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Formation Flying in Earth, Libration, and Distant Retrograde Orbits

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Agenda

I. Formation flying – current and future

II. LEO Formations

- Background on perturbation theory / accelerations
  - Two body motion
  - Perturbations and accelerations
- LEO formation flying
  - Rotating frames
  - Review of CW equations,
  - Lambert problems, Shuttle
  - The EO-1 mission
  - Realities of operations

III. Control strategies for formation flight in the vicinity of the libration points

- Libration missions
- Natural and controlled libration orbit formations
  - Natural motion
  - Non-Natural motion

IV. Distant Retrograde Orbit Formations

V. References

- All references are textbooks and published papers
- Reference(s) used listed on each slide, lower left, as *ref#*

NASA Themes and Libration Orbits

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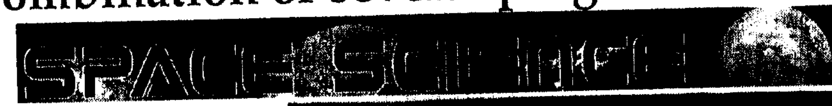
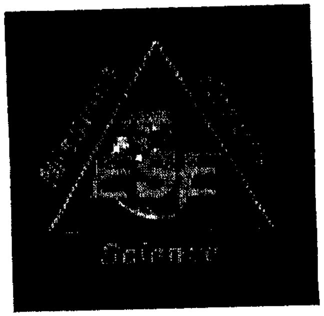
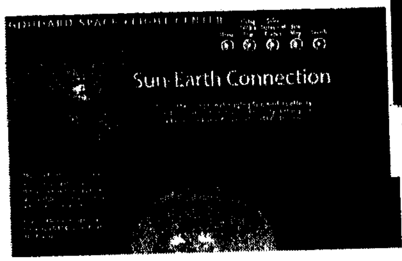
NASA Enterprises of Space Sciences (SSE) and Earth Sciences (ESE) are a combination of several programs and themes



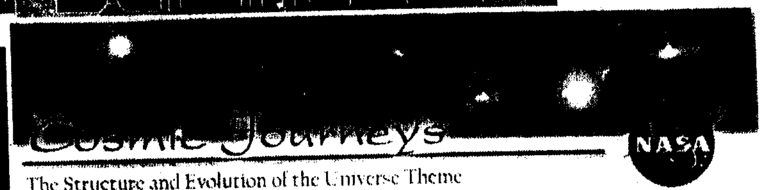
SSE

ESE

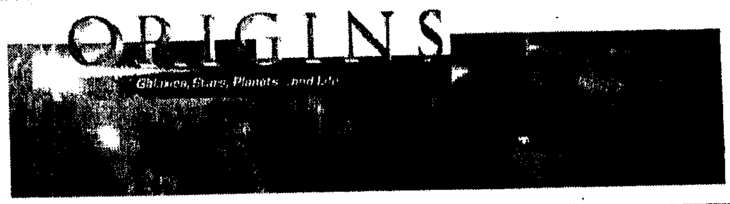




SEC



SEU



Origins

- Recent SEC missions include ACE, SOHO, and the L_1/L_2 WIND mission. The Living With a Star (LWS) portion of SEC may require libration orbits at the L_1 and L_3 Sun-Earth libration points.
- Structure and Evolution of the Universe (SEU) currently has MAP and the future Micro Arc-second X-ray Imaging Mission (MAXIM) and Constellation-X missions.
- Space Sciences' Origins libration missions are the James Webb Space Telescope (JWST) and The Terrestrial Planet Finder (TPF).
- The Triana mission is the lone ESE mission not orbiting the Earth.
- A major challenge is formation flying components of Constellation-X, MAXIM, TPF, and Stellar Imager.

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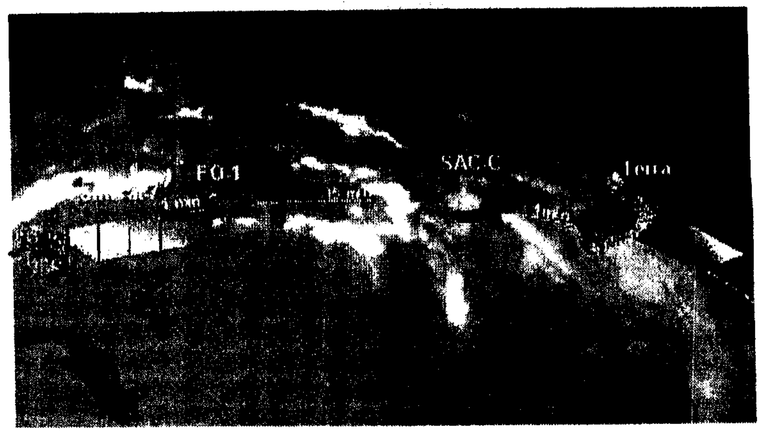
Earth Science Launches
Low Earth Orbit Formations

<u>The 'a.m.' train</u>

~ 705km, 98^0 inclination,

10:30 .pm. Descending node sun-sync

- -Terra (99): Earth Observatory
- - Landsat-7(99): Advanced land imager
- -SAC-C(00): Argentina s/c
- *-EO-1(00) : Hyperspectral inst.*



<u>The 'p.m.' train</u>

~ 705km, 98^0 inclination,

1:30 .pm. Ascending node sun-sync

- - Aqua (02)
- - Aura (04)
- - *Calipso (05)*
- - *CloudSat (05)*
- - Parasol (04)
- - OCO (tbd)

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Space Science Launches
Possible Libration Orbit missions

- FKSI (Fourier Kelvin Stellar Interferometer): near IR interferometer
- JWST (James Webb Space Telescope): deployable, ~6.6 m, L2
- Constellation – X: formation flying in librations orbit
- SAFIR (Single Aperture Far IR): 10 m deployable at L2,
- Deep space robotic or human-assisted servicing
- Membrane telescopes
- Very Large Space Telescope (UV-OIR): 10 m deployable or assembled in LEO, GEO or libration orbit
- MAXIM: Multiple X ray s/c
- Stellar Imager: multiple s/c form a fizeau interferometer
- TPF (Terrestrial Planet Finder): Interferometer at L2
- 30 m single dish telescopes
- SPECS (Submillimeter Probe of the Evolution of Cosmic Structure): Interferometer 1 km at L2

r #1

Future Mission Challenges
Considering science and operations

- ***Orbit Challenges***
  - Biased orbits when using large sun shades
  - Shadow restrictions
  - Very small amplitudes
  - Reorientation and Lissajous classes
  - Rendezvous and formation flying
  - Low thrust transfers
  - Quasi-stationary orbits
  - Earth-moon libration orbits
  - Equilateral libration orbits: L_4 & L_5

- ***Science Challenges***
  - Interferometers
  - Environment
  - Data Rates
  - Limited Maneuvers

- ***Operational Challenges***
  - Servicing of resources in libration orbits
  - Minimal fuel
  - Constrained communications
  - Limited ΔV directions
  - Solar sail applications
  - Continuous control to reference trajectories
  - Tethered missions
  - Human exploration

Background on perturbation theory / accelerations

- Two Body Motion
- Atmospheric Drag
- Potential Models Forces
- Solar Radiation Pressure

Two – Body Motion

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$$m_1\vec{r}_1'' = -\frac{Gm_1m_2}{|\vec{r}_1-\vec{r}_2|^2}\frac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|} = -\frac{Gm_1m_2(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|^3}$$

- Newton's law of gravity of force is inversely proportional to distance

$$m_2\vec{r}_2'' = -\frac{Gm_1m_2}{|\vec{r}_2-\vec{r}_1|^2}\frac{\vec{r}_2-\vec{r}_1}{|\vec{r}_2-\vec{r}_1|} = -\frac{Gm_1m_2(\vec{r}_2-\vec{r}_1)}{|\vec{r}_1-\vec{r}_2|^3}$$

- Vector direction from r_2 to r_1 and r_1 to r_2
- Subtract one from other and define vector r, and gravitational constants

$$\vec{r}_1'' = -\frac{Gm_2(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|^3} \qquad \vec{r}_2'' = -\frac{Gm_1(\vec{r}_2-\vec{r}_1)}{|\vec{r}_1-\vec{r}_2|^3}$$

$$\vec{r}_1''-\vec{r}_2'' = -\frac{Gm_2(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|^3} + \frac{Gm_1(\vec{r}_2-\vec{r}_1)}{|\vec{r}_1-\vec{r}_2|^3} = -\frac{G(m_2+m_1)(\vec{r}_1-\vec{r}_2)}{|\vec{r}_1-\vec{r}_2|^3}$$

$$\vec{r} = \vec{r}_1 - \vec{r}_2$$

Fundamental Equation of Motion

$$\boxed{\vec{r}'' = -\frac{G(m_2+m_1)\vec{r}}{r^3}}$$

$$\mu = G(m_1+m_2)$$

$$\mu = Gm_{earth} \approx 3.986\times10^{14}\,m^3/sec^2$$

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FORCES ON PROPAGATED ORBIT

- Equation Of Motion Propagated.

$$m\frac{d^2\vec{r}}{dt^2} = -\frac{\mu\hat{r}}{r^3} \quad + \text{ accelerations}$$

- External Accelerations Caused By Perturbations

$$a = a_{nonspherical} + a_{drag} + a_{3body} + a_{srp} + a_{tides} + a_{other}$$

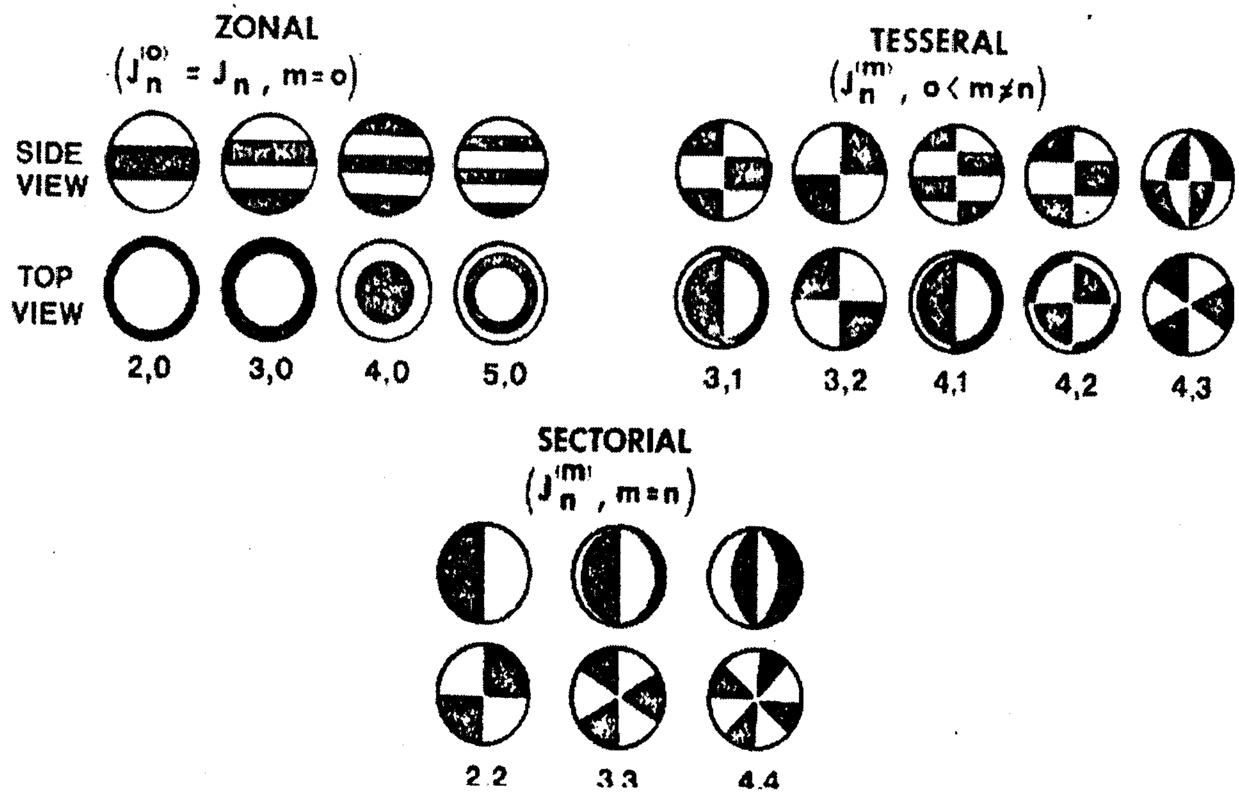
Gaussian Lagrange Planetary Equations

- Changes in Keplerian motion due to perturbations in terms of the applied force. These are a set of differential equations in orbital elements that provide analytic solutions to problems involving perturbations from Keplerian orbits. For a given disturbing function, *R*, they are given by

$$
\begin{aligned}
\dot{a} &= \frac{2}{na}\frac{\partial R}{\partial M} \\
\dot{e} &= \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial M} - \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial \omega} \\
\dot{I} &= \frac{\cos I}{na^2\sqrt{1-e^2}\sin I}\frac{\partial R}{\partial \omega} - \frac{1}{na^2\sqrt{1-e^2}\sin I}\frac{\partial R}{\partial \Omega} \\
\dot{\omega} &= -\frac{\cos I}{na^2\sqrt{1-e^2}\sin I}\frac{\partial R}{\partial I} + \frac{\sqrt{1-e^2}}{na^2 e}\frac{\partial R}{\partial e} \\
\dot{\Omega} &= \frac{1}{na^2\sqrt{1-e^2}\sin I}\frac{\partial R}{\partial I} \\
\dot{M} &= n - \frac{1-e^2}{na^2 e}\frac{\partial R}{\partial e} - \frac{2}{na}\frac{\partial R}{\partial a}
\end{aligned}
$$

Geopotential

- Spherical Harmonics break down into three types of terms
  Zonal – symmetrical about the polar axis
  Sectorial – longitude variations
  Tesseral – combinations of the two to model specific regions
- J2 accounts for most of non-spherical mass
- Shading in figures indicates additional mass



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Potential Accelerations

$$\Phi = \frac{\mu}{r}\left[1 - \sum_{1=2}^{\infty} J_1 \left(\frac{R}{r}\right)^1 P_1(\sin\phi) + \sum_{1=1}^{\infty} \sum_{m=1}^{1} \frac{1}{r^{1+1}} P_{1m}(\sin\phi)\{C_{1m}\cos m\lambda + S_{1m}\sin m\lambda\}\right]$$

The coordinates of P are now expressed in spherical coordinates (r, ϕ, λ) where ϕ is the geocentric latitude and λ is the longitude. R is the equatorial radius of the primary body and $P_{1m}(\sin\phi)$ is the Legendre's Associated Functions of degree and λ order m.

The coefficients and are referred to as spherical harmonic coefficients. If m = 0 the coefficients are referred to as zonal harmonics. If $1 \neq m \neq 0$ they are referred to as tesseral harmonics, and if $1 = m \neq 0$, they are called sectoral harmonics.

Simplified J2 acceleration model for analysis with acceleration in inertial coordinates

$$\mathbf{a} = \nabla \phi = \frac{\delta\phi}{\delta x}\mathbf{i} \;\frac{+\delta\phi}{+\delta y}\mathbf{j} \;\frac{+\delta\phi}{+\delta z}\mathbf{k}$$

$$a_i = \frac{-3J_2\mu R_e^2 r_i}{2r^5}\left(1 - \frac{5r_k^2}{r^2}\right)$$

$$a_j = \frac{-3J_2\mu R_e^2 r_j}{2r^5}\left(1 - \frac{5r_k^2}{r^2}\right)$$

$$a_k = \frac{-3J_2\mu R_e^2 r_j}{2r^5}\left(3 - \frac{5r_k^2}{r^2}\right)$$

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Atmospheric Drag

- Atmospheric Drag Force On The Spacecraft Is A Result Of Solar Effects On The Earth's Atmosphere
- The Two Solar Effects:
  - Direct Heating of the Atmosphere
  - Interaction of Solar Particles (Solar Wind) with the Earth's Magnetic Field
- NASA / GSFC Flight Dynamics Analysis Branch Uses Several models:
  - Harris-Priester
    - Models direct heating only
    - Converts flux value to density
  - Jacchia-Roberts or MSIS
    - Models both effects
    - Converts to exospheric temp. And then to atmospheric density
    - Contains lag heating terms

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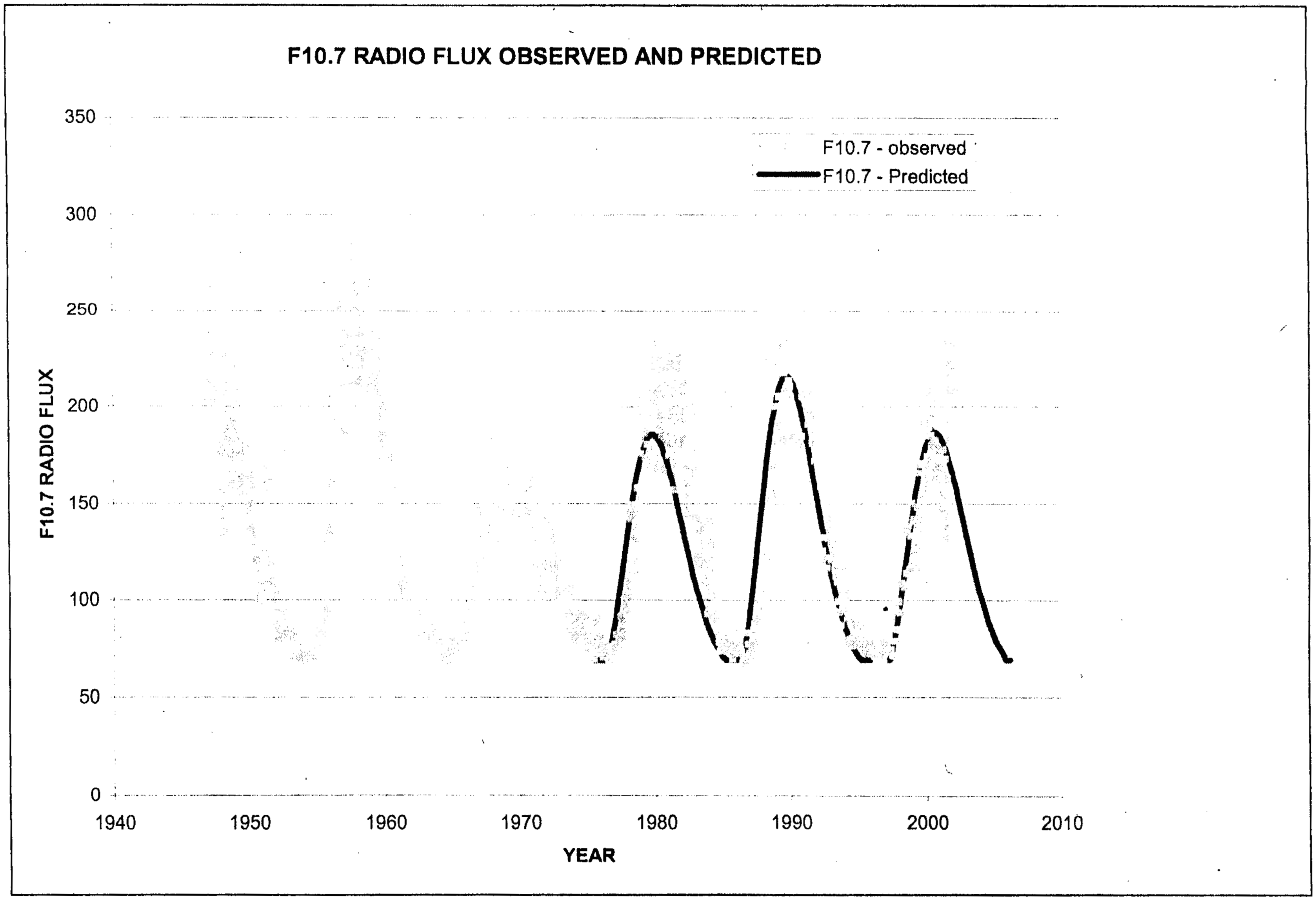
Solar Flux Prediction

Historical Solar Flux, F10.7cm values Observed and Predicted (+2s) 1945-2002



Drag Acceleration

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- Acceleration defined as

$$a = \frac{1}{2} \frac{C_d A \rho v_a^2 \hat{v}}{m}$$

A = Spacecraft cross sectional area, (m^2)
C_d = Spacecraft Coefficient of Drag, unitless
m = mass, (kg)
ρ = atmospheric density, (kg/m^3)
v_a = s/c velocity wrt to atmosphere, (km/s)
$\hat{v}$ = inertial spacecraft velocity unit vector
$\frac{C_d A}{m}$ = Spacecraft ballistic property

- Planetary Equation for semi-major axis decay rate of circular orbit (Wertz/Vallado p629), small effect in e

$$\Delta a = \frac{- 2\pi C_d A \rho a^2}{m}$$

Solar Radiation Pressure Acceleration

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$$\bar{F} = -\frac{1}{c} G A_{s/c} \hat{s}, \text{ but}$$

$$\frac{G}{c} = \frac{1350\,\text{watts}/m^2}{3x10^8\,m/s} = 4.5x10^{-6}\,\text{watt sec}/m^3$$

$$= 4.5x10^{-6}\,N/m^2 \equiv P_{SR}$$

Therefore,

$$\bar{F} = -P_{SR} A_{s/c} \hat{s} \qquad (3)$$

Where G is the incident solar radiation per unit area striking the surface, $A_{s/c}$. G at 1 AU = 1350 watts/m^2 and $A_{s/c}$ = area of the spacecraft normal to the sun direction. In general we break the solar pressure force into the component due to absorption and the component due to reflection

$$\bar{F} = \bar{F}_s + \bar{F}_n$$

Where $\bar{F}_s$ is the force in the solar direction and $\bar{F}_n$ is the force normal to the surface

$$\bar{F} = -P_{SP} A_{s/c} [\alpha \hat{s} + 2\gamma \hat{n}]$$

where

$\alpha \equiv$ absorptivity coefficient, $0 \le \alpha \le 1$, $\alpha = 1 - \gamma$

$\gamma \equiv$ reflectivity coefficient of specular reflection, $0 \le \gamma \le 1$

$\hat{n}$ is a unit vector normal to the surface, $A_{s/c}$

$A_{s/c}$ is the area normal to the sun direction

From Lagrange's planetary equations

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}} \{ e \sin \nu \bar{F}_R + \frac{a(1-e^2)}{r} \bar{F}_I \}$$

Where $\bar{F}_R$ and $\bar{F}_I$ are the radial and in-track solar pressure forces.

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Other Perturbations

- Third Body

  $a_{3b} = -\mu/r^3 \bar{r} = \mu(\bar{r}_j/r_j^3 - \bar{r}_k/r_k^3)$

  - Where r_j is distance from s/c to body and r_k is distance from body to Earth

- Thrust – from maneuvers and out gassing from instrumentation and materials

  Inertial acceleration: $\ddot{x} = T_x/m$, $\ddot{y}=T_y/m$, $\ddot{z}=T_z/m$

- Tides, others

Ballistic Coefficient

- Area (A) is calculated based on spacecraft model.
  - Typically held constant over the entire orbit
  - Variable is possible, but more complicated to model
  - Effects of fixed vs. articulated solar array
- Coefficient of Drag (C_d)is defined based on the shape of an object.
  - The spacecraft is typically made up of many objects of different shapes.
  - We typically use 2.0 to 2.2 (C_d for A sphere or flat plate) held constant over the entire orbit because it represents an average
- For 3 axis, 1 rev per orbit, earth pointing s/c: A and C_d do not change drastically over an orbit wrt velocity vector
  - Geometry of solar panel, antenna pointing, rotating instruments
- Inertial pointing spacecraft could have drastic changes in B_c over an orbit

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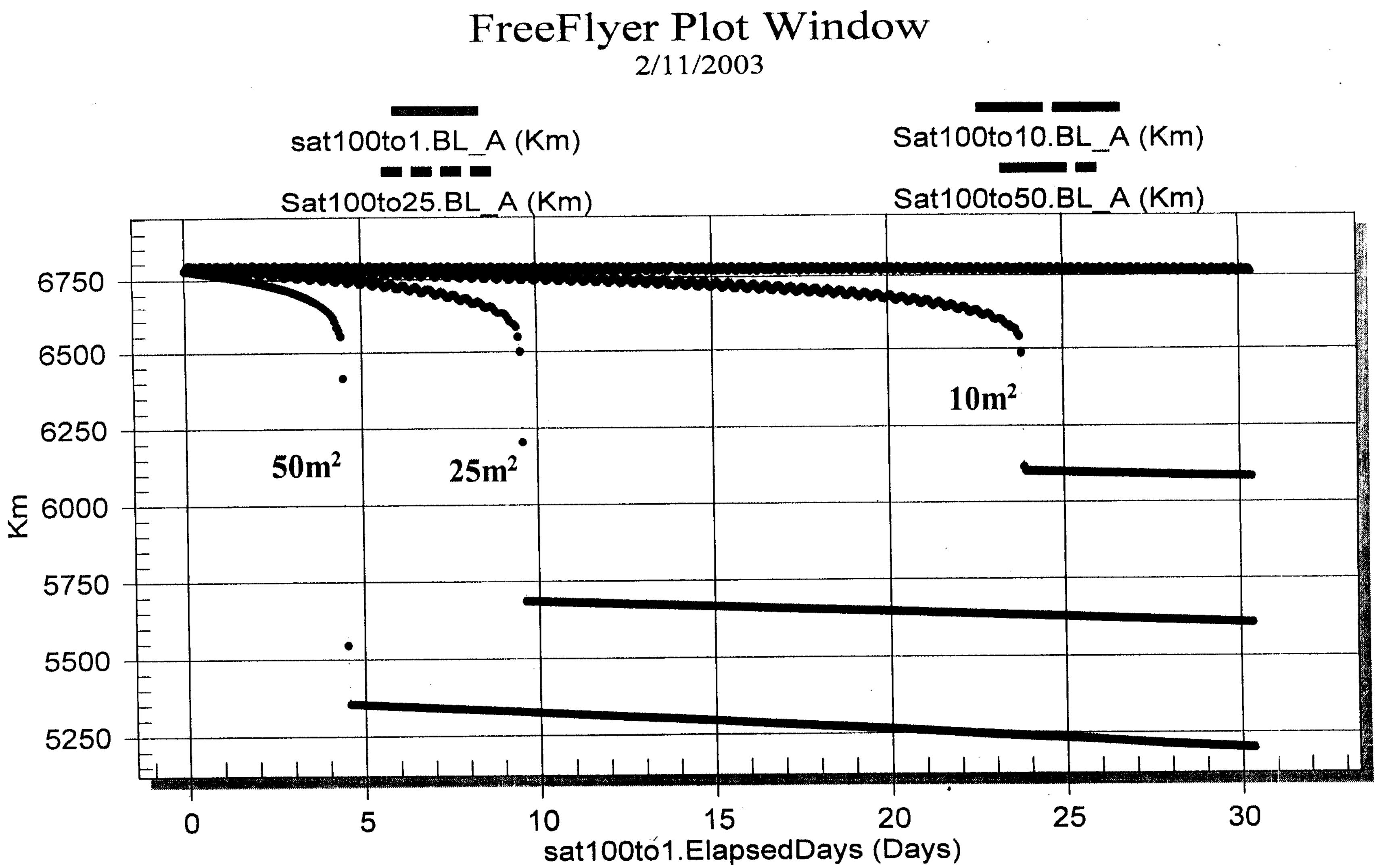
Ballistic Effects

Varying the mass to area yields different decay rates

Sample: 100kg with area of 1, 10, 25, and 50m^2, C_d=2.2



Numerical Integration

- Solutions to ordinary differential equations (ODEs) to solve the equations of motion.
- Includes a numerical integration of all accelerations to solve the equations of motion
- Typical integrators are based on
  - Runge-Kutta

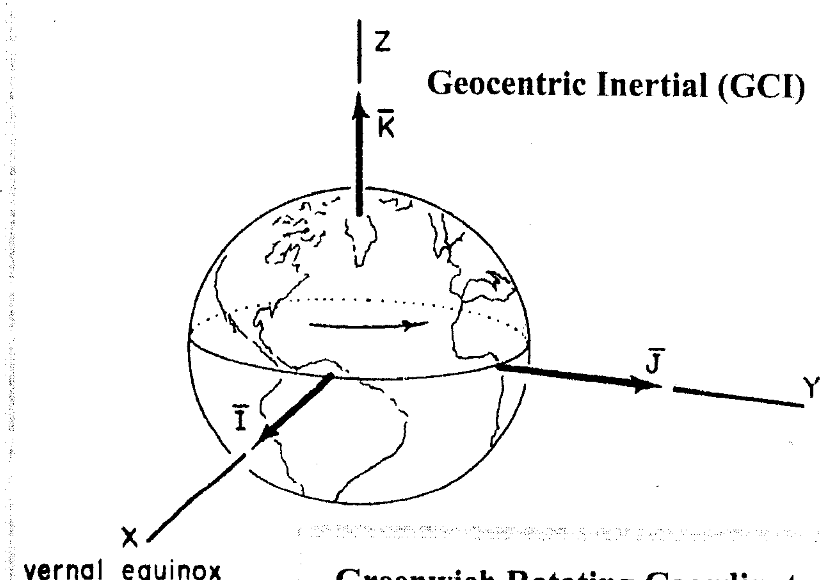
    formula for y_{n+1}, namely:

    $$y_{n+1} = y_n + (1/6)(k_1 + 2k_2 + 2k_3 + k_4)$$

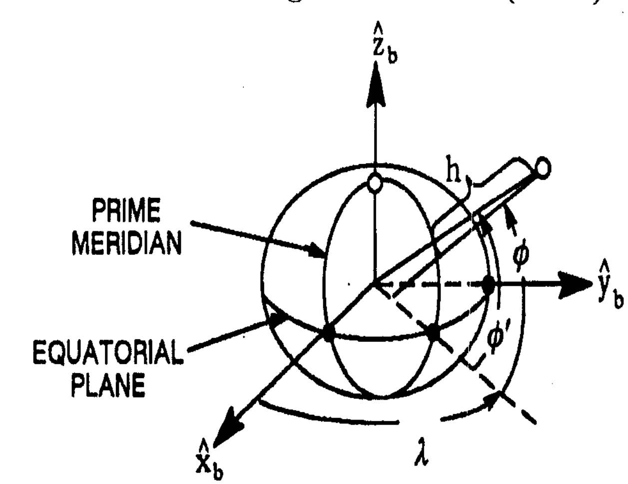
    is simply the y-value of the current point plus a weighted average of four different y-jump estimates for the interval, with the estimates based on the slope at the midpoint being weighted twice as heavily as the those using the slope at the end-points.
  - Cowell-Moulton
  - Multi-Conic (patched)
  - Matlab ODE 4/5 is a variable step RK

Coordinate Systems

- Origin of reference frames:
  - Planet
  - Barycenter
  - Topographic
- Reference planes:
  - Equator – equinox
  - Ecliptic – equinox
  - Equator – local meridian
  - Horizon – local meridian



Geocentric Inertial (GCI)



Greenwich Rotating Coordinates (GRC)

Most used systems

GCI – Integration of EOM

ECEF – Navigation

Topographic – Ground station

Describing Motion Near a Known Orbit

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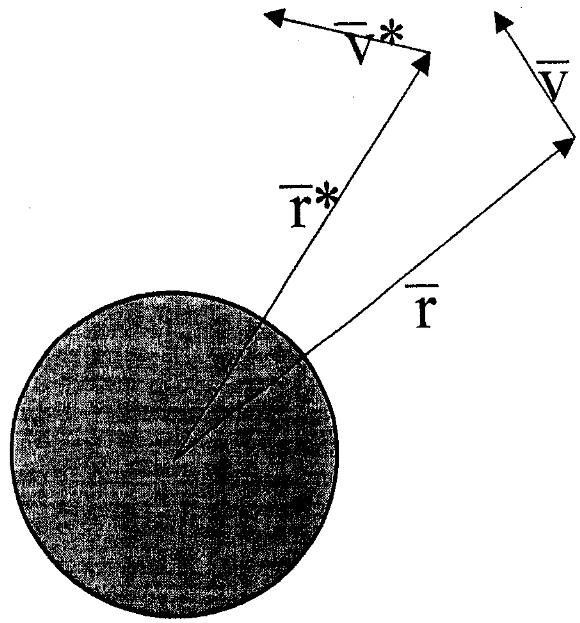
A local system can be established by selection of a central s/c or center point and using the Cartesian elements to construct the local system.that rotates with respect to a fixed point (spacecraft)



$$\bar{r}^* = \bar{r}^*(t)$$
$$\bar{v}^* = \bar{v}^*(t)$$
$$\bar{r} = \bar{r}(t)$$
$$\bar{v} = \bar{v}(t)$$

Known (reference Orbit)

$$\delta\bar{r}(t) = \bar{r}(t) - \bar{r}^*(t)$$
$$\delta\bar{v}(t) = \bar{v}(t) - \bar{v}^*(t)$$

Relative Motion

$$\frac{d^2}{dt^2}\bar{r}(t) = 1/m\bar{F}(\bar{r},\bar{v}) \equiv \bar{f}(\bar{r},\bar{v})$$

What equations of motion does the relative motion follow?

$$\frac{d^2}{dt^2}\delta\bar{r} = \frac{d^2}{dt^2}(\bar{r}(t) - \bar{r}^*(t)) = \frac{d^2}{dt^2}\bar{r}(t) - \frac{d^2}{dt^2}\bar{r}^*(t) = \bar{f}(\bar{r},\bar{v}) - \bar{f}(\bar{r}^*,\bar{v}^*)$$

$$\frac{d^2}{dt^2}\delta\bar{r}(t) = \bar{f}(\bar{r},\bar{v}) - \bar{f}(\bar{r}^*,\bar{v}^*)$$

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Describing Motion Near a Known Orbit

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As it stands (1) is exact. However if $\delta\bar{r}$ is sufficiently close, the term $\bar{f}(\bar{r})$an be expanded via Taylor's series …

$$\bar{f}(\bar{r}) = \bar{f}(\bar{r}^* + \delta\bar{r}) = \bar{f}(\bar{r}^*) + \frac{\partial \bar{f}}{\partial \bar{r}}\Big|_{\bar{r}=\bar{r}^*} \delta\bar{r} + \ldots$$

Substituting in yields a linear set of ODEs

$$\frac{d^2}{dt^2}\delta\bar{r}(t) = \bar{f}(\bar{r}) - \bar{f}(\bar{r}^*) = \bar{f}(\bar{r}^*) + \frac{\partial \bar{f}}{\partial \bar{r}}\Big|_{\bar{r}=\bar{r}^*} \delta\bar{r} - \bar{f}(\bar{r}^*)$$

$$\boxed{\frac{d^2}{dt^2}\delta\bar{r} = \frac{\partial \bar{f}}{\partial \bar{r}}\Big|_{\bar{r}=\bar{r}^*} \delta\bar{r}}$$

This is important since it will be our starting point for everything that follows

Describing Motion Near a Known Orbit

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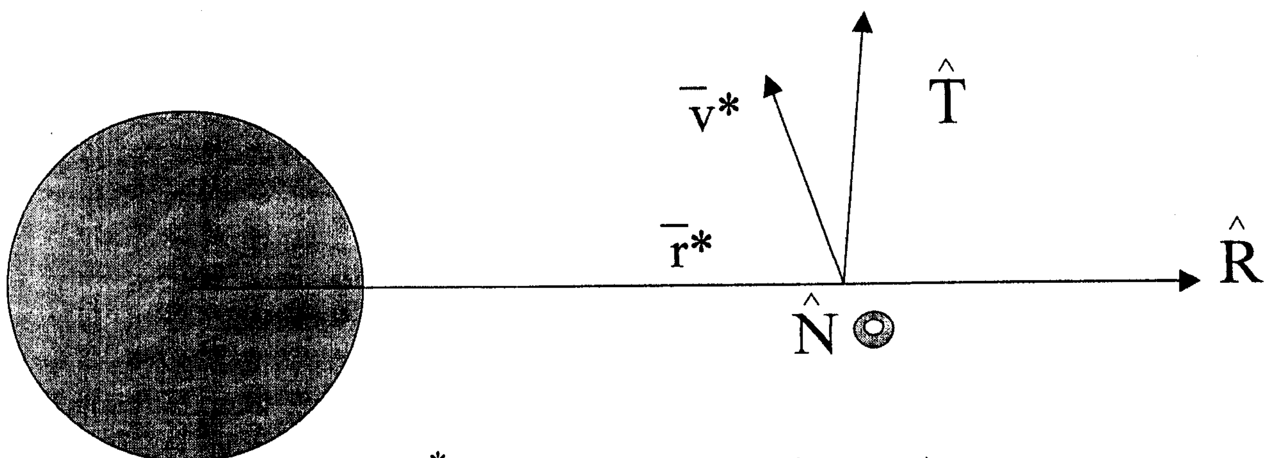
- Describe motion taking place near a circular orbit
- A natural coordinate frame is one that rotates with the circular orbit



$$\hat{R} = \frac{\bar{r}^*}{\left|\bar{r}^*\right|} \qquad \hat{N} = \frac{\bar{r}^* \times \bar{v}^*}{\left|\bar{r}^* \times \bar{v}^*\right|} \qquad \hat{T} = \hat{N} \times \hat{R}$$

The frame described is known as

- Hill's	- RTN
- Clohessy-Wiltshire	- RAC
- LVLH	- RIC

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Describing Motion Near a Known Orbit

Any vector will now be given by: $\bar{A} = x\hat{R} + y\hat{T} + z\hat{N} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Now we can evaluate $\bar{f} = (\frac{-\mu}{r^3}\bar{r}) \Rightarrow f_i = (\frac{-\mu}{r^3}x_i)$

$$\frac{\partial \bar{f}}{\partial \bar{r}} = \frac{\partial f_i}{\partial x_j} = \frac{\partial}{\partial x_j}(\frac{-\mu}{r^3}x_i) = \frac{3\mu}{r^5}x_i x_j - \frac{\mu}{r^3}\delta_{ij}$$

$$\frac{\partial \bar{f}}{\partial \bar{r}} = \frac{\mu}{r^5}(x_i x_j - r^2\delta_{ij})$$

In RTN =RR

x=r=r ^

y=z=0 $\bar{r}$*

$$= \frac{\mu}{r^5}\begin{pmatrix} 2x^2 - y^2 - z^2 & 3xy & 3xz \\ 3xy & 2y^2 - x^2 - z^2 & 3yz \\ 3xz & 3yz & 2z^2 - y^2 - x^2 \end{pmatrix}$$

$$\boxed{\frac{\partial \bar{f}}{\partial \bar{r}}\Big|_{\bar{r}^*} \frac{\mu}{r^3}\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = n^2\begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}$$

Transforming the EOM

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Now convert Newton's 2nd law to RTN frame

$$\frac{d}{dt}\Big)_{moving} = \frac{d}{dt}\Big)_{fixed} - \omega \times$$

Newton's law involves 2nd derivatives:

$$\frac{D^2}{Dt^2}\delta\bar{r} = \frac{d^2}{dt^2}\delta\bar{r} - \frac{d\varpi^2}{dt^2} \times \delta\bar{r} - 2\varpi \times \frac{d}{dt}\delta\bar{r} + \varpi \times (\varpi \times \delta\bar{r})$$

$$\frac{D^2}{Dt^2}\delta\bar{r} = \frac{d^2}{dt^2}\delta\bar{r} - \frac{d\varpi^2}{dt^2} \times \delta\bar{r} - 2\varpi \times \frac{D}{Dt}\delta\bar{r} + \varpi \times (\varpi \times \delta\bar{r})$$

$$\boxed{\varpi = \begin{pmatrix} 0 \\ 0 \\ n \end{pmatrix}} \quad \boxed{\delta\bar{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}} \quad \boxed{\frac{d\varpi}{dt} = \frac{D\varpi}{Dt} = 0} \quad \boxed{\frac{D}{Dt}\delta\bar{r} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}} \quad \boxed{\varpi \times \frac{D}{Dt}\delta\bar{r} = \begin{pmatrix} -n\dot{y} \\ n\dot{x} \\ 0 \end{pmatrix}} \quad \boxed{\varpi \times (\varpi \times \delta\bar{r}) = \begin{pmatrix} -n^2 x \\ -n^2 y \\ 0 \end{pmatrix}}$$

$$\frac{d^2}{dt^2}\delta\bar{r} = \frac{\partial \bar{f}}{\partial \bar{r}}\Big|_{\bar{r}^*} \delta\bar{r} = \begin{pmatrix} 2n^2 x \\ -n^2 y \\ -n^2 z \end{pmatrix}$$

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Transforming the EOM yields Clohessy-Wiltshire Equations

$$\frac{D^2}{Dt^2}\delta\bar{r} = \begin{pmatrix} -2n^2x \\ -n^2y \\ -n^2z \end{pmatrix} - 2\begin{pmatrix} -n^2\dot{y} \\ -n^2\dot{x} \\ 0 \end{pmatrix} - \begin{pmatrix} -n^2x \\ -n^2y \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = \begin{pmatrix} 3n^2x + 2n^2\dot{y} \\ -2n\dot{x} \\ -nz \end{pmatrix}$$

$$\ddot{y} = -2n\dot{x} \Rightarrow \dot{y} = -2nx + k_1$$

$$\ddot{x} = -n^2x + 2nk_1$$

$$x = x_0\cos(nt) + \frac{v_0}{n}\sin(nt) + \frac{2k_1}{n}$$

$$y = -2x_0\sin(nt) + \frac{2v_0}{n}\cos(nt) + \frac{2k_1}{n}t + y_0$$

$$z = z_0\cos(nt) + \frac{\omega_0}{n}\sin(nt)$$

A "balance" form will have no secular growth, k_1=0

Note that the y-motion (associated with T) has twice the amplitude of the x motion (R)

5,6,7

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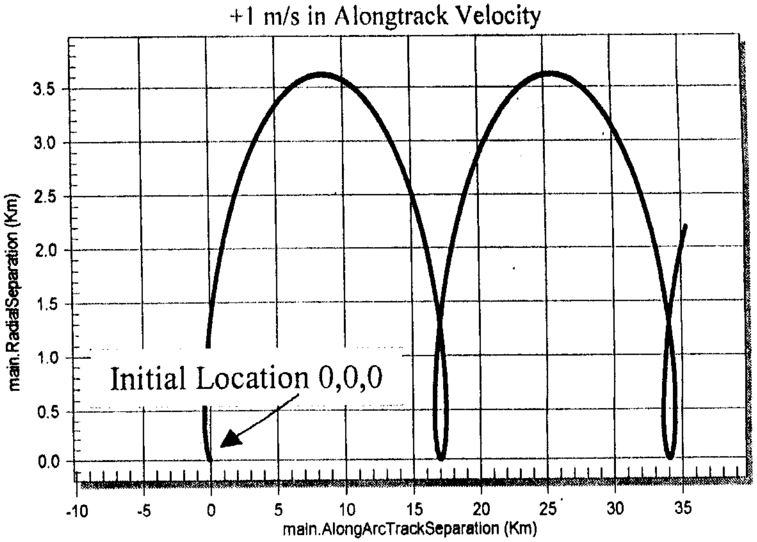
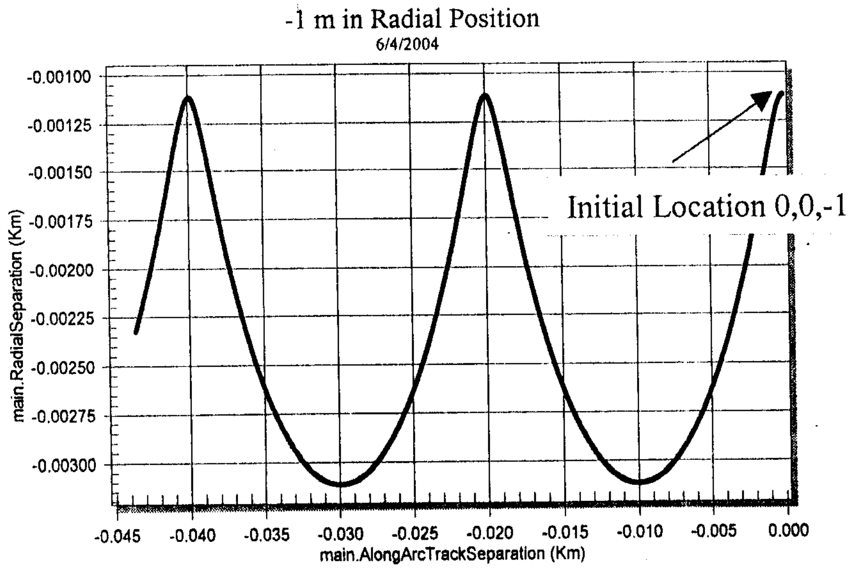
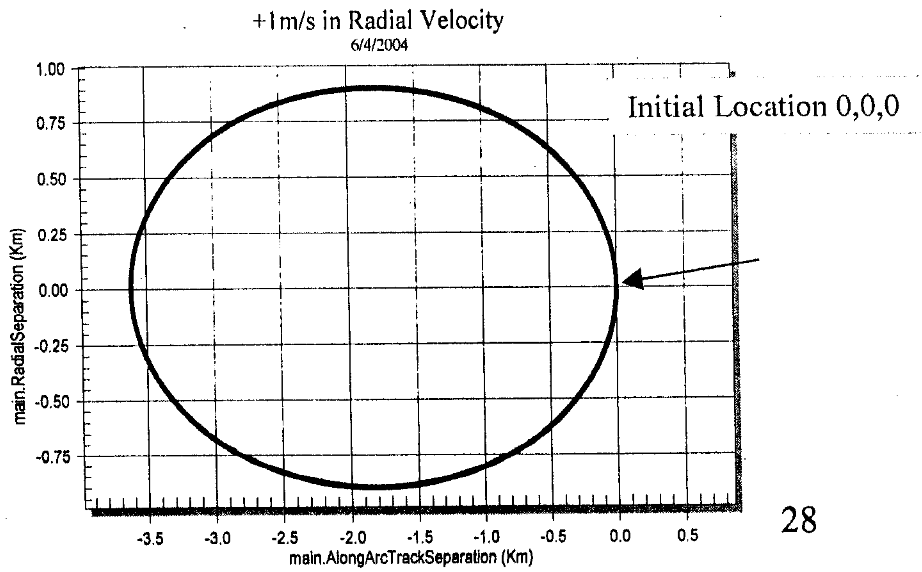
Relative Motion

A numerical simulation using RK8/9 and point mass Effect of Velocity (1 m/s) or Position(1 m) Difference

- An Along-track separation remain constant
- A 1 m radial position difference yields an along-track motion
- A 1 m/s along-track velocity yields an along-track motion
- A 1 m/s radial velocity yields a shifted circular motion







Shuttle Vbar / Rbar

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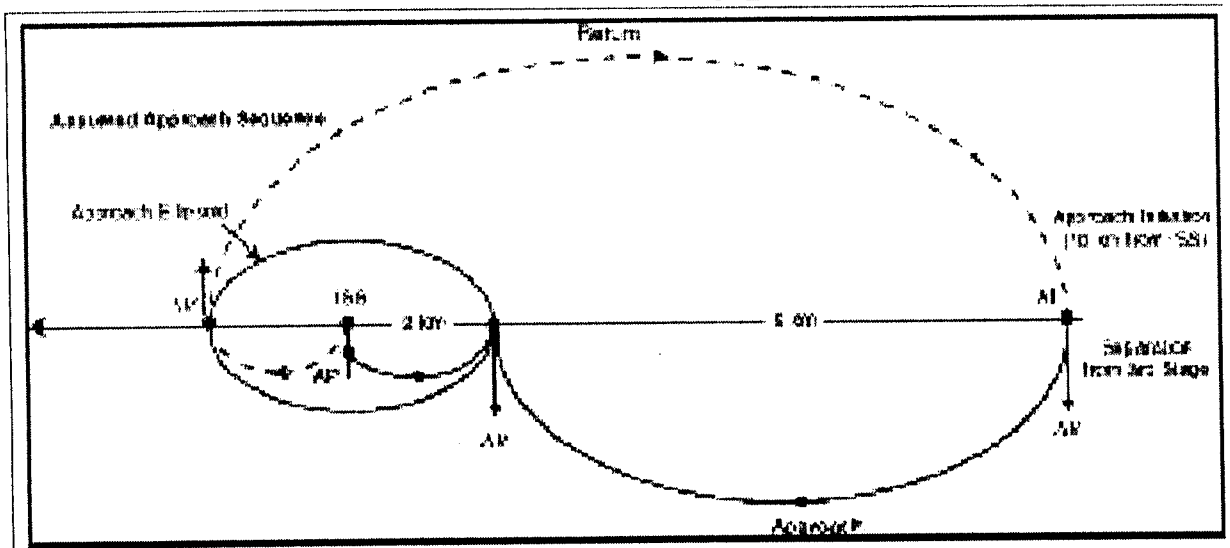
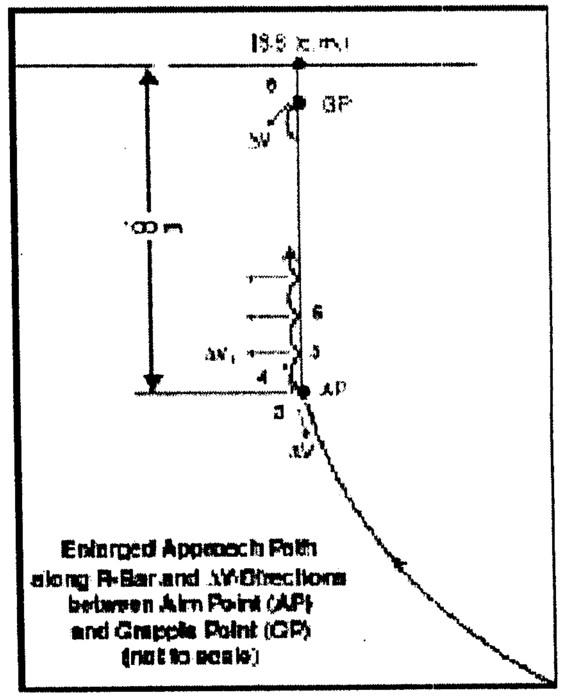
- Shuttle approach strategies
  - Vbar – Velocity vector direction in an LVLH (CW) coordinate system
  - Rbar – Radial vector direction in an LVLH (CW) coordinate system

- Passively safe trajectories – Planned trajectories that make use of predictable CW motion if a maneuver is not performed.

- Consideration of ballistic differences – Relative CW motion considering the difference in the drag profiles.





Graphics Ref: Collins, Meissinger, and Bell, Small Orbit Transfer Vehicle (OTV) for On-Orbit Satellite Servicing and Resupply, 15th USU Small Satellite Conference, 2001

7,9

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What Goes Wrong with an Ellipse

In stable – space notation, linearization is written as $\frac{d}{dt}\delta\overline{S} = A\delta\overline{S}$

$$A = \begin{pmatrix} 0 & \mathrm{I} \\ \frac{\partial \bar{f}}{\partial \bar{x}} & 0 \end{pmatrix}$$

Since the equation is linear

$$\delta\overline{S} = \Phi\,\delta S_0 \Rightarrow \frac{d}{dt}\Phi = A\Phi$$

$$\Phi = \mathrm{I} + \int_{t_0}^{t} dt' A(\bar{r}^*(t')\Phi(t',t_0)$$

Which has no closed form solution if

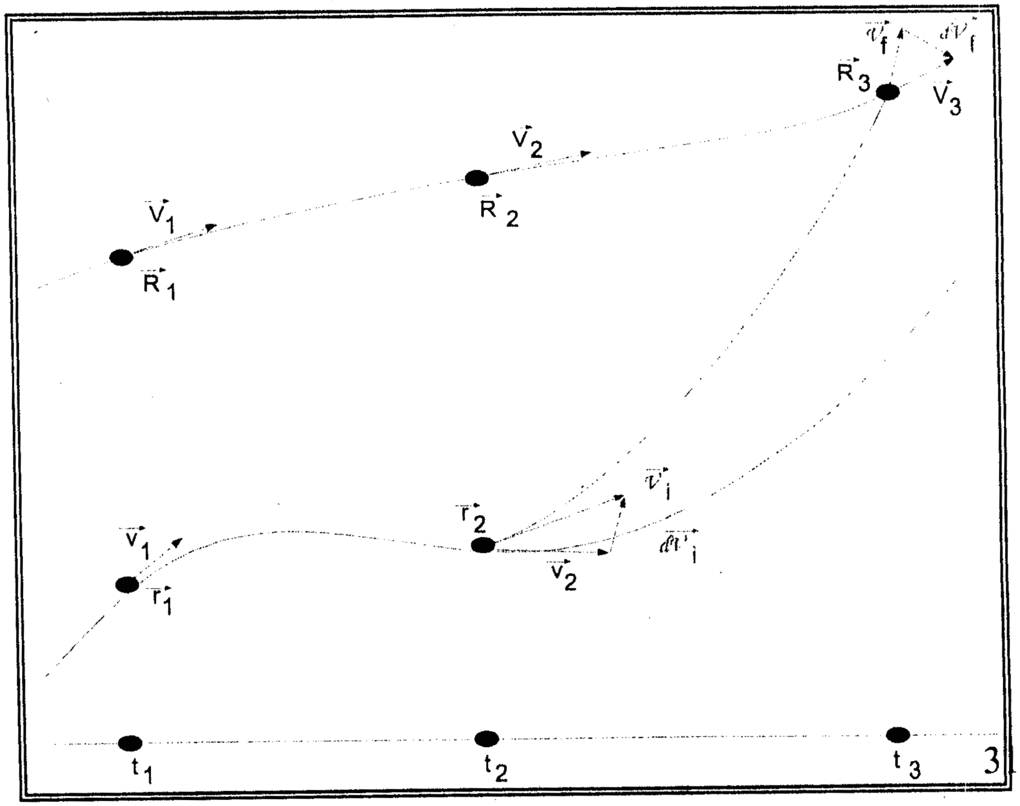
$$\left[A(t_1), A(t_2)\right] = 0$$

ef # 5,6,7

Lambert Problem

- Consider two trajectories **r**(t) and **R**(t).
- Transfer from **r**(t) to **R**(t) is affected by two ΔVs
  - First $d\mathcal{V}_i$ is designed to match the velocity of a transfer trajectory $\mathcal{R}$(t) at time t_2
  - Second $d\mathcal{V}_f$ is designed to match the velocity of **R**(t) where the transfer intersects at time t_3
- Lambert problem:

  Determine the two ΔVs



f # 5,6,7

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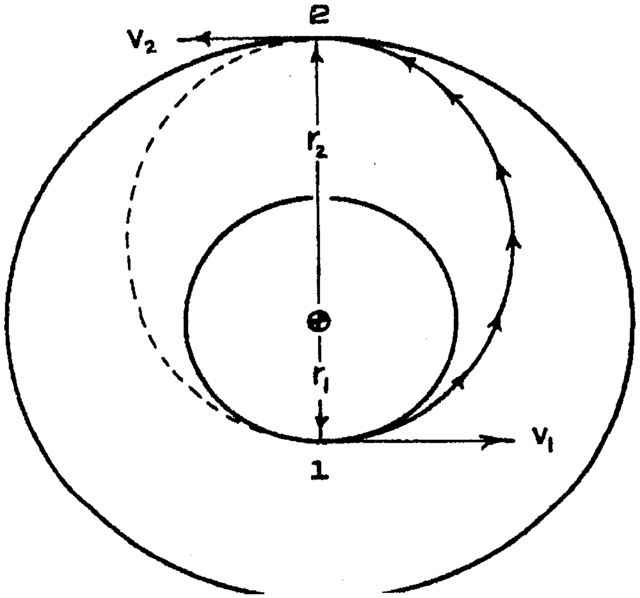
Lambert Problem

- The most general way to solve the problem is to use to numerically integrate $\mathbf{r}(t)$, $\mathbf{R}(t)$, and $\mathcal{R}(t)$ using a shooting method to determine $d\mathcal{V}_i$ and then simply subtracting to determine $d\mathcal{V}_f$
- However this is relatively expensive (prohibitively onboard) and is not necessary when $\mathbf{r}(t)$ and $\mathbf{R}(t)$ are close
- For the case when $\mathbf{r}(t)$ and $\mathbf{R}(t)$ are nearby, say in a stationkeeping situation, then linearization can be used.
- Taking $\mathbf{r}(t)$, $\mathbf{R}(t)$, and $\Phi(t_3,t_2)$ as known, we can determine $d\mathcal{V}_i$ and $d\mathcal{V}_f$ using simple matrix methods to compute a 'single pass'.

5,6,7

The Hohmann Transfer



$$V^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right)$$

where:

$$a = \frac{r_a + r_p}{2}$$

$$V_p = \sqrt{\frac{\mu}{r_p}}\sqrt{\frac{2r_a}{r_a + r_p}} \qquad V_a = \sqrt{\frac{\mu}{r_a}}\sqrt{\frac{2r_p}{r_a + r_p}}$$

$$V_c = \sqrt{\frac{\mu}{r}}$$

$$\Delta V_1 = \sqrt{\frac{\mu}{r_1}}\sqrt{\frac{2r_2}{r_1 + r_2}} - \sqrt{\frac{\mu}{r_1}} = \sqrt{\frac{\mu}{r_1}}\left[\sqrt{\frac{2r_2}{r_1 + r_2}} - 1\right]$$

$$\Delta V_2 = \sqrt{\frac{\mu}{r_2}}\sqrt{\frac{2r_1}{r_1 + r_2}} - \sqrt{\frac{\mu}{r_2}} = \sqrt{\frac{\mu}{r_2}}\left[1 - \sqrt{\frac{2r_1}{r_1 + r_2}}\right]$$

$$\mathbf{\Delta V_T = \Delta V_1 + \Delta V_2}$$

EO-1 GSFC Formation Flying
New Millennium Requirements

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- **Enhanced Formation Flying (EFF)**

The Enhanced Formation Flying (EFF) technology shall provide the autonomous capability of flying over the same ground track of another spacecraft at a fixed separation in time.

- **Ground track Control**

EO-1 shall fly over the same ground track as Landsat-7. EFF shall predict and plan formation control maneuvers or D*a* maneuvers to maintain the ground track if necessary.

- **Formation Control**

Predict and plan formation flying maneuvers to meet a nominal 1 minute spacecraft separation with a +/- 6 seconds tolerance. Plan maneuver in 12 hours with a 2 day notification to ground.

- **Autonomy**

The onboard flight software, called the EFF, shall provide the interface between the ACS / C&DH and the AutoCon™ system for Autonomy for transfer of all data and tables.

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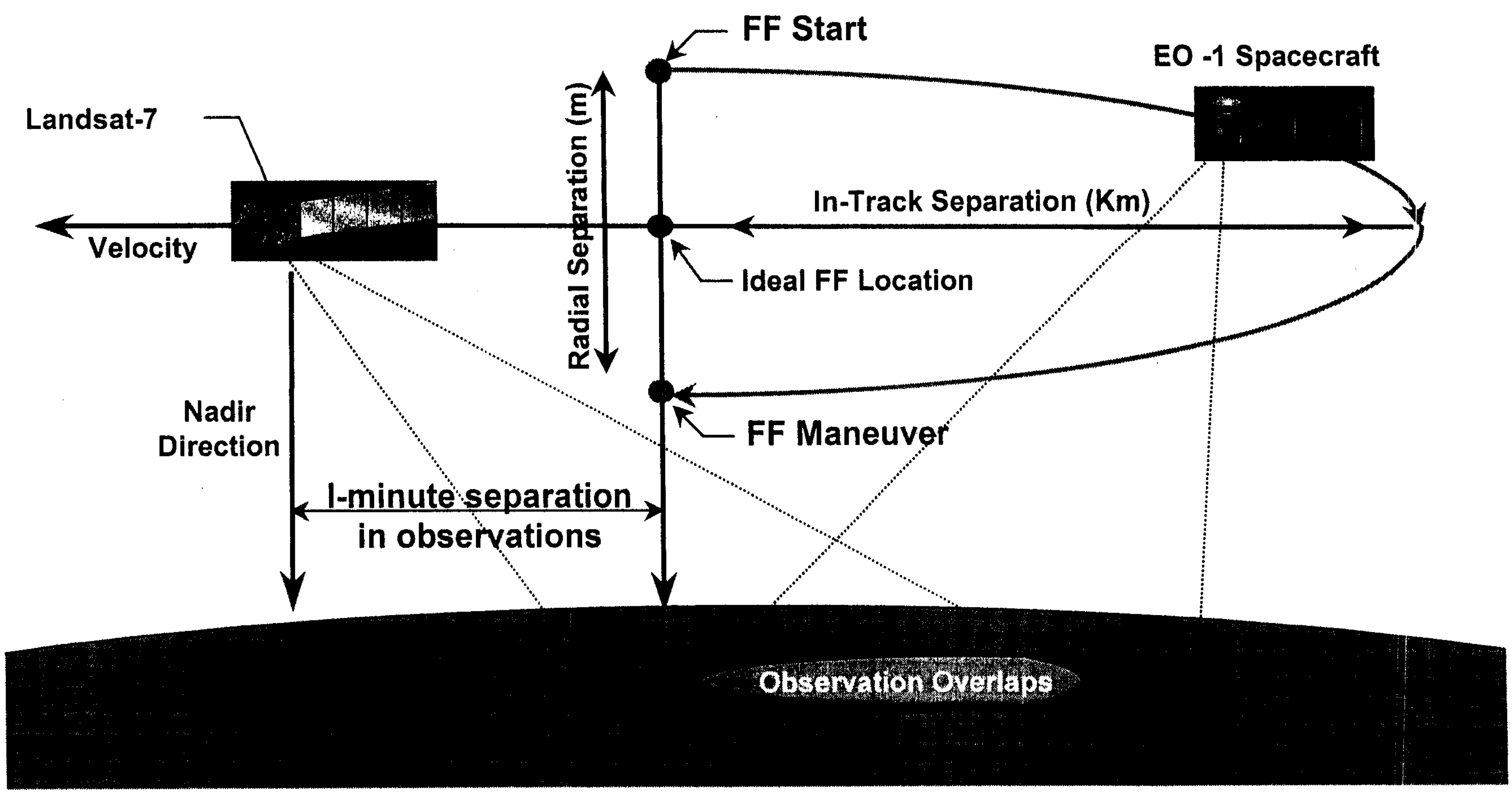
Formation Flying Maintenance Description
Landsat-7 and EO-1

Different Ballistic Coefficients and Relative Motion



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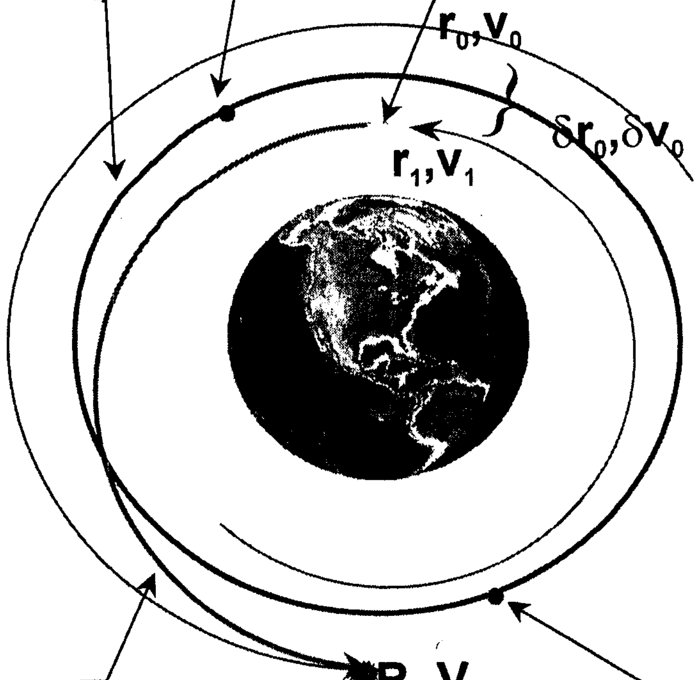
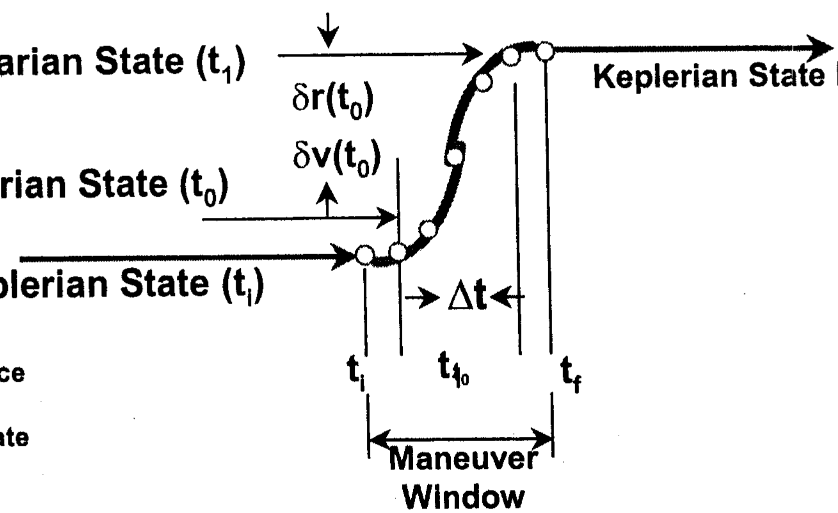
EO-1 Formation Flying Algorithm



- ***Determine*** (r_1, v_1) ***at*** t_0 *(where you are at time* t_0*).*
- ***Determine*** (R_1, V_1) ***at*** t_1 *(where you want to be at time* t_1*).*
- ***Project*** (R_1, V_1) ***through*** $-\Delta t$ ***to determine*** (r_0, v_0) *(where you should be at time* t_0*).*
- ***Compute*** $(\delta r_0, \delta v_0)$ *(difference between where you are and where you want be at* t_0*).*



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State Transition Matrix

A state transition matrix, $F(t_1,t_0)$, can be constructed that will be a function of both t_1 and t_0 while satisfying matrix differential equation relationships. The initial conditions of $F(t_1,t_0)$ are the identity matrix. Having partitioned the state transition matrix, $F(t_1,t_0)$ for time $t_0 < t_1$:

$$\Phi(t_1,t_0) \equiv \begin{bmatrix} \Phi_1(t_1,t_0) & \Phi_2(t_1,t_0) \\ \Phi_3(t_1,t_0) & \Phi_4(t_1,t_0) \end{bmatrix}$$

We find the inverse may be directly obtained by employing symplectic properties:

$$\Phi^{-1}(t_1,t_0) \equiv \begin{bmatrix} (\Phi_4(t_1,t_0))^T & (\Phi_2(t_1,t_0))^T \\ (\Phi_3(t_1,t_0))^T & (\Phi_1(t_1,t_0))^T \end{bmatrix} \quad \Phi^{-1}(t_1,t_0) \equiv \Phi(t_0,t_1) \equiv \begin{bmatrix} \Phi_1(t_0,t_1) & \Phi_2(t_0,t_1) \\ \Phi_3(t_0,t_1) & \Phi_4(t_0,t_1) \end{bmatrix}$$

$F(t_0,t_1)$ is based on a propagation forward in time from t_0 to t_1 (the navigation matrix)
$F(t_1,t_0)$ is based on a propagation backward in time from t_1 to t_0, (the guidance matrix).
We can further define the elements of the transition matrices as follows:

$$\begin{aligned} \tilde{R}(t_1) &\equiv \Phi_1(t_1,t_0) & \tilde{R}^*(t_0) &\equiv \Phi_1(t_0,t_1) \\ R(t_1) &\equiv \Phi_2(t_1,t_0) & R^*(t_0) &\equiv \Phi_2(t_0,t_1) \\ \tilde{V}(t_1) &\equiv \Phi_3(t_1,t_0) & \tilde{V}^*(t_0) &\equiv \Phi_3(t_0,t_1) \\ V(t_1) &\equiv \Phi_4(t_1,t_0) & V^*(t_0) &\equiv \Phi_4(t_0,t_1) \end{aligned}$$

$$\begin{bmatrix} \tilde{R}^*(t_0) & R^*(t_0) \\ \tilde{V}^*(t_0) & V^*(t_0) \end{bmatrix} = \begin{bmatrix} V^T(t_1) & -R(t_1) \\ -\tilde{V}^T(t_1) & \tilde{R}(t_1) \end{bmatrix}$$

10,11

Enhanced Formation Flying Algorithm

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The Algorithm is found from the STM and is based on the simplectic nature (navigation and guidance matrices) of the STM)

- ***Compute the matrices*** $\left[\mathrm{R}(t_1)\right]$, $\left[\tilde{\mathrm{R}}(t_1)\right]$ ***according to the following:***

  Given

  $$\delta \mathbf{r}_0 \equiv (\mathbf{r}_1 - \mathbf{r}_0) \qquad \delta \mathbf{v}_0 \equiv (\mathbf{v}_1 - \mathbf{v}_0)$$

  Compute

  $$\left[\mathbf{R}(t_1)\right] = \frac{\mathrm{r}_0}{\mu}(1-\mathrm{F})\left[(\mathbf{R}_1 - \mathbf{r}_0)\mathbf{v}_0^{\mathrm{T}} - (\mathbf{V}_1 - \mathbf{v}_0)\mathbf{r}_0^{\mathrm{T}}\right] + \frac{C}{\mu}\left[\mathbf{V}_1\mathbf{v}_0^{\mathrm{T}}\right] + G[\mathbf{I}]$$

  $$\left[\tilde{\mathbf{R}}(t_1)\right] = \frac{\mathrm{R}_1}{\mu}\left[(\mathbf{V}_1 - \mathbf{v}_0)(\mathbf{V}_1 - \mathbf{v}_0)^{\mathrm{T}}\right] + \frac{1}{\mathrm{r}_0^{\,3}}\left[\mathrm{r}_0(1-\mathrm{F})\mathbf{R}_1\mathbf{r}_0^{\mathrm{T}} + C\mathbf{V}_1\mathbf{r}_0^{\mathrm{T}}\right] + F[\mathbf{I}]$$

- ***Compute the 'velocity-to-be-gained'*** ($\Delta \mathrm{v}_0$) ***for the current cycle.***

$$\boxed{\Delta \mathbf{v}_0 = \left\{\left[\tilde{\mathrm{R}}^{\mathrm{T}}(t_1)\right]\left[-\mathrm{R}^{\mathrm{T}}(t_1)\right]\right\} \delta \mathbf{r}_0 - \delta \mathbf{v}_0}$$

where F and G are found from Gauss problem and the f & g series and C found through universal variable formulation

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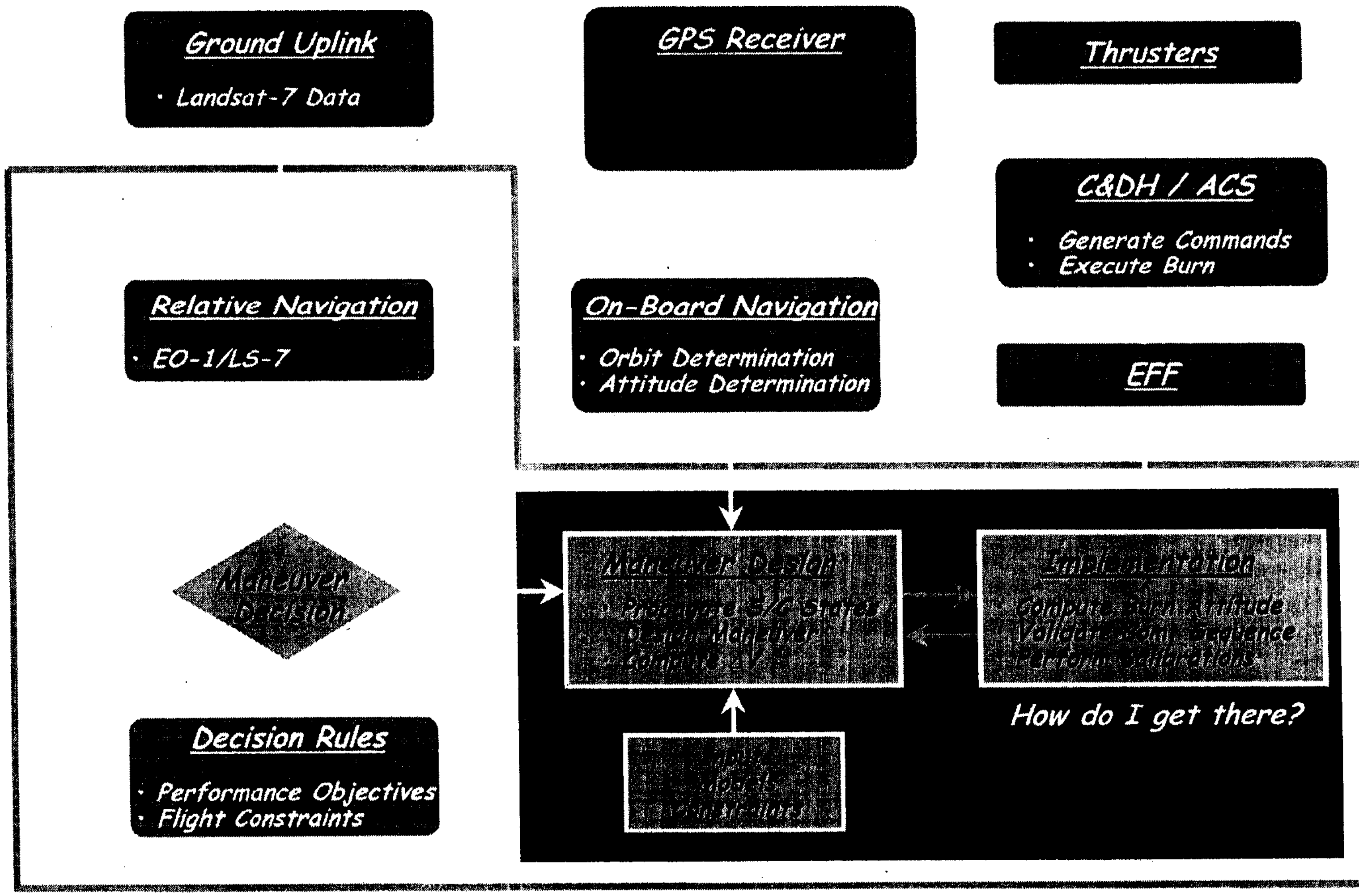
EO-1 AutoCon™ Functional Description



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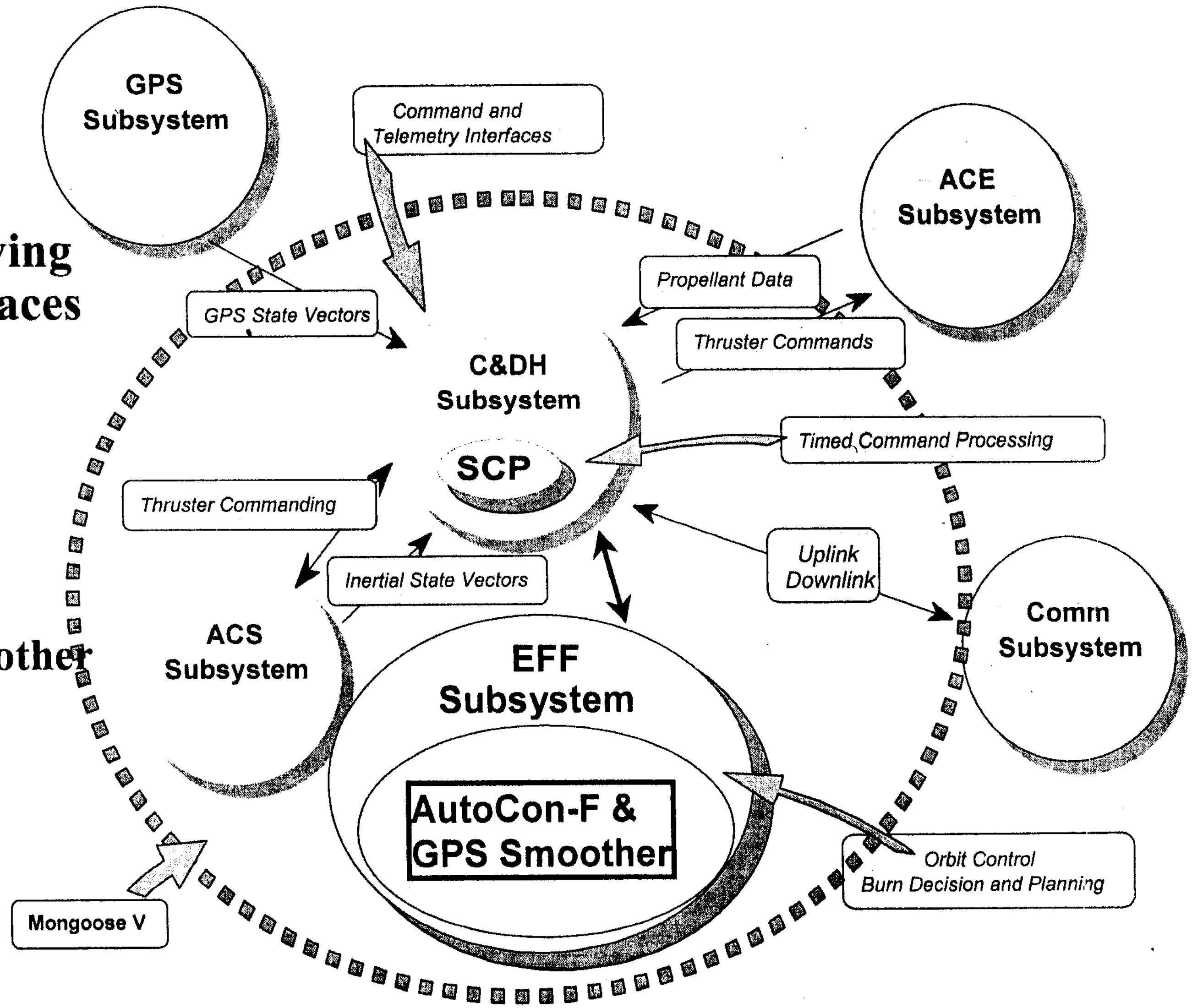
EO-1 Subsystem Level

EO-1 Formation Flying Subsystem Interfaces

- EFF Subsystem
- AutoCon-F
  - GSFC
  - JPL
  - GPS Data Smoother
- Stored Command Processor
- Cmd Load



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Difference in EO-1 Onboard and Ground Maneuver Quantized DVs

Quantized - EO-1 rounded maneuver durations to nearest second

Mode	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
Auto-GPS	[illegible]	1.85	[illegible]	2e-1	.0001	12.50
Auto-GPS	5.35	4.33	3e-7	2e-1	.0005	5.883
Auto	4.98	0.00	1e-7	0.0	.0001	0.0
Auto	2.43	3.79	3e-7	2e-7	.0001	.0005
Semi-auto	1.08	1.62	6e-6	3e-3	.0588	14.23
Semi-auto	2.38	0.26	1e-7	1e-7	.0001	.0007
Semi-auto	5.29	1.85	8e-4	3e-4	1.569	1.572
Manual	2.19	5.20	4e-7	3e-3	.0016	.0002
Manual	3.55	7.93	3e-7	3e-3	.0008	3.57

<u>Inclination Maneuver Validation</u>: Computed ΔV at node crossing, of ~ 24 cm/s (114 sec duration), Ground validation gave same results

10,11

Difference in EO-1 Onboard and Ground Maneuver Three-Axis ΔVs

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EO-1 maneuver computations in all three axis

[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Algorithm ΔV1 Diff %
Auto-GPS	2.8[illegible]	1.122	[illegible]	[illegible]	-1.794
Auto-GPS	8.45[illegible]	68.33	-8.08	0.462	-.0054
Auto	10.85	-5e-4	.0000	.0003	0
Auto	11.86	0.178	.0015	-.0102	-.0008
Semi-auto	12.64	0.312	.0024	.00091	.0002
Semi-auto	14.76	0.188	.0013	0.000	.0001
Semi-auto	15.38	-.256	-.0016	-.0633	-.0045
Manual	15.58	10.41	.6682	-.0117	-.0007
Manual	15.47	0.002	.0001	-.0307	-.0021

f # 10,11

Formation Data from Definitive Navigation Solutions

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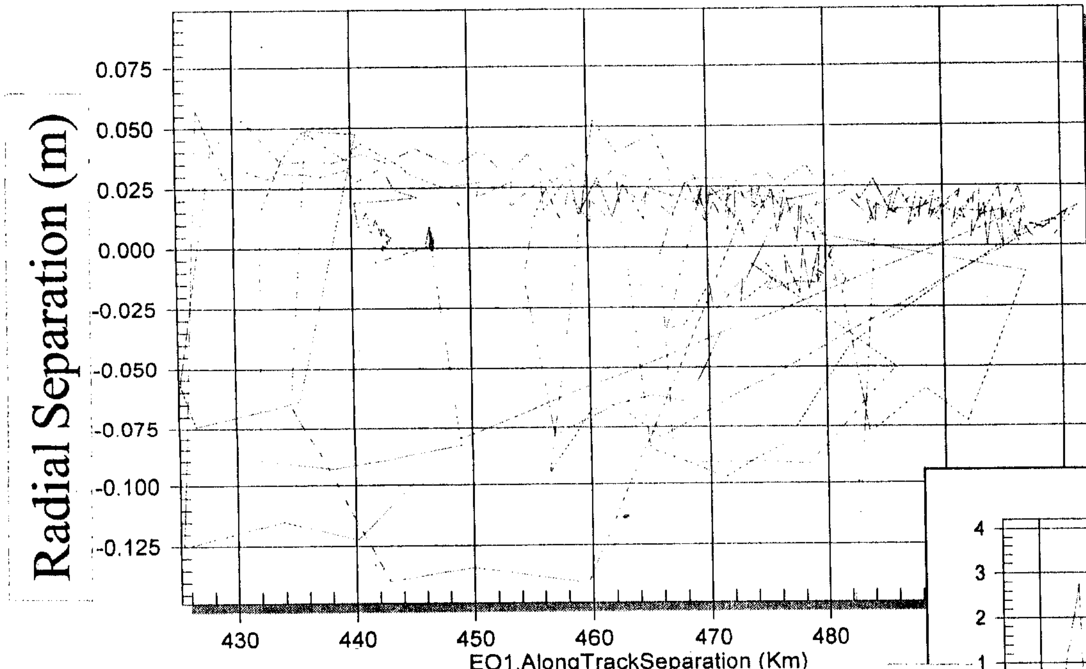
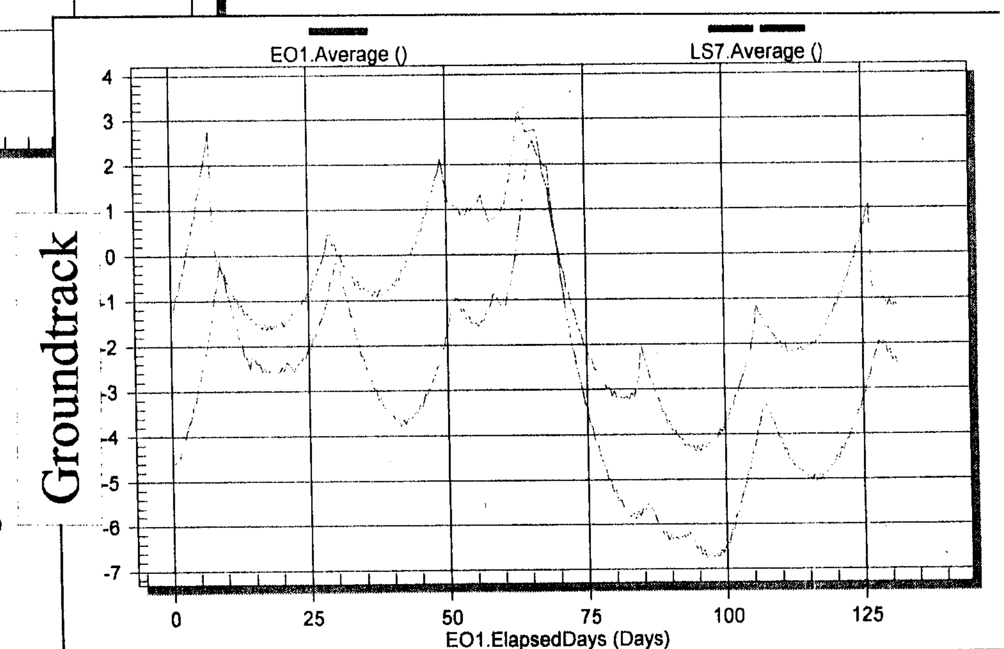


Radial vs. along-track separation over all formation maneuvers (range of 425-490km)



Ground-track separation over all formation maneuvers maintained to 3km

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Formation Data from Definitive Navigation Solutions

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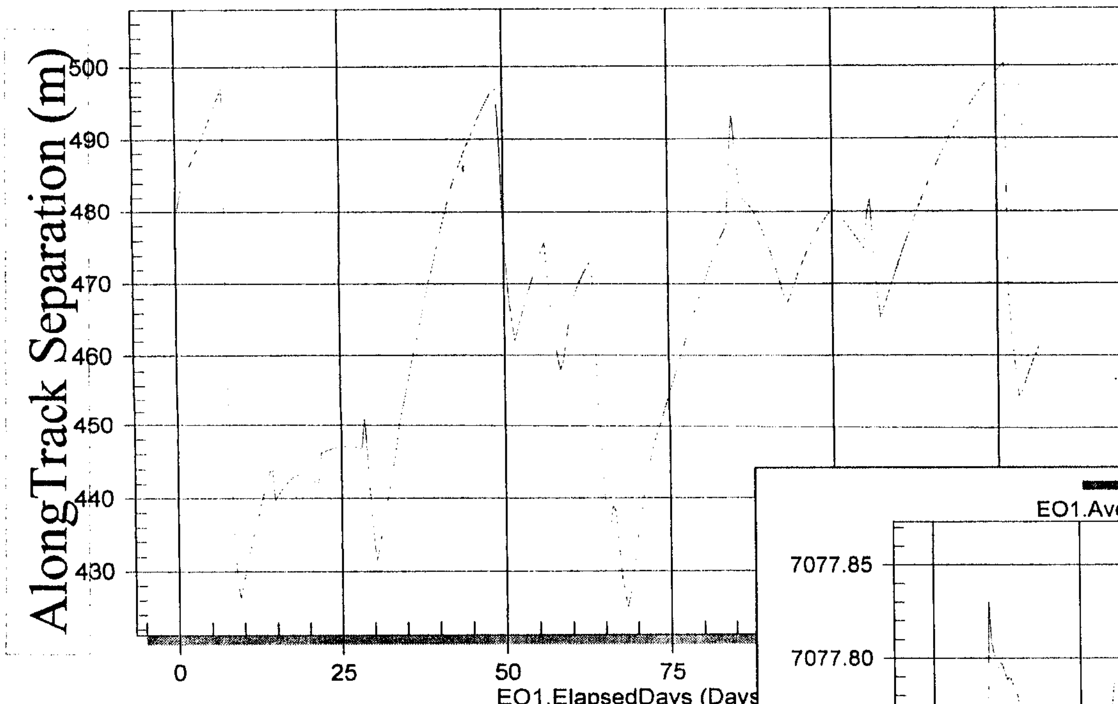
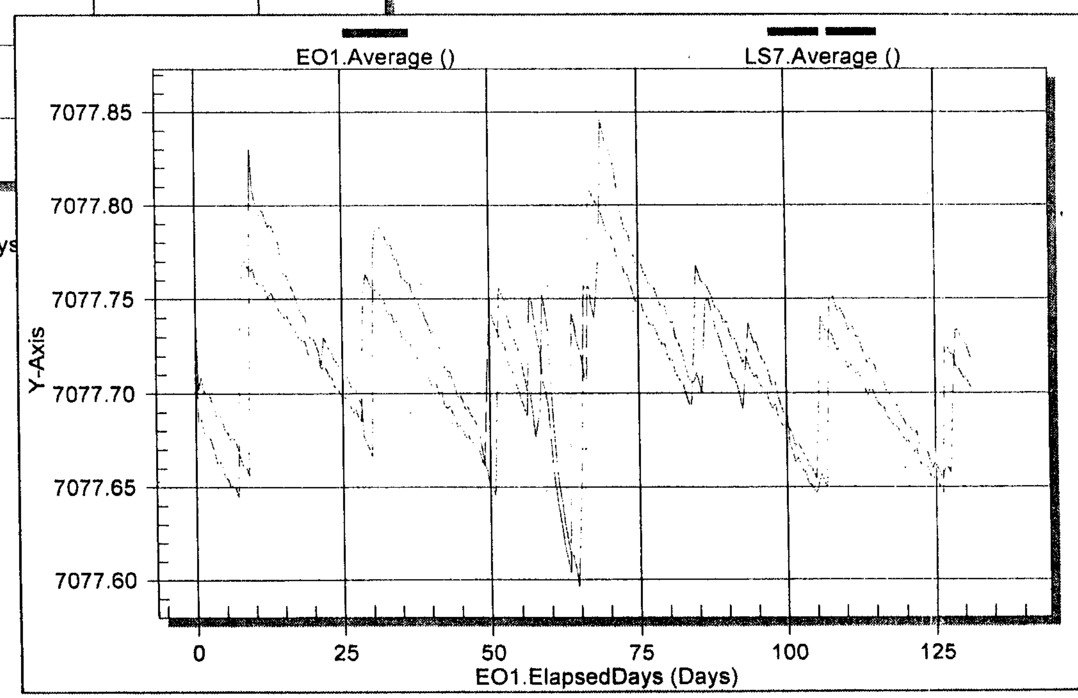


Along-track separation vs. Time over all formation maneuvers (range of 425-490km)



Semi-major axis of EO-1 and LS-7 over all formation maneuvers

f # 10,11

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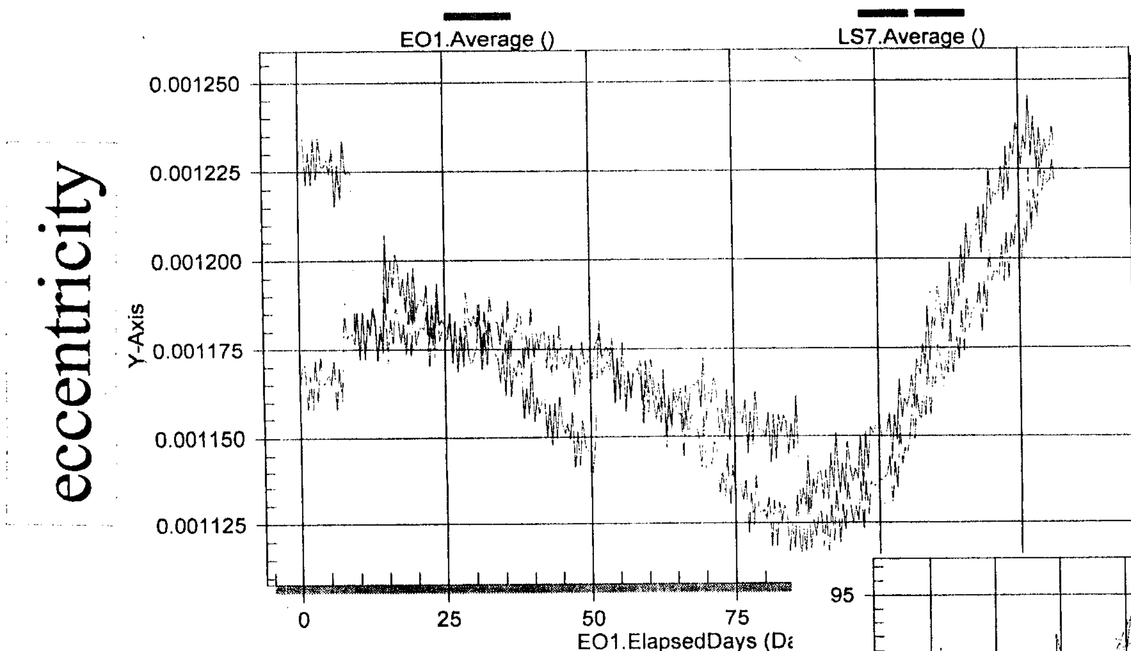
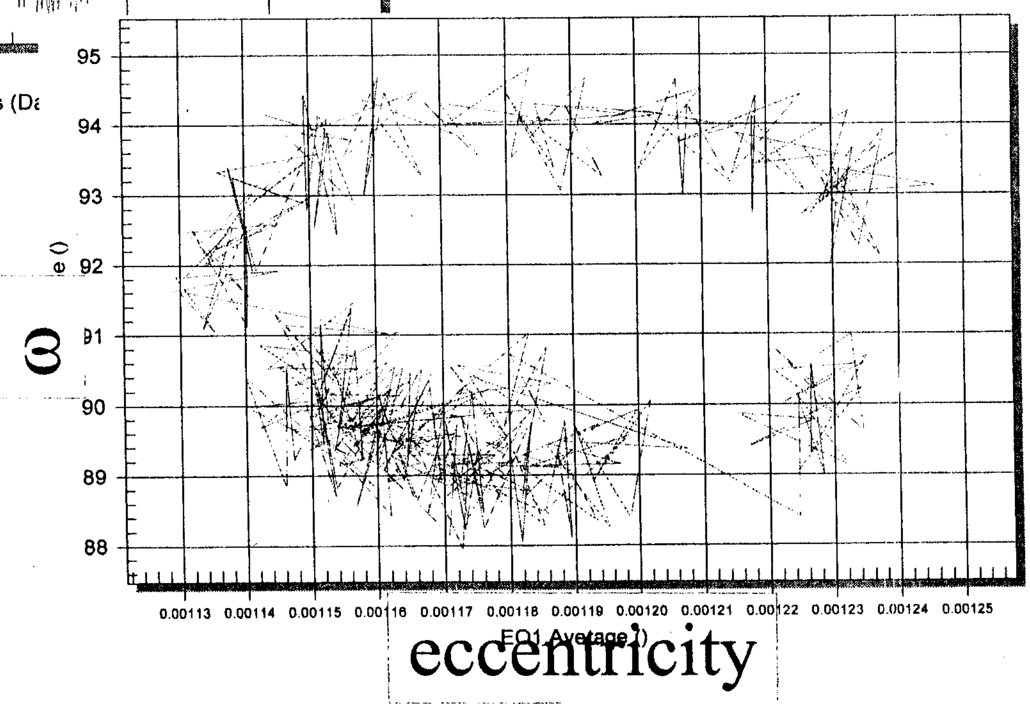
Formation Data from Definitive Navigation Solutions



Frozen Orbit eccentricity over all formation maneuvers (range of .001125 - 0.001250)

Frozen Orbit ω vs. ecc. over all formation maneuvers. ω range of 90+/- 5 deg.



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EO-1 Summary / Conclusions

- o A demonstrated, validated fully non-linear autonomous system
- o A formation flying algorithm that incorporates
  - o Intrack velocity changes for semi-major axis ground-track control
  - o Radial changes for formation maintenance and eccentricity control
  - o Crosstrack changes for inclination control or node changes
  - o Any combination of the above for maintenance maneuvers

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Summary / Conclusions

- Proven executive flight code
- Scripting language alters behavior w/o flight software changes
- I/F for Tlm and Cmds
- Incorporates fuzzy logic for multiple constraint checking for maneuver planning and control
- Single or multiple maneuver computations.
- Multiple or generalized navigation inputs (GPS, Uplinks).
- Attitude (quaternion) required of the spacecraft to meet the ΔV components
- Maintenance of combinations of Keperlian orbit requirements sma, inclination, eccentricity,etc.

Enables Autonomous StationKeeping,
Formation Flying and Multiple Spacecraft Missions

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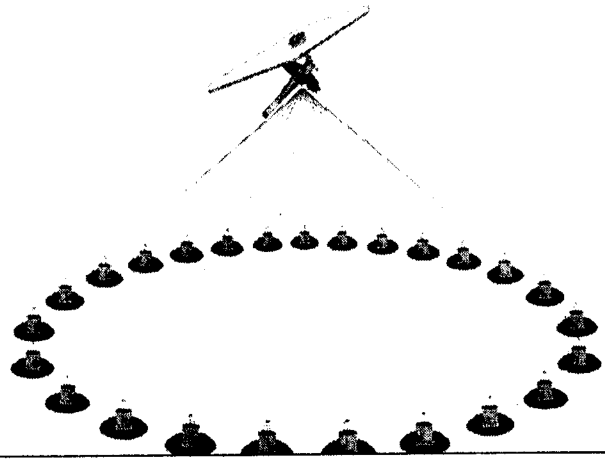
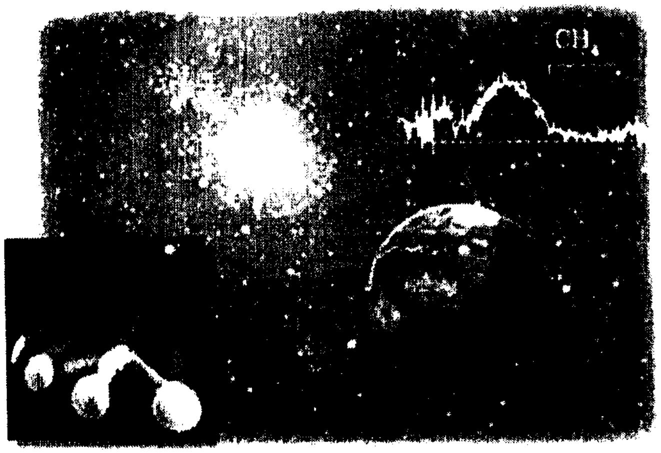
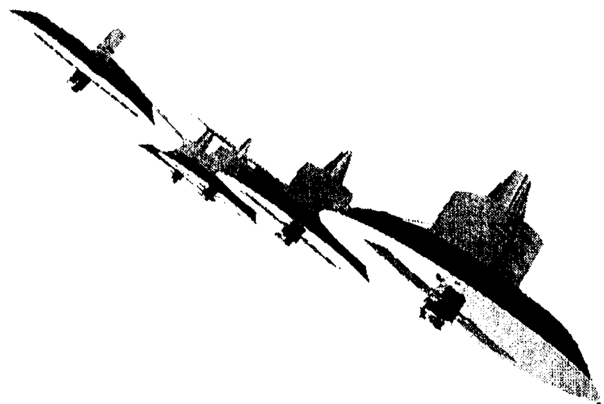


CONTROL STRATEGIES FOR FORMATION FLIGHT IN THE VICINITY OF THE LIBRATION POINTS





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NASA Libration Missions

L1 Missions

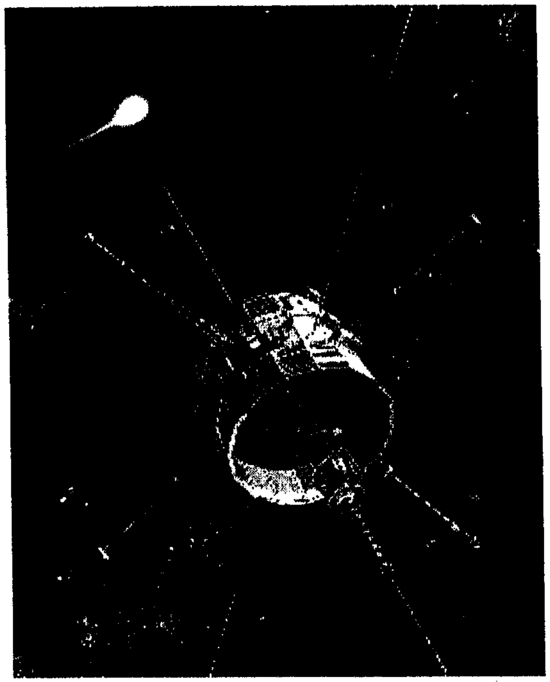
• ISEE-3/ICE(78-85)	L1 Halo Orbit, Direct Transfer, L2 Pseudo Orbit, Comet Mission
• WIND (94-04)	Multiple Lunar Gravity Assist - Pseudo-L1/2 Orbit
• SOHO(95-04)	Large Halo, Direct Transfer
• ACE (97-04)	Small Amplitude Lissajous, Direct Transfer
• GENESIS(01-04)	Lissajous Orbit, Direct Transfer,Return Via L2 Transfer
• TRIANA	L1 Lissajous Constrained, Direct Transfer

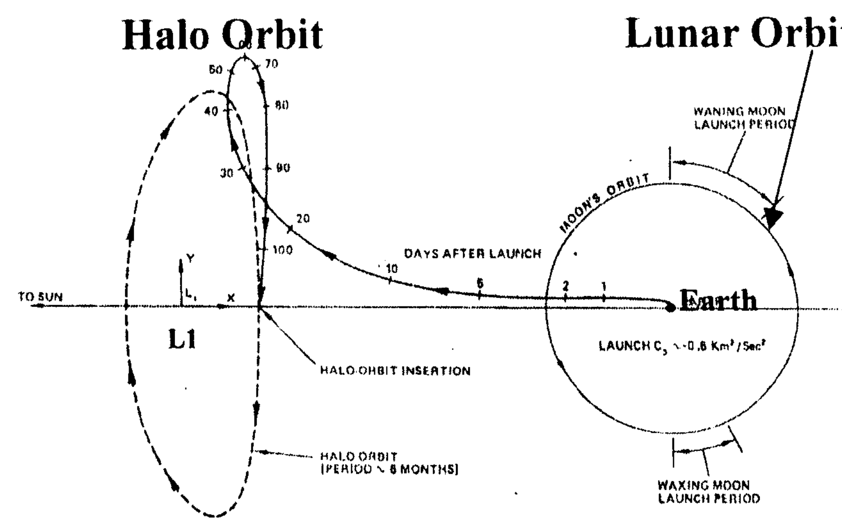
L2 Missions

• GEOTAIL(1992)	L2 Pseudo Orbit, Gravity Assist
• MAP(2001-04)	Orbit, Lissajous Constrained, Gravity Assist
• JWST (~2012)	Large Lissajous, Direct Transfer
• CONSTELLATION-X	Lissajous Constellation, Direct Transfer?, Multiple S/C
• SPECS	Lissajous, Direct Transfer?, Tethered S/C
• MAXIM	Lissajous, Formation Flying of Multiple S/C
• TPF	Lissajous, Formation Flying of Multiple S/C

(Previous missions marked in blue)

ISEE-3 / ICE





Solar-Rotating Coordinates
Ecliptic Plane Projection

Mission:	**Investigate Solar-Terrestrial relationships, Solar Wind, Magnetosphere, and Cosmic Rays**
Launch:	**Sept., 1978, Comet Encounter Sept., 1985**
Lissajous Orbit:	**L1 Libration Halo Orbit, Ax=~175,000km, Ay = 660,000km, Az~ 120,000km, Class I**
Spacecraft:	**Mass=480Kg, Spin stabilized,**
Notable:	**First Ever Libration Orbiter, First Ever Comet Encounter**

Farquhar et al [1985] Trajectories and Orbital Maneuvers for the ISEE-3/ICE, Comet Mission, JAS 33, No. 3

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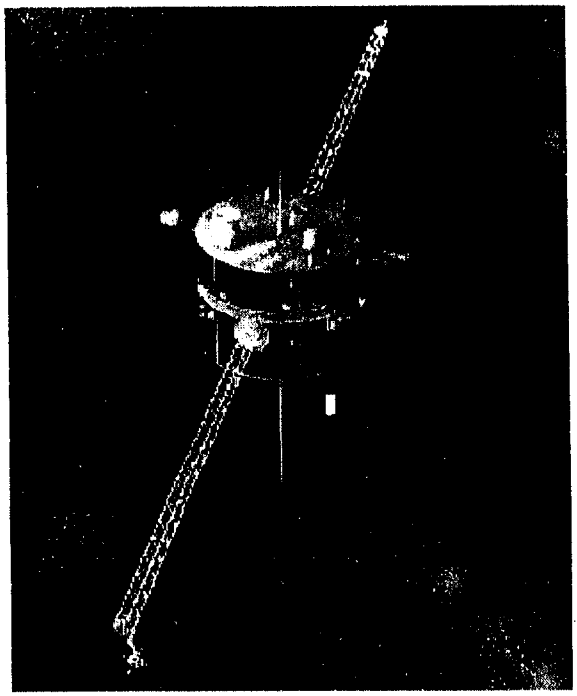
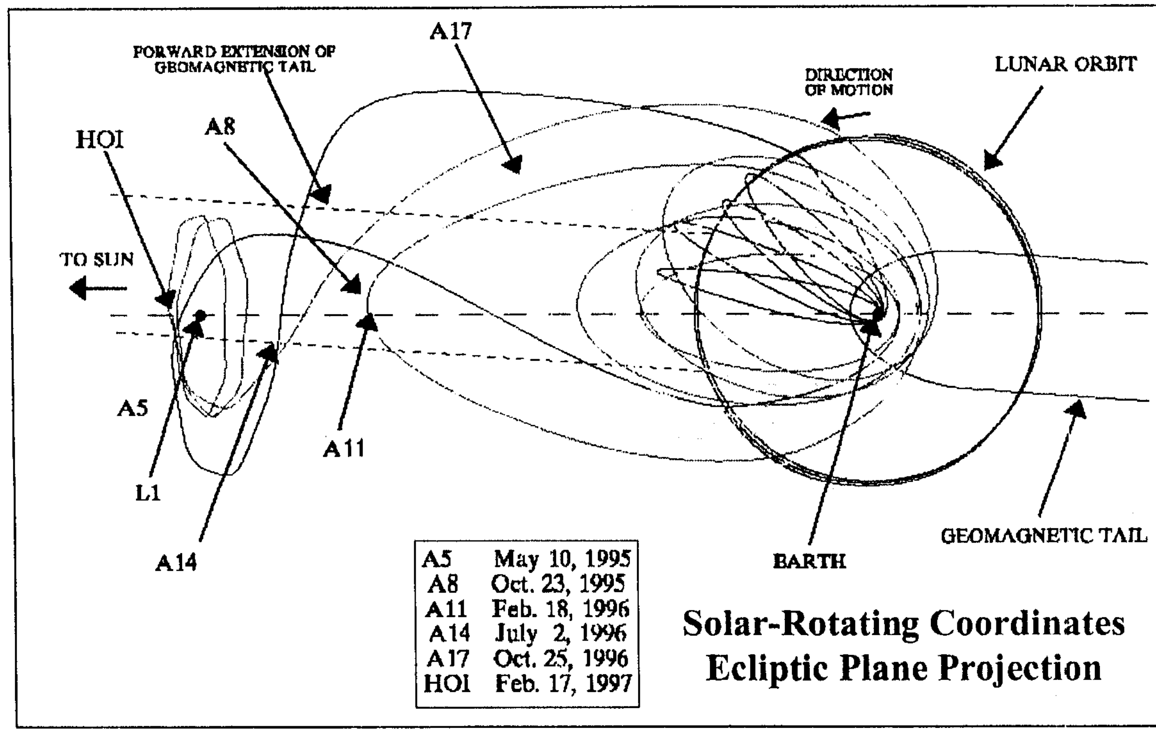
WIND





Solar-Rotating Coordinates
Ecliptic Plane Projection

Mission: Investigate Solar-Terrestrial Relationships, Solar Wind, Magnetosphere

Launch: Nov., 1994, Multiple Lunar Gravity Assist

Lissajous Orbit: Originally an L1 Lissajous Constrained Orbit, Ax~10,000km, Ay ~ 350,000km, Az~ 250,000km, Class I

Spacecraft: Mass=1254kg, Spin Stabilized,

Notable: First Ever Multiple Gravity Assist Towards L1

T # 1,12

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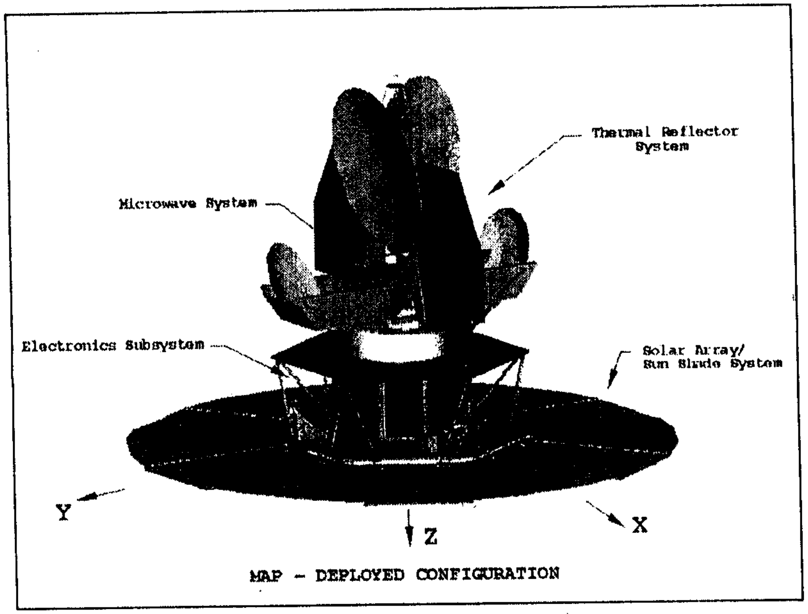
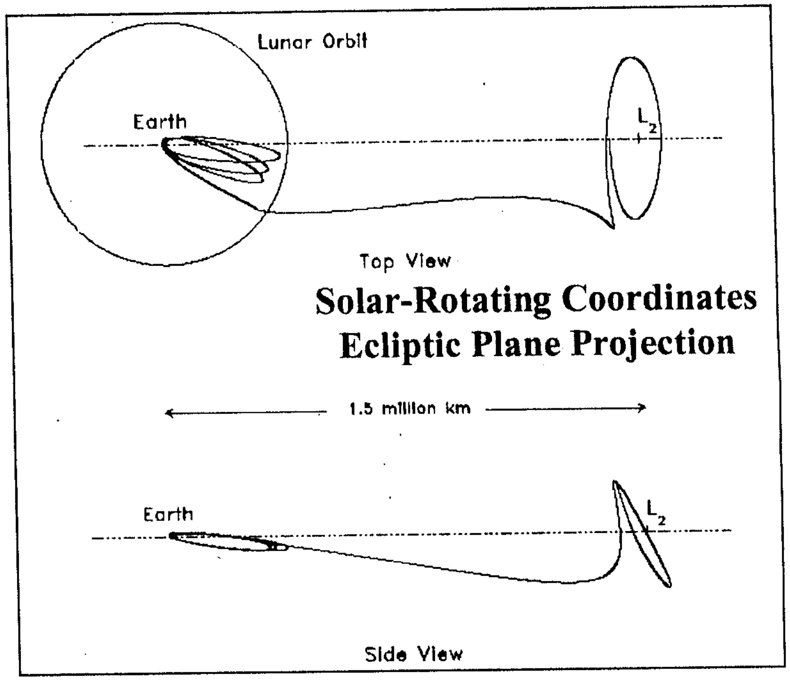
MAP





Mission: Produce an Accurate Full-sky Map of the Cosmic Microwave Background Temperature Fluctuations (Anisotropy)

Launch: Summer 2001, Gravity Assist Transfer

Lissajous Orbit: L2 Lissajous Constrained Orbit Ay~ 264,000km, Ax~tbd, Ay~ 264,000km, Class II

Spacecraft: Mass=818kg, Three Axis Stabilized,

Notable: First Gravity Assisted Constrained L2 Lissajous Orbit; Map-earth Vector Remains Between 0.5° and 10° off the Sun-earth Vector to Satisfy Communications Requirements While Avoiding Eclipses

1,12

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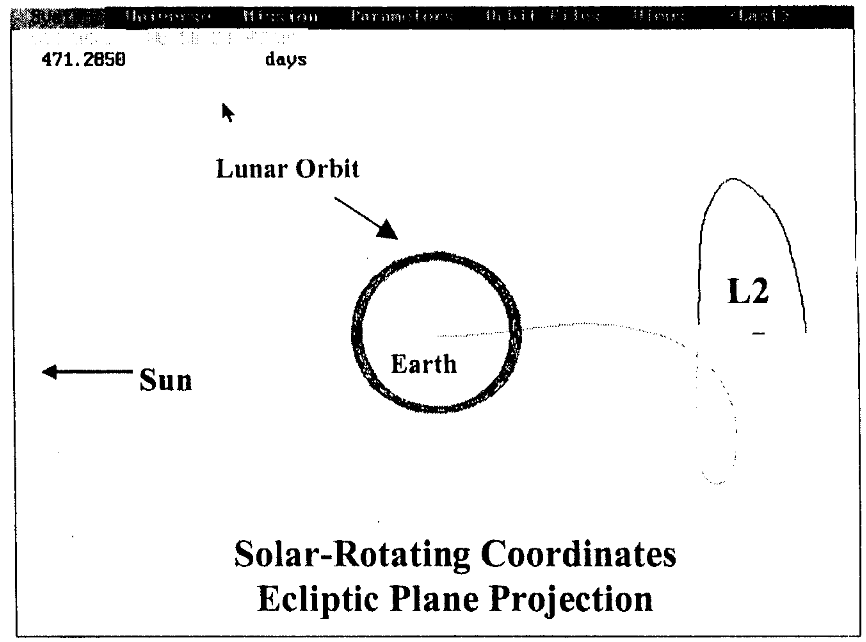
JWST



Mission:	**JWST Is Part of Origins Program. Designed to Be the Successor to the Hubble Space Telescope. JWST Observations in the Infrared Part of the Spectrum.**
Launch:	**JWST~2010, Direct Transfer**
Lissajous Orbit:	**L2 large lissajous, Ay~ 294,000km, Ax~800,000km, Az~ 131,000km, Class I or II**
Spacecraft:	**Mass~6000kg, Three Axis Stabilized, 'Star' Pointing**
Notable:	**Observations in the Infrared Part of the Spectrum. Important That the Telescope Be Kept at Low Temperatures, ~3^0K. Large Solar Shade/Solar Sail**

1,12

A State Space Model

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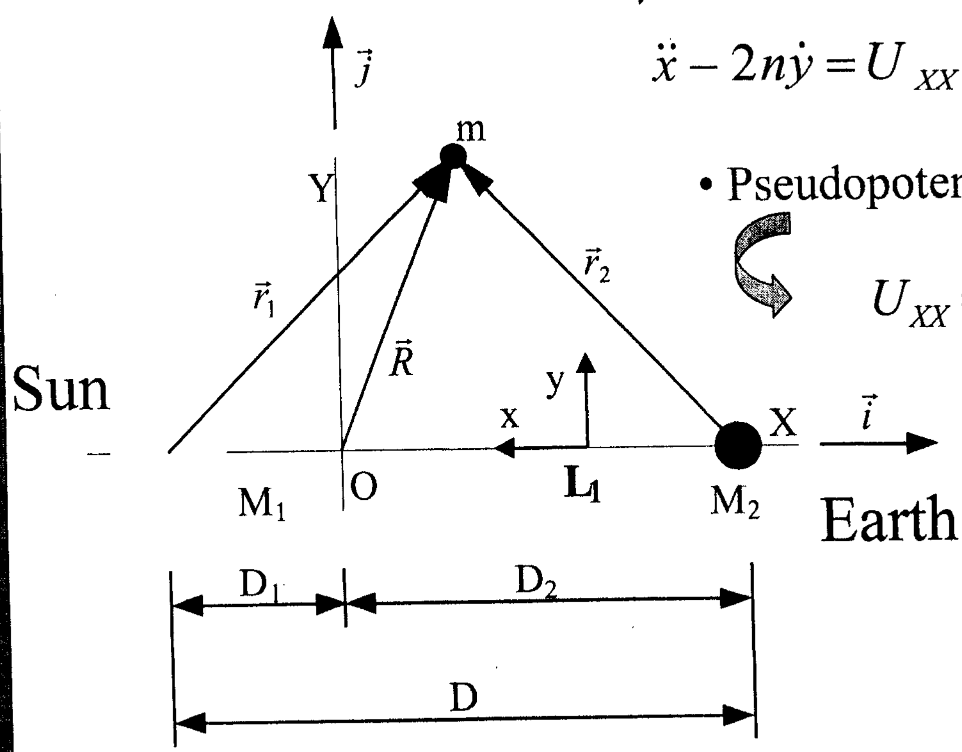
The linearized equations of motion for a S/C close to the libration point are calculated at the respective libration point.



- Linearized Eq. Of Motion Based on Inertial X, Y, Z Using

$X = X_0 + x,$ $\quad Y = Y_0 + y,$ $\quad Z = Z_0 + z$

$$\ddot{x} - 2n\dot{y} = U_{XX}\,x\,, \quad \ddot{y} + 2n\dot{x} = U_{YY}\,y\,, \quad \ddot{z} = U_{ZZ}\,z$$

- Pseudopotential:

$$U_{XX} = \frac{\partial^2 U}{\partial X^2}, \quad U_{YY} = \frac{\partial^2 U}{\partial Y^2}, \quad U_{ZZ} = \frac{\partial^2 U}{\partial Z^2}.$$

1,12,13,17,20

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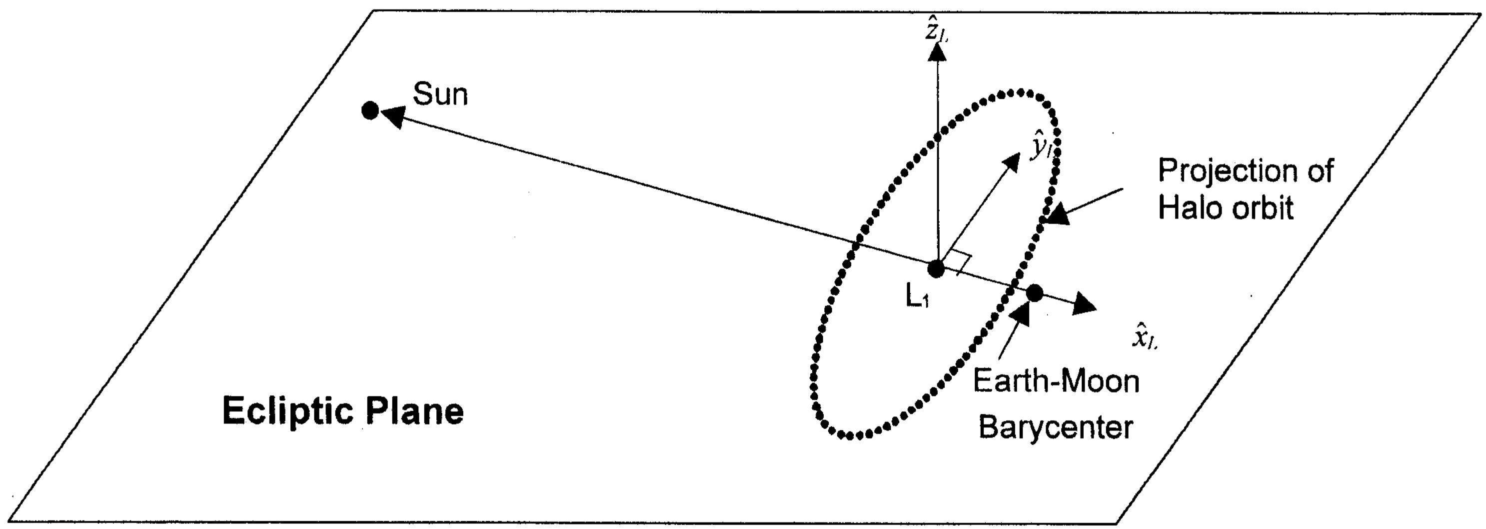
A State Space Model

$$\dot{\mathbf{x}}^j = A^j \mathbf{x}^j \qquad \text{where} \qquad \mathbf{x}^j = \begin{bmatrix} x^j \\ y^j \\ z^j \\ \dot{x}^j \\ \dot{y}^j \\ \dot{z}^j \end{bmatrix}, \quad A^j = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline U_{XX} & 0 & 0 & 0 & 2n & 0 \\ 0 & U_{YY} & 0 & -2n & 0 & 0 \\ 0 & 0 & U_{ZZ} & 0 & 0 & 0 \end{array}\right]$$

$$n = \sqrt{\frac{G(M_1 + M_2)}{D}}$$



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Reference Motions

- Natural Formations
  - String of Pearls
  - Others: Identify via Floquet controller (CR3BP)
    - Quasi-Periodic Relative Orbits (2D-Torus)
    - Nearly Periodic Relative Orbits
    - Slowly Expanding Nearly Vertical Orbits

    + Stable Manifolds
- Non-Natural Formations
  - Fixed Relative Distance and Orientation
    - RLP
    - Inertial
  - Fixed Relative Distance, Free Orientation
  - Fixed Relative Distance & Rotation Rate
  - Aspherical Configurations (Position & Rates)

Natural Formations

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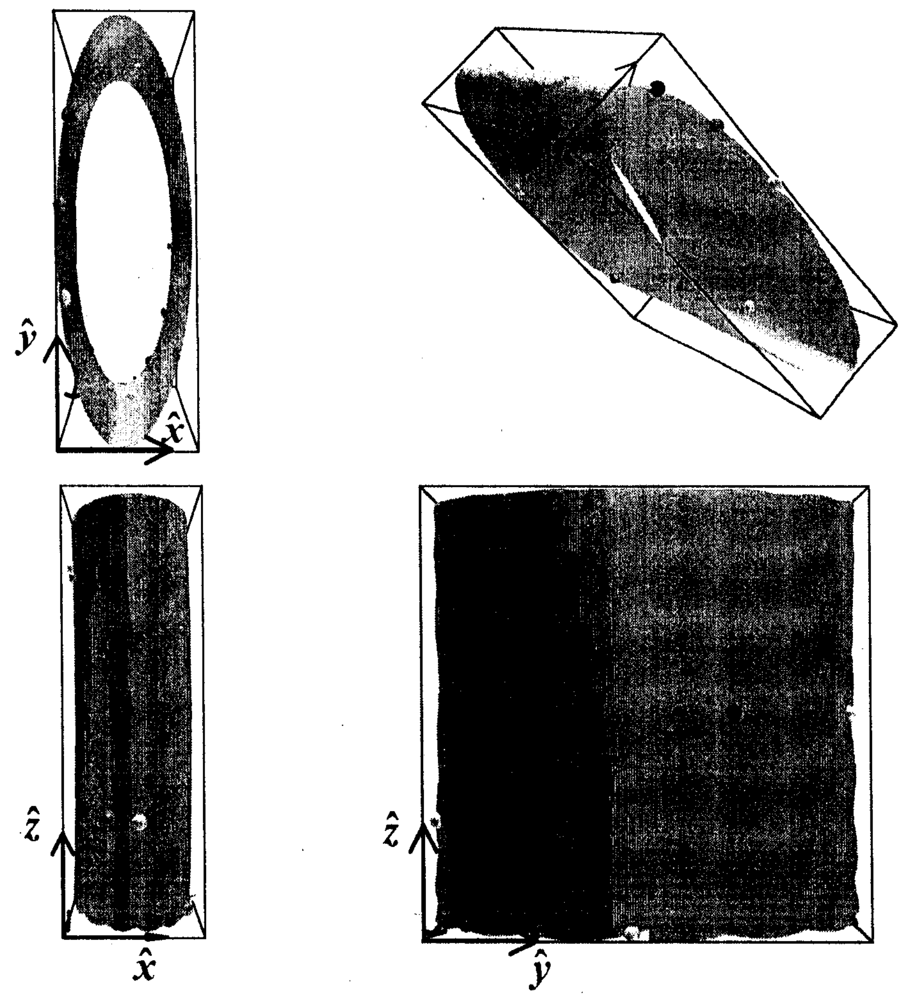
Natural Formations: String of Pearls



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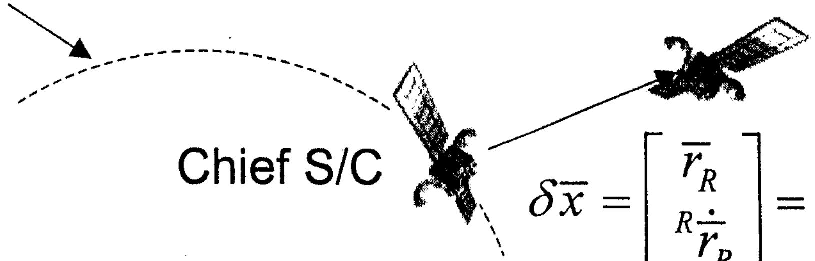
CR3BP Analysis of Phase Space Eigenstructure Near Halo Orbit



$$\delta\overline{x} = \begin{bmatrix} \overline{r}_R \\ {}^R\dot{\overline{r}}_R \end{bmatrix} = \Phi(t,0)\,\delta\overline{x}(0)$$

Floquet Decomposition of $\Phi(t,0)$:

$$\Phi(t,0) = P(t)e^{\Lambda t}P(0) = \{P(t)S\}e^{Jt}\{P(0)S\}^{-1}$$

Floquet Modal Matrix:

$$E(t) = P(t)S = \Phi(t,0)E(0)e^{-Jt}$$

Solution to Variational Eqn. in terms of Floquet Modes:

$$\delta\overline{x}(t) = \sum_{j=1}^{6}\delta\overline{x}_j(t) = \sum_{j=1}^{6}c_j(t)\overline{e}_j(t) = E(t)\overline{c}$$

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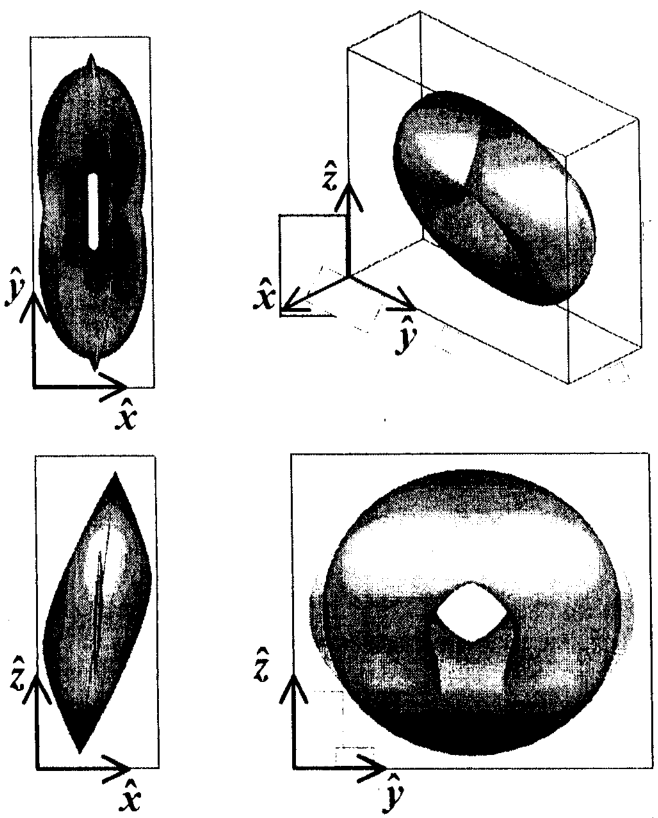
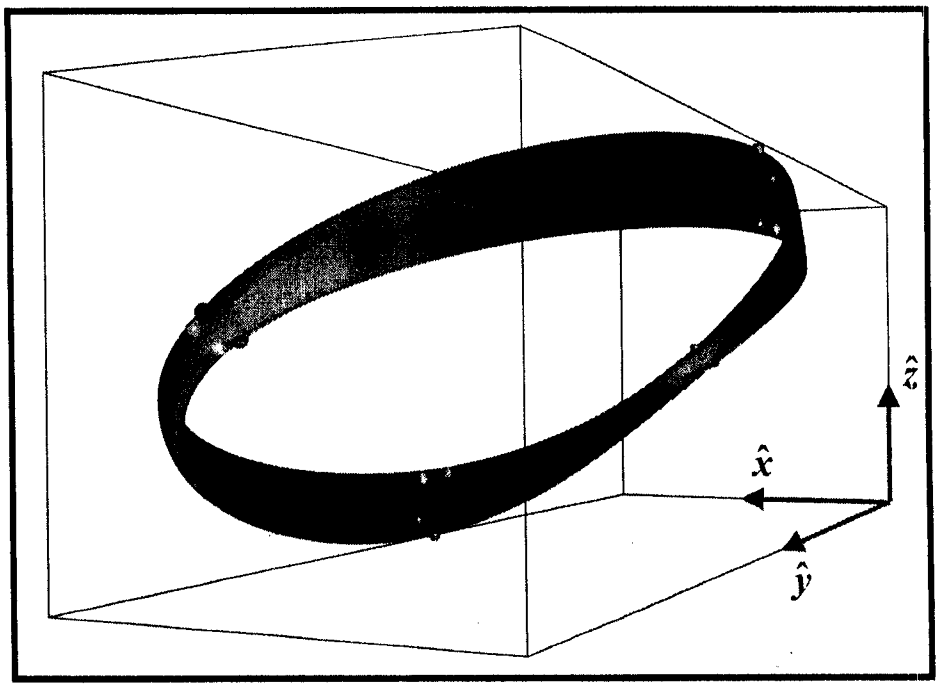
Natural Formations: Quasi-Periodic Relative Orbits → 2-D Torus

Chief S/C Centered View (RLP Frame)





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Floquet Controller
(Remove Unstable + 2 of the 4 Center Modes)

Find $\Delta\bar{v}$ that removes undesired response modes:

$$\sum_{j=1}^{6} \delta\bar{x}_j + \begin{bmatrix} 0_3 \\ I_3 \end{bmatrix} \Delta\bar{v} = \sum_{\substack{j=2,3,4 \\ \text{or} \\ j=2,5,6}} \left(1+\alpha_j\right)\delta\bar{x}_j$$

Remove Modes 1, 3, and 4:

$$\begin{bmatrix} \bar{\alpha} \\ \Delta\bar{v} \end{bmatrix} = \begin{bmatrix} \delta\bar{x}_{2\bar{r}} & \delta\bar{x}_{5\bar{r}} & \delta\bar{x}_{6\bar{r}} & 0_3 \\ \delta\bar{x}_{2\bar{v}} & \delta\bar{x}_{5\bar{v}} & \delta\bar{x}_{6\bar{v}} & -I_3 \end{bmatrix}^{-1} \left(\delta\bar{x}_1 + \delta\bar{x}_3 + \delta\bar{x}_4\right)$$

Remove Modes 1, 5, and 6:

$$\begin{bmatrix} \bar{\alpha} \\ \Delta\bar{v} \end{bmatrix} = \begin{bmatrix} \delta\bar{x}_{2\bar{r}} & \delta\bar{x}_{3\bar{r}} & \delta\bar{x}_{4\bar{r}} & 0_3 \\ \delta\bar{x}_{2\bar{v}} & \delta\bar{x}_{3\bar{v}} & \delta\bar{x}_{4\bar{v}} & -I_3 \end{bmatrix}^{-1} \left(\delta\bar{x}_1 + \delta\bar{x}_5 + \delta\bar{x}_6\right)$$

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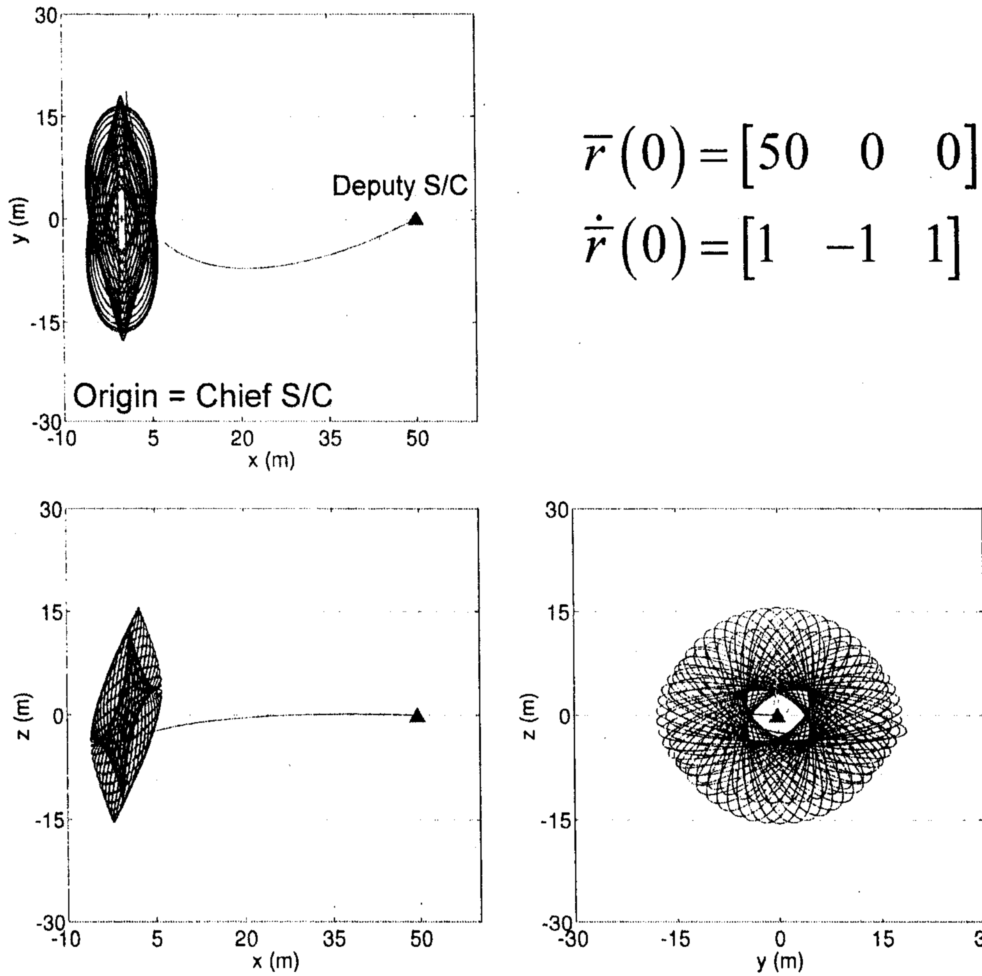
Deployment into Torus (Remove Modes 1, 5, and 6)



$$\bar{r}(0) = [50 \quad 0 \quad 0]\ \text{m}$$

$$\dot{\bar{r}}(0) = [1 \quad -1 \quad 1]\ \text{m/sec}$$

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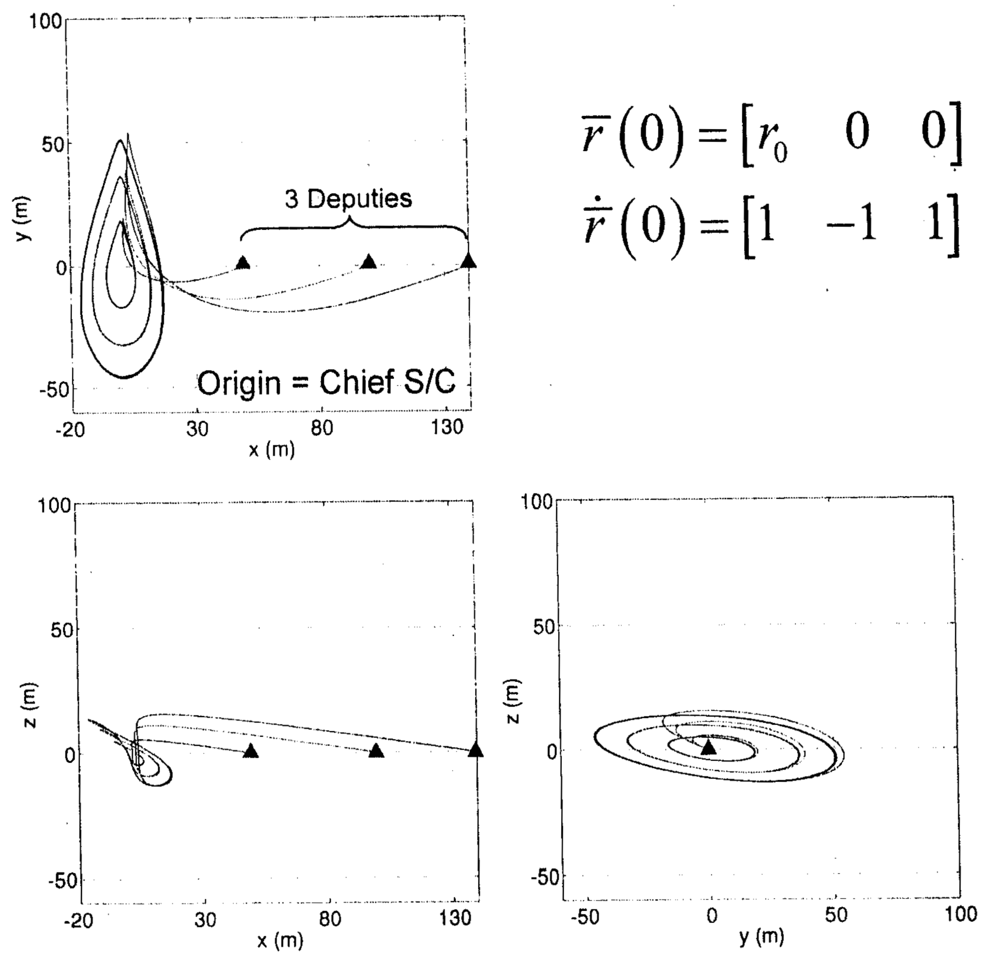
Deployment into Natural Orbits (Remove Modes 1, 3, and 4)



$$\bar{r}(0) = \begin{bmatrix} r_0 & 0 & 0 \end{bmatrix} \text{ m}$$

$$\dot{\bar{r}}(0) = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \text{ m/sec}$$

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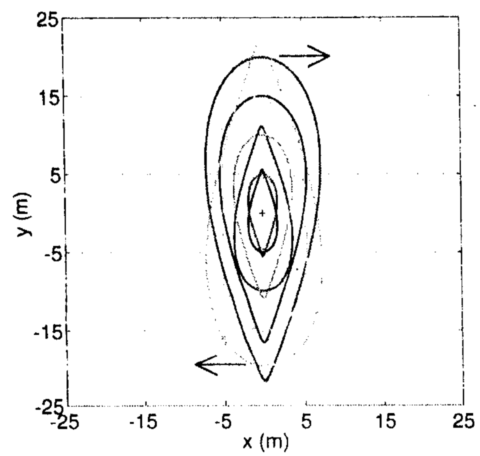
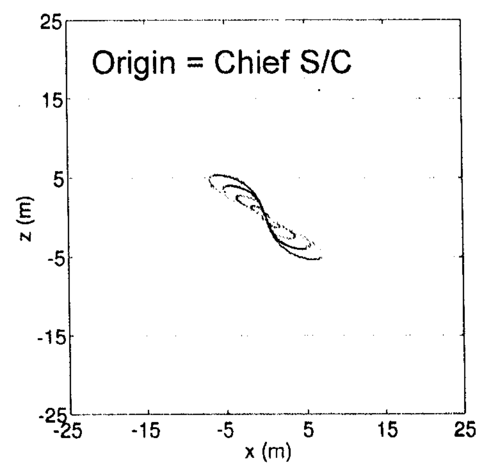
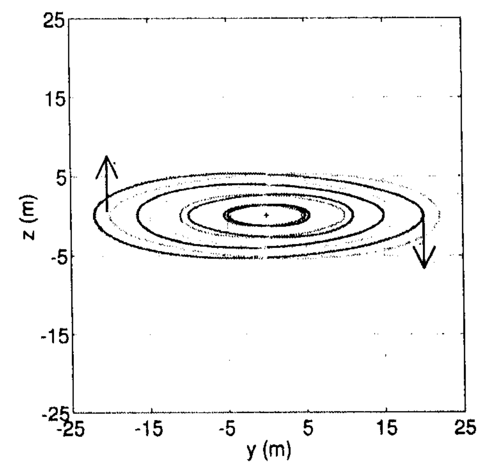
Natural Formations: Nearly Periodic Relative Motion



10 Revolutions = 1,800 days





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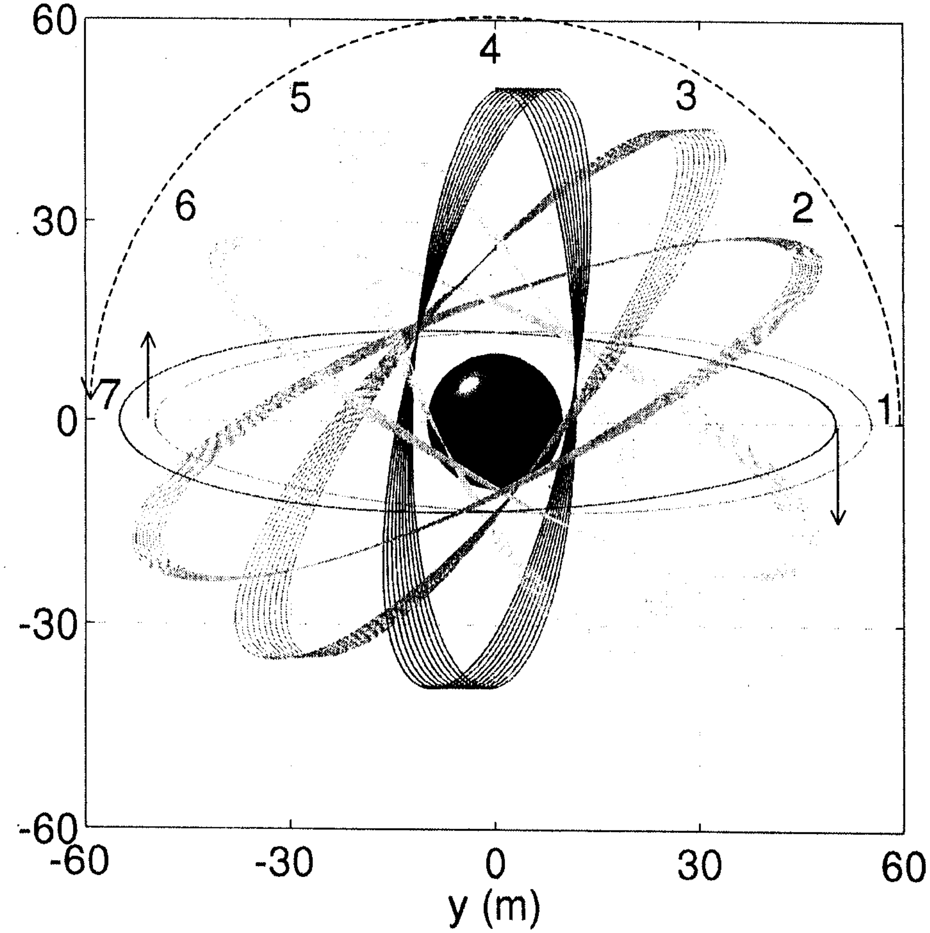
Evolution of Nearly Vertical Orbits Along the *yz*-Plane



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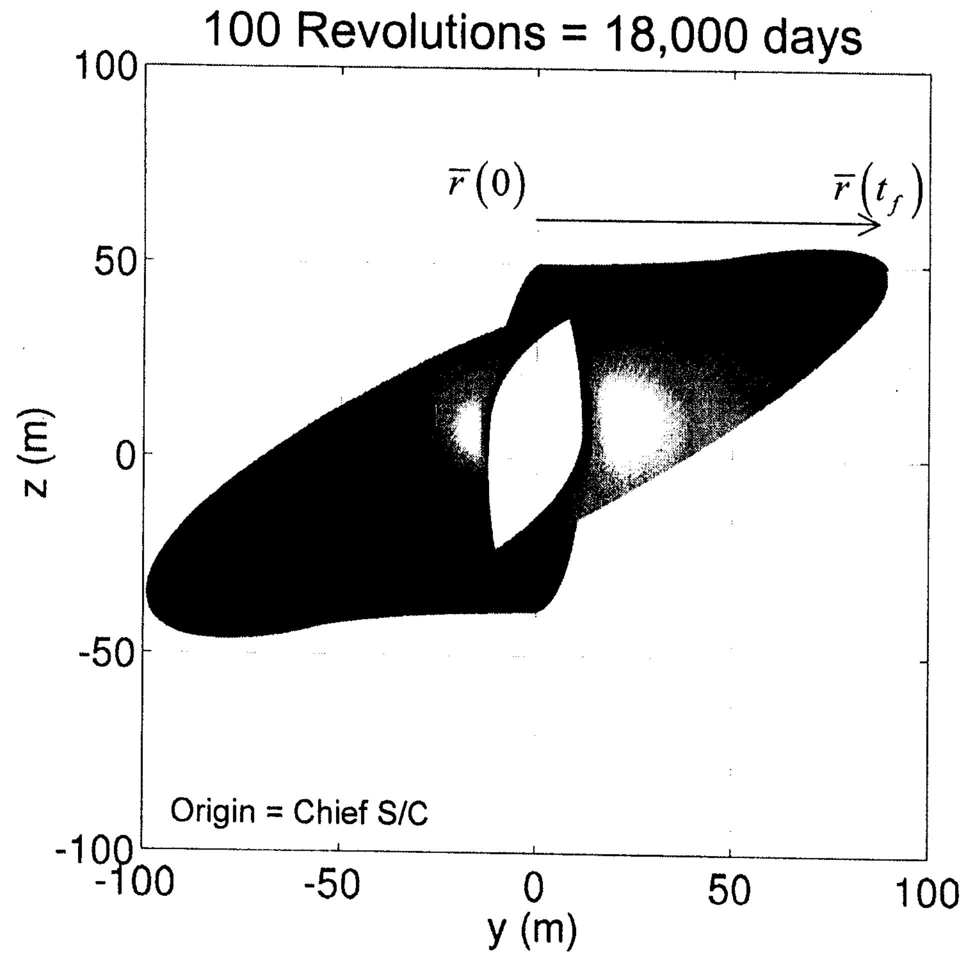
Natural Formations: Slowly Expanding Vertical Orbits



Non-Natural Formations

- Fixed Relative Distance and Orientation
  - Fixed in Inertial Frame
  - Fixed in Rotating Frame
- Spherical Configurations (Inertial or RLP)
  - Fixed Relative Distance, Free Orientation
  - Fixed Relative Distance & Rotation Rate
- Aspherical Configurations (Position & Rates)
  - Parabolic
  - Others

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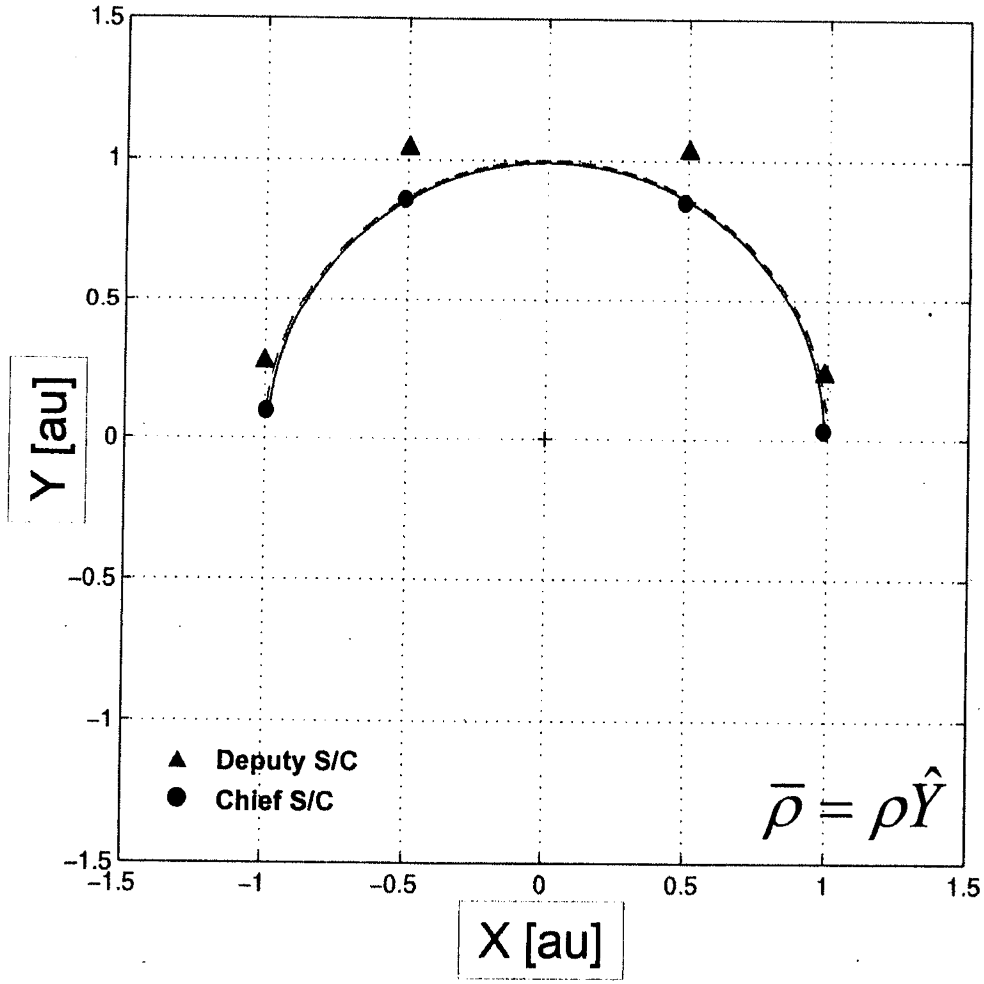
Formations Fixed in the Inertial Frame



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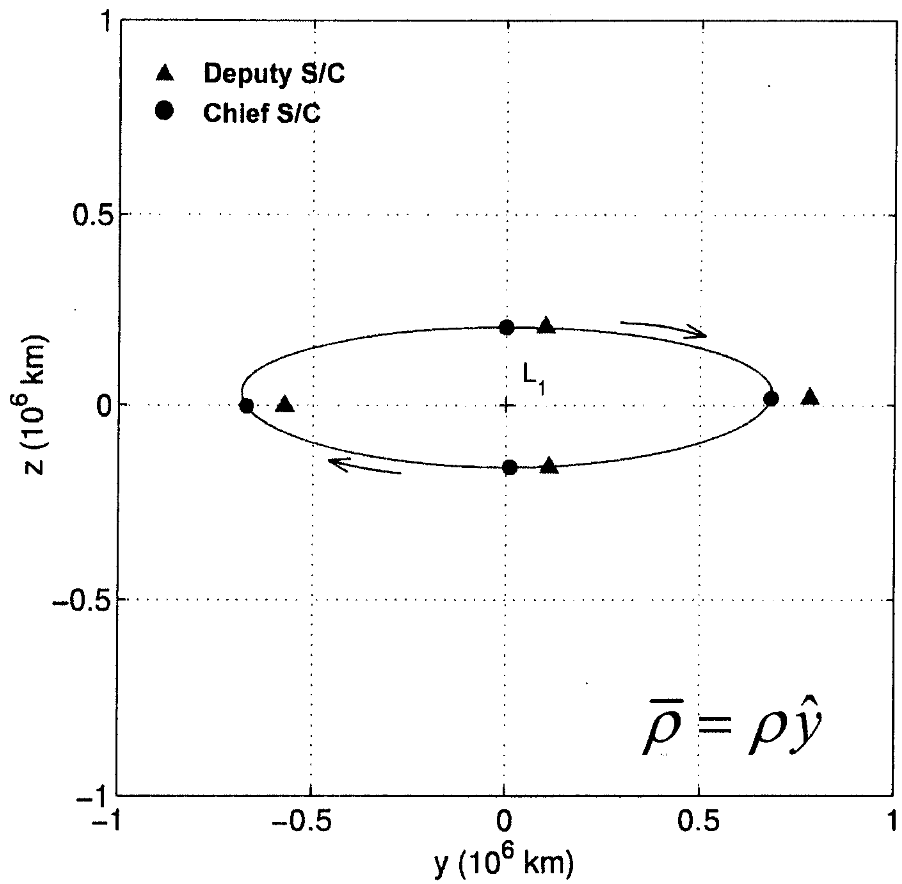
Formations Fixed in the Rotating Frame



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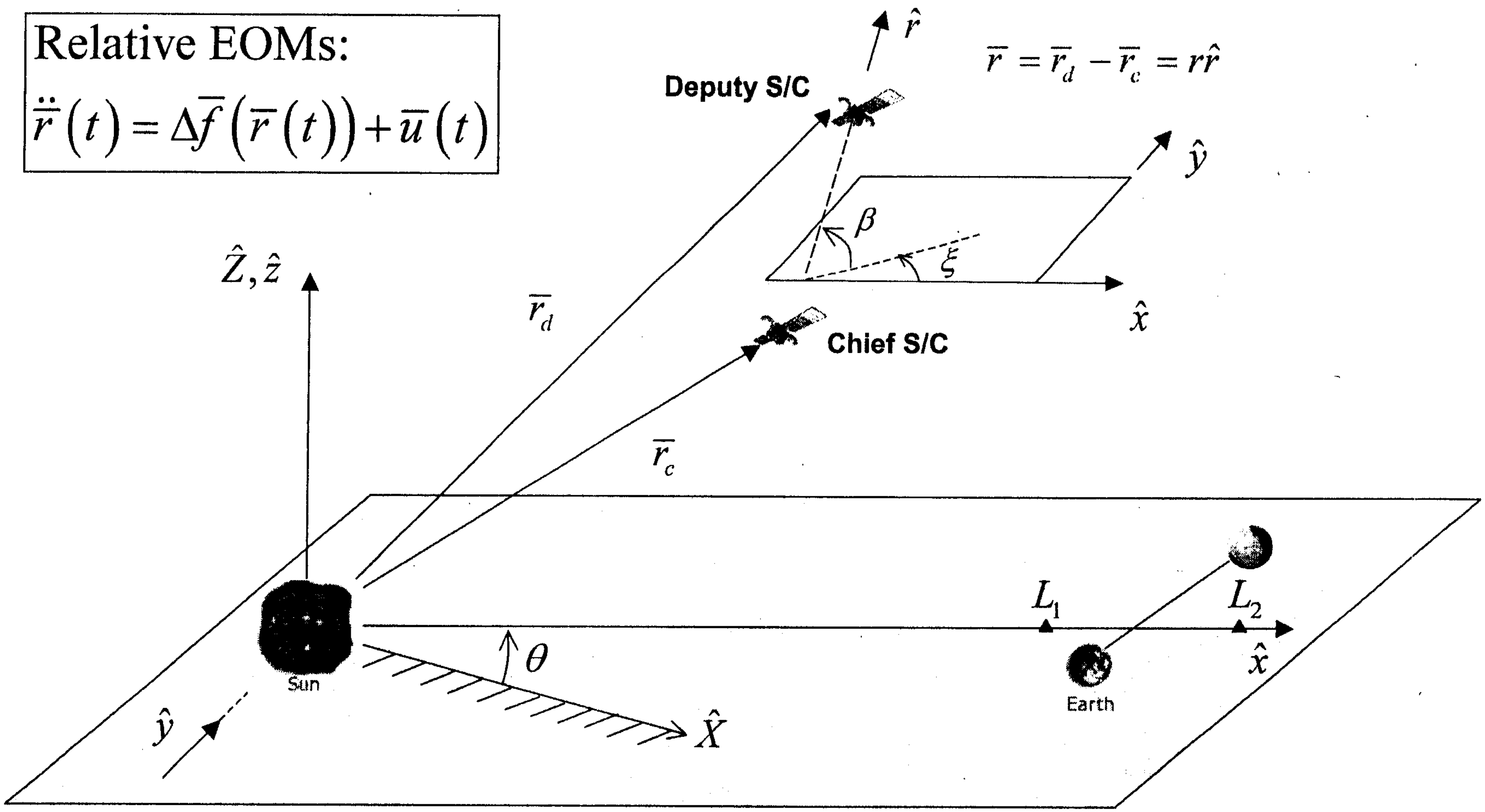
2-S/C Formation Model in the Sun-Earth-Moon System

Relative EOMs:

$$\ddot{\bar{r}}(t) = \Delta\bar{f}\left(\bar{r}(t)\right) + \bar{u}(t)$$



Ephemeris System = Sun+Earth+Moon Ephemeris + SRP

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Discrete & Continuous Control

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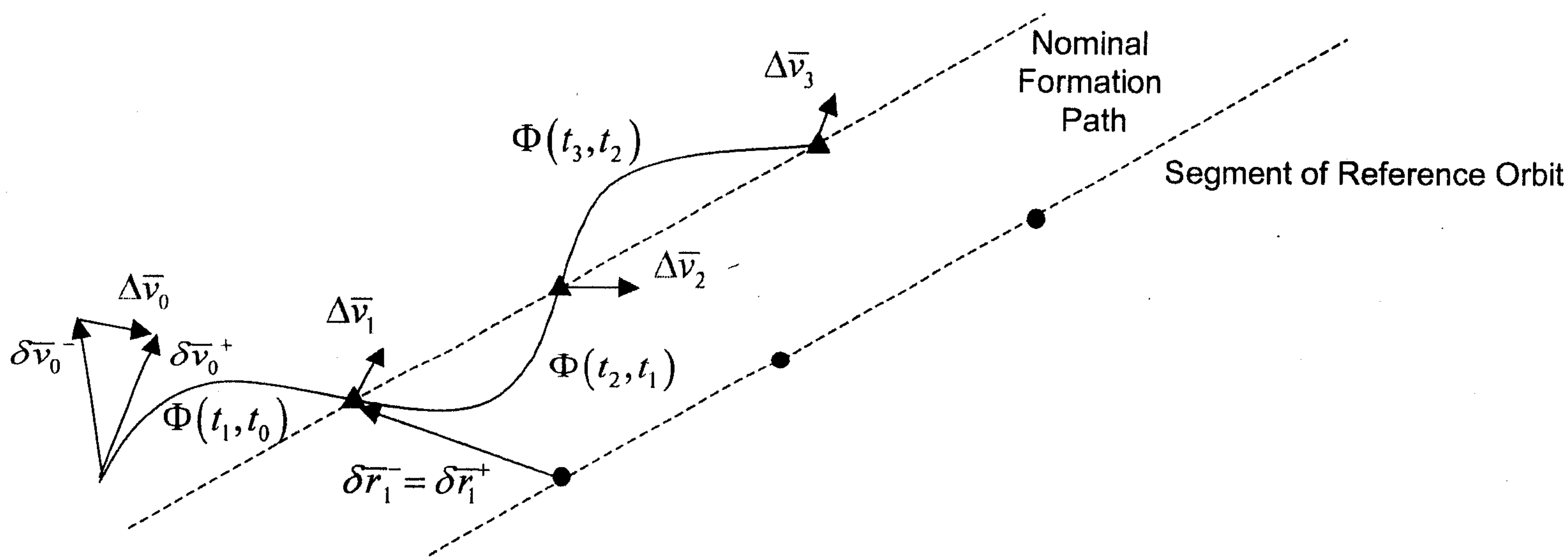
Linear Targeter



$$\begin{bmatrix} \delta\overline{r}_{k+1}^- \\ \delta\overline{v}_{k+1}^- \end{bmatrix} = \Phi\left(t_{k+1}, t_k\right) \begin{bmatrix} \delta\overline{r}_{k+1}^+ \\ \delta\overline{v}_{k+1}^+ \end{bmatrix} = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \begin{bmatrix} \delta\overline{r}_{k+1}^- \\ \delta\overline{v}_{k+1}^- + \Delta\overline{v}_k \end{bmatrix}$$

$$\boxed{\Delta\overline{v}_k = B_k^{-1}\left(\delta\overline{r}_{k+1} - A_k\delta\overline{r}_k\right) - \delta\overline{v}_k^-}$$

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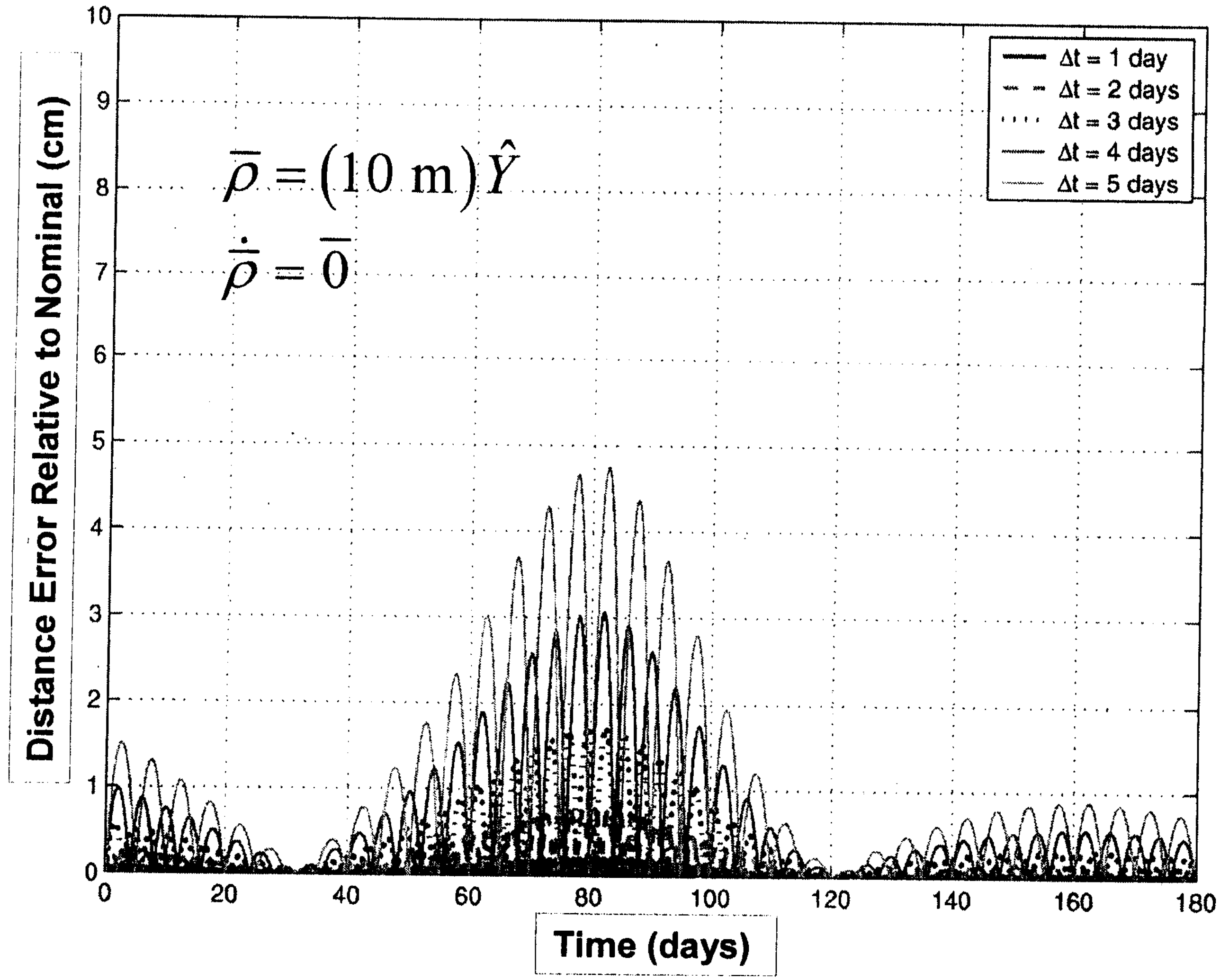
Discrete Control: Linear Targeter



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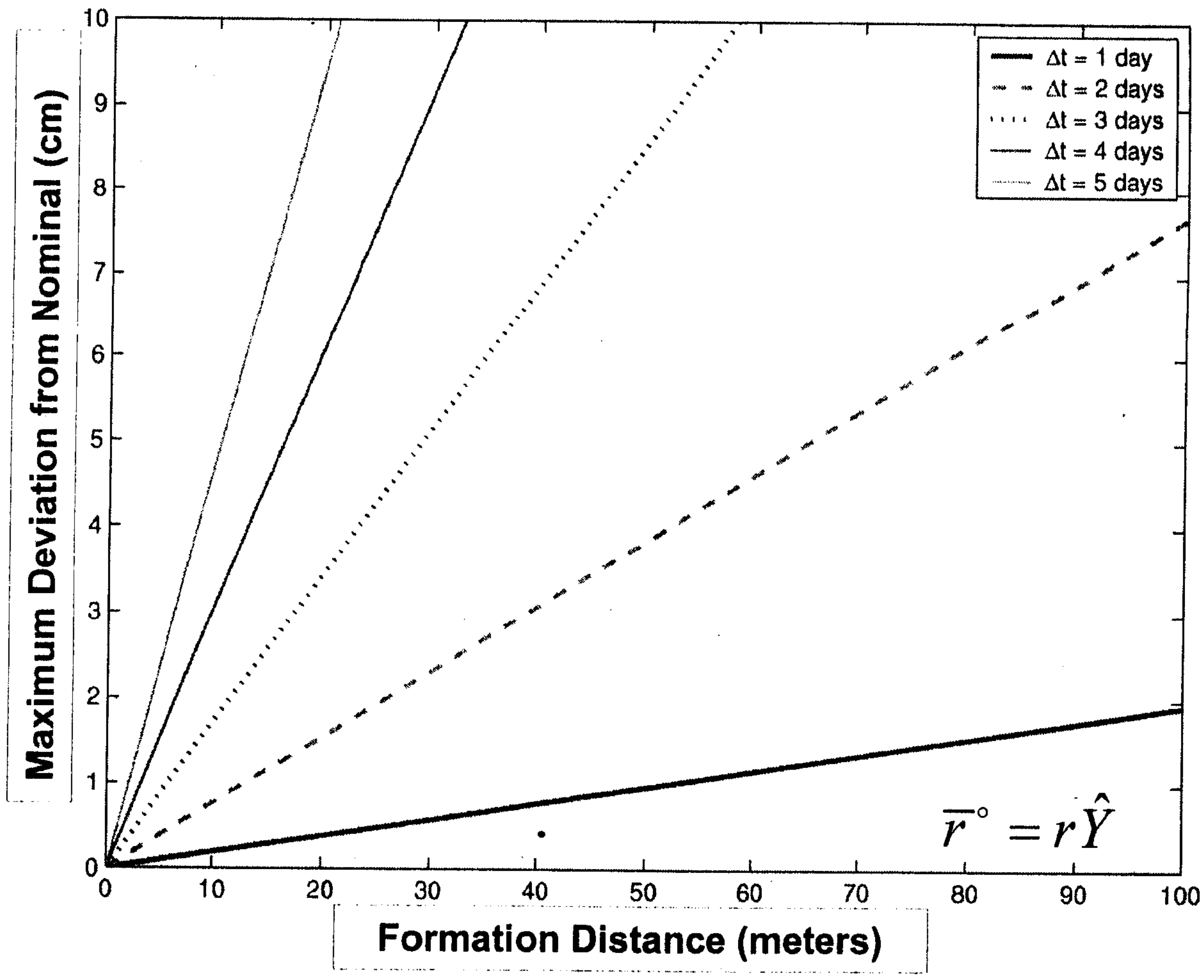
Achievable Accuracy via Targeter Scheme



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Continuous Control: LQR vs. Input Feedback Linearization

- LQR for <u>Time-Varying</u> Nominal Motions

$$\dot{\bar{x}}(t) = \begin{bmatrix} \dot{\bar{r}} & \ddot{\bar{r}} \end{bmatrix}^T = \bar{f}\left(t, \bar{x}(t), \bar{u}(t)\right) \qquad \rightarrow \bar{x}(0) = \bar{x}_0$$

$$\dot{P} = -A^T(t)P(t) - P(t)A(t) + P(t)B(t)R^{-1}B^T(t)P(t) - Q \rightarrow P(t_f) = 0$$

Optimal Control Law:

$$\bar{u}(t) = \overbrace{\bar{u}^{\circ}(t)}^{\text{Nominal Control Input}} + \left\{ \underbrace{-R^{-1}B^T P(t)\left(\bar{x}(t) - \bar{x}^{\circ}(t)\right)}_{\text{Optimal Control, Relative to Nominal, from LQR}} \right\}$$

- Input Feedback Linearization (IFL)

$$\ddot{\bar{r}}(t) = \bar{F}\left(\bar{r}(t)\right) + \bar{u}(t) \quad \Rightarrow \quad \bar{u}(t) = \underbrace{-\bar{F}\left(\bar{r}(t)\right)}_{\text{Anihilate Natural Dynamics}} + \overbrace{\bar{g}\left(\bar{r}(t), \dot{\bar{r}}(t)\right)}^{\text{Desired Dynamic Response}}$$

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LQR Goals

$$\min J = \frac{1}{2}\int_{t_0}^{t_f}\left[\delta\overline{x}(t)^T Q\delta\overline{x}(t) + \delta\overline{u}_d(t)^T R\delta\overline{u}_d(t)\right]dt$$

$$\delta\overline{x}(t) = \overline{x}(t) - \overline{x}^{\circ}(t)$$

$$\delta\overline{u}_d(t) = \overline{u}_d(t) - \overline{u}_d^{\circ}(t)$$

$$Q = diag\left(10^{12}, 10^{12}, 10^{12}, 10^{5}, 10^{5}, 10^{5}\right)$$

$$R = diag(1,1,1)$$

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LQR Process

Step 1: Evaluate the Jacobian matrix, at time t_i, associated with nominal path

$\rightarrow A(t_i)$ evaluated on $\overline{x}^{\circ}(t_i)$

Step 2: Numerically integrate the differential Riccati Equation backwards in time

from $t_i \rightarrow t_{i-1}$, subject to $P(t_N) = 0$.

$$\dot{P}(t_i) = -A^T(t_i)P(t_i) - P(t_i)A(t_i) + P(t_i)BR^{-1}B^TP(t_i) - Q$$

Step 3: Compute and store the controller gain matrix

$$K(t_i) = R^{-1}B^TP(t_i)$$

Step 4: Repeat steps 1-3 until $t_{i-1} = t_0$

Step 5: Numerically integrate the perturbed trajectory forward in time

from $t_0 \rightarrow t_N$, subject to $\overline{x}(t_0) = \overline{x}_0$.

$\rightarrow$ Recall $K(t)$ from stored data

$\rightarrow$ The new integration step size is defined by the sampled gain data

$\rightarrow$ Substitute $\overline{x}^{\circ}(t)$ into EOMs and solve for $\overline{u}_d^{\circ}(t)$

$\rightarrow \overline{u}_d(t) = \overline{u}_d^{\circ}(t) - K(t)\left(\overline{x}(t) - \overline{x}^{\circ}(t)\right)$

$\rightarrow$ Apply the computed control input to the perturbed EOMs

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IFL Process

Step 1: Define, analytically, the desired response characteristics

$$\rightarrow \ddot{\bar{r}} = \ddot{\bar{r}}^{\circ} - 2\omega_n\left(\dot{\bar{r}} - \dot{\bar{r}}^{\circ}\right) - \omega_n^{\;2}\left(\bar{r} - \bar{r}^{\circ}\right) = g\left(\bar{r}, \dot{\bar{r}}\right)$$

Step 2: Begin numerical integration of perturbed path

Step 3: At each point in time, compute and apply the control input necessary to achieve the desired response characteristics:

$$\bar{u}_d(t) = \underbrace{-\bar{f}\left(\bar{r}^{P_2D}, \dot{\bar{r}}^{P_2D}\right)}_{\text{Annihilate Natural Dynamics}} + \underbrace{g\left(\bar{r}, \dot{\bar{r}}\right)}_{\text{Reflects desired response}}$$

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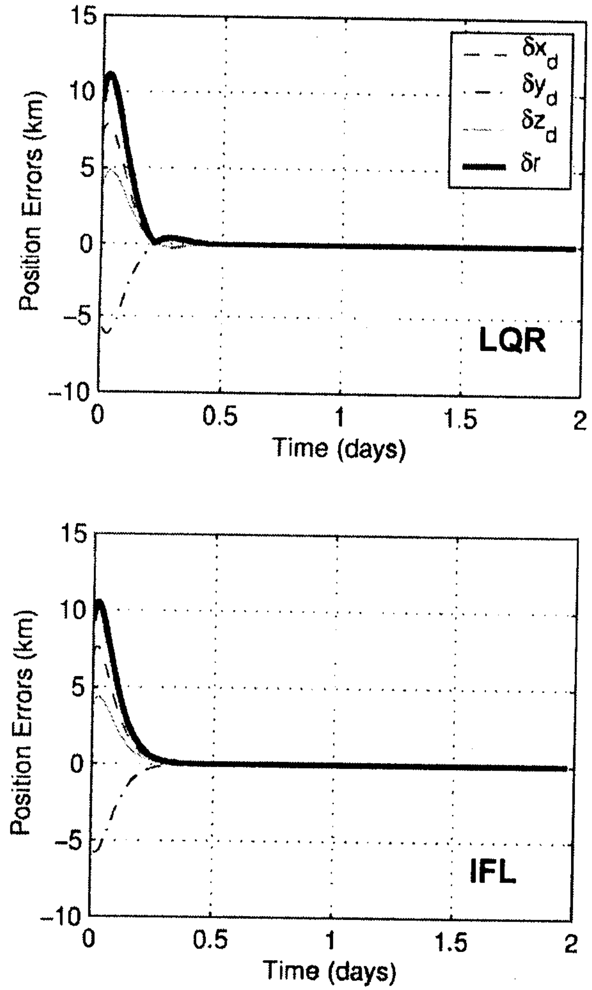
LQR vs. IFL Comparison

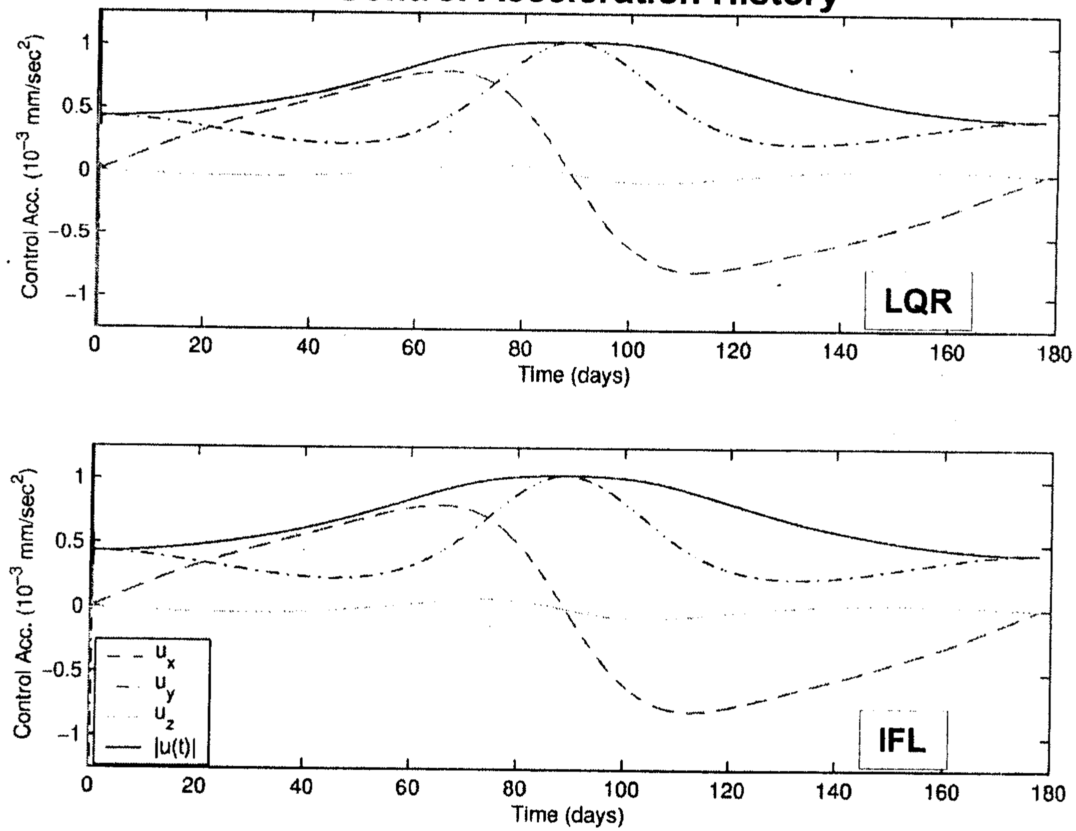
$$\rho = 5000 \text{ km}, \xi = 90^\circ, \beta = 0^\circ$$

$$\delta\bar{x}(0) = \begin{bmatrix} 7 \text{ km} & -5 \text{ km} & 3.5 \text{ km} & 1 \text{ mps} & -1 \text{ mps} & 1 \text{ mps} \end{bmatrix}^T$$

Dynamic Response



Control Acceleration History



Dynamic Response Modeled in the CR3BP
Nominal State Fixed in the Rotating Frame

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Output Feedback Linearization (Radial Distance Control)

Formation Dynamics

$$\ddot{\bar{r}} = \Delta\bar{f}(\bar{r}) + \bar{u}(t) \quad \rightarrow \text{Generalized Relative EOMs}$$

$$y = l(\bar{r}) \quad \rightarrow \text{Measured Output}$$

Measured Output Response (Radial Distance)

$$\ddot{y} = \frac{d^2 l}{dt^2} = \overbrace{p(\bar{r},\dot{\bar{r}}) + q(\bar{r},\dot{\bar{r}})\bar{u}^T\bar{r}}^{\textbf{Actual Response}} = \overbrace{g(\bar{r},\dot{\bar{r}})}^{\textbf{Desired Response}}$$

Scalar Nonlinear Functions of $\bar{r}$ and $\dot{\bar{r}}$

Scalar Nonlinear Constraint on Control Inputs

$$\boxed{h(\bar{r}(t),\dot{\bar{r}}(t)) - \bar{u}(t)^T\bar{r}(t) = 0}$$

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Output Feedback Linearization (OFL) (Radial Distance Control in the Ephemeris Model)

$y = l\left(\bar{r}, \dot{\bar{r}}\right)$	Control Law
r	$\bar{u}(t) = \frac{h\left(\bar{r}, \dot{\bar{r}}\right)}{r}\hat{r}$ Geometric Approach: Radial inputs only
r	$\bar{u}(t) = \left\{ \frac{g\left(\bar{r}, \dot{\bar{r}}\right)}{r} - \frac{\dot{\bar{r}}^T \dot{\bar{r}}}{r^2} \right\} \bar{r} + \left(\frac{\dot{r}}{r}\right) \dot{\bar{r}} - \Delta \bar{f}(\bar{r})$
r^2	$\bar{u}(t) = \left\{ \frac{1}{2} \frac{g\left(\bar{r}, \dot{\bar{r}}\right)}{r^2} - \frac{\dot{\bar{r}}^T \dot{\bar{r}}}{r^2} \right\} \bar{r} - \Delta \bar{f}(\bar{r})$
$1/r$	$\bar{u}(t) = \left\{ -rg\left(\bar{r}, \dot{\bar{r}}\right) - \frac{\dot{\bar{r}}^T \dot{\bar{r}}}{r^2} \right\} \bar{r} + 3\left(\frac{\dot{r}}{r}\right) \dot{\bar{r}} - \Delta \bar{f}(\bar{r})$

- Critically damped output response achieved in all cases
- Total ΔV can vary significantly for these four controllers

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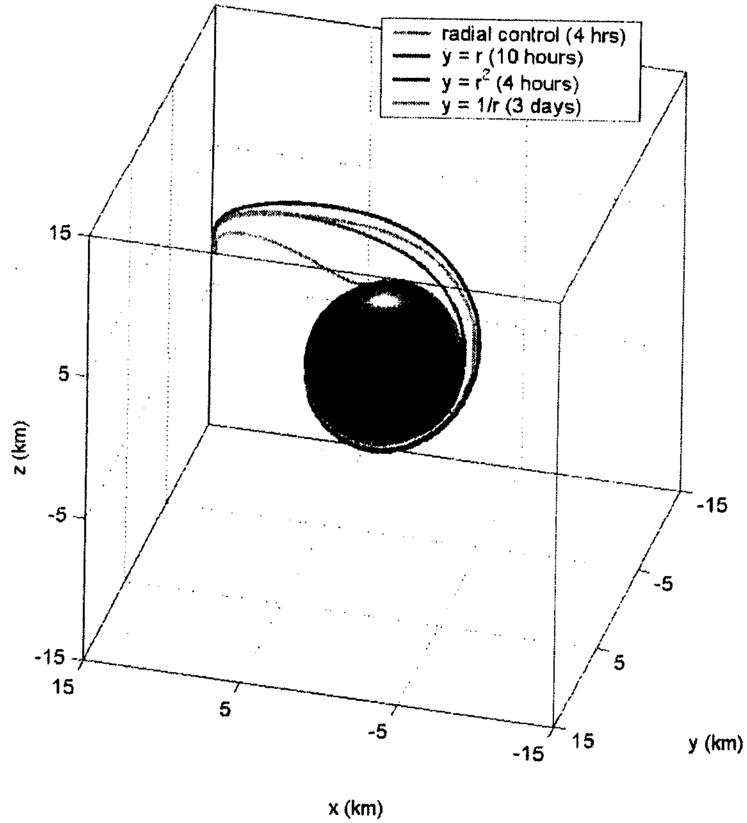
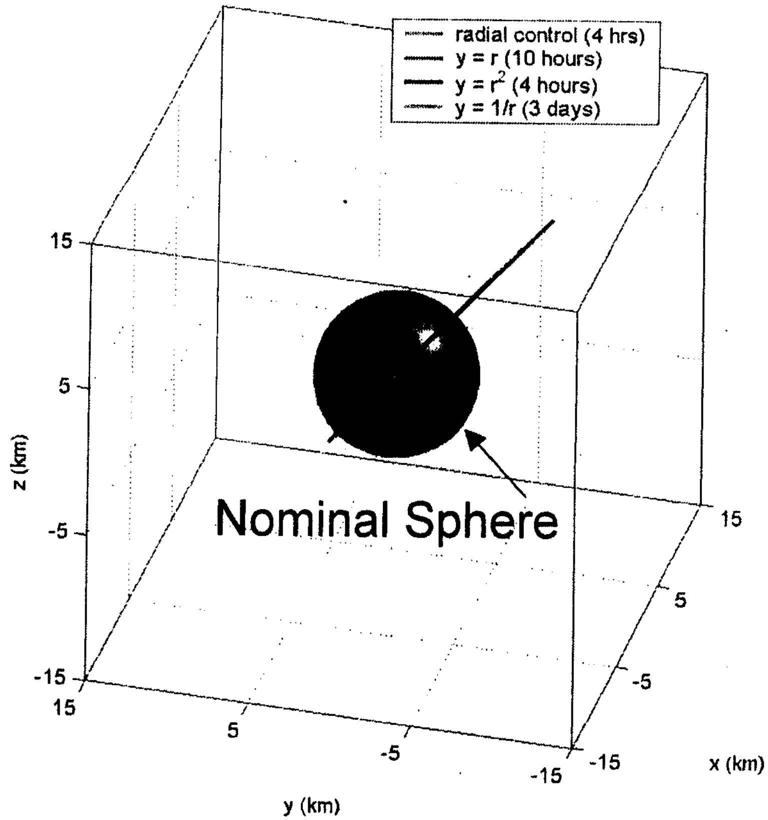
OFL Control of Spherical Formations in the Ephemeris Model

$$\bar{r}(0) = [12 \quad -5 \quad 3]\ \text{km} \qquad \dot{\bar{r}}(0) = [1 \quad -1 \quad 1]\ \text{m/sec}$$





Relative Dynamics as Observed in the Inertial Frame

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OFL Controlled Response of Deputy S/C Radial Distance + Rotation Rate Tracking

Radial Error Response (Critically Damped):

$$\delta\ddot{r} = -2\omega_n \delta\dot{r} - \omega_n^2 \delta r$$

$$\boxed{\ddot{r} = g_r(t) = \ddot{r}_n - 2\omega_n(\dot{r} - \dot{r}_n) - \omega_n^2(r - r_n)}$$

Rotation Rate Error Response (Exponential Decay):

$$\delta\dot{\theta} = \delta\dot{\theta}_0 e^{-t/T} \quad \rightarrow \quad \delta\ddot{\theta} = -\left(\delta\dot{\theta}_0 / T\right) e^{-t/T} = -\delta\dot{\theta}/T$$

$$\boxed{\ddot{\theta} = g_\theta(t) = \ddot{\theta}_n - \left(\dot{\theta} - \dot{\theta}_n\right)/T = \ddot{\theta}_n - k\omega_n\left(\dot{\theta} - \dot{\theta}_n\right)}$$

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OFL Controlled Response of Deputy S/C

Equations of Motion in the Relative Rotating Frame

$$\ddot{r} - r\dot{\theta}^2 = f_r + u_r \qquad f_r = \Delta\overline{f} \cdot \hat{r}, \quad f_\theta = \Delta\overline{f} \cdot \hat{\theta}, \quad f_h = \Delta\overline{f} \cdot \hat{h}$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = f_\theta + u_\theta \qquad u_r = \overline{u} \cdot \hat{r}, \quad u_\theta = \overline{u} \cdot \hat{\theta}, \quad u_h = \overline{u} \cdot \hat{h}$$

Rearrange to isolate the radial and rotational accelerations:

$$\ddot{r} = f_r + u_r + r\dot{\theta}^2 = g_r(t)$$

$$r\ddot{\theta} = f_\theta + u_\theta - 2\dot{r}\dot{\theta} = r g_\theta(t)$$

Solve for the Control Inputs:

$$\boxed{\begin{aligned} u_r(t) &= g_r(t) - f_r - r\dot{\theta}^2 \\ u_\theta(t) &= r g_\theta(t) - f_\theta + 2\dot{r}\dot{\theta} \\ u_h(t) &= -f_h \ \ (\text{constraint}) \end{aligned}}$$

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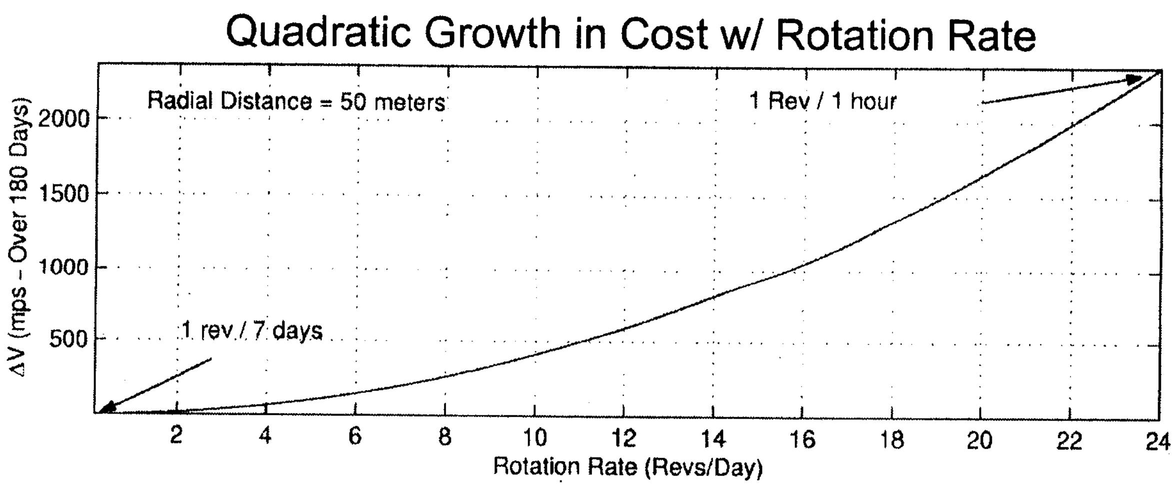
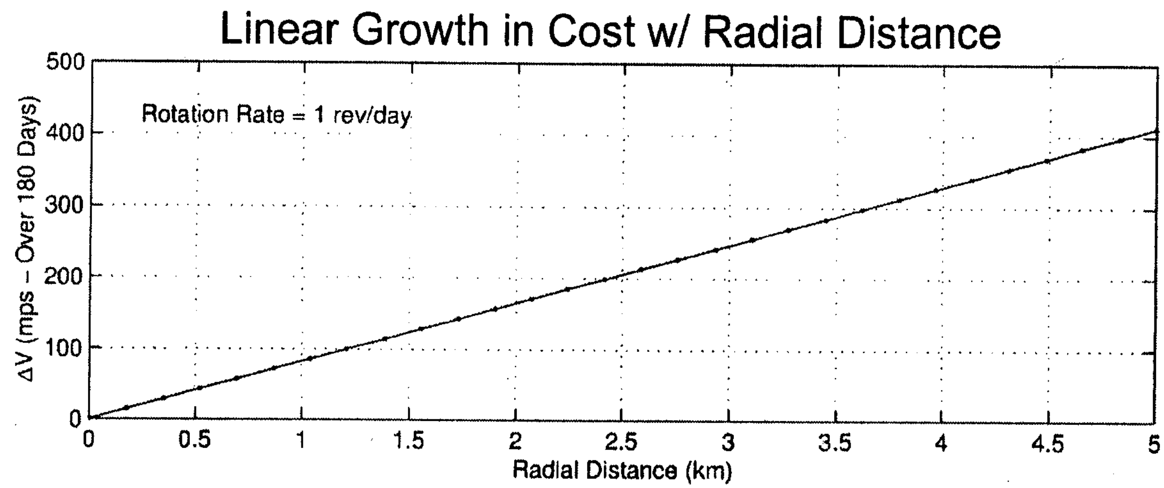
OFL Control of Spherical Formations
Radial Dist. + Rotation Rate





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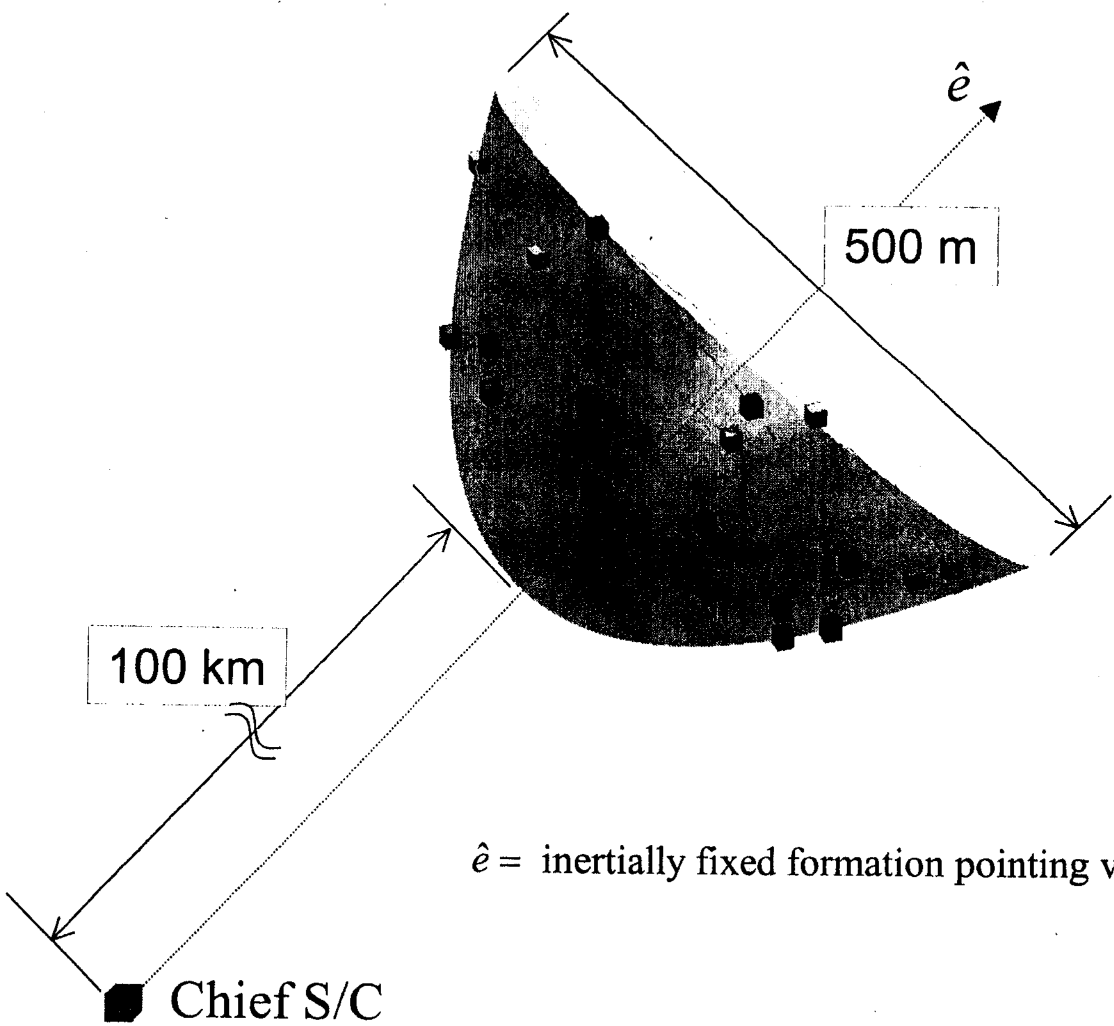
Inertially Fixed Formations in the Ephemeris Model



$\hat{e}$ = inertially fixed formation pointing vector (focal line)

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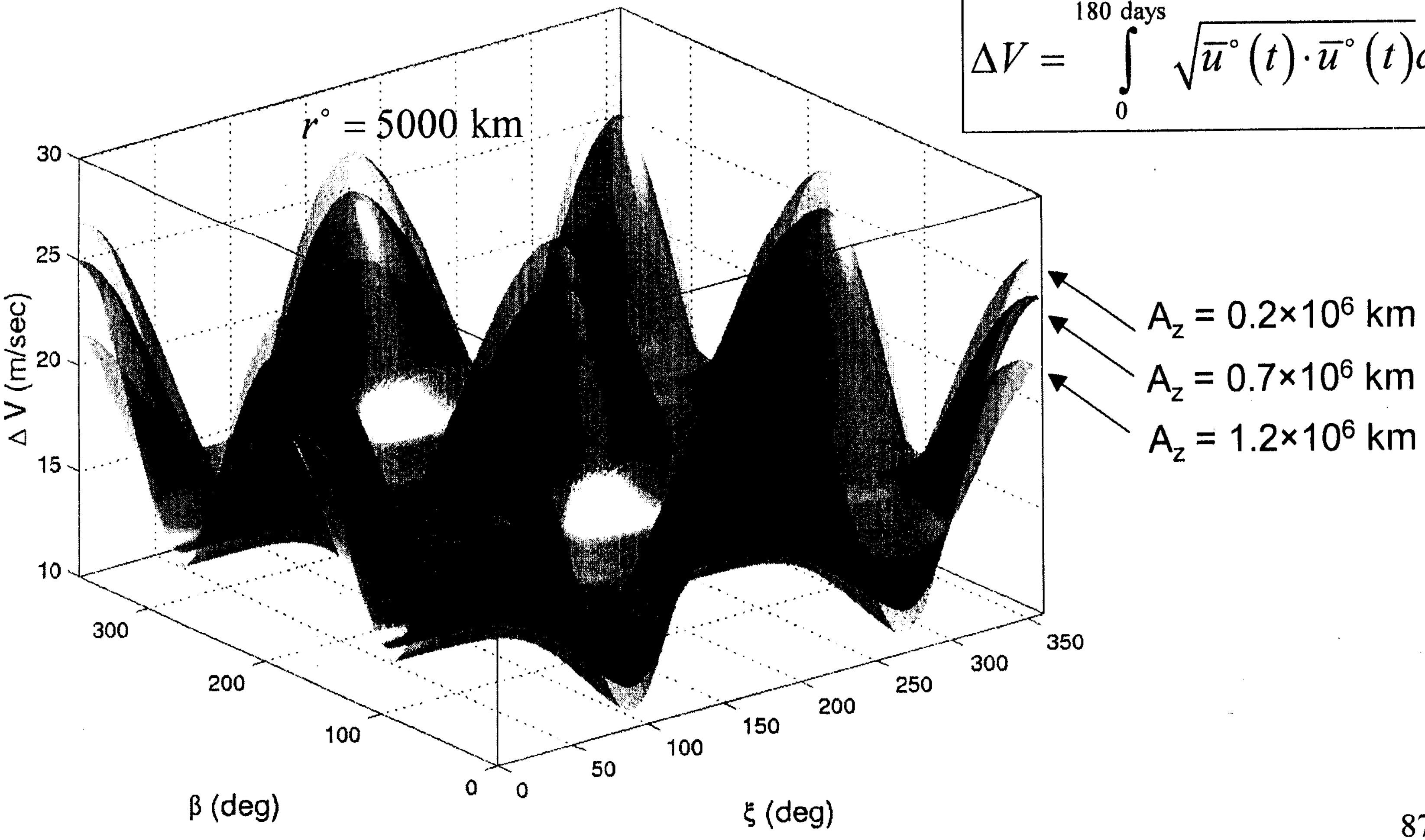
Nominal Formation Keeping Cost (Configurations Fixed in the RLP Frame)

$$\Delta V = \int_0^{180\text{ days}} \sqrt{\bar{u}^\circ(t)\cdot\bar{u}^\circ(t)}\,dt$$



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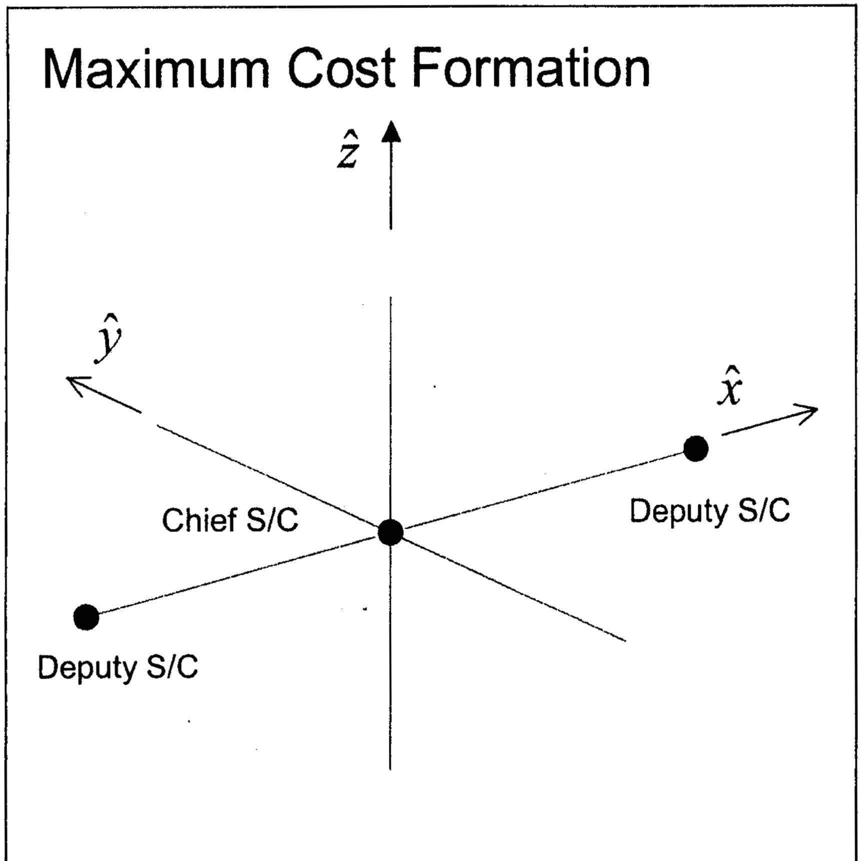
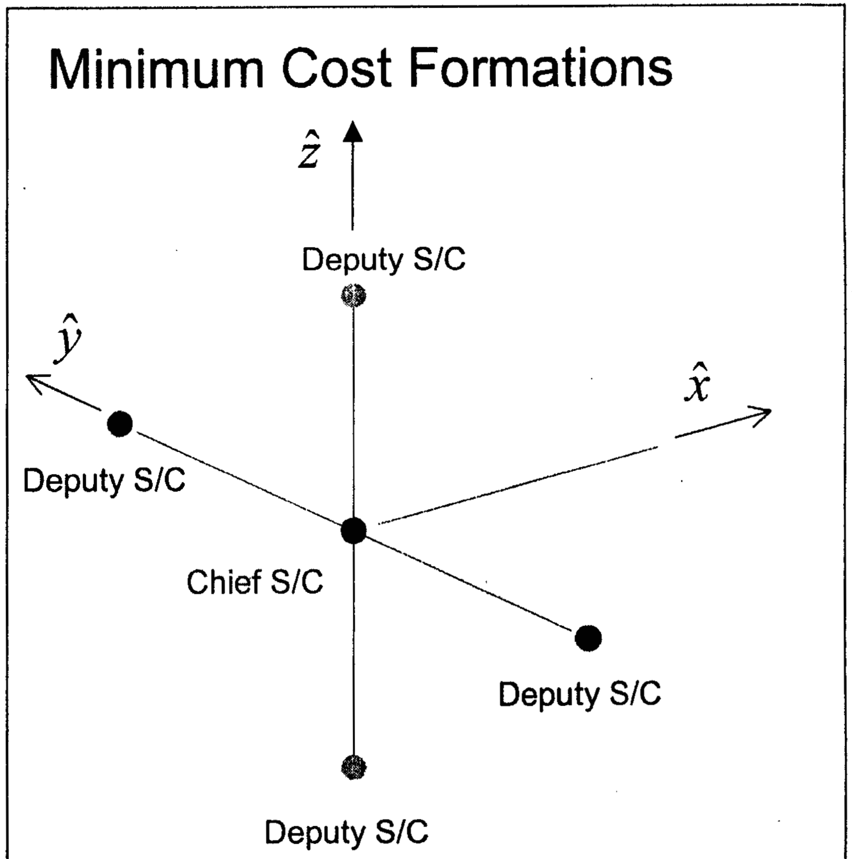
Max./Min. Cost Formations (Configurations Fixed in the RLP Frame)





Nominal Relative Dynamics in the Synodic Rotating Frame

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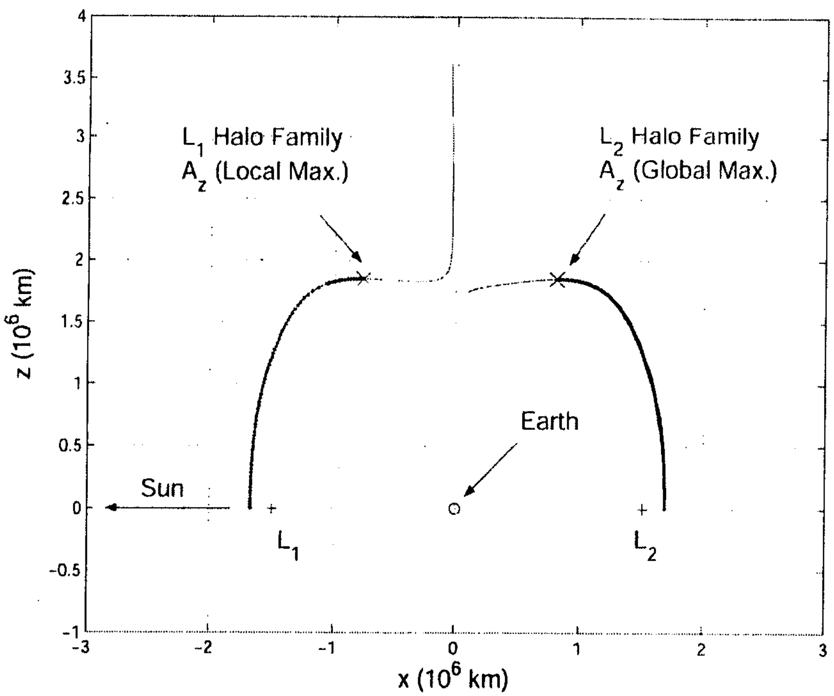
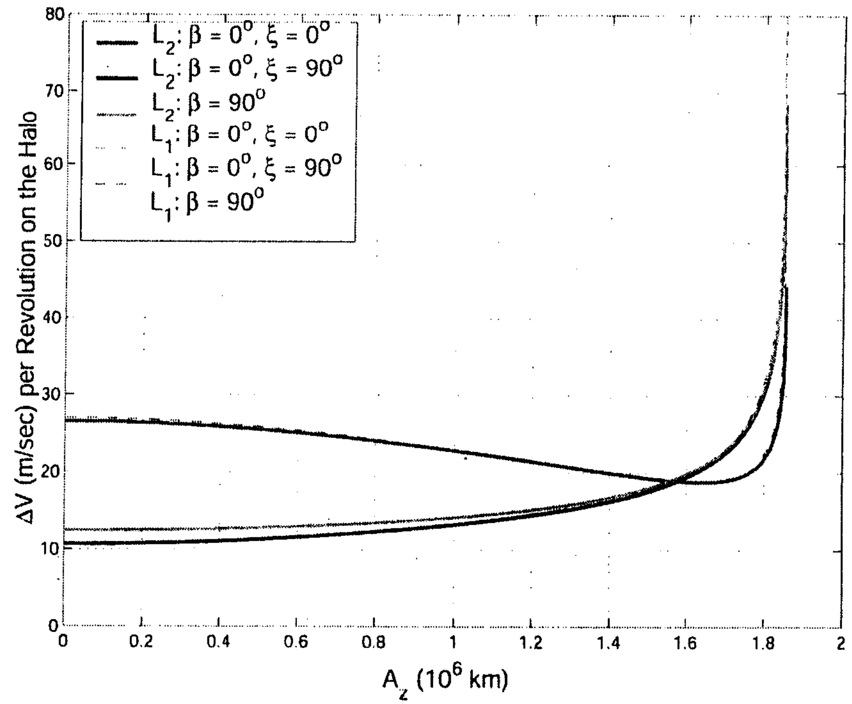
Formation Keeping Cost Variation Along the SEM L_1 and L_2 Halo Families (Configurations Fixed in the RLP Frame)





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Conclusions

- Continuous Control in the Ephemeris Model:
  - Non-Natural Formations
    - LQR/IFL → essentially identical responses & control inputs
    - IFL appears to have some advantages over LQR in this case
    - OFL → spherical configurations + unnatural rates
    - Low acceleration levels → Implementation Issues
- Discrete Control of Non-Natural Formations
  - Targeter Approach
    - Small relative separations → Good accuracy
    - Large relative separations → Require nearly continuous control
    - Extremely Small ΔV's (10^{-5} m/sec)
- Natural Formations
  - Nearly periodic & quasi-periodic formations in the RLP frame
  - Floquet controller: numerically ID solutions + stable manifolds

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Some Examples from Simulations

- A simple formation about the Sun-Earth L1
  - Using CRTB based on L1 dynamics
  - Errors associated with perturbations

- A more complex Fizeau-type interferometer fizeau interferometer.
  - Composed of 30 small spacecraft at L2
  - Formation maintenance, rotation, and slewing

16, 17, 20

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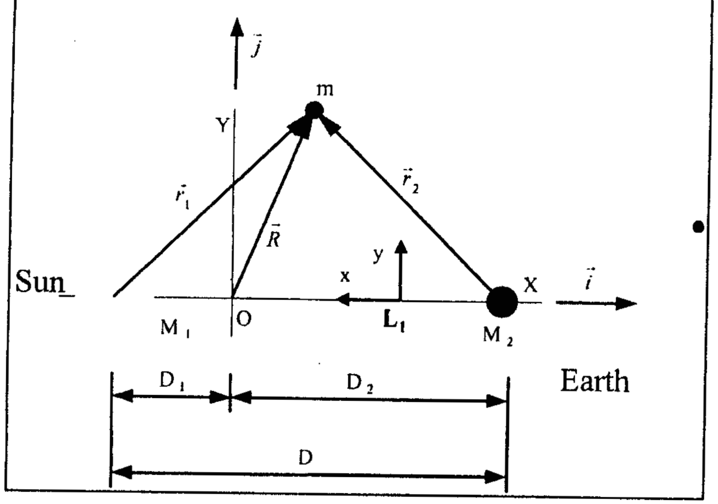
A State Space Model

- A common approximation in research of this type of orbit models the dynamics using CRTB approximations
- The Linearized Equations of Motion for a S/C Close to the Libration Point Are Calculated at the Respective Libration Point.

CRTB problem rotating frame



- Linearized Eq. Of Motion Based on Inertial X, Y, Z Using

$$X = X_0 + x, \qquad Y = Y_0 + y, \qquad Z = Z_0 + z$$

$$\ddot{x} - 2n\dot{y} = U_{XX}\,x, \quad \ddot{y} + 2n\dot{x} = U_{YY}\,y, \quad \ddot{z} = U_{ZZ}\,z$$

- Pseudopotential:

$$U_{XX} = \frac{\partial^2 U}{\partial X^2}, \quad U_{YY} = \frac{\partial^2 U}{\partial Y^2}, \quad U_{ZZ} = \frac{\partial^2 U}{\partial Z^2}.$$

$$\dot{\mathbf{x}}^j = A^j \mathbf{x}^j \quad \text{where} \quad \mathbf{x}^j = \begin{bmatrix} x^j \\ y^j \\ z^j \\ \dot{x}^j \\ \dot{y}^j \\ \dot{z}^j \end{bmatrix}, \quad A^j = \left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \hline U_{XX} & 0 & 0 & 0 & 2n & 0 \\ 0 & U_{YY} & 0 & -2n & 0 & 0 \\ 0 & 0 & U_{ZZ} & 0 & 0 & 0 \end{array}\right]$$

$$n = \sqrt{\frac{G(M_1 + M_2)}{D}}$$

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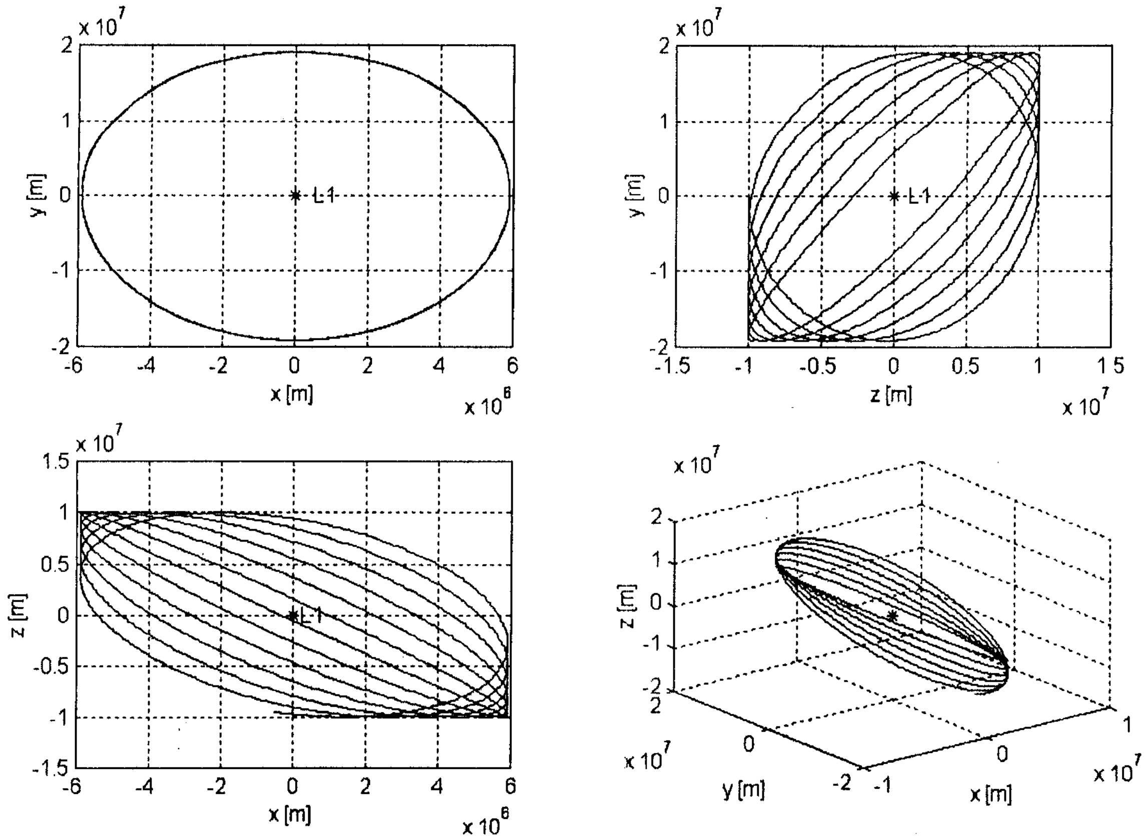
Periodic Reference Orbit

$$
\begin{array}{ll}
x = -A_x \sin(\omega_{xy} t + \varphi_{xy}) & \dot{x} = -A_x \omega_{xy} \cos(\omega_{xy} t + \varphi_{xy}) \\
y = -A_y \cos(\omega_{xy} t + \varphi_{xy}) & \dot{y} = A_y \omega_{xy} \sin(\omega_{xy} t + \varphi_{xy}) \\
z = A_z \sin(\omega_z t + \varphi_z) & \dot{z} = A_z \omega_z \cos(\omega_z t + \varphi_z)
\end{array}
$$



A = Amplitude

ω = frequency

ϕ = Phase angle

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Centralized LQR Design

State Dynamics ⟶ $\dot{\mathbf{x}}^j = A^j \mathbf{x}^j + B^j \mathbf{u}^j + \mathbf{a}_{solpres} + \mathbf{a}_{FB}$

Performance Index to Minimize ⟶ $J = \frac{1}{2}\int_0^\infty \{(\mathbf{x}-\mathbf{x}^R)^T Q(\mathbf{x}-\mathbf{x}^R) + \mathbf{u}^T R\,\mathbf{u}\}dt$

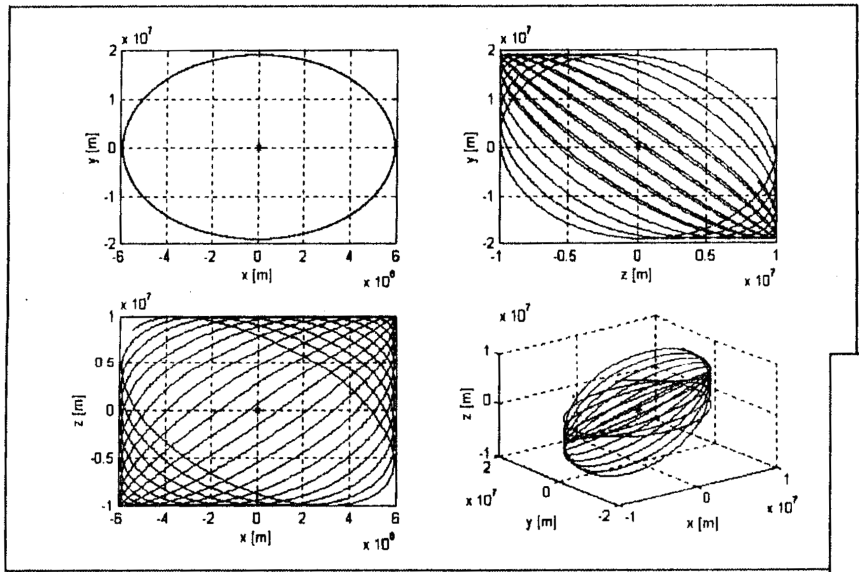
Control ⟶ $\mathbf{u}^j = -\left[R^j\right]^{-1}\left[B^j\right]^T S\,\mathbf{x}$

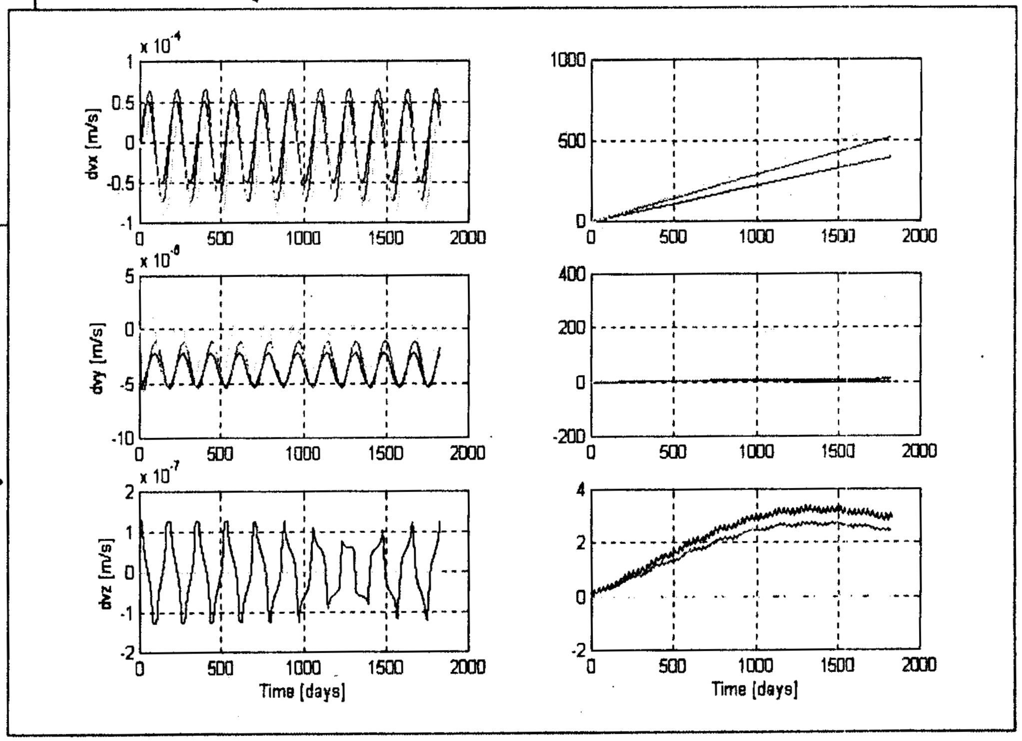
Algebraic Riccatic Eq. time invariant system ⟶ $S(B R^{-1} B^T)S - SA - A^T S - Q = 0$

B Maps Control Input From Control Space to State Space	Q Is Weight of State Error	R Is Weight of Control
$B^j = \begin{bmatrix} O_{3\times3} \\ I_3 \end{bmatrix}$	$Q^j = \begin{bmatrix} \frac{1}{p} I_3 & O_{3\times3} \\ O_{3\times3} & \frac{1}{q} I_3 \end{bmatrix}$	$R^j = \frac{1}{r} I_3$

16, 17, 20

Centralized LQR Design





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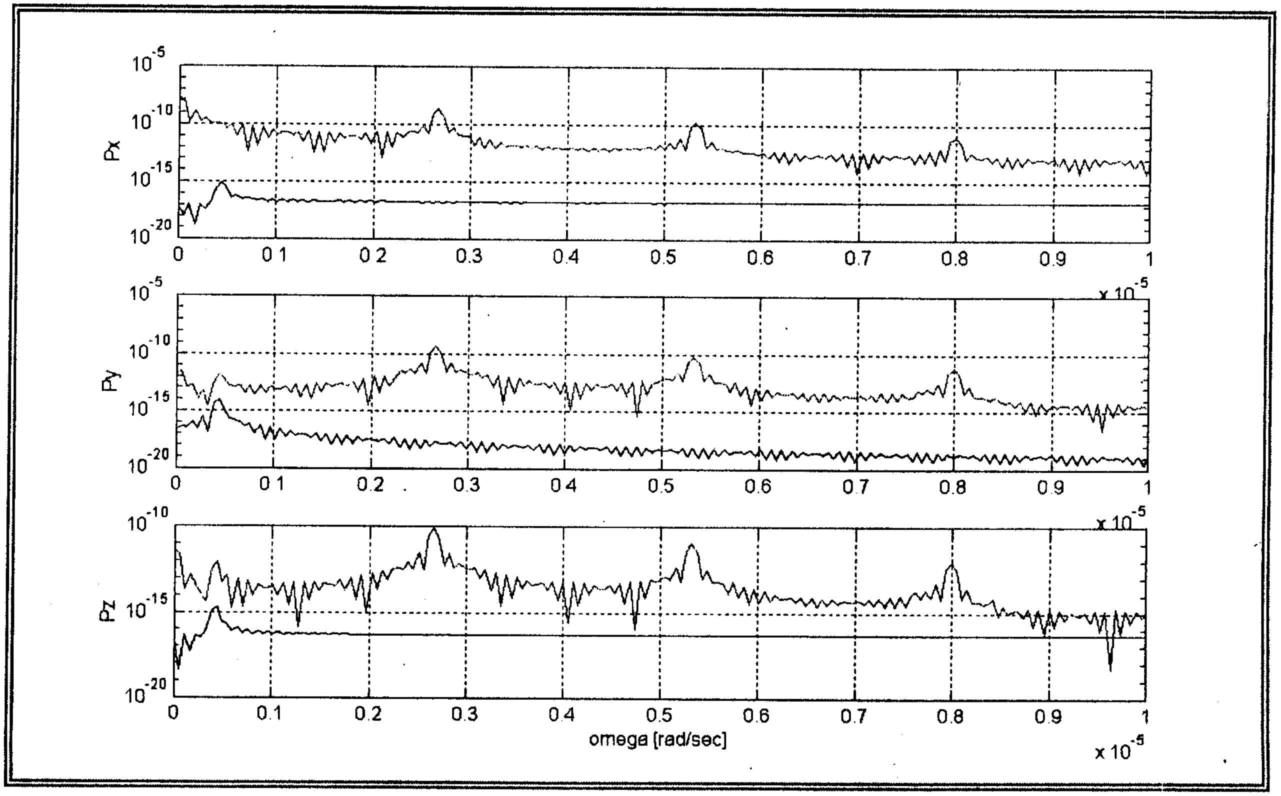
Disturbance Accommodation Model

- The *A* Matrix Does Not Include the Perturbation Disturbances nor Exactly Equal the Reference
- Disturbance Accommodation Model Allows the States to Have Non-zero Variations From the Reference in Response to the Perturbations Without Inducing Additional Control Effort
- The Periodic Disturbances Are Determined by Calculating the Power Spectral Density of the Optimal Control [Hoff93] To Find a Suitable Set of Frequencies.



Unperturbed

$\omega_{x,y,z} = 4.26106e\text{-}7$ *rad/s*

Perturbed

$\omega_x = 1.5979e\text{-}7$ *rad/s*

$2.6632e\text{-}6$ *rad/s*

$\omega_y = 2.6632e\text{-}6$ *rad/s*

$\omega_z = 2.6632e\text{-}6$ *rad/s*

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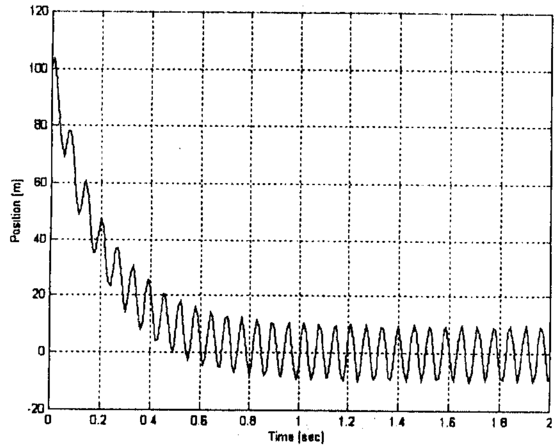
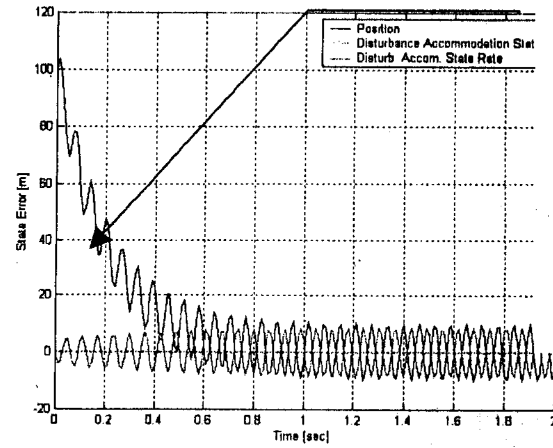
Disturbance Accommodation Model

Without Disturbance Accommodation

With Disturbance Accommodation

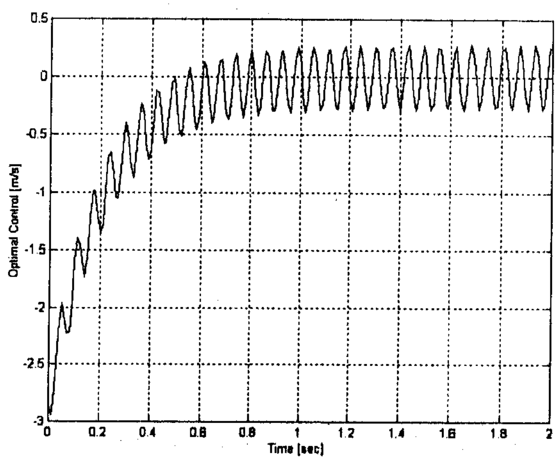
State Error

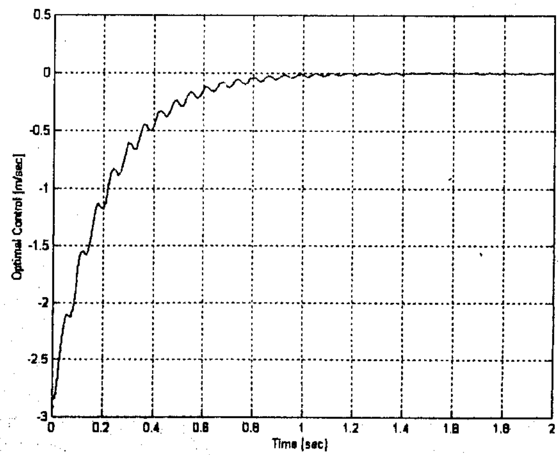




Disturbance accommodation state is out of phase with state error, absorbing unnecessary control effort

Control Effort





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Motion of Formation Flyer With Respect to Reference Spacecraft, in Local (S/C-1) Coordinates

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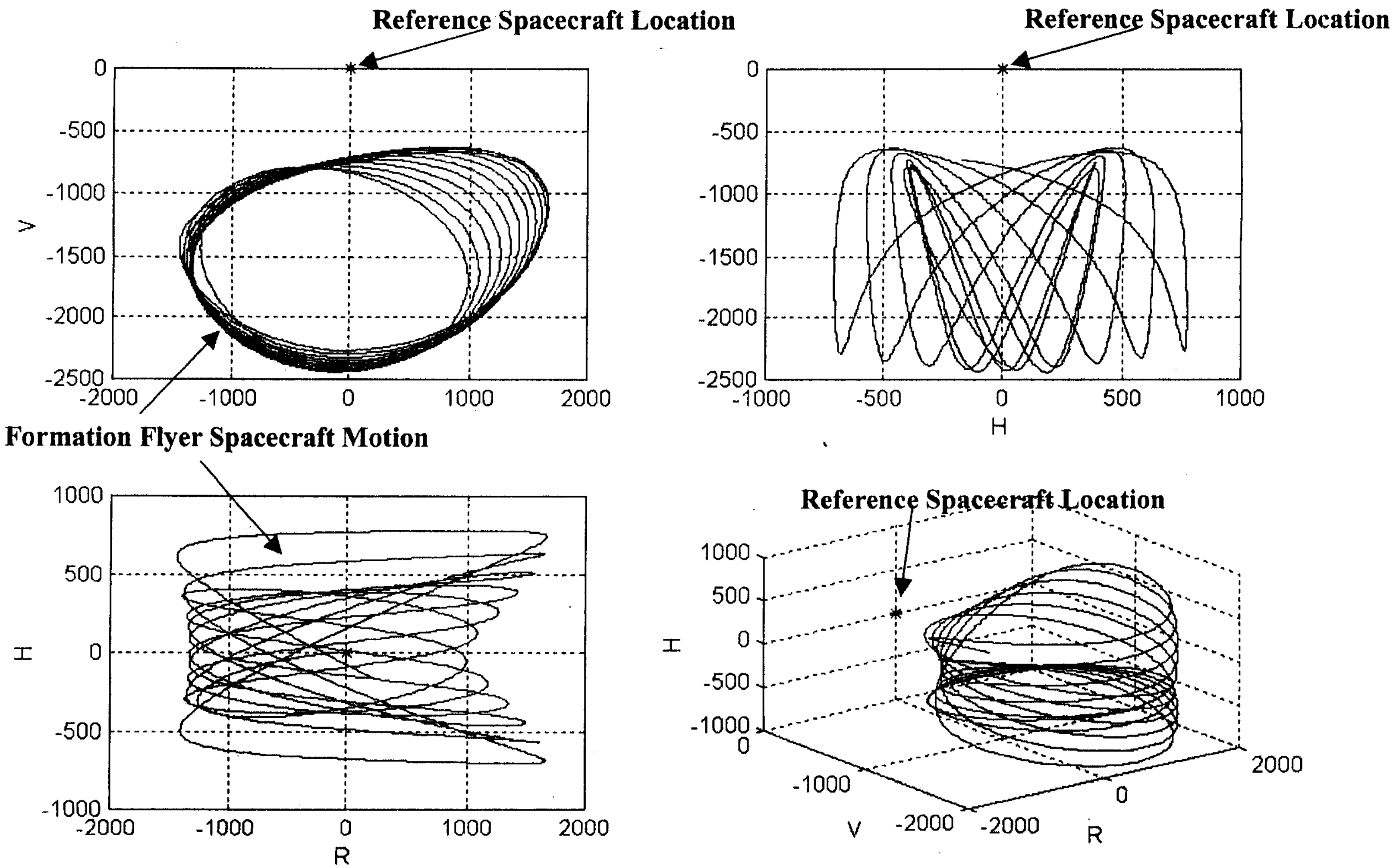

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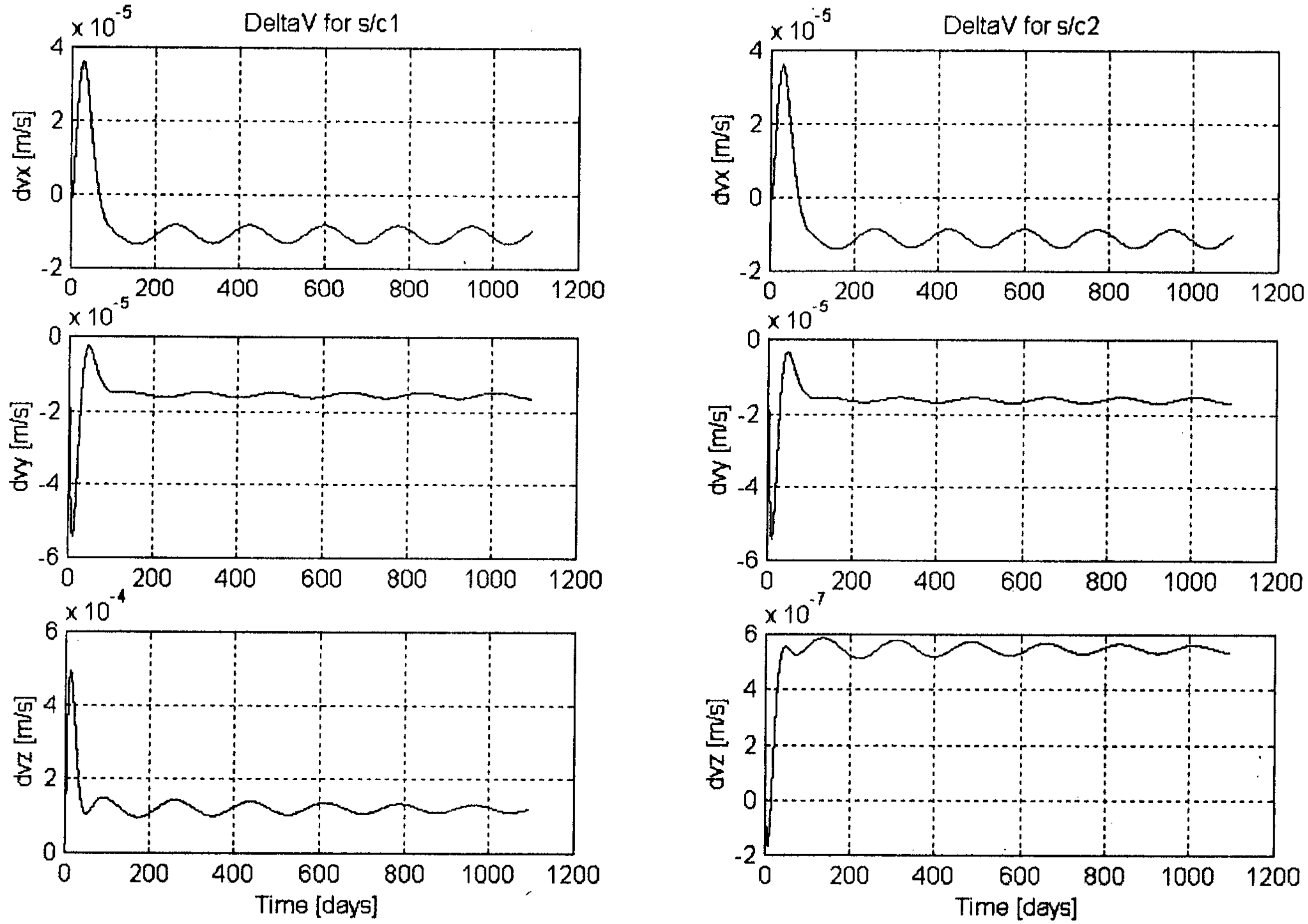
ΔV Maintenance in Libration Orbit Formation



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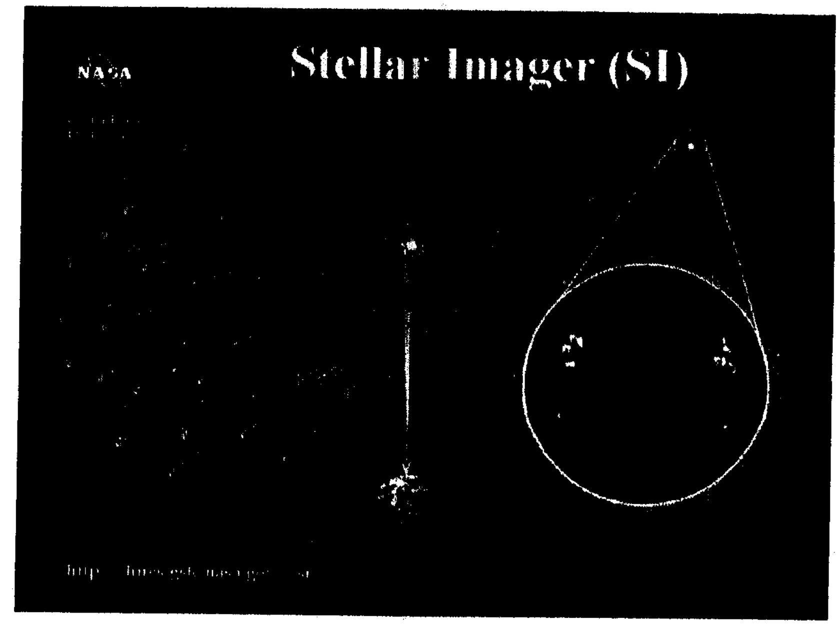
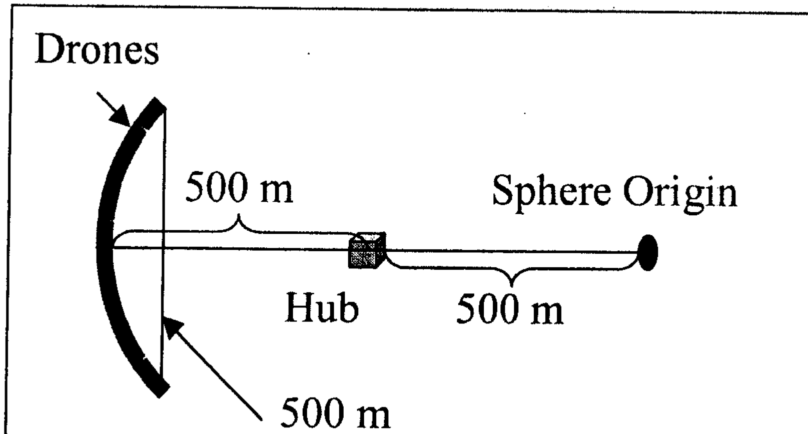
Stellar Imager Concept

(Using conceptual distances and control requirements to analyze formation possibilities)

- ✓ Stellar Imager (SI) concept for a space-based, UV-optical interferometer, proposed by Carpenter and Schrijver at NASA / GSFC (Magnetic fields, Stellar structures and dynamics)
- ✓ 500-meter diameter Fizeau-type interferometer composed of 30 small drone satellites
- ✓ Hub satellite lies halfway between the surface of a sphere containing the drones and the sphere origin.
- ✓ Focal lengths of both 0.5 km and 4 km, with radius of the sphere either 1 km or 8 km.
- ✓ L2 Libration orbit to meet science, spacecraft, and environmental conditions





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Stellar Imager

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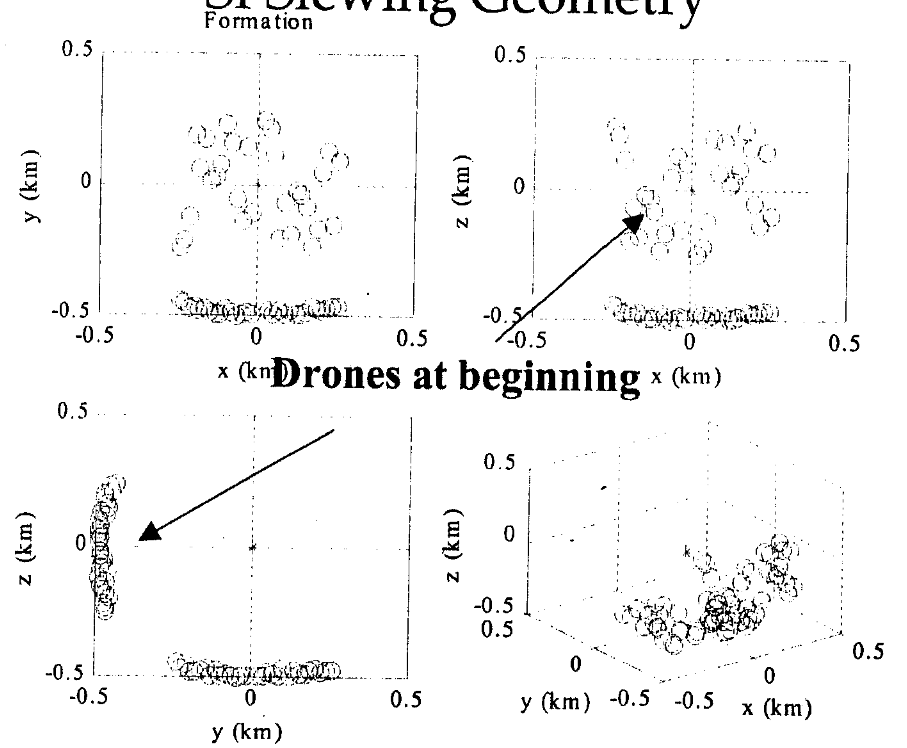
- Three different scenarios make up the SI formation control problem; maintaining the Lissajous orbit, slewing the formation, and reconfiguring
- Using a LQR with position updates, the hub maintains an orbit while drones maintain a geometric formation

The magenta circles represent drones at the beginning of the simulation, and the red circles represent drones at the end of the simulation. The hub is the black asterisk at the origin.

SI Slewing Geometry



Formation ΔV Cost per slewing maneuver

Focal Length (km)	Slew Angle (deg)	Hub (m/s)	Drone 2 (m/s)	Drone 31 (m/s)
0.5	30	1.0705	0.8271	0.8307
0.5	90	1.1355	0.9395	0.9587
1	30	1.2688	1.1189	1.1315
1	90	1.8570	2.1907	2.1932

Stellar Imager Mission Study Example Requirements

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- Maintain an orbit about the Sun-Earth L2 co-linear point
- Slew and rotate the Fizeau system about the sky, movement of few km and attitude adjustments of up to 180deg
- While imaging 'drones' must maintain position
  - ~ 3 nanometers radially from Hub
  - ~ 0.5 millimeters along the spherical surface
- Accuracy of pointing is 5 milli-arcseconds, rotation about axis < 10 deg

3-Tiered Formation Control Effort:

Coarse -	RF ranging, star trackers, and thrusters	~ centimeters
Intermediate -	Laser ranging and micro-N thrusters control	~ 50 microns
Precision -	Optics adjusted, phase diversity, wave front.	~ nanometers

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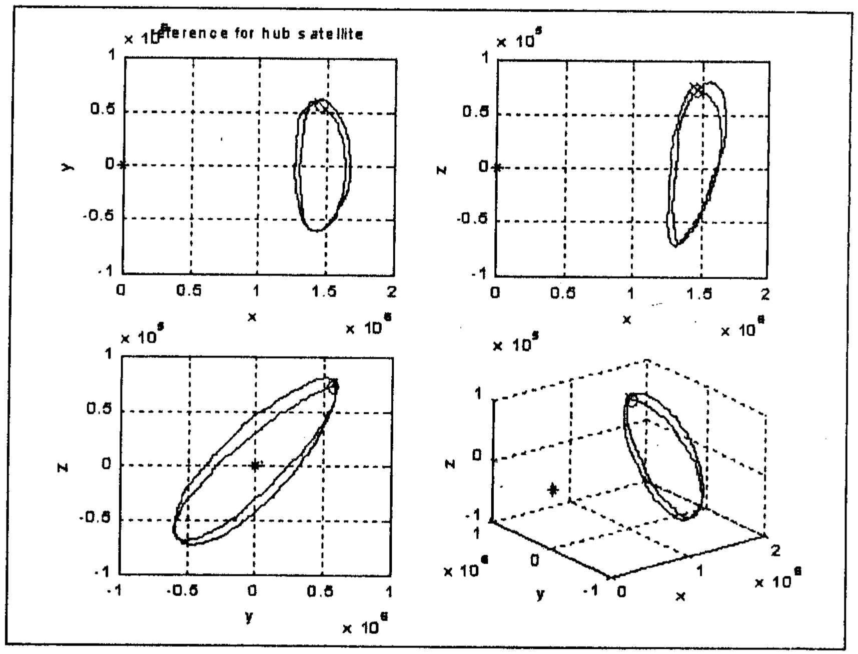
State Space Controller Development

- This analysis uses high fidelity dynamics based on a software named Generator that Purdue University has developed along with GSFC

- **Creates realistic lissajous orbits as compared to CRTB motion.**
- **Uses sun, Earth, lunar, planetary ephemeris data**
- **Generator accounts for eccentricity and solar radiation pressure.**
- **Lissajous orbit is more an accurate reference orbit.**
- **Numerically computes and outputs the linearized dynamics matrix, *A*, for a single**
- **satellite at each epoch.**
- **Data used onboard for autonomous**
  - **computation by simple uploads or**
  - **onboard computation as a backgrou**
  - **task of the 36 matrix elements and**
  - **the state vector.**



•Origin in figure is Earth
•Solar rotating coordinates

State Space Controller Development, LQR Design

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- **Rotating Coordinates of SI:** $X = X_0 + x$, $Y = Y_0 + y$, $Z = Z_0 + z$
  where the open-loop linearized EOM about L2 can be expressed as $\dot{\mathbf{x}} = A\mathbf{x}$
  and the A **matrix is the Generator Output** $\mathbf{x} = [x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z}]^T$

- **The STM is created from the dynamics partials output from Generator and assumes to be constant over an analysis time period** $\Phi(t - t_0) = e^{A(t-t_0)} = \sum_{k=0}^{\infty} \frac{A^k (t-t_0)^k}{k!}$

State Dynamics and Error ➡ $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$ $\quad \tilde{\mathbf{x}}(t) = \mathbf{x}(t) - \mathbf{x}_{ref}(t)$

Performance Index to Minimize ➡ $J = \int_{t_0}^{t_f} \left\{ \tilde{\mathbf{x}}^T(\tau) W \tilde{\mathbf{x}}(\tau) + \mathbf{u}^T(\tau) V \mathbf{u}(\tau) \right\} d\tau$

Control and Closed Loop Dynamics ➡ $\mathbf{u} = -K(t)\tilde{\mathbf{x}}$ $\quad \dot{\tilde{\mathbf{x}}} = (A - BK(t))\tilde{\mathbf{x}}$

Algebraic Riccatic Eq. ➡ $S(BR^{-1}B^T)S - SA - A^T S - Q = 0$
time invariant system

State Space Controller Development, LQR Design

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- Expanding for the SI collector (hub) and mirrors (drones) yields a controller

$$\dot{\tilde{\mathbf{x}}}_2 = A\tilde{\mathbf{x}}_2 + B\mathbf{u}_2 - B\mathbf{u}_1$$

- Redefine A and B such that

$$A = \begin{bmatrix} A_1 & & & \\ & A_2 & & \\ & & \ddots & \\ & & & A_j \end{bmatrix} \qquad B = \begin{bmatrix} B_1 & & & \\ -B_1 & B_i & & \\ -B_1 & & B_i & \\ \vdots & & & \ddots \\ -B_1 & & & & B_j \end{bmatrix}$$

B Maps Control Input From Control Space to State Space

$$B^j = \begin{bmatrix} O_{3\times3} \\ I_3 \end{bmatrix}$$

W Is Weight of State Error

$$W = \begin{bmatrix} \frac{1}{p} I_3 & O_{3\times3} \\ O_{3\times3} & \frac{1}{q} I_3 \end{bmatrix}$$

V Is Weight of Control

$$V^j = \frac{1}{r} I_3$$

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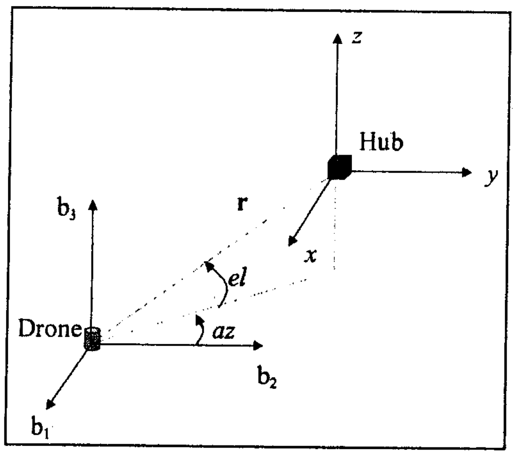
Simplified extended Kalman Filter

- Using dynamics described and linear measurements augmented by zero-mean white Gaussian process and measurement noise
- Discretized state dynamics for the filter are: $\tilde{\mathbf{x}}_{k+1} = A_d \tilde{\mathbf{x}}_k + B_d \mathbf{u}_k + w$ where w is the random process noise
- The non-linear measurement model is $\mathbf{y}_k = m(\tilde{\mathbf{x}}_k) + v$ and the covariances of process and measurement noise are

$$E\left[ww^T\right] = Q$$
$$E\left[vv^T\right] = R$$

- Hub measurements are range(r) and azimuth(az) / elevation(el) angles from Earth Drone measurements are r, az, el from drone to hub



$$r = \sqrt{x^2 + y^2 + z^2}$$

$$el = \sin^{-1}\left(\frac{-z}{r}\right), \quad and \quad az = \sin^{-1}\left(\frac{x}{r\cos(el)}\right)$$

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Simulation Matrix Initial Values

- Continuous state weighting and control chosen as

$$V = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix} \quad W = \begin{bmatrix} 1e6 & & & & & \\ & 1e6 & & & & \\ & & 1e6 & & & \\ & & & 1e3 & & \\ & & & & 1e3 & \\ & & & & & 1e3 \end{bmatrix}$$

- The process and measurement noise Covariance (hub and drone) are

$$Q_c = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 1e-6 & & \\ & & & & 1e-6 & \\ & & & & & 1e-6 \end{bmatrix} \quad R = \begin{bmatrix} 0.1^2 & & \\ & \left(\frac{0.3}{1500000}\right)^2 & \\ & & \left(\frac{0.3}{1500000}\right)^2 \end{bmatrix} \quad R = \begin{bmatrix} 0.0001^2 & & \\ & \left(\frac{0.0003}{0.5}\right)^2 & \\ & & \left(\frac{0.0003}{0.5}\right)^2 \end{bmatrix}$$

- Initial covariances

$$P_1(+) = \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 86.4^2 & & \\ & & & & 86.4^2 & \\ & & & & & 86.4^2 \end{bmatrix} \quad P_1(+) = \begin{bmatrix} .001^2 & & & & & \\ & .001^2 & & & & \\ & & .001^2 & & & \\ & & & .0864^2 & & \\ & & & & .0864^2 & \\ & & & & & .0864^2 \end{bmatrix}$$

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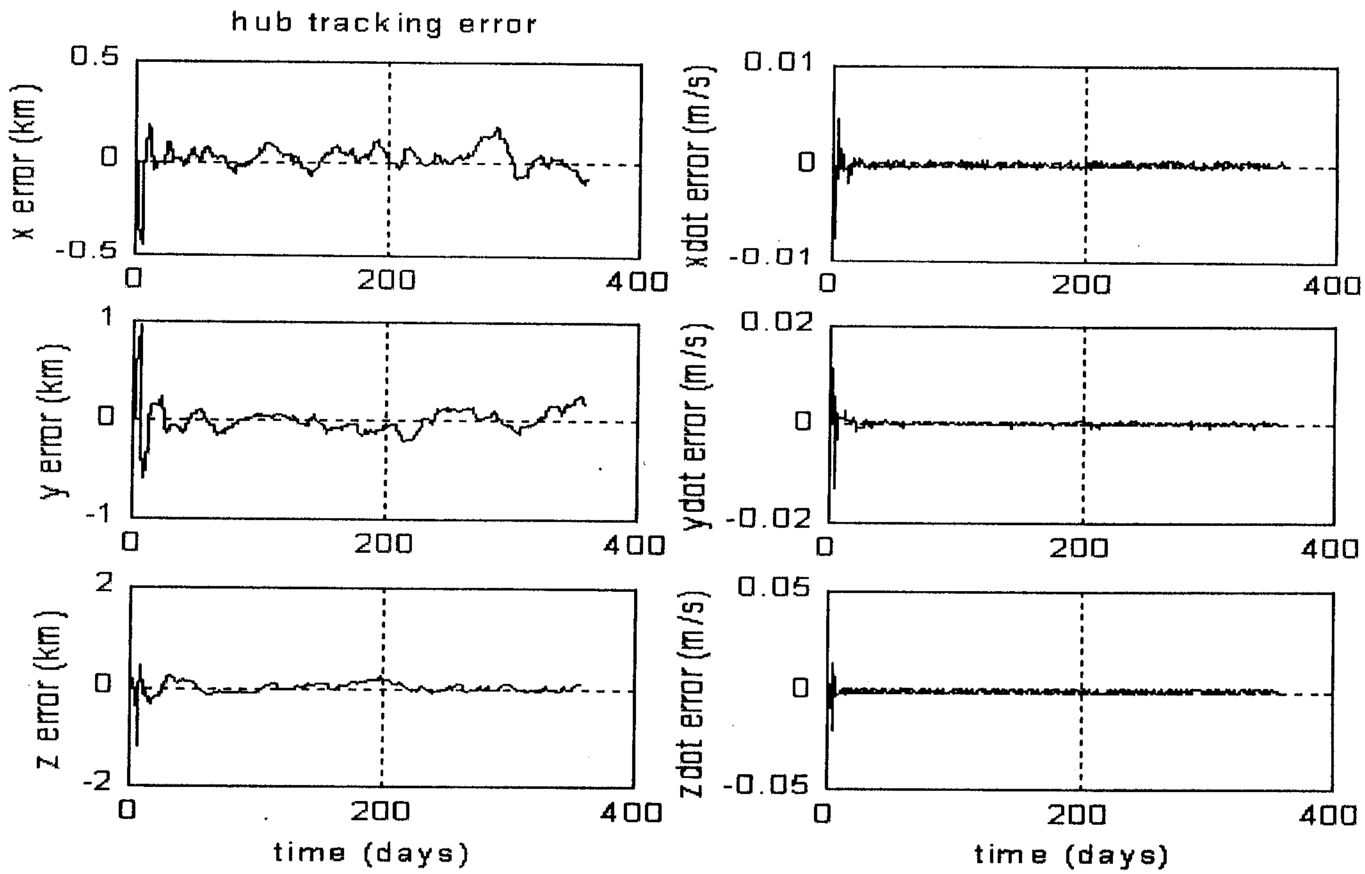
Results – Libration Orbit Maintenance

- Only Hub spacecraft was simulated for maintenance
- Tracking errors for 1 year: Position and Velocity
- Steady State errors of 250 meters and .075 cm/s



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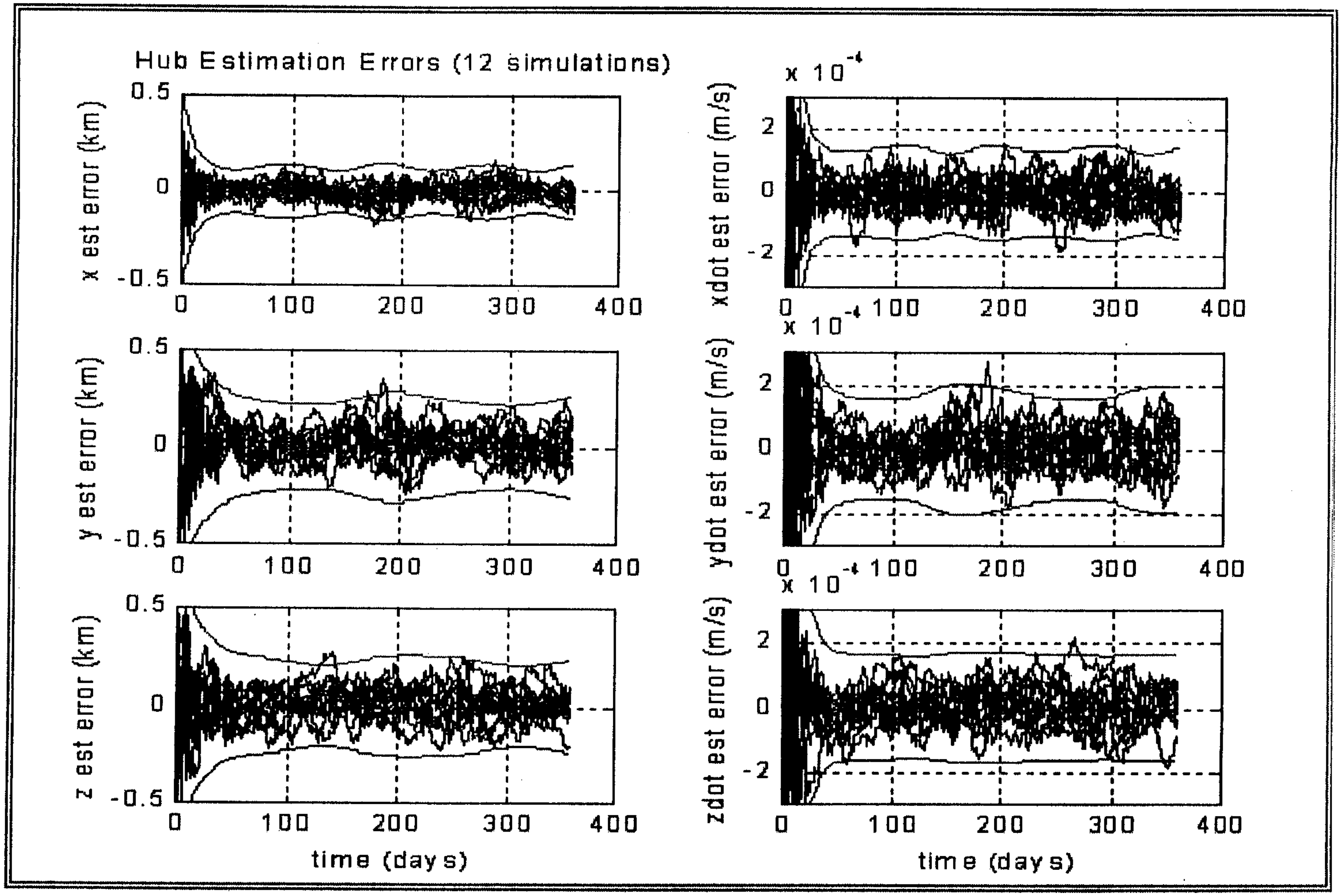
Results – Libration Orbit Maintenance

- Estimation errors for 12 simulations for 1 year: Position and Velocity
- Estimation errors of 250 meters and 2e-4 m/s in each component

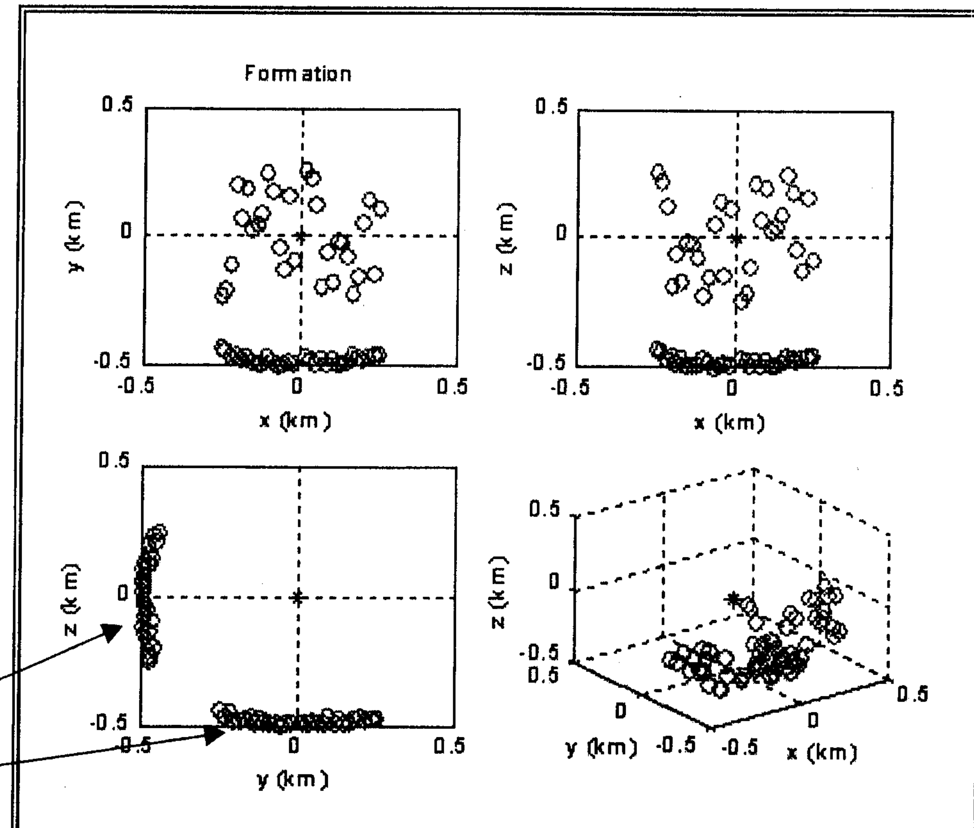


Results – Formation Slewing

- Length of simulation is 24 hours
- Maneuver frequency is 1 per minute
- Using a constant A from day-2 of the previous simulation
- Tuning parameters are same but strength of process noise is

$$Q_c = \begin{bmatrix} 0 & & & & & \\ & 0 & & & & \\ & & 0 & & & \\ & & & 1e-24 & & \\ & & & & 1e-24 & \\ & & & & & 1e-24 \end{bmatrix}$$

Formation Slewing:
90^0 simulation shown



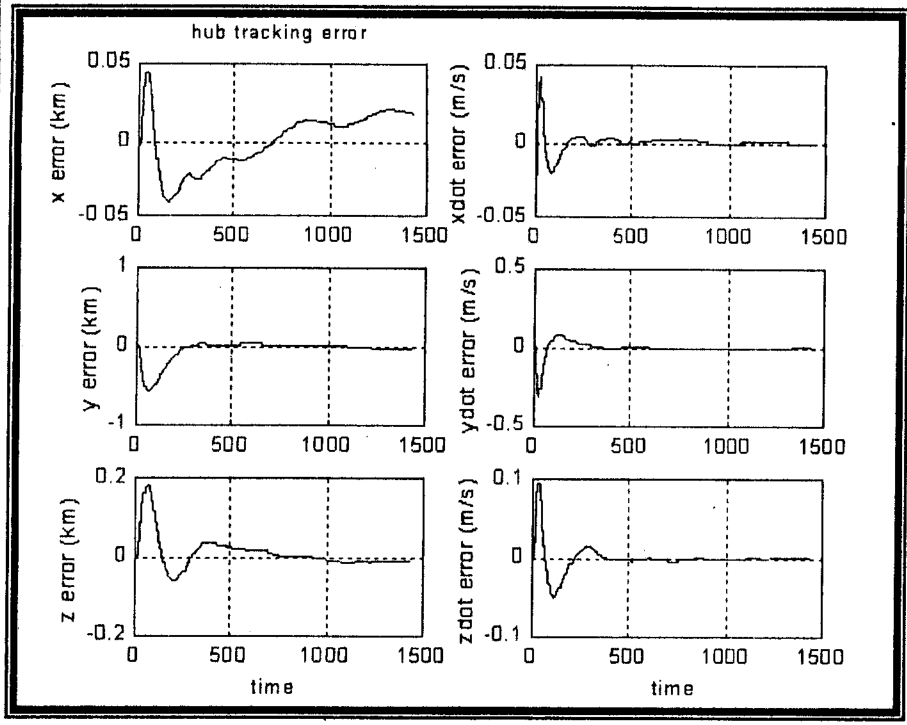
Purple - Begin
Red - End

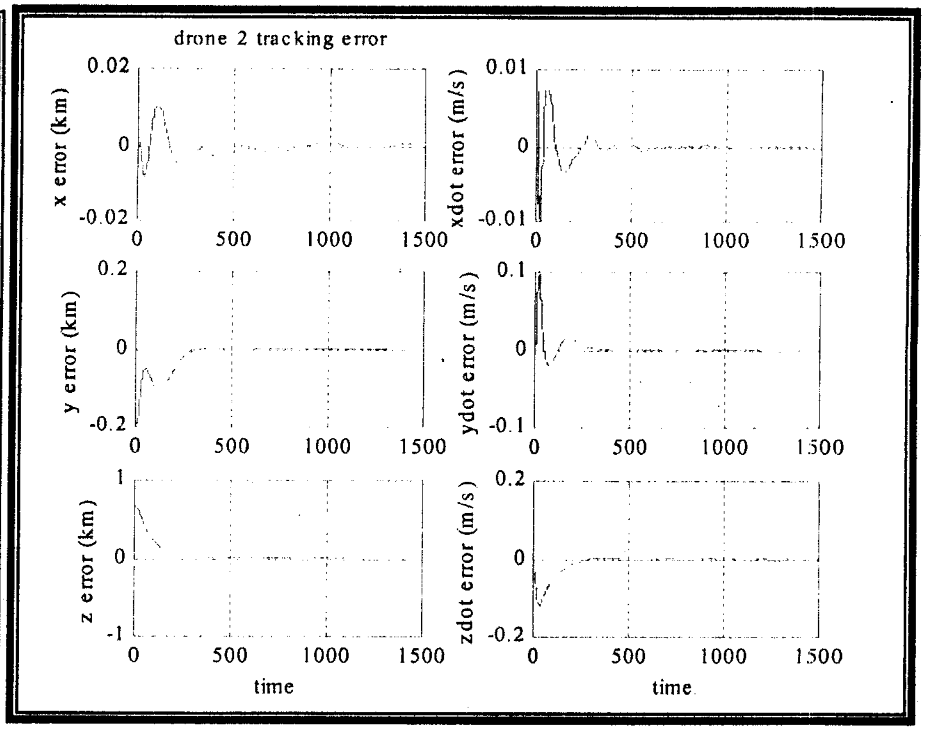
Results – Formation Slewing

- Tracking errors for 24 hours: Position and Velocity
- Steady State errors of 50 meters - hub, 3 meters - drone and 5 millimeters/sec – hub, and 1 millimeter/s - drone

HUB

DRONE





Represents both 0.5 and 4 km focal lengths

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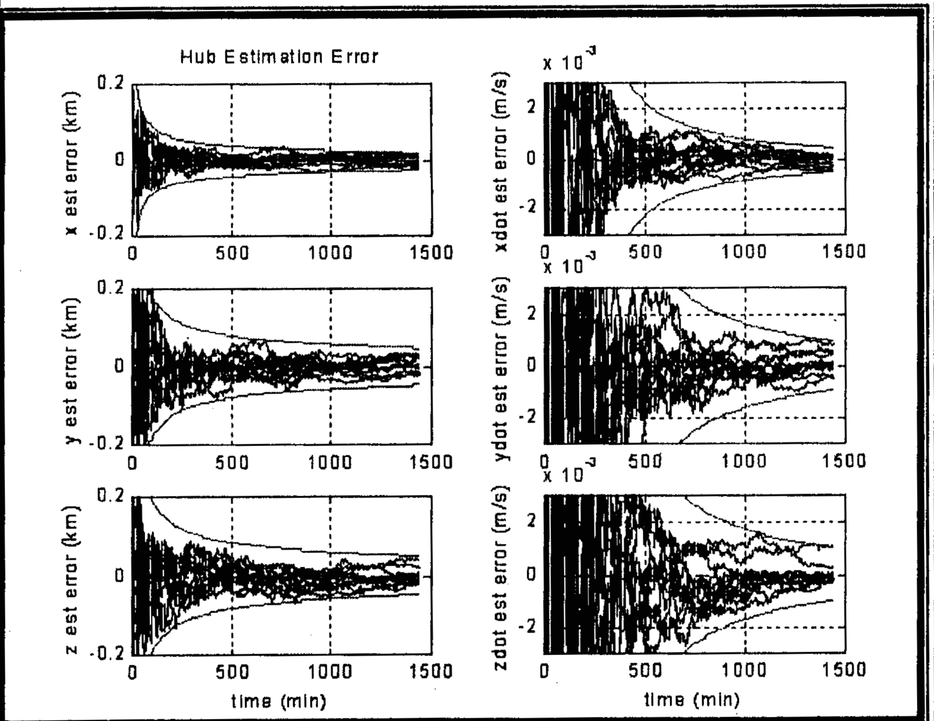
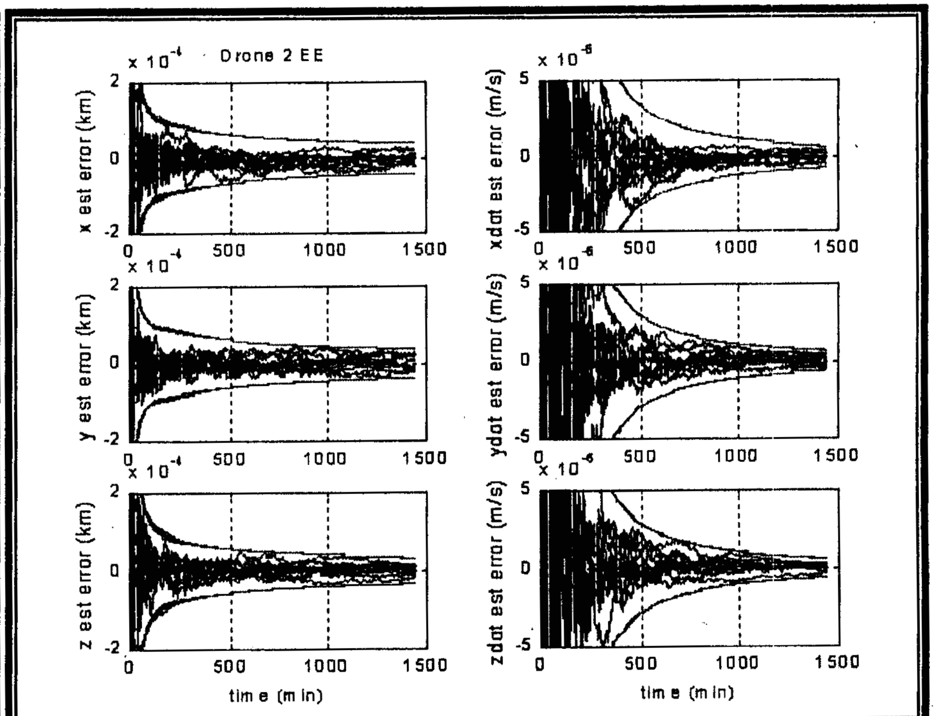
Results – Formation Slewing

- Estimation errors;12 simulations for 24 hours: position and velocity
- Estimation 3σ errors of ~50km and ~1 millimeter/s for all scenarios



Hub estimation 0.5 km separation / 90 deg slew



Drone estimation 0.5 km separation / 90 deg slew

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Results – Formation Slewing

Formation Slewing Average ΔVs (12 simulations)

Focal Length (km)	Slew Angle (deg)	Hub (m/s)	Drone 2 (m/s)	Drone 31 (m/s)
0.5	30	1.0705	0.8271	0.8307
0.5	90	1.1355	0.9395	0.9587
4	30	1.2688	1.1189	1.1315
4	90	1.8570	2.1907	2.1932

Formation Slewing Average Propellant Mass

Focal Length (km)	Slew Angle (deg)	Hub mass-prop (g)	Drone 2 mass-prop (g)	Drone 31 mass-prop (g)
0.5	30	6.0018	0.8431	0.8468
0.5	90	6.3662	0.9577	0.9773
4	30	7.1135	1.1406	1.1534
4	90	10.4112	2.2331	2.2357

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Results – Formation Slewing

Formation Slewing Average ΔVs (without noise)

Focal Length (km)	Slew Angle (deg)	Hub (m/s)	Drone 2 (m/s)	Drone 31 (m/s)
0.5	30	0.0504	0.0853	0.0998
0.5	90	0.1581	0.2150	0.2315
4	30	0.4420	0.5896	0.6441
4	90	1.3945	1.9446	1.9469

Formation Slewing Average Propellant Mass (without noise)

Focal Length (km)	Slew Angle (deg)	Hub mass-prop (g)	Drone 2 mass-prop (g)	Drone 31 mass-prop (g)
0.5	30	0.2826	0.0870	0.1017
0.5	90	0.8864	0.2192	0.2360
4	30	2.4781	0.6010	0.6566
4	90	7.8182	1.9822	1.9846

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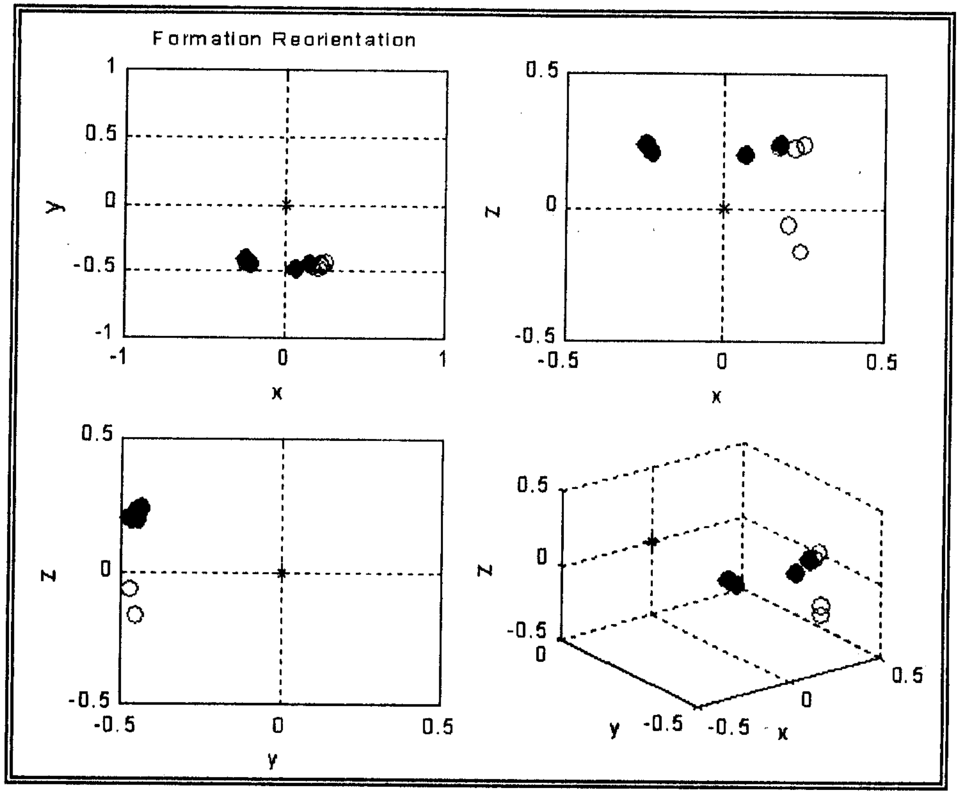
Results – Formation Reorientation

- Rotation about the line of sight
- Length of simulation is 24 hours
- Maneuver frequency is 1 per minute
- Using a constant *A* from day-2 of the previous simulation
- Tuning parameters are same as slewing
- Reorientation of 4 drones 90^0



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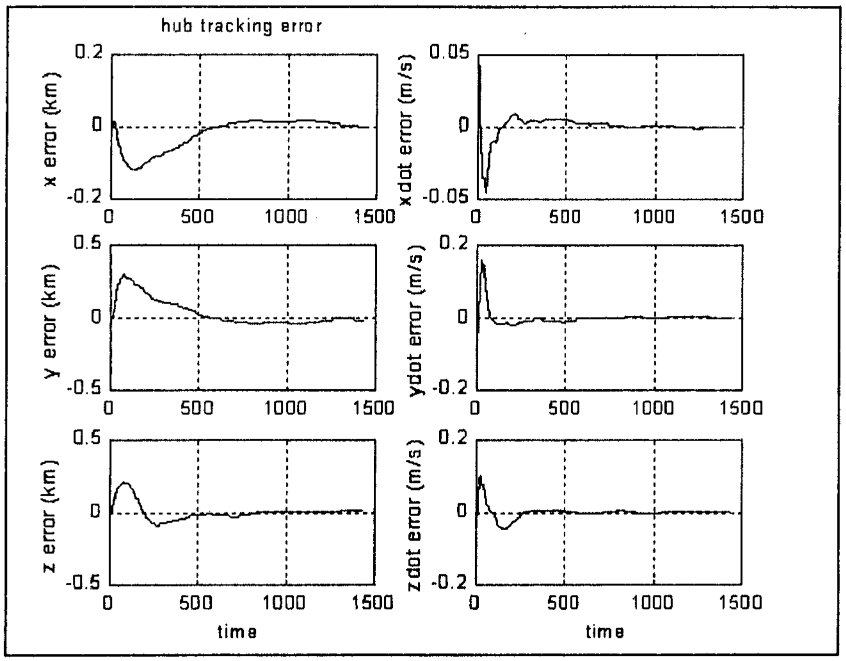
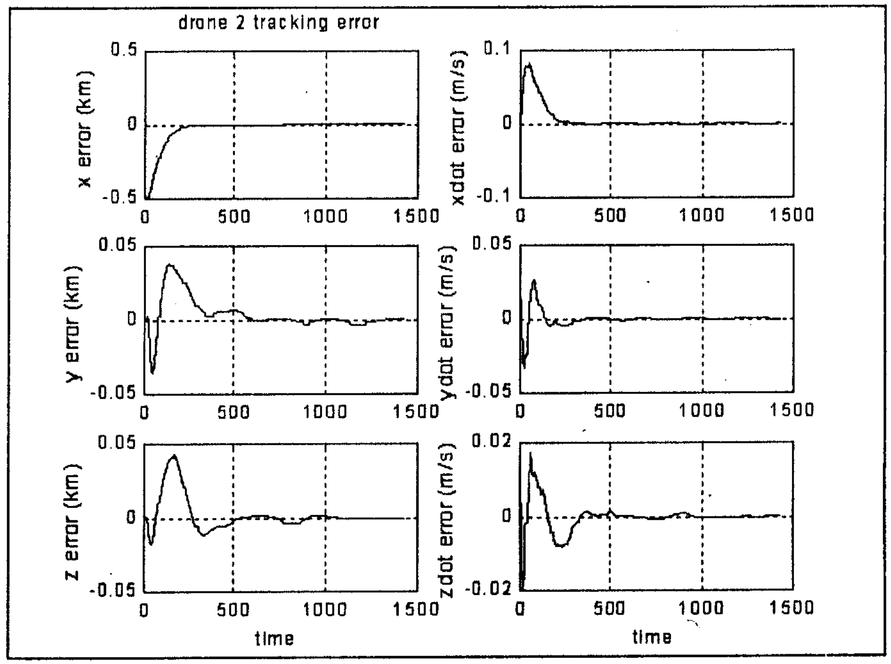
Results – Formation Reorientation

- Tracking errors: Position and Velocity
- Steady State errors of 40 meters - hub, 4 meters - drone and 8 millimeters/sec – hub, and 1.5 millimeter/s - drone

HUB



DRONE



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Results – Formation Reorientation

- The steady-state estimation 3σ values are: x ~30 meters, y and z ~ 50 meters
- The steady-state estimation 3σ velocity values are about 1 millimeter per second.
- For any drone and either focal length, the steady-state 3σ position values are less than 0.1 meters, and the steady-state velocity 3σ values are less than 1e-6 meters per second.

Formation Reorientation Average ΔVs

Focal Length (km)	Slew Angle (deg)	Hub (m/s)	Drone 2 (m/s)	Drone 31 (m/s)
0.5	90	1.0126	0.8421	0.8095
4	90	1.0133	0.8496	0.8190

Formation Reorientation Average Propellant Mass

Focal Length (km)	Slew Angle (deg)	Hub mass-prop (g)	Drone 2 mass-prop (g)	Drone 31 mass-prop (g)
0.5	90	5.6771	0.8584	0.8252
4	90	5.6811	0.8661	0.8349

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Results – Formation Reorientation

Formation Reorientation Average ΔVs (without noise)

Focal Length (km)	Slew Angle (deg)	Hub (m/s)	Drone 2 (m/s)	Drone 31 (m/s)
0.5	90	0.0408	0.1529	0.1496
4	90	0.0408	0.1529	0.1495

Formation Reorientation Average Propellant Mass (without noise)

Focal Length (km)	Slew Angle (deg)	Hub mass-prop (g)	Drone 2 mass-prop (g)	Drone 31 mass-prop (g)
0.5	90	0.2287	0.1623	0.1525
4	90	0.2287	0.1623	0.1524

Summary
(using example requirements and constraints)

o The control strategy and Kalman filter using higher fidelity dynamics provides satisfactory results.

o The hub satellite tracks to its reference orbit sufficiently for the SI mission requirements. The drone satellites, on the other hand, track to only within a few meters.

o Without noise, though, the drones track to within several micrometers.

o Improvements for first tier control scheme (centimeter control) for SI could be accomplished with better sensors to lessen the effect of the process and measurement noise.

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Summary
(using example requirements and constraints)

o **Tuning the controller and varying the maneuver intervals should provide additional savings as well. Future studies must integrate the attitude dynamics and control problem**

o **The propellant mass and results provide a minimum design boundary for the SI mission. Additional propellant will be needed to perform all attitude maneuvers, tighter control requirement adjustments, and other mission functions.**

o **Other items that should be considered in the future include;**

- **Non-ideal thrusters,**
- **Collision avoidance,**
- **System reliability and fault detection**
- **Nonlinear control and estimation**
- **Second and third control tiers and new control strategies and algorithms**

- A Distant Retrograde Formation

- Decentralized control

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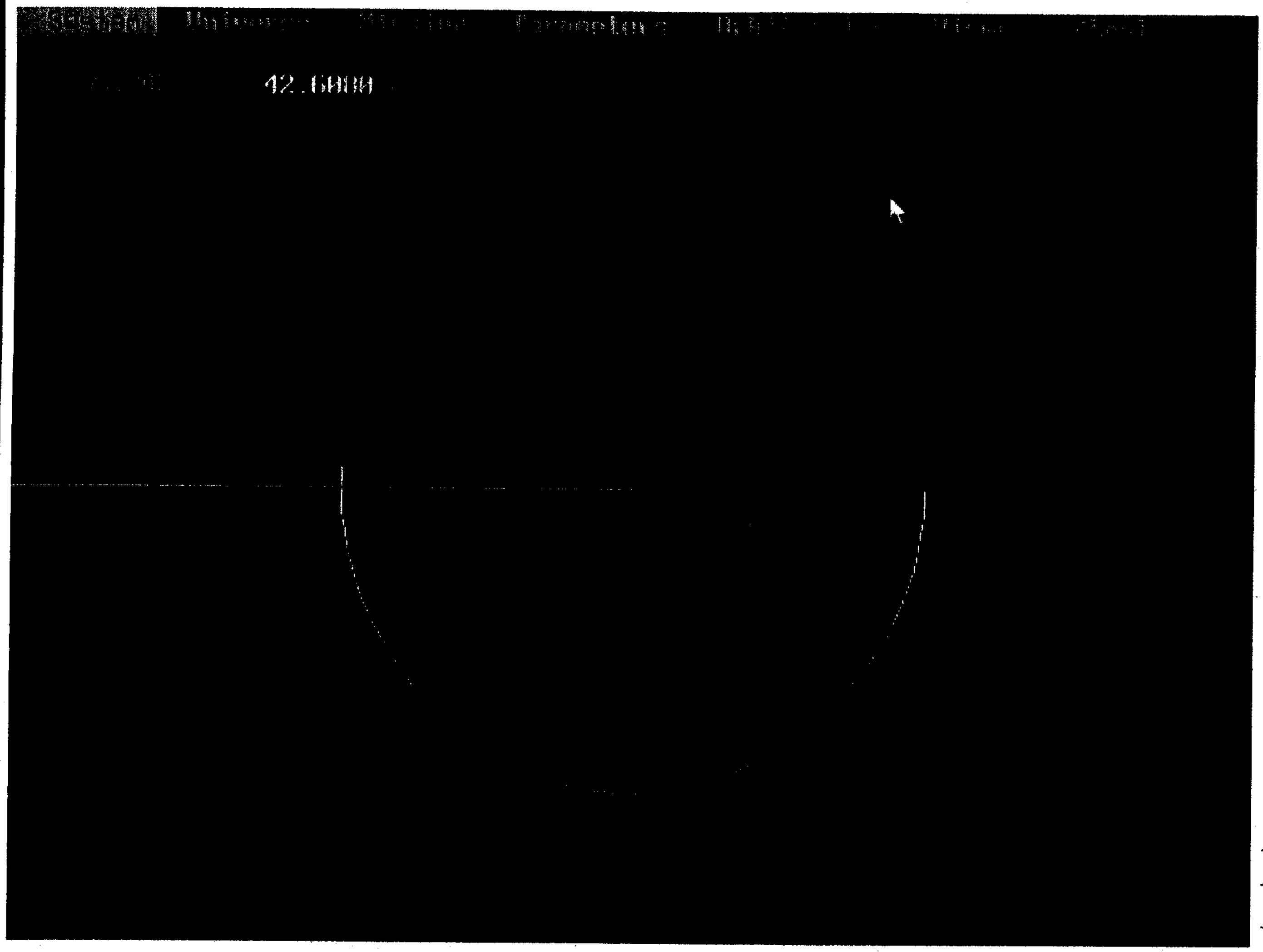
DRO Mission Metrics

- Earth-constellation distance: > 50 Re (less interference) and< 100 Re (link margin).
  - **Closer than 100 Re would be desirable to improve the link margin requirement**
  - **A retrograde orbit of <160 Re (10^6 km), for a stable orbit would be ok**
- The density of "baselines" in the u-v plane should be uniformly distributed. Satellites randomly distributed on a sphere will produce this result.
- Formation diameter: ~50 km to achieve desired angular resolution
- The plan is to have up to 16 microsats, each with it's own "downlink".
- Satellites will be "approximately" 3-axis stabilized.
- Lower energy orbit insertion requirements are always appreciated.
- Eclipses should be avoided if possible.
- Defunct satellites should not "interfere" excessively with operational satellites.

Distant Retrograde Orbit (DRO)



Why DRO?

- Stable Orbit
- No Skp ΔV
- Not as distant as L1
- Mult. Transfers
- No Shadows?
- Good Environment

✓Really a Lunar Periodic Orbit

✓Classified as a Symmetric Doubly Asymptotic Orbit in the Restricted Three-Body Problem

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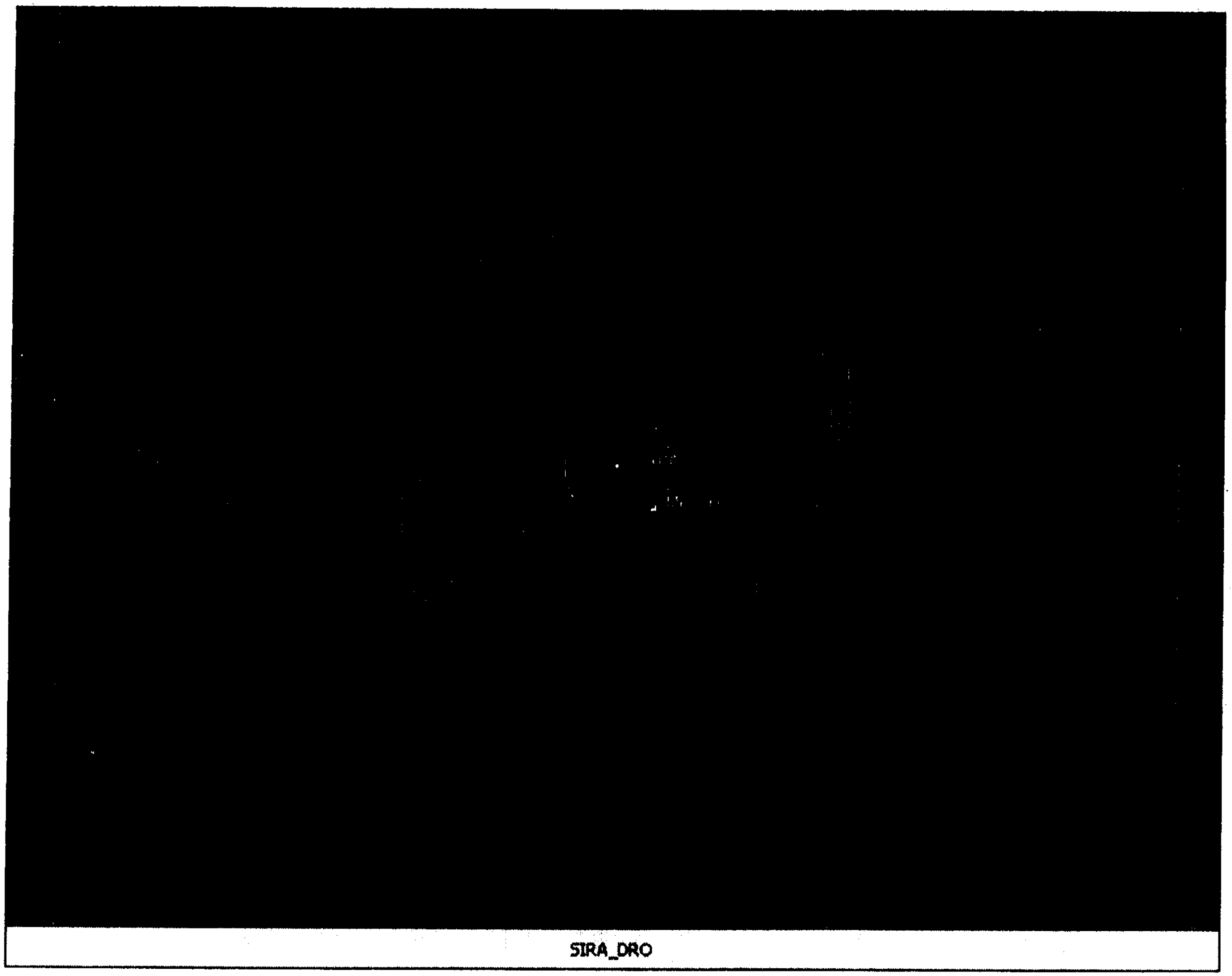
Earth Distant Retrograde Orbit (DRO) Orbit



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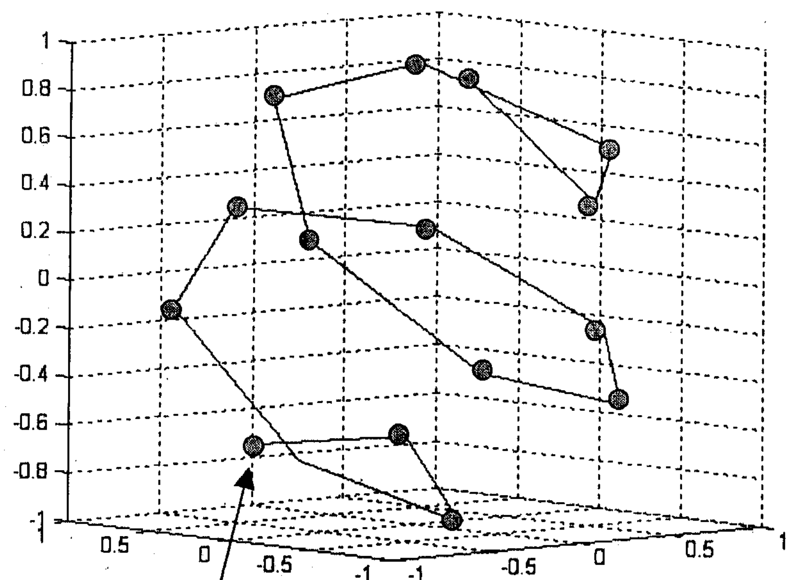
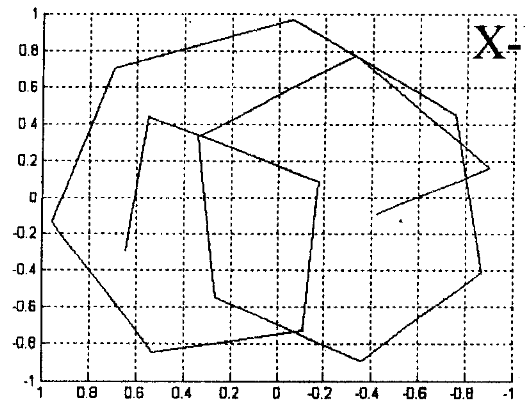
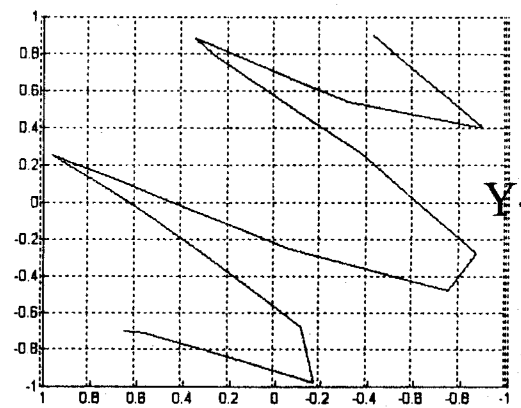
DRO Formation Sphere

- Matlab generated sphere based on S03 algorithm
  - ✓ Uniform distribution of points on a unit sphere
  - ✓ 16 points at vertices represents spacecraft locations

3-D view







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DRO Formation Control Analysis

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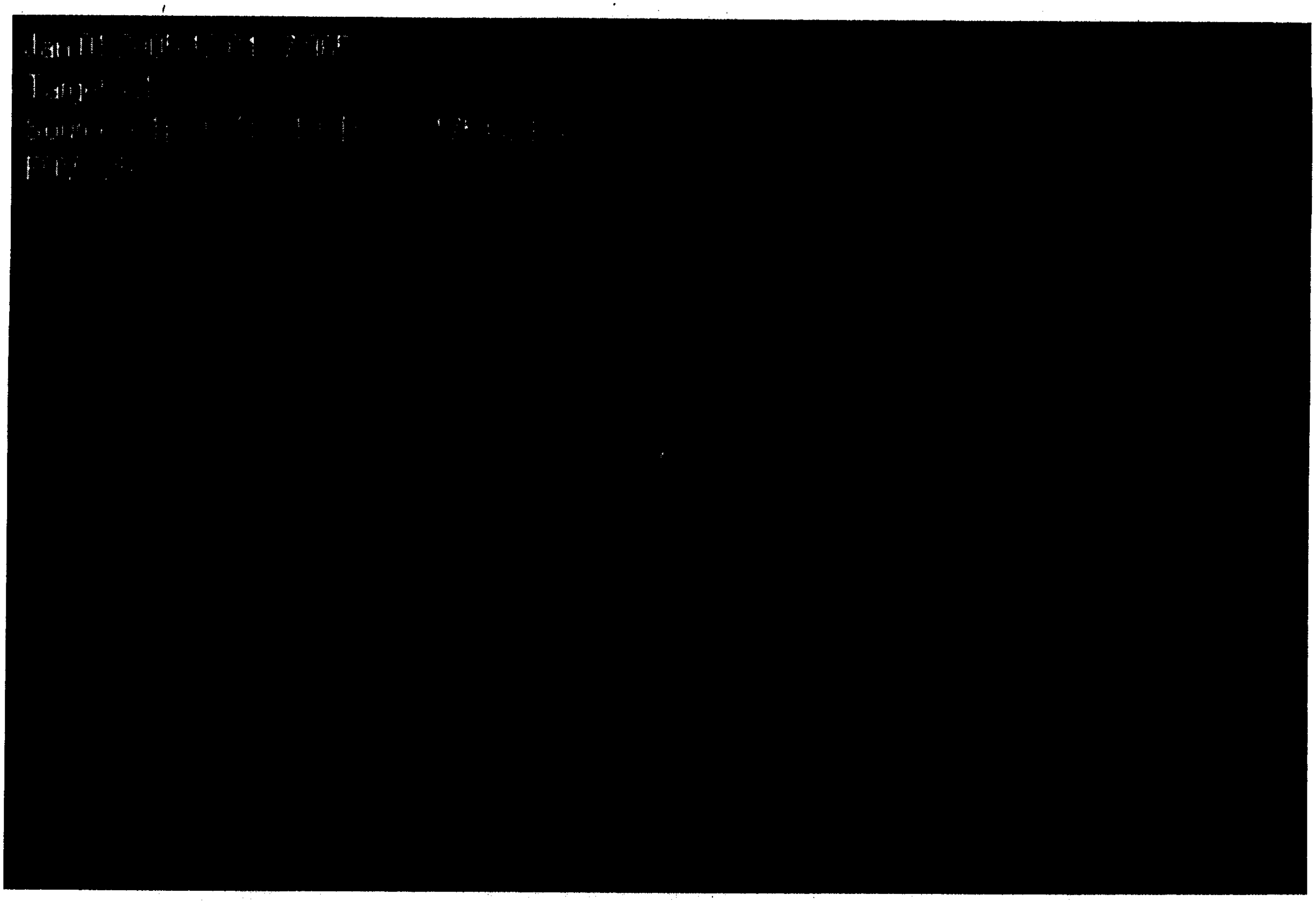

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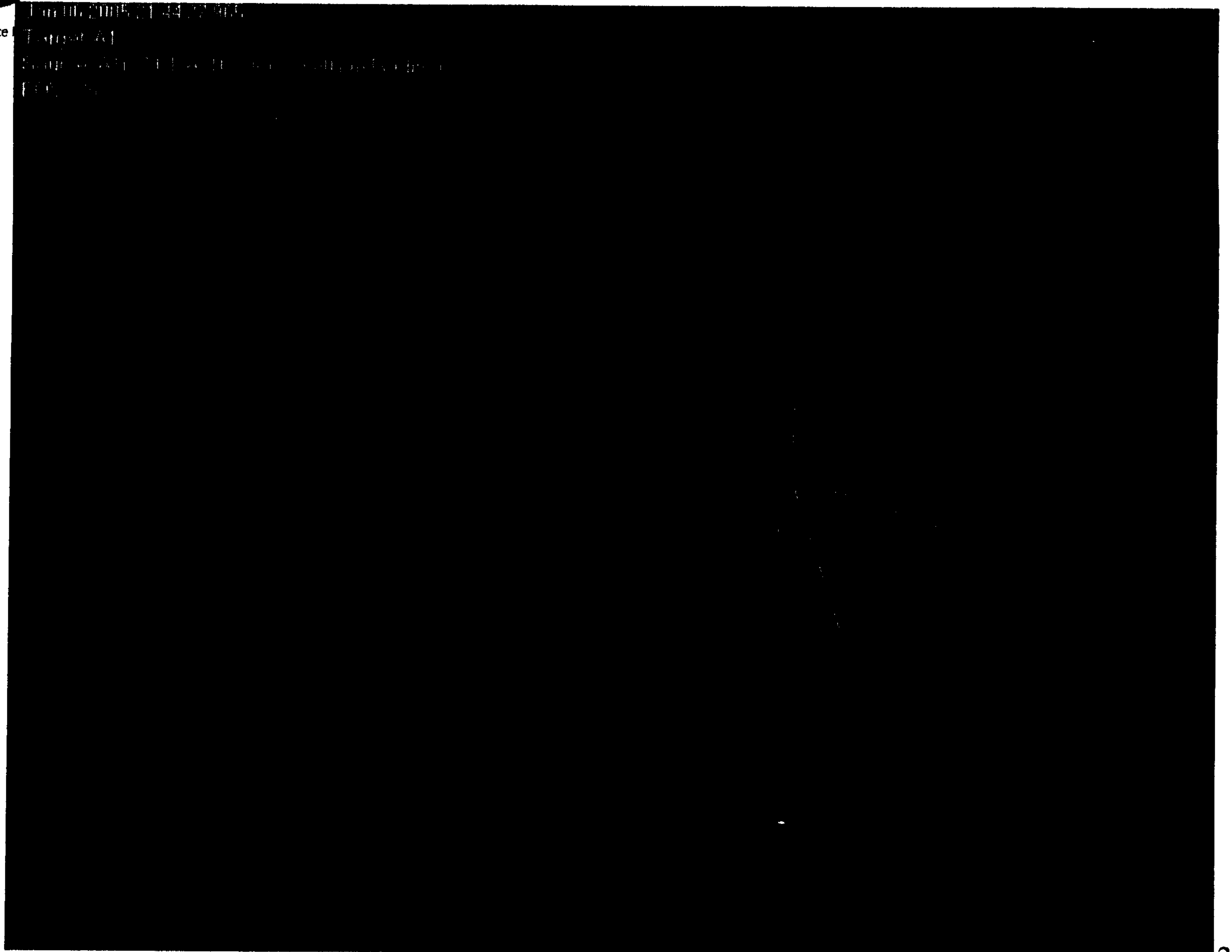
DRO Formation Control Analysis



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DRO Formation Control Analysis

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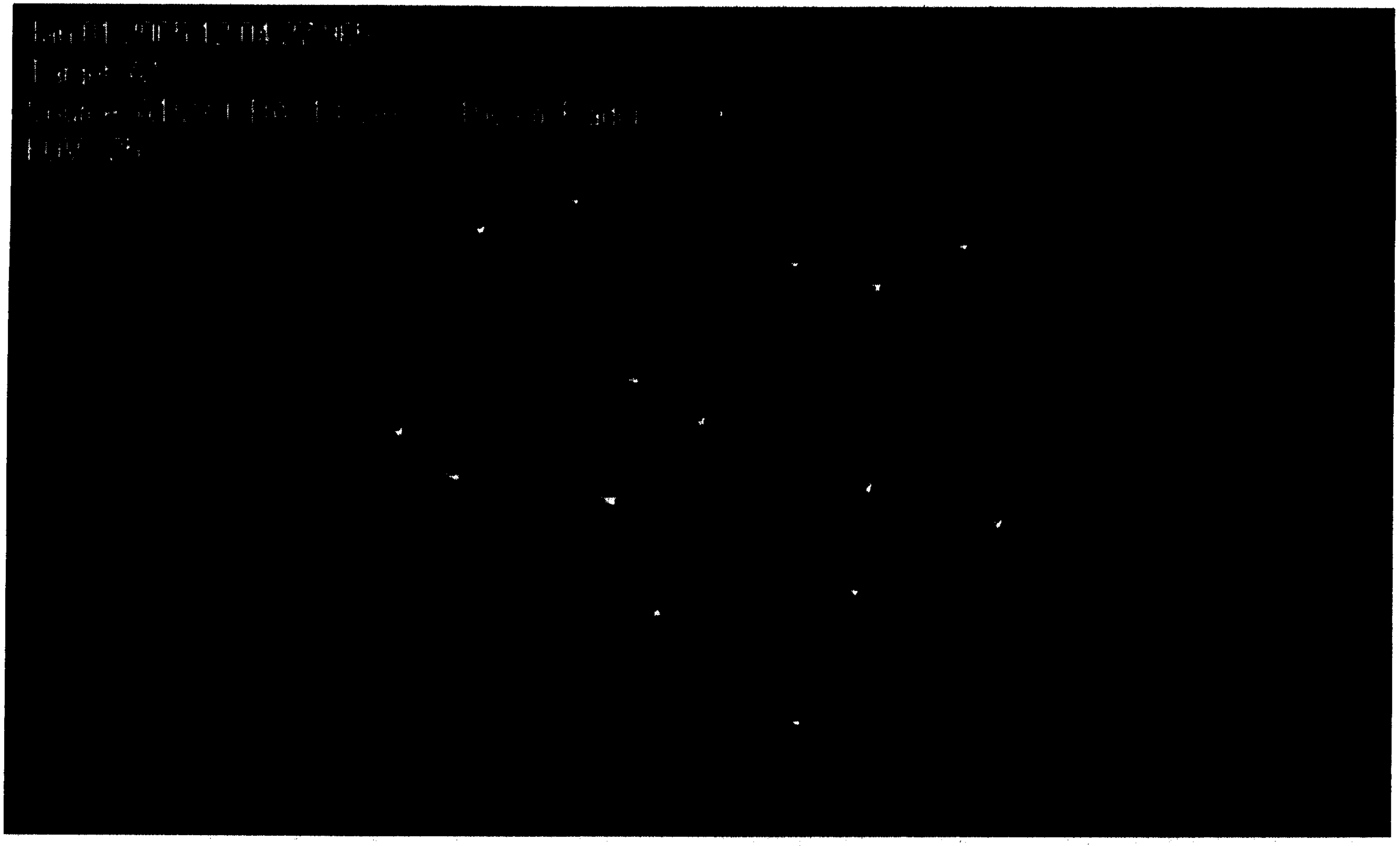

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Formation Control Analysis

How much ΔV to initialize, maintain, and resize?

Phase	Max	Mean	Min	Std
	[m/s]	[m/s]	[m/s]	[m/s]
a	0.635	0.607	0.592	0.014
b	1.323	0.792	0.392	0.293
c	0.757	0.674	0.541	0.069
d	0.679	0.616	0.582	0.031
e	1.201	0.721	0.367	0.263
f	0.679	0.608	0.503	0.056

Phase Description

a) Init 25km sphere
b) Maintain 25km sphere (strict PD control) one month
c) Maintain 25km sphere (loose control) one month
d) Resize from 25km to 50km
e) Maintain 50km sphere (strict PD control) one month
f) Maintain 50km sphere (loose control) one month

Examples:

Initialize & maintain 2 yr:
= 33 m/s

Initialize, Maintain 2yr, & four resizes:
= 36 m/s

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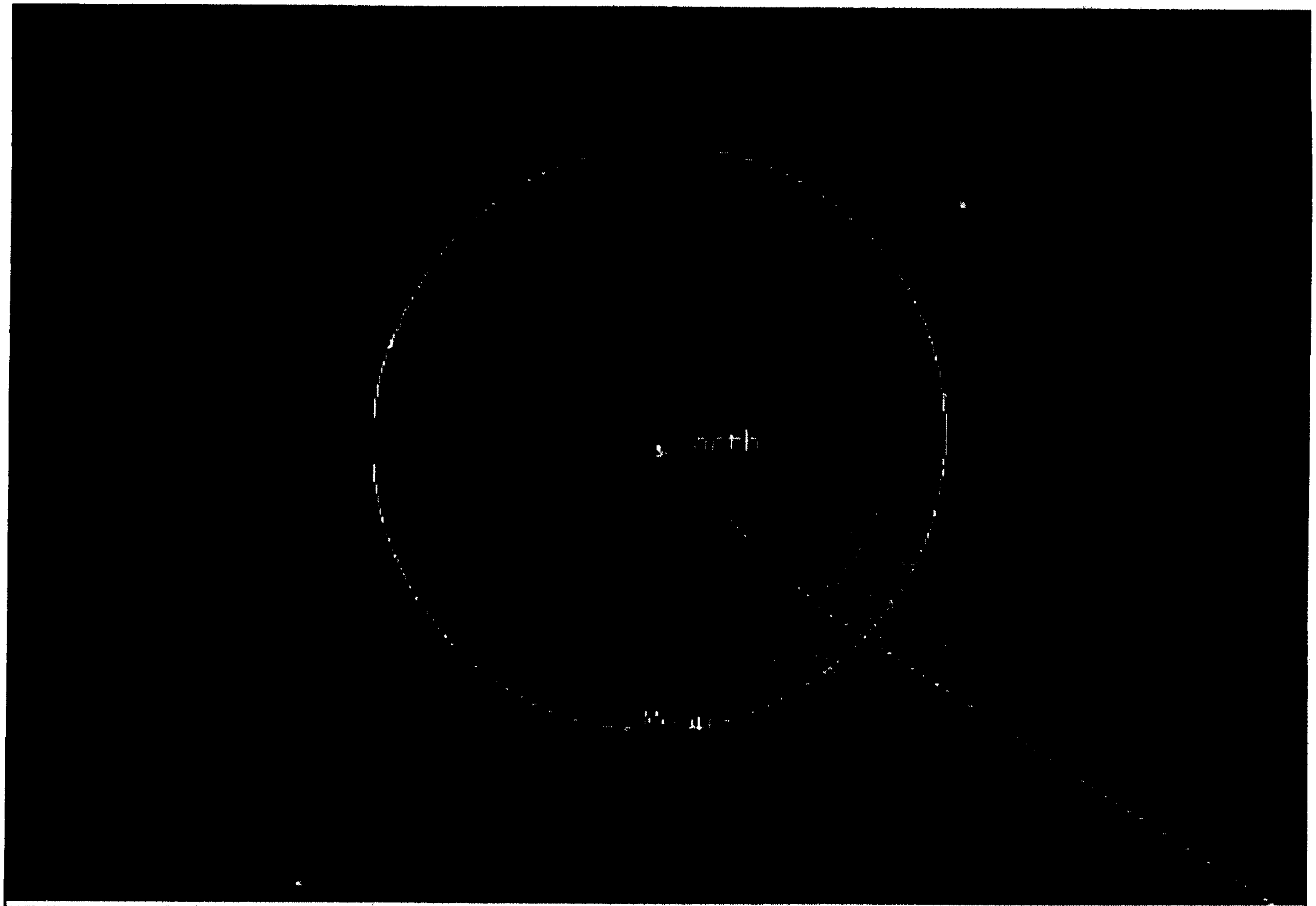
Earth - Moon L4 Libration Orbit
an alternate orbit location



SIRA_EM_L4

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DRO Formation Control Analysis

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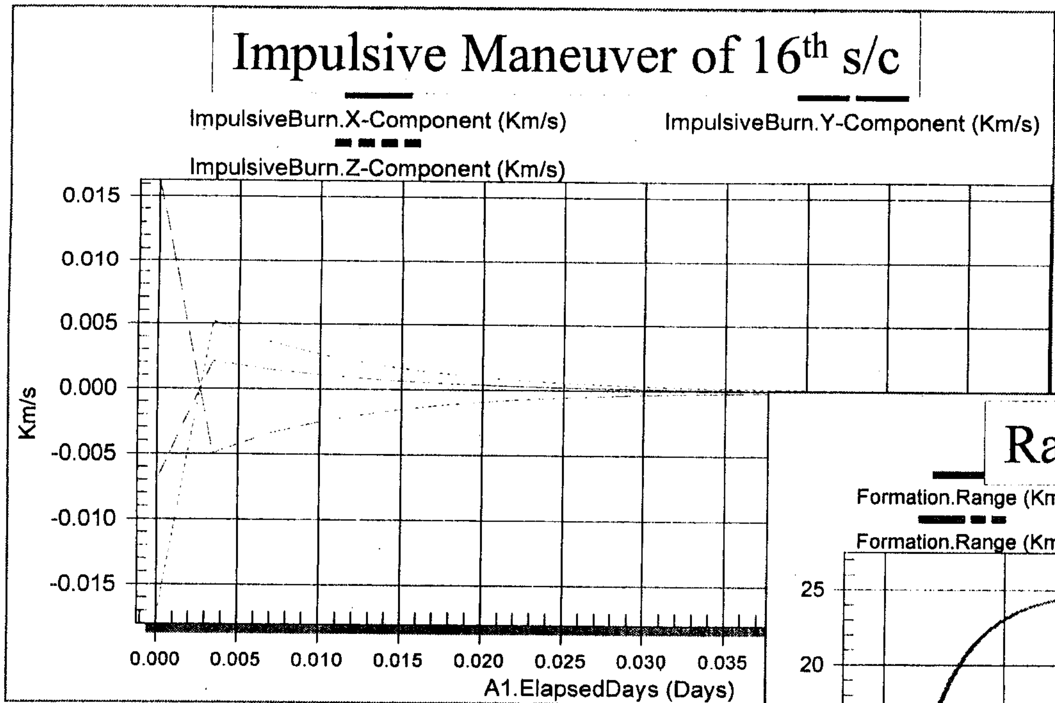
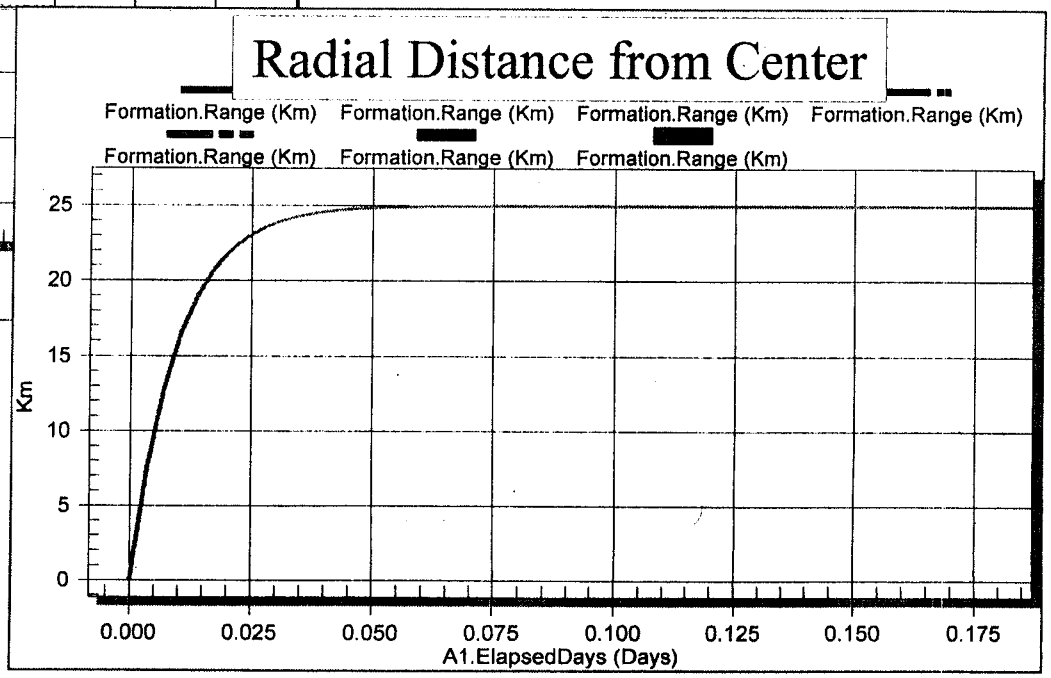
- Earth/Moon L4 Libration Orbit
- Spacecraft controlled to maintain only relative separations
- Plots show formation position and drift (sphere represent 25km radius)
- Maneuver performed in most optimum direction based on controller output





General Theory of Decentralized Control

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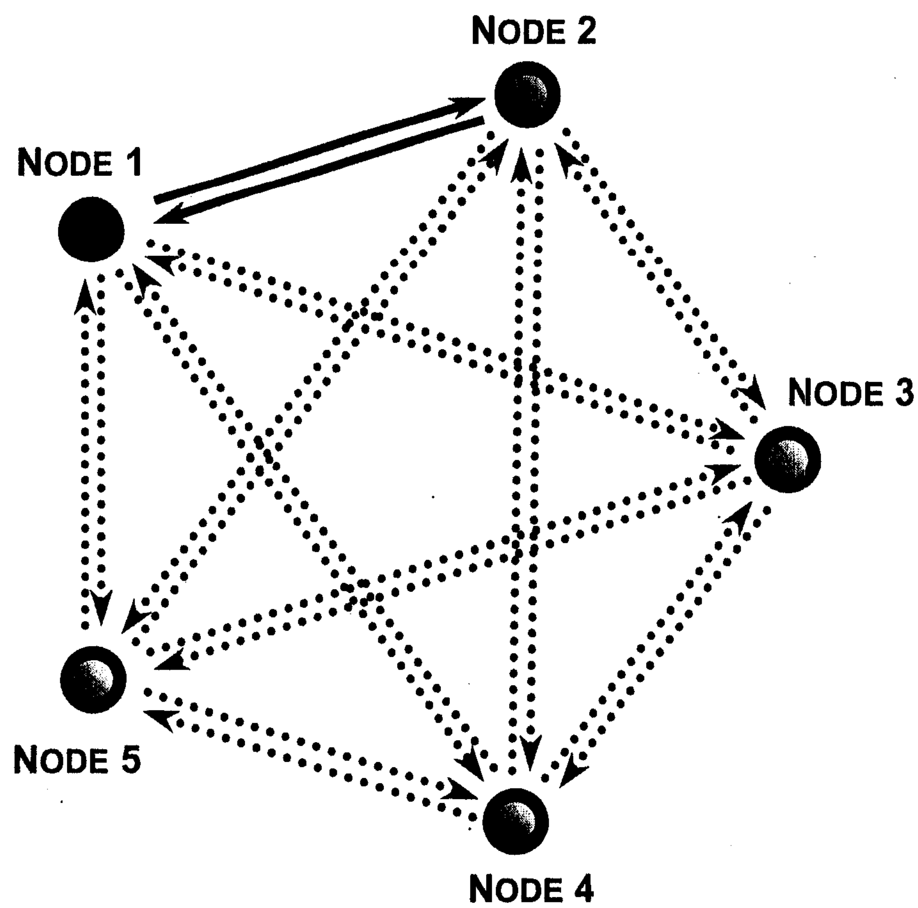
MANY NODES IN A NETWORK CAN COOPERATE TO BEHAVE AS SINGLE VIRTUAL PLATFORM:

- REQUIRES A FULLY CONNECTED NETWORK OF NODES.
- EACH NODE PROCESSES ONLY ITS OWN MEASUREMENTS.
- NON-HIERARCHICAL MEANS NO LEADS OR MASTERS.
- NO SINGLE POINTS OF FAILURE MEANS DETECTED FAILURES CAUSE SYSTEM TO DEGRADE GRACEFULLY.
- BASIC PROBLEM PREVIOUSLY INVESTIGATED BY SPEYER.
- BASED ON LQG PARADIGM.
- DATA TRANSMISSION REQUIREMENTS ARE MINIMIZED.



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Backup and other slides

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DST/Numerical Comparisons

Numerical Systems

- Limited Set of Initial Conditions
- Perturbation Theory
- Single Trajectory
- Intuitive DC Process
- Operational

Dynamical Systems

- Qualitative Assessments
- Global Solutions
- Time Saver / Trust Results
- Robust
- Helps in choosing numerical methods

(e.g., Hamiltonian => Symplectic Integration Schemes?)

Libration Point Trajectory Generation Process

Phase and Lissajous Utilities Generate Lissajous of Interest	Lissajous	Patch Points and Lissajous
Compute Monodromy Matrix And Eigenvalues/Eigenvectors For Half Manifold of Interest	Monodromy	Fixed Points and Stable and Unstable Manifold Approximations
Globalize the Stable Manifold	Manifold	1-D Manifold
Use Manifold Information for a Differential Corrector Step To Achieve Mission Constraints.	Transfer	Transfer Trajectory to Earth Access Region

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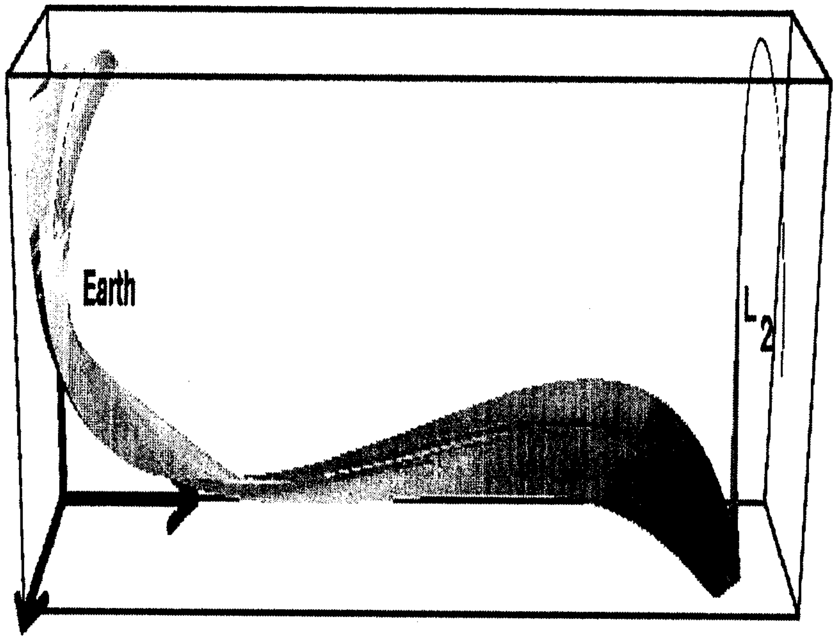
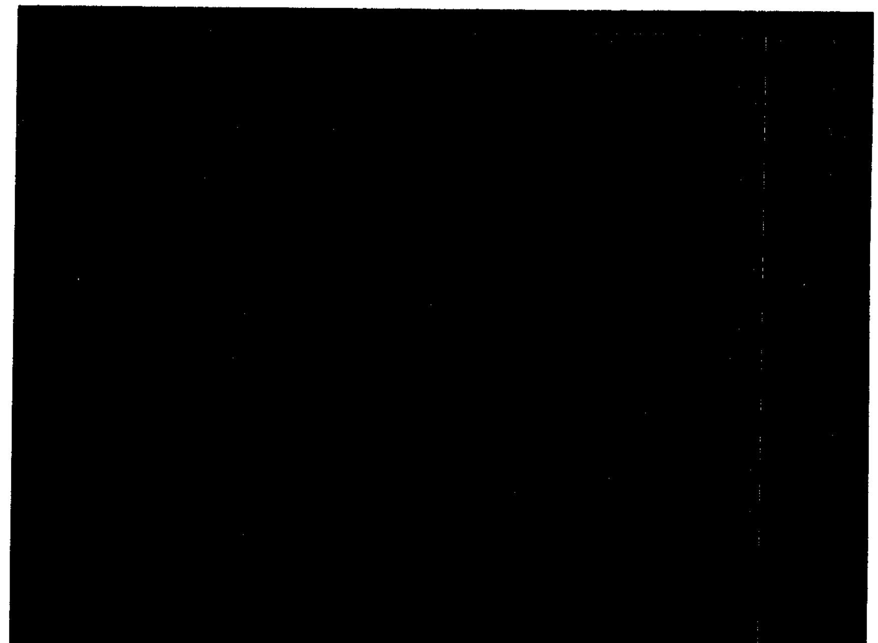
MAP Mission Design: DST Perspective



- MAP Manifold and Earth Access
- Manifold Generated Starting with Lissajous Orbit



- Swingby Numerical Propagation
- Trajectory Generated Starting with Manifold States

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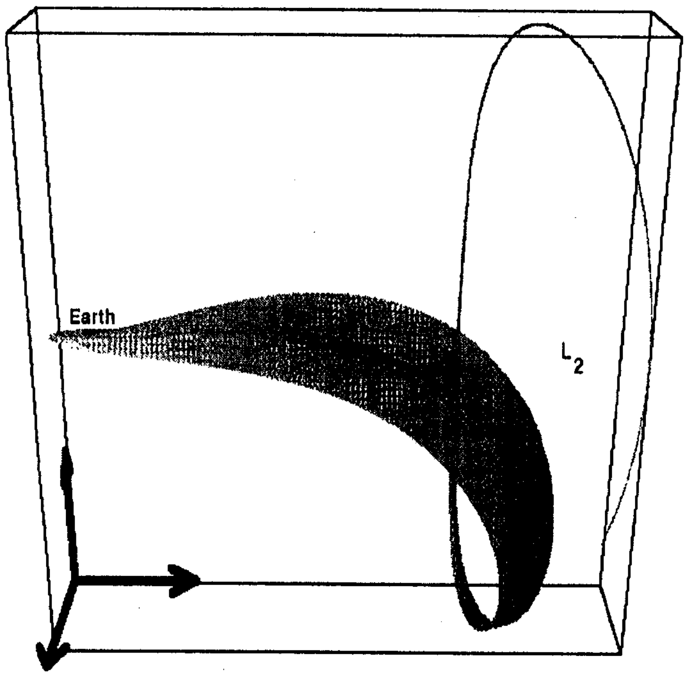
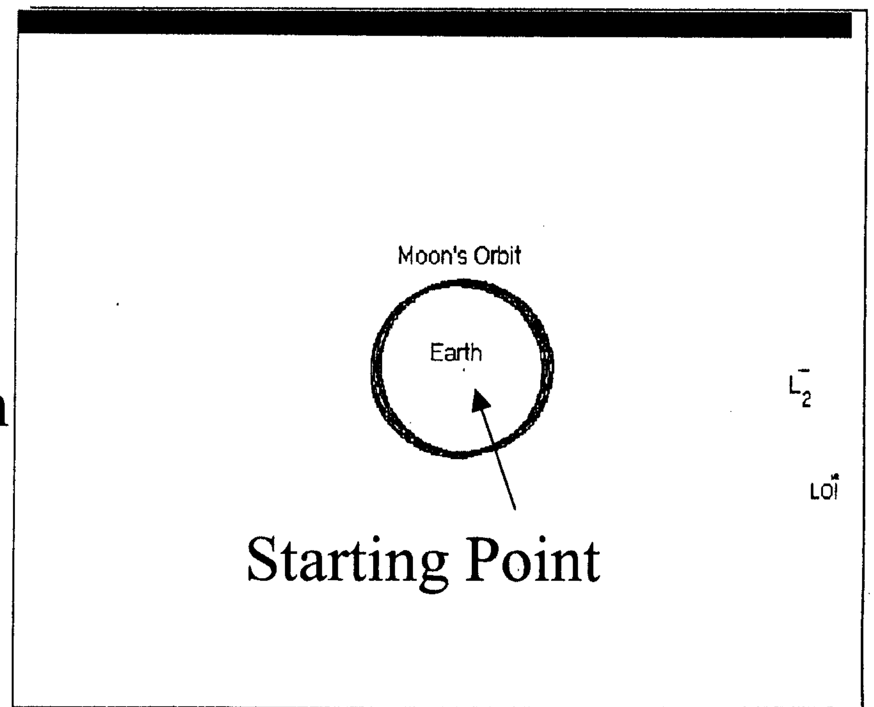
JWST DST Perspective



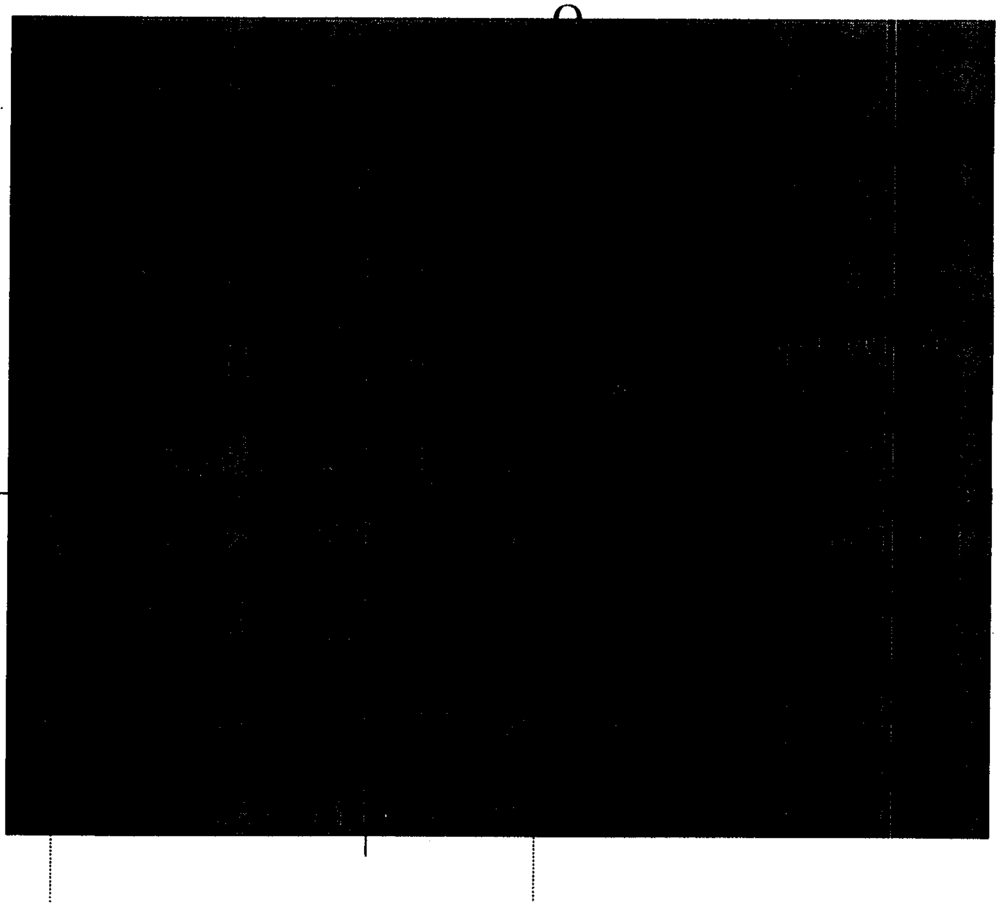
- JWST Manifold and Earth Access
- Manifold Generated Starting with Halo Orbit



- Swingby Numerical Propagation
- Trajectory Generated Starting with Manifold States

Two – Body Motion

- Motion of spacecraft in elliptical orbit
- Counter-clockwise
- x and y correspond to $\bar{P}$ and $\bar{Q}$ axes in PQW frame



Two angles are defined

E is Eccentric Anomaly

θ is True Anomaly

x and y coordinates are

$x = r \cos\theta$

$y = r \sin\theta$

In terms of Eccentric anomaly, E

$a \cos E = ae + x$

$x = a(\cos E - e)$

From eqn. of ellipse: $r = a(1 - e^2)/(1 + e\cos\theta)$

Leads to: $r = a(1 - e\cos E)$

Can solve for y: $y = a(1 - e^2)^{1/2} \sin E$

Coordinates of spacecraft in plane of motion are then

$r = a(1 - e\cos E) = a(1 - e^2)/(1 + e\cos\theta)$

$x = a(\cos E - e) = r\cos\theta$ $\qquad v_x = (\mu/p)^{1/2}[-\sin\theta]$

$y = a(1 - e^2)^{1/2}\sin E = r\sin\theta$ $\qquad v_y = (\mu/p)^{1/2}[e + \cos\theta]$ where $p = a(1 - e^2)$

3-7

Two – Body Motion

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Differentiate x, y, and r wrt time in terms of E to get

$$dx/dt = -a\sin E \; de/dt$$

$$dy/dt = a(1-e^2)^{1/2} \cos E \; dE/dt$$

$$dr/dt = ae \sin E \; dE/dt$$

From the definition of angular momentum h = r cross dr/dt and expand to get $h = a^2(1-e^2)^{1/2}\, dE/dt[1-e\cos E]$ in direction perpendicular to orbit plane

Knowing $h^2 = ma(1-e^2)$, equate the expressions and cancel common factor to yielc

$$(\mu)^{1/2}/a^{3/2} = (1-e\cos E)\; dE/dt$$

Multiple across by dt and integrate from the perigee passage time yields

$$n(t_0 - t_p) = E - e\sin E$$

Where $n = (\mu)^{1/2}/a^{3/2}$ is the mean motion

We can also compute the period: $P=2p(a^{3/2} / (\mu)^{1/2})$ which can be associated with Kepler's 3rd law

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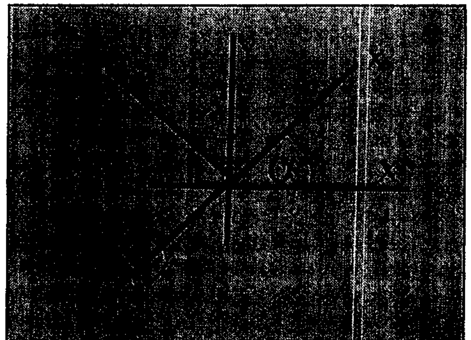
Coordinate system transformation
Euler Angle Rotations

Suppose we rotate the x-y plane about the z-axis by an angle α and call the new coordinates x',y',z'

$$x' = x\cos\alpha + y\sin\alpha$$
$$y' = -x\sin\alpha + y\cos\alpha$$
$$z' = z$$



Rotate about Z

Rotate about Y

Rotate about X

Coordinate system transformation
Orbital to inertial coordinates

Inertial to/from Orbit plane

$\bar{r} = \bar{r}(x,y,z) \qquad \bar{r} = \bar{r}(P,Q,W)$

In orbit plane system, position $r = (r\cos f)P + (r\sin f)Q + (0)W$ where P,Q,W are unit vectors

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} P_x & Q_x & W_x \\ P_y & Q_y & W_y \\ P_z & Q_z & W_z \end{pmatrix} \begin{pmatrix} r\cos f \\ r\sin f \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos i & -\sin i \\ 0 & -\sin i & -\cos i \end{pmatrix} \begin{pmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} r\cos f \\ r\sin f \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i & -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i & \sin\Omega\sin i \\ \sin\Omega\cos\omega - \cos\Omega\sin\omega\cos i & -\sin\Omega\sin\omega - \cos\Omega\cos\omega\cos i & -\cos\Omega\sin i \\ \sin\Omega\sin i & \cos\Omega\sin i & \cos i \end{pmatrix} \begin{pmatrix} r\cos f \\ r\sin f \\ 0 \end{pmatrix}$$

For Velocity transformation, $v_x = (\mu/p)^{1/2}[-\sin\theta]$ and $v_y = (\mu/p)^{1/2}[e + \cos\theta]$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the position and $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$ is the velocity

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Principles Behind Decentralized Control

- THE STATE VECTOR IS DECOMPOSED INTO TWO PARTITIONS:

  [redacted] DEPENDS ONLY ON THE CONTROL.

  [redacted] DEPENDS ONLY ON THE LOCAL MEASUREMENT DATA NODE-*j*.

- A LOCALLY OPTIMAL KALMAN FILTER OPERATES ON [redacted]
- A GLOBALLY OPTIMAL CONTROL IS COMPUTED, USING [redacted] AND GLOBALLY OPTIMAL DATA THAT IS RECONSTRUCTED LOCALLY USING TWO ADDITIONAL VECTORS:

  A "DATA VECTOR" [redacted], IS MAINTAINED LOCALLY IN ADDITION TO THE STATE.

  A TRANSMISSION VECTOR [redacted] THAT MINIMIZES THE DIMENSIONS OF THE DATA WHICH MUST BE EXCHANGED BETWEEN NODES.

- EACH NODE-*j* COMPUTES AND TRANSMITS [redacted] TO AND RECEIVES [redacted] FROM ALLTHE OTHER NODES IN THE NETWORK.

The LQG Decentralized Controller Overview