, it is impossible for decision makers to assess their risk assumption level.

2. **Uncertainty management**. At this stage, considerable effort is made to keep the error of code predictions within target bounds but any uncertainty statements are best characterized as qualitative. For example, if an uncertainty band is given, the precise coverage level and how well it is known are left unstated. It is probable that, when resources are not available to drive grid convergence into the asymptotic region, this is the best level attainable unless the processes have been demonstrated to be in statistical control [Wheeler and Chambers 1992]. The Boeing Company has developed a particular philosophy for this stage and shown its value for the airframe aero design process [Rubbert 1998]. At this level, a decision maker would typically have precedent as a guide in assessing the risk assumption level. It is also likely that the decision maker will include safety factors and/or margins to reduce the consequences of the insufficiently known uncertainty. For this stage there is basic project management to repeat earlier successes on projects with similar applications.

3. **Reported uncertainty**. At this stage, the CA uncertainty process is capable of reporting a quantitative uncertainty band, together with the coverage level and how well it is known, for the domain of the validation effort, past and present. It is unlikely that this level can be achieved without grid convergence to the asymptotic region and without experimental efforts that are specifically designed for code validation [Oberkampf and Trucano 2002]. Achieving this level would seem to require process documentation, standardization and integration across the organization with all projects using approved, tailored versions of the organization's standard process for maintaining the CA uncertainty process. Achieving this stage would allow decision makers to reduce the safety factors and/or margins required at stage 2, particularly for predictions which are somewhat close to the previously explored inference space.

4. **Reported prediction error**. This stage is distinguished from stage 3 by the ability to estimate the uncertainty band associated with a

prediction outside the validation domain, together with a statement of coverage level and how well it is known. Clearly, this would require, in addition to the requirements of stage 3, thorough understanding of model form error in the validation region as well as in the prediction inference space [Easterling 2001]. (See the Inference Space section above for possible ways to carry out such studies.) For this stage, it may be possible for decision makers to use the uncertainty bands to assess risk without safety factors or margins. However, so little work has been done in this area that it is not clear if such a change will ever be warranted. But, for this stage, it seems appropriate to consider that the organization that is producing the predictions is assuming most of the risk.

5. **Certification and Protocols**. At this stage, the decision maker should be assuming none of the risk because the producing organization is guaranteeing that the prediction uncertainty band and its coverage level are known. See Lee [1998] and Pilch [2000] for descriptions of such activities.

It is important to realize that moving from a lower to higher CA uncertainty stage requires considerable effort on the part of all stakeholders. It actually requires a paradigm shift that is impossible to appreciate and understand until the stage has been achieved. Training and education of all of the stakeholders is essential to make such a shift possible. Achieving the higher stages requires attempting to work at those levels and providing the necessary will and resources.

CLOSING REMARKS

Computational aerodynamics is a rapidly evolving field that has been widely accepted in engineering analysis and design. However, uncertainty estimates are required for computational aerodynamics users and decision makers to analyze and manage risk. In this paper we have enumerated the stages of CA maturity. Each organization will be required to determine their own needs and work to accomplish the maturity level adequate to meet their objectives.

Various elements of CA uncertainty have been outlined. A method to estimate uncertainty at flow conditions when there is no experimental validation data has been outlined using the concept of physics based inference space and the method of strained coordinates. A tremendous amount of work remains to develop techniques to calculate model form uncertainty.

