Modeling compressibility effects in high-speed turbulent flows

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1 Introduction

Man has strived to make objects fly faster, first from subsonic to supersonic and then to hypersonic speeds. Spacecraft and high-speed missiles routinely fly at hypersonic Mach numbers, $M > 5$. In defense applications, aircraft reach hypersonic speeds at high altitude and so may civilian aircraft in the future. Hypersonic flight, while presenting opportunities, has formidable challenges that have spurred vigorous research and development, mainly by NASA and the Air Force in the USA. Although NASP, the premier hypersonic concept of the eighties and early nineties, did not lead to flight demonstration, much basic research and technology development was possible. There is renewed interest in supersonic and hypersonic flight with the HyTech program of the Air Force and the Hyper-X program at NASA being examples of current thrusts in the field.

At high-subsonic to supersonic speeds, fluid compressibility becomes increasingly important in the turbulent boundary layers and shear layers associated with the flow around aerospace vehicles. Changes in thermodynamic variables: density, temperature and pressure, interact strongly with the underlying vortical, turbulent flow. The ensuing changes to the flow may be qualitative such as shocks which have no incompressible counterpart, or quantitative such as the reduction of skin friction with Mach number, large heat transfer rates

due to viscous heating, and the dramatic reduction of fuel/oxidant mixing at high convective Mach number. The peculiarities of compressible turbulence, so-called compressibility effects, have been reviewed by Fernholz and Finley (1976); Bradshaw (1977); Lele (1994); Smits & Dussauge (1996); Chassaing *et al.* (2002). Predictions of aerodynamic performance in high-speed applications require accurate computational modeling of these 'compressibility effects' on turbulence.

During the course of the project we have made fundamental advances in modeling the pressure-strain correlation and developed a code to evaluate alternate turbulence models in the compressible shear layer.

2   The pressure-strain correlation

Pressure fluctuations, through the pressure strain correlation, impact Reynolds stress levels. The pressure strain correlation is defined by

$$\Pi_{ij} = \overline{p'(u''_{i,j} + u''_{j,i})} \,, \tag{1}$$

where p' is the fluctuating pressure and $\partial u''_i / \partial x_j$ the fluctuating strain rate. This term is important in a turbulent shear flow because it transfers kinetic energy from the stream-wise component, $\widetilde{u''^2}$, to the cross-stream component, $\widetilde{v''^2}$, and thus, indirectly, through the Reynolds shear stress production term, $\widetilde{v''^2}\partial \widetilde{u}/\partial y$, it effects the all-important turbulent shear stress, $\widetilde{u''v''}$. The finite propagation speed of pressure signals, the speed of sound, can be comparable to other velocity scales in flows when the Mach number is not small. Furthermore, the entropy mode maintained by large variations in mean density/temperature can couple with the pressure fluctuations. Therefore, Reynolds stress levels may be expected to be significantly different in compressible flow and, indeed, newer experimental and simulation studies show such behavior.

Unlike laboratory experiments, statistics such as pressure strain and root-mean-square thermodynamic fluctuations are available in DNS so that analysis of the pressure/turbulence

coupling is facilitated. The available DNS data in free shear layers show that the pressure-strain correlation is strongly affected and, in particular, all components decrease with Mach number. With such clear evidence, it is clear that a sound strategy for the accurate prediction of compressible flows must involve modeling compressibility effects on the pressure-strain term within the framework of a full Reynolds stress closure.

Compressible uniform shear flow with an imposed linear mean velocity gradient was studied by Sarkar (1995). The gradient Mach number, $M_g = Sl/\bar{c}$, was varied in the subsonic-to-supersonic regime, with values ranging between $M_g = 0.5$ and $M_g = 5$. Here $S = \partial \tilde{u}/\partial y$ is the constant mean shear, l the turbulence integral length scale in the cross-stream direction and $\bar{c}$ the mean speed of sound. The growth rate of turbulent kinetic energy was found to decrease with increasing values of Mach number. The reduction of turbulence levels was related to the inhibited turbulence production, $P = -S\widetilde{u''v''}$, and not to the explicit dilatational terms: the compressible dissipation and pressure dilatation. Sarkar (1996) noted that pressure fluctuations were reduced in cases with high gradient Mach number and stated that a change in the pressure gradient term in the momentum equations (and the pressure-strain term in the Reynolds stress equations) leads to reduced levels of turbulence.

Pantano and Sarkar (2001) studied the temporally-evolving shear layer between two streams with different velocities. Simulations were performed for the air-air shear layer with three values of the convective Mach number, $M_c = (U_1 - U_2)/(c_1 + c_2) = 0.3, 0.7$, and 1.1. Fig. 1 shows that the DNS results capture the dramatic reduction of thickness growth rate and agree well with the Langley experimental curve which is a consensus of experiments with various air-air shear layers. Interestingly, the growth rates in Fig. 1 which are lower than the Langley curve were measured in experiments with dissimilar gas streams. This observation prompted additional simulations by Pantano and Sarkar (2001) who found an effect of freestream density ratio over and above that of the convective Mach number.

All components of the Reynolds stress tensor were measured in the shear layer DNS and found to decrease with increasing values of M_c. During the late-time evolution when the shear layer was in an approximately self-similar state, the decrease was similar in all

components so that the Reynolds stress anisotropy remained relatively unaffected by M_c. However, during the early-time evolution, when the shear number, SK/ϵ, was larger than its equilibrium value, the ratio of $\widetilde{v''^2}/\widetilde{u''^2}$ showed a systematic decrease with increasing M_c.

As alluded to in the introduction, the pressure-strain, defined by Eq. (1) can be expected to change as a function of M_c. Fig. 2 shows the pressure-strain correlation, integrated across the shear layer, as a function of M_c and, indeed, all components of the pressure-strain correlation as well as the r.m.s pressure fluctuations decrease with increasing M_c. A reason for this "fluctuating pressure" effect of compressibility was identified by Pantano and Sarkar (2001) who, by performing an analysis of the wave equation for pressure at the center of the shear layer assuming locally-homogeneous turbulence, were able to predict a monotone reduction of the pressure-strain with increasing M_c. The associated physical mechanism is that the acoustic time delay, l/c, of signals traversing adjacent points of an "eddy" implies additional decorrelation between these points and, therefore, a contribution to the pressure-strain term which is smaller than in the incompressible, infinite-signal speed case.

Laboratory data of Elliott & Samimy (1990), Barre *et al.* (1994), and more recently, Chambres *et al.* (1999) show that all measured turbulence intensities and the Reynolds shear stress decrease with increasing values of the convective Mach number M_c. However, Goebel & Dutton (1991) find experimentally that the streamwise turbulence intensity remains relatively unchanged while all other components of the Reynolds stress tensor decrease. Despite this disagreement, based on the experimental data, it is certain that compressibility does effect the turbulence structure. There are some effects on turbulence structure in wall-bounded high-speed flows as discussed by Smits & Dussauge (1996) but these effects are less clear because of the difficulties in making measurements. However, it does appear that, for the same value of Mach number based on outer variables, the effect on turbulence structure is perhaps weaker in wall-bounded flows relative to free shear flows.

4

gradients. M_t is not the main contributor to compressibility effects in shear flows because applications of M_t-based modifications that work in the compressible shear layer are found to cause underprediction of skin friction in the high-speed boundary layer (especially the cooled case).

Calibration of M_g and M_t dependences in the pressure-strain model must be performed taking the Langley growth rate curve into account. Under the assumptions of a temporally-evolving shear layer and high Reynolds number, the momentum thickness growth rate is given by

$$\dot{\delta}_\theta \simeq \bar{\epsilon}_{11} - \bar{\Pi}_{11} \tag{3}$$

where the overbar indicates quantities obtained by integrating self-similar profiles over the width of the shear layer and $\bar{\epsilon}_{11}$, $\bar{\Pi}_{11}$ denote the streamwise components of the turbulent dissipation rate and pressure-strain correlation. If $\bar{\epsilon}_{11}$ is assumed to be a constant fraction of $\bar{\Pi}_{11}$ given by the baseline incompressible model, then the experimental curve, $\dot{\delta}_\theta(M_c)$, can be directly used to obtain $\Pi_{11}(M_c)$. Now, the convective Mach number can be related to M_g at the centerline following Sarkar (1995),

$$M_g \simeq 2.2 M_c.$$

With knowledge of the functional relationship, $\Pi_{11}(M_c)$, the required compressibility modifications can be accomplished.

4   The pressure fluctuation equation

The starting point of analysis of the pressure-strain correlation is an equation for the fluctuating pressure, p'. After taking the divergence of the momentum equation and using the continuity equation, the following wave equation for the fluctuating pressure ensues,

$$\left(\frac{1}{c^2}\frac{D^2}{Dt^2} - \frac{\partial^2}{\partial x_j^2} \right) p' = 2\frac{\partial \widetilde{u}_i}{\partial x_j}\frac{\partial \rho u_j''}{\partial x_i} + 2\frac{\partial \widetilde{u}_i}{\partial x_i}\frac{\partial \rho u_j''}{\partial x_j} + 2\frac{\partial^2 \widetilde{u}_i}{\partial x_i \partial x_j}(\rho u_j'')$$

$$+ \frac{\partial^2}{\partial x_i \partial x_j}(\rho u_i'' u_j'' - \widetilde{\rho u_i'' u_j''})$$

$$+ \rho' \left[(\frac{\partial \widetilde{u}_i}{\partial x_i})^2 + \frac{\partial \widetilde{u}_i}{\partial x_j}\frac{\partial \widetilde{u}_j}{\partial x_i} \right]$$

$$+ \frac{\partial^2 \tau_{ij}}{\partial x_i \partial x_j}, \tag{4}$$

where we use the notation

$$\frac{D^2}{Dt^2} = \frac{\partial^2}{\partial t^2} + 2\widetilde{u}_j\frac{\partial^2}{\partial x_j \partial t} + \widetilde{u}_j^2\frac{\partial^2}{\partial x_j^2}. \tag{5}$$

On the r.h.s. (the source terms) of Eq. 4, the first line involves mean velocity gradients and the total density, the second line involves fluctuating velocity gradients and the total density, the third line contains density fluctuations and mean velocity gradients, and the final term is the viscous term, usually small relative to the others. It is clear that deviations in p' from the incompressible solution originate from three effects: (1) a wave operator instead of the Laplacian, (2) mean density gradients in the source, and (3) fluctuating density gradients in the source. The relative importance of these effects depend on the flow in question.

5   Green's function analysis

The pressure equation, Eq. 4, has a formal solution in terms of the Green's function convolved with the forcing terms which can then be used to obtain the pressure-strain term. The Green's function in the case of zero mean velocity gradient is a simple harmonic function that is isotropic, that is, without any dependence on the angle of wave propagation. The resulting compressibility effect on pressure strain was shown by Pantano and Sarkar (2001) to be a monotone reduction with increasing Mach number. The Green's function, G, for uniform shear flow has now been derived by Thacker, Sarkar & Gatski (2004) and involves parabolic cylinder functions exhibiting oscillatory behavior having, at long time, increasing

7

3 Computational model

The compressible Navier-Stokes equations for the mean velocity, density and temperature along with equations for the Reynolds stress tensor and dissipation rate are applied. This compressible second-order closure has a framework similar to the studies of Gatski *et al.* (1990); Speziale & Sarkar (1991). The baseline pressure strain model applicable for the limit of zero Mach number will be the SSG model developed by Speziale, Sarkar & Gatski (1991) that has been found to work well in a variety of complex turbulent flows in the low-speed case. We are developing a compressibility modification of this model valid for high speeds. The basis of the new pressure strain model is that the parameters that determines the effect of compressibility are the gradient Mach number, $M_g = Sl/c$, and the turbulent Mach number, M_t. We emphasize that using M_g is suggested by both DNS of uniform shear flow, Sarkar (1995), and DNS of the shear layer, Pantano and Sarkar (2001). The simplest-possible compressible pressure strain model is

$$\Pi_{ij} = \Pi_{ij}^I f(M_g) \tag{2}$$

where $f(M_g)$ is an arbitrary function of the gradient Mach number and Π_{ij}^I is the SSG pressure strain model. As discussed later, our analysis suggests that such a model is likely too simplified and, instead, each term in the the SSG model has a dependence on M_g and M_t.

An alternative to the gradient Mach number is the mean Mach number based on a characteristic mean velocity difference, ΔU in the shear layer or U_∞ in the boundary layer. The mean Mach number is not a universal correlating parameter because of the following experimental observation: When the mean Mach number is unity in the free shear layer, the Reynolds stresses are strongly reduced; however, at unity Mach number, the Reynolds stresses are hardly affected in the boundary layer. Another parameter is the turbulent Mach number based on the local magnitude of the fluctuation velocity, for example, $q = \sqrt{2K}$, where K is the turbulent kinetic energy. The turbulent Mach number, $M_t = q/\overline{U}$, determines compressibility of the velocity fluctuations in contrast to M_g which is related to mean velocity

frequency and decaying amplitude. $\dot{G}$ depends on the mean shear, $S = d\widetilde{u}_1/dx_2$, the speed of sound, c, and the wave number vector, $\mathbf{k}$, of the forcing through the following nondimensional quantities: M_t, $M_g k_1$, and k_2. Here, k_1 and k_2 denote the nondimensional streamwise and cross-stream (in direction of shear) wave numbers.

The Green's function has the following characteristics. First, it has pronounced anisotropy as long as M_g is not too small. For example, during the initial temporal evolution, both the Green's function amplitude and time period show monotone decrease if the wave propagates downstream (x_1-component parallel to the mean flow) while, propagating upstream, both the amplitude and time period may decrease or increase depending on the circumstances. Thus, for some propagation angles, the maximum amplitude of the Green's function is not at $t = 0$. Second, with increasing M_t, the anisotropy is not diminished while both the amplitude and frequency of the oscillations decrease. Third, with increased wave number, k, there is less anisotropy; also, the amplitude of the Green's function decreases while the frequency increases.

The Green's function analysis has been used to obtain compressibility effects on the rapid pressure-strain term, the component having explicit dependence on the mean velocity gradient, in uniform shear flow. A monotone reduction with increasing M_g and M_t of all the diagonal terms are found in agreement with DNS data. The off-diagonal term, Π_{12}, of the rapid component is relatively unaffected in contrast to DNS data suggesting that, to have a complete picture of compressibility effects in uniform shear flow, the slow component explicitly dependent on velocity fluctuations must also be taken into account.

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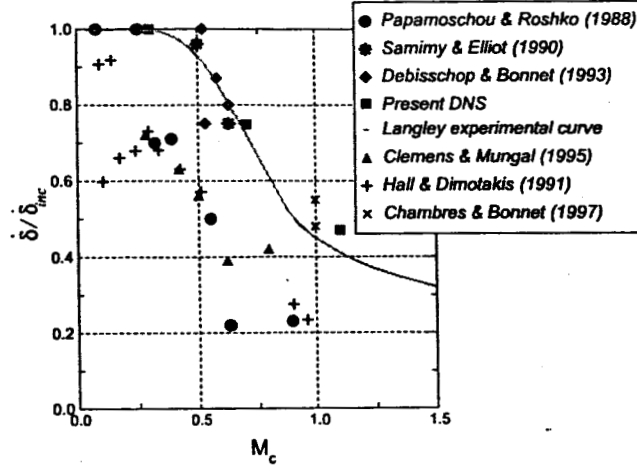
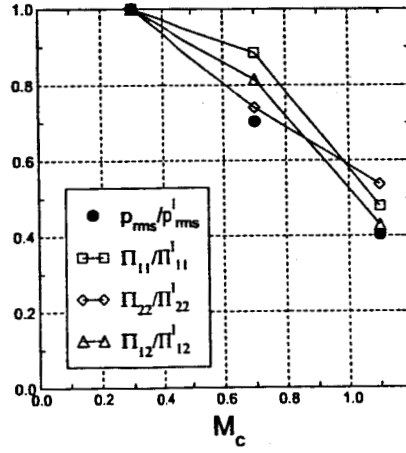Figure 1: Dependence of shear layer growth rate on M_c.

Figure 2: Open symbols $\bar{\Pi}_{ij}/\bar{\Pi}_{ij}^I$ and filled symbols p_{rms}/p_{rms}^I. Each quantity is normalized by its incompressible counterpart.