Minimum-time and vibration avoidance attitude maneuver for spacecraft with torque and momentum limit constraints in redundant reaction wheel configuration

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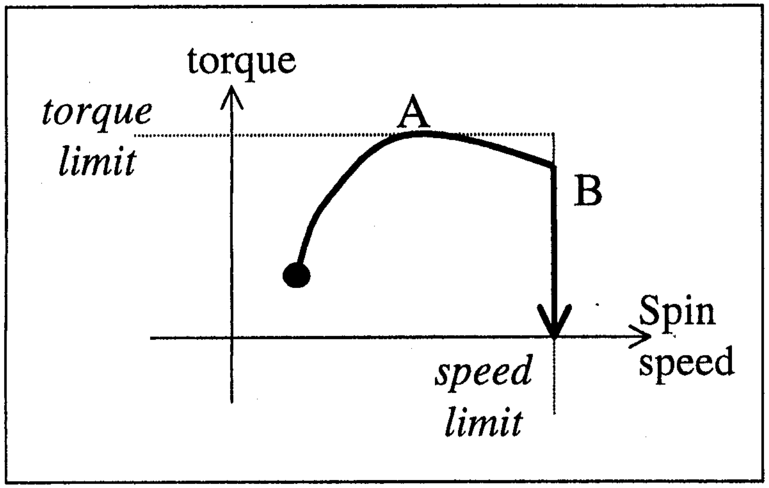
Goals

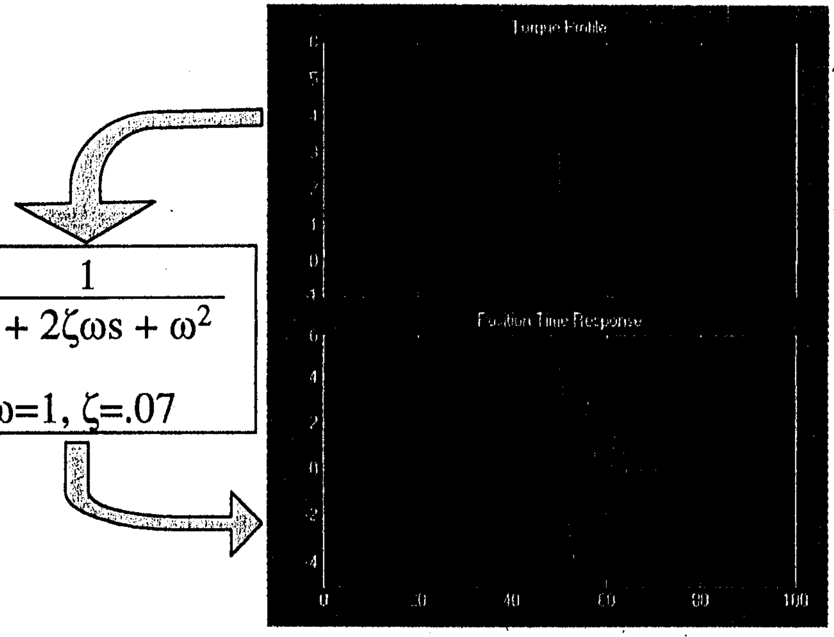
To develop an optimal open-loop trajectory algorithm for slew with constant eigen-axis and spacecraft with reaction wheels as control actuators. To do this requires:

- Formulate a trajectory profile that has continuous derivatives of all orders, involves only low order polynomials, and satisfies slew boundary conditions, e.g. zero speed and acceleration at end points.
- Derive slew trajectory parameters as functions of initial wheel speeds and wheel torque and momentum limits, to ensures that wheel momentum and torque profiles approach tangentially reaction wheel limits.
- Minimize maneuver time by maximizing the use of available wheel torque.

Wheel limits and Impact on jitter

Torque-Speed Curve





$$\frac{1}{s^2 + 2\zeta\omega s + \omega^2}$$

$\omega=1, \zeta=.07$

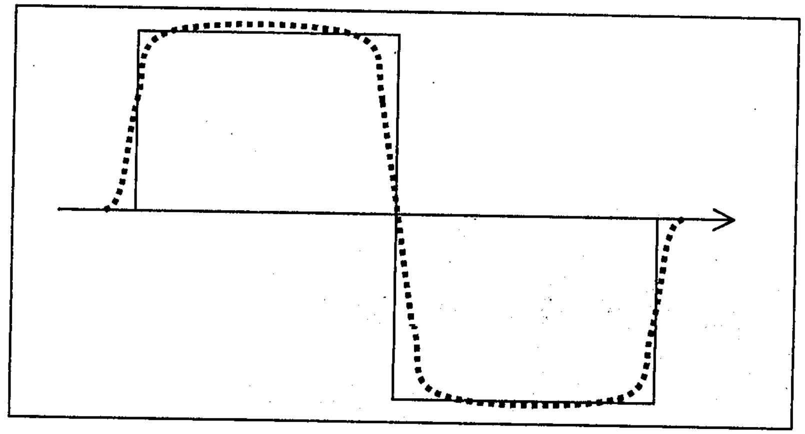
- Non-tangent saturation leads to discontinuity that causes control failure in tracking commanded motion and produces high frequency torque components capable of exciting structural modes.
- Momentum limit saturation and torque limit saturation induce wide-band step-function response.

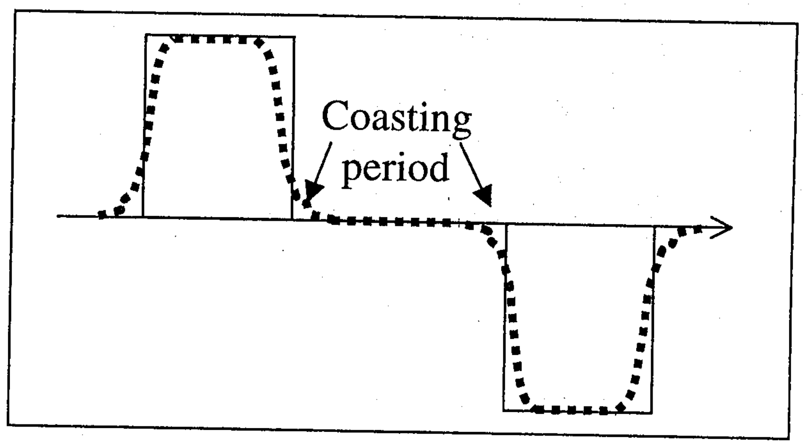
Slew Profile and Minimum Maneuver Time

- Optimal bang-bang control principle gives best slew trajectory with torque profile that achieve minimum time for a maximum allowable acceleration. Dashed curve is its smooth version. Ref.[4].

- For a system that not only has finite torque but also finite momentum, the above profile must be modified to include the option for coasting period.

- A smooth version of this modified bang-bang torque profile is needed to prevent flexible mode excitation.





Slew Trajectory functions

Slew acceleration, velocity, and angle functions:

$$\alpha(t) = \alpha_d \, \hat{e} \frac{d}{dt} f\left(t, \Delta t_a, \Delta t_c\right)$$

$$\omega(t) = \omega_d \, \hat{e} \, f\left(t, \Delta t_a, \Delta t_c\right)$$

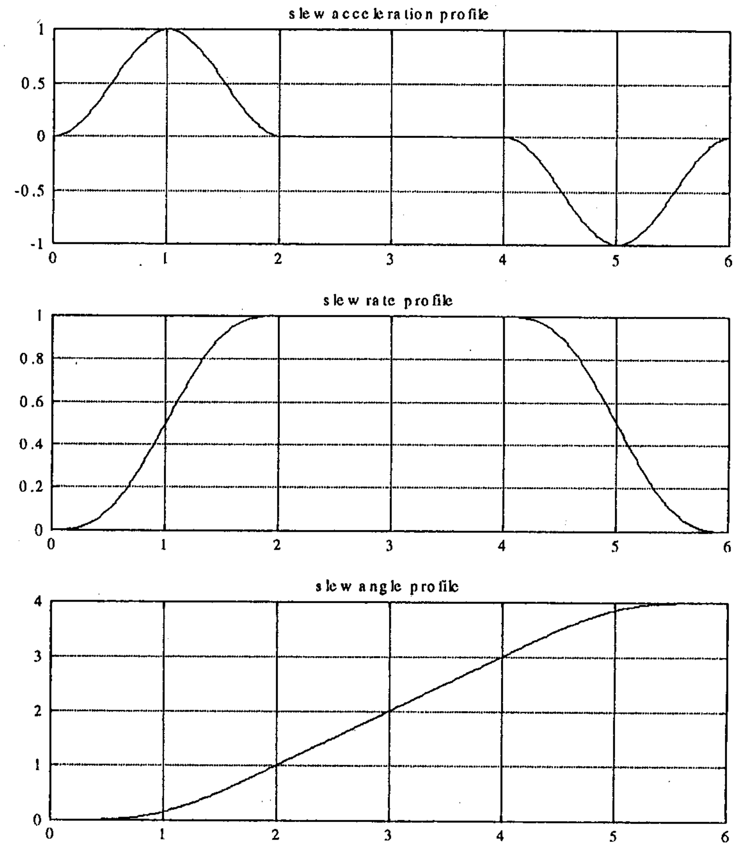
$$\theta(t) = \omega_d \, \hat{e} \int_0^t f\left(t', \Delta t_a, \Delta t_c\right) dt'$$

Where α_d and $\boldsymbol{\omega}_d$ are the desired maximum acceleration and rate amplitudes, $\hat{e}$ is the constant slew rotation axis, *f(t)* is a positive scalar time-function defining the desired spacecraft rate profile, and Δt_a and Δt_c are acceleration and coasting time periods.

To complete the slew trajectory description, compute α_d, ω_d, Δt_a, and Δt_c as functions of reaction wheel initial spin rates, and the reaction wheel torque and momentum limits.

Slew Trajectory Profiles

$$\frac{d}{dt}f(t,\Delta t_a,\Delta t_c)=\begin{cases}\left(\frac{t-t_0}{\Delta t_a}\right)^2\left(3-2\left(\frac{t-t_0}{\Delta t_a}\right)\right) & t<t_1=t_0+\Delta t_a\\ \left(\frac{t_2-t}{\Delta t_a}\right)^2\left(3-2\left(\frac{t_2-t}{\Delta t_a}\right)\right) & t<t_2=t_1+\Delta t_a\\ 0 & t<t_3=t_2+\Delta t_c\\ \left(\frac{t-t_3}{\Delta t_a}\right)^2\left(3-2\left(\frac{t-t_3}{\Delta t_a}\right)\right) & t<t_4=t_3+\Delta t_a\\ \left(\frac{t_5-t}{\Delta t_a}\right)^2\left(3-2\left(\frac{t_5-t}{\Delta t_a}\right)\right) & t<t_5=t_4+\Delta t_a\\ 0 & t\geq t_5\end{cases}$$



Some Remarks

- Slew parameters to be derived are not all independent, with function $f(t)$ given above, their relations can be explicitly derived and used in the formulation, e.g.

$\Delta t_a = \frac{\omega_d}{\alpha_d}$, $\Delta t_c = \frac{\theta(T_s)}{\omega_d} - 2\frac{\omega_d}{\alpha_d}$ Where $\Delta T_s = \Delta t_c + 4\Delta t_a$ the total slew time.

- In the derivation, it is also assumed that for redundant reaction wheel configuration, the transformation between the body and the wheel frame is already established. For example, a 4-wheel configuration optimized for minimum spin rate magnitude will have as the transformation the left pseudo-inverse of a 3×4 matrix whose columns are reaction wheel spin axes given in the body frame.

Wheel Spin Rates

- Assume over a slew period, external disturbance is negligible.

- With the total system momentum in inertial frame denoted as H_0, the transformation from the body to the wheel frame as A_b^w, and the inertial to body transformation as $R(t)$, the wheel momentum vector can be given as:

$$h_w = A_b^w\left(R(t)H_0 - I_s\omega(t)\right)$$

- With $\pm h_{\lim}$ denoted the reaction wheel momentum limits, the maximum allowable body rate, specified by ω_d, is the one that satisfied the following inequality for all wheels in the system:

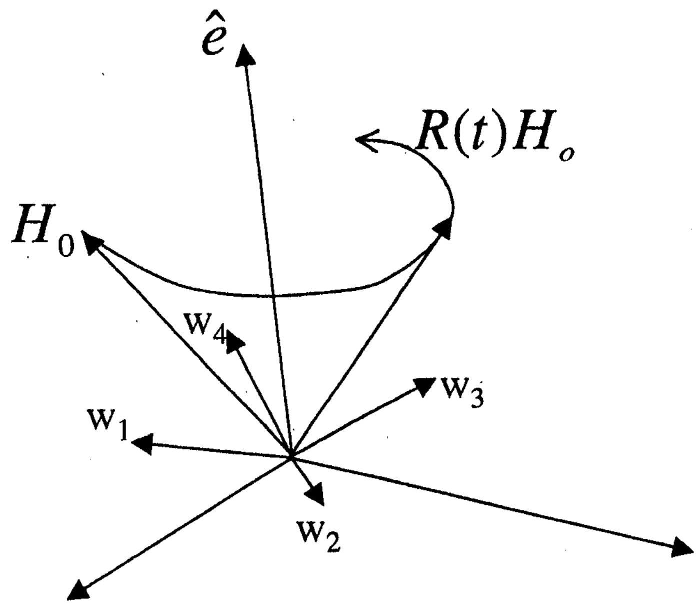
$$-h_{\lim} \leq h_{wi} \equiv \left(A_b^w\right)_i\left(R(t)H_0 - I_s\hat{e}\omega_d f(t)\right) \leq h_{\lim}$$

i = 1..N number of wheels

Maximum Spin Rate

- The problem leads to:

$$\omega_{d_i} = Max\left\{ \frac{-h_{\lim} + \underset{t\in\Delta T_s}{\text{Max}}\left\{\left(\hat{A}_b^w\right)_i R(t)H_0\right\}}{\left(\hat{A}_b^w\right)_i I_s\hat{e}}, \frac{h_{\lim} + \underset{t\in\Delta T_s}{\text{Min}}\left\{\left(\hat{A}_b^w\right)_i R(t)H_0\right\}}{\left(\hat{A}_b^w\right)_i I_s\hat{e}} \right\}$$



- The maximum and minimum momentum carried by a wheel in the course of a slew from the total momentum occur when the total momentum vector is closest and farthest from the wheel's rotational axis.

- The maximum allowable body rate is given as:

$$\omega_d = \underset{i}{Min}\{\omega_{di}\}$$

Wheel Spin Accelerations

- This process is not as straightforward as for deriving the desired maximum body rate due to the cross term in the wheel acceleration vector:

$$\frac{d}{dt}h_w = \hat{A}_b^w\left([\omega(t)]_\times R(t)H_0 - I_s \frac{d}{dt}\omega(t) \right)$$

- Although, it should be noted that by using similar approach as for body rate, a good but sub-optimal solution is easily obtained:

$$\alpha_{d\,i} = Min\left\{ \left| \frac{-\tau_{\lim} + \omega_{\mathrm{d}} \operatorname*{Max}_{t\in\Delta T_s}\left\{ \left(\hat{A}_b^w\right)_i [\hat{e}]_\times R(t)H_0 \right\}}{\left(\hat{A}_b^w\right)_i I_s \hat{e}} \right|, \left| \frac{\tau_{\lim} + \omega_{\mathrm{d}} \operatorname*{Min}_{t\in\Delta T_s}\left\{ \left(\hat{A}_b^w\right)_i [\hat{e}]_\times R(t)H_0 \right\}}{\left(\hat{A}_b^w\right)_i I_s \hat{e}} \right| \right\}$$

$$\alpha_d = \operatorname*{Min}_i\left\{\alpha_{di}\right\}$$

General Approach

- The general approach accounts for the difference between the wheel acceleration in the ramp-up and ramp-down regions, and select the minimum of the maximum accelerations in the two region as the solution.
- For each reaction wheel, determining maximum acceleration simultaneously yields the required time period, Δt_a, for the wheel to reach this maximum acceleration value. This is done by parameterized these parameters in terms of slew angle, and used the dependent relations to solve for the solution.
- The method to determine the solution is an iterative search process since it involves solution of transcendental functions.

Maximum Spin Acceleration

- With $\pm\tau_{\lim}$ denoted the reaction wheel torque limits, the maximum wheel momentum during the ramp-up and ramp-down can be given by:

$$\alpha_{di}(\pm) \begin{matrix} \le \\ \ge \end{matrix} \left| \frac{\tau_{\lim}}{\left(\hat{A}_b^w\right)_i I_s \hat{e}} \right| \pm \frac{\frac{1}{2}\omega_d \left(\hat{A}_b^w\right)_i [\hat{e}] R(\Delta) H_0}{\left(\hat{A}_b^w\right)_i I_s \hat{e}} \quad \text{Where } \Delta = \begin{cases} \Delta t_a & \text{for } + \\ \Delta t_s - \Delta t_a & \text{for } - \end{cases}$$

- Expressing α_{di} and Δt_a in terms of $\theta(\Delta t_a)$ and $\theta(\Delta t_s - \Delta t_a)$ which are solutions of functions of the form:

$$\frac{A}{\Delta\theta_a} = B + C\sin(\Delta\theta_a + \phi)$$

- The maximum allowable body acceleration is given as:

$$\alpha_d = \underset{i}{Min}\left\{\alpha_{di}(-), \alpha_{di}(+)\right\}$$

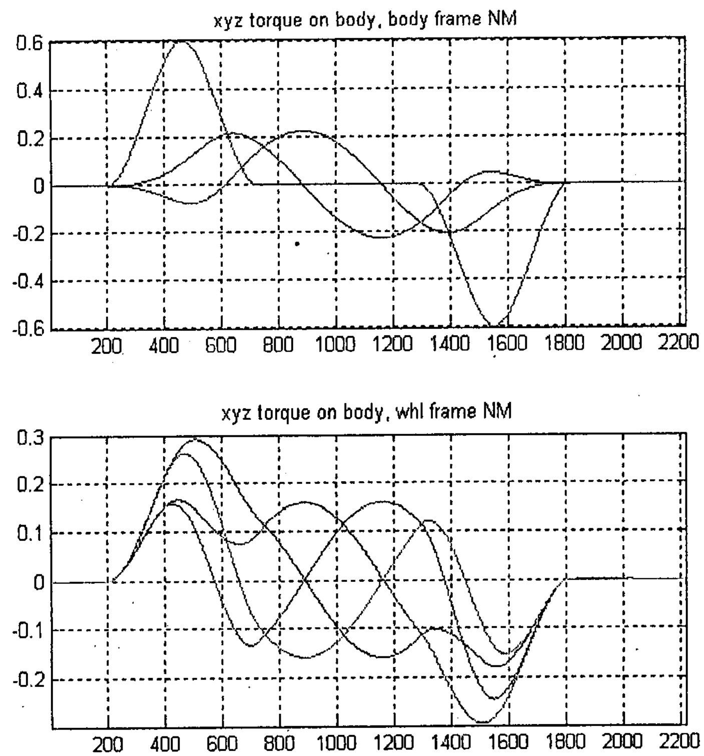
Simple Rigid Body Simulation