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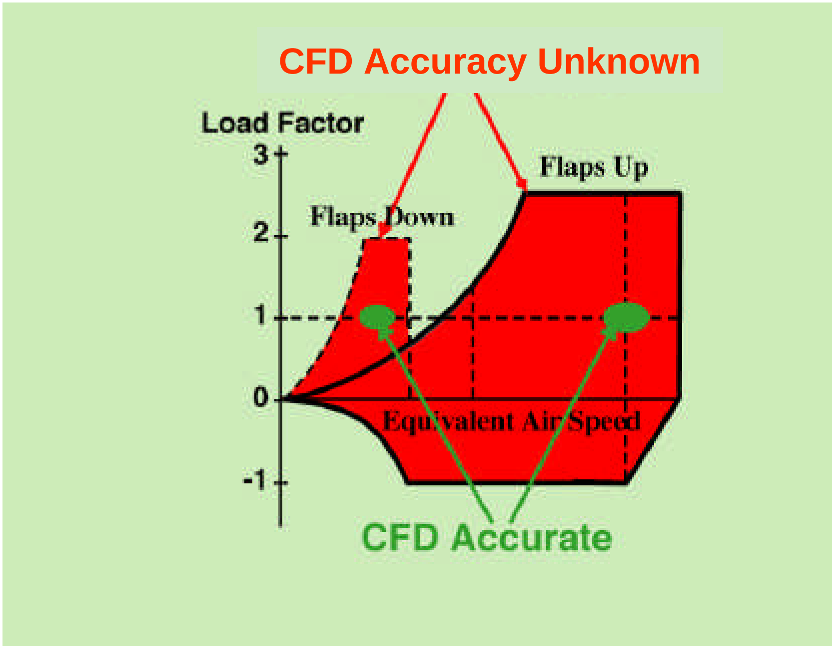
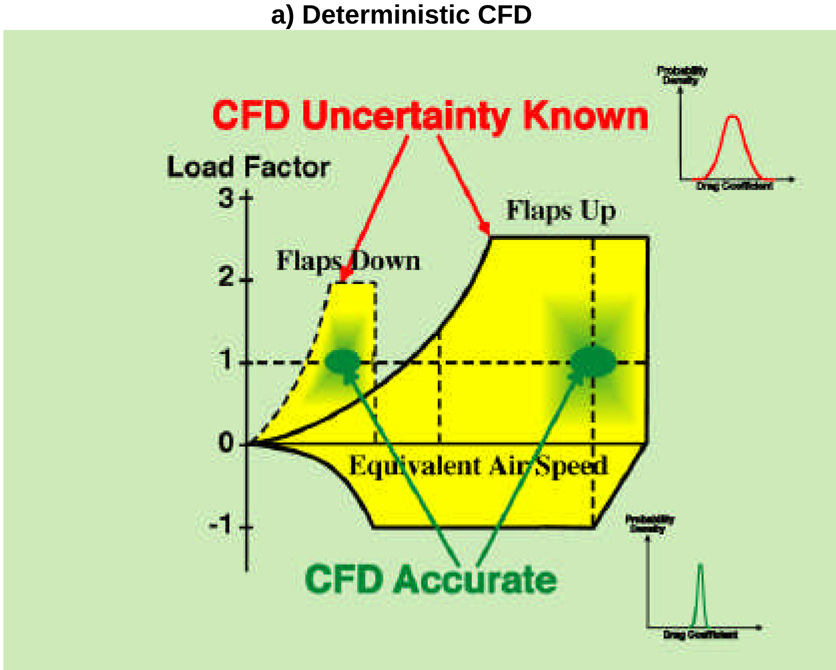
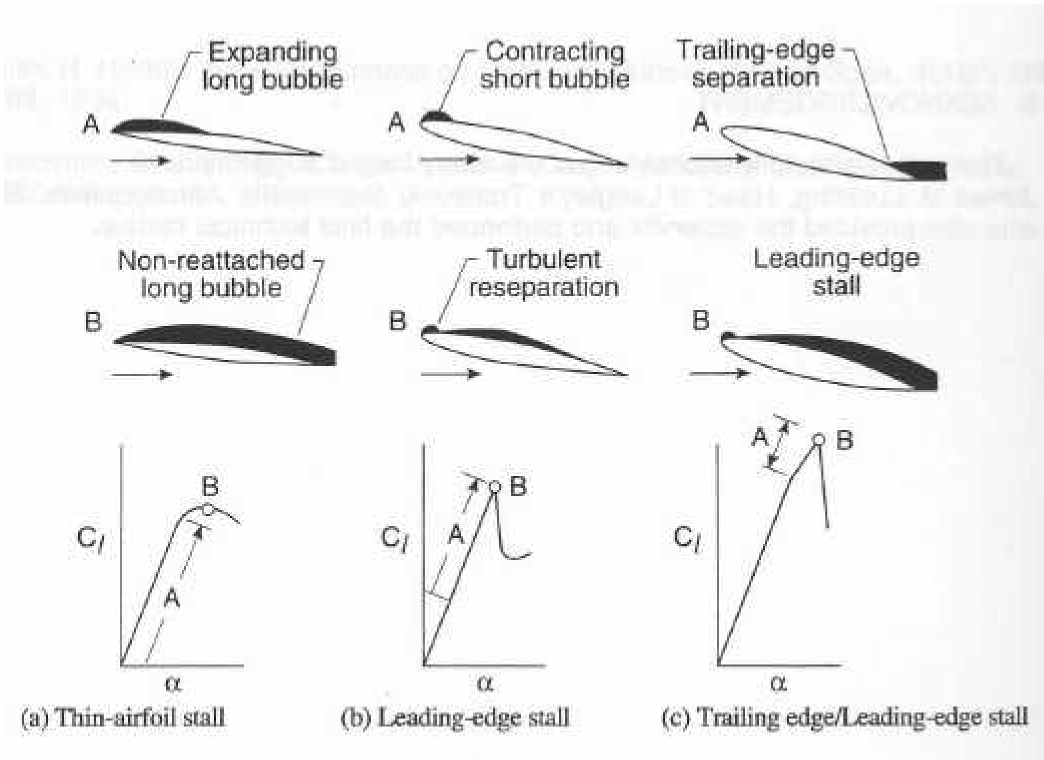


a) Deterministic CFD

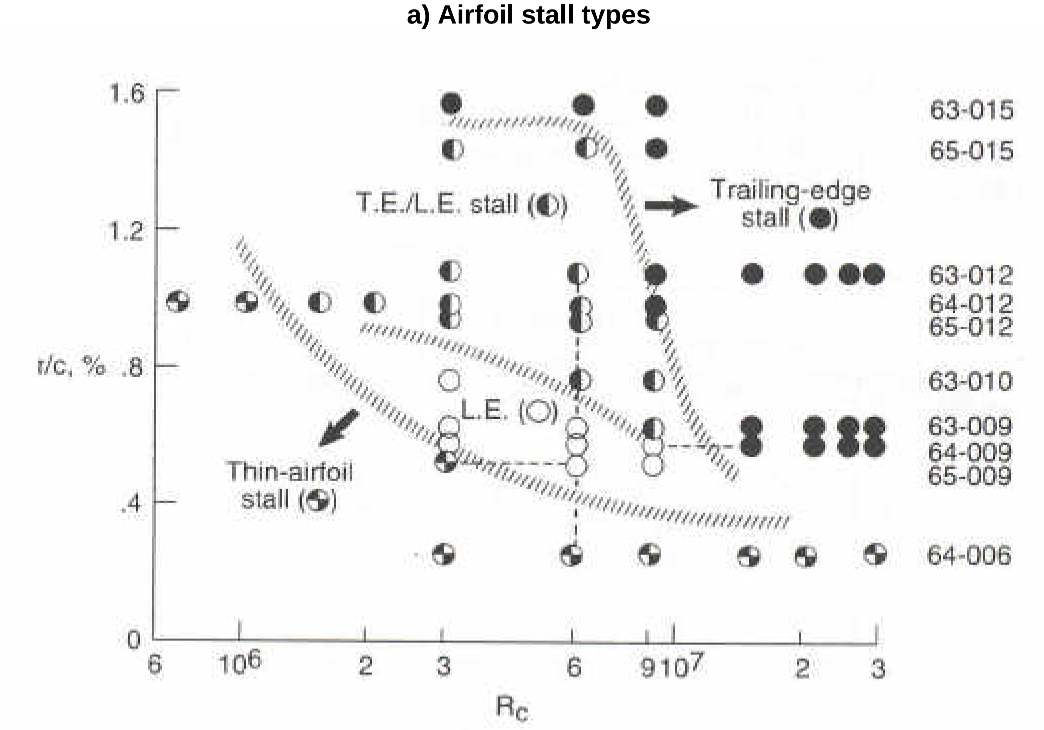


b) Uncertainty-based CFD

Figure 1.- Alternate views toward computational aerodynamics.
From Zang [2002]and Wahls [2002]



a) Airfoil stall types



b) Stall classification

Figure 2.- Airfoil stall inference spaces. From Polhamus [1996]

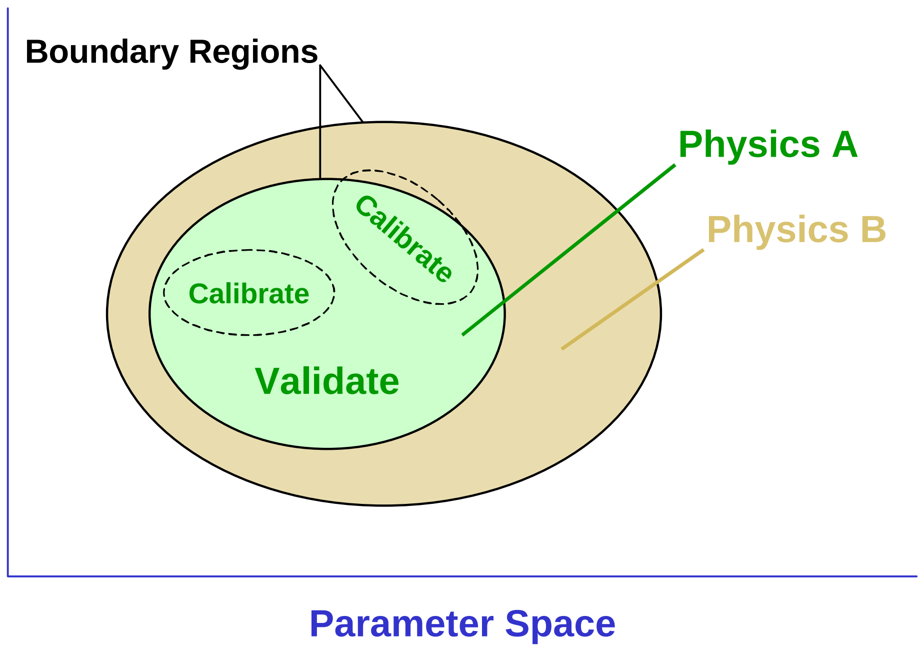
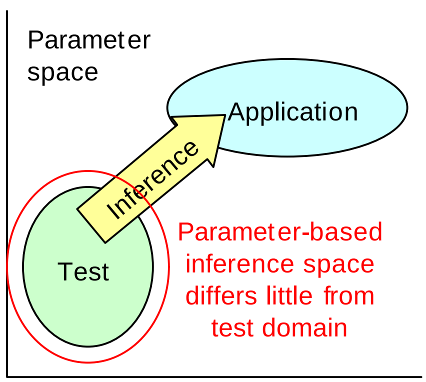
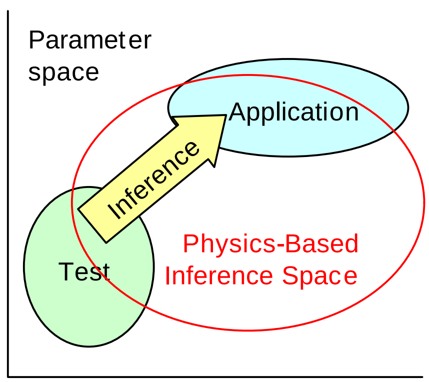


Figure 3.- Relationships among calibration, validation and inference spaces.





a) Parameter-based extrapolation **b) Physics-based extrapolation**

Figure 4.- Computational prediction.

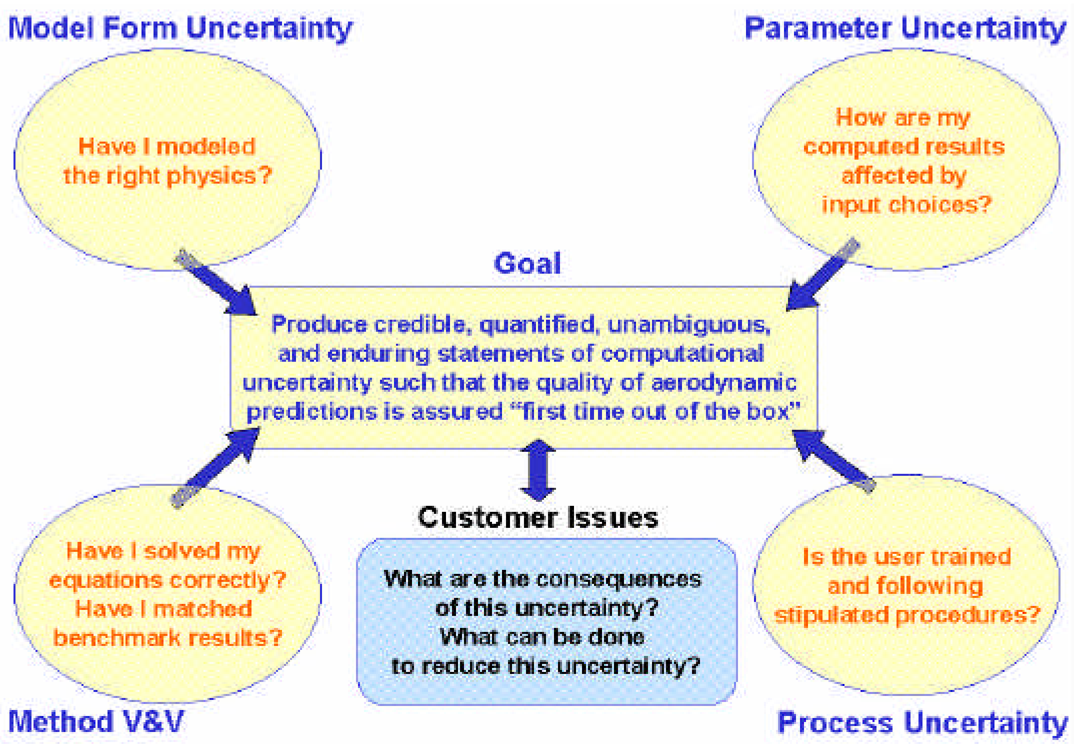


Figure 5.- Verification and Validation elements for computational aerodynamics.

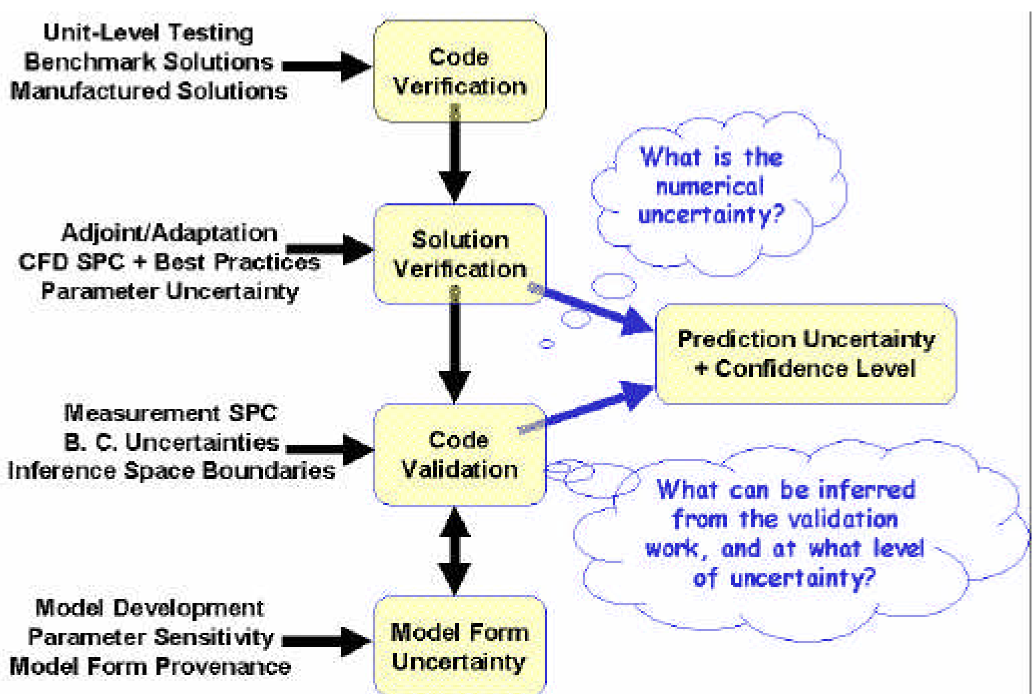


Figure 6.- Uncertainty element interactions.

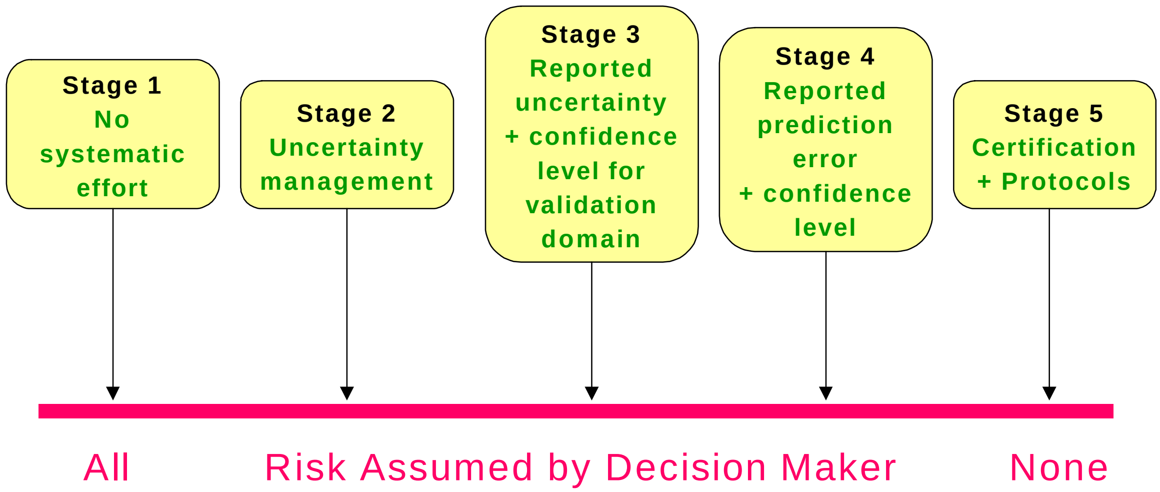


Figure 7.- Mapping computational uncertainty stages onto decision-maker risk.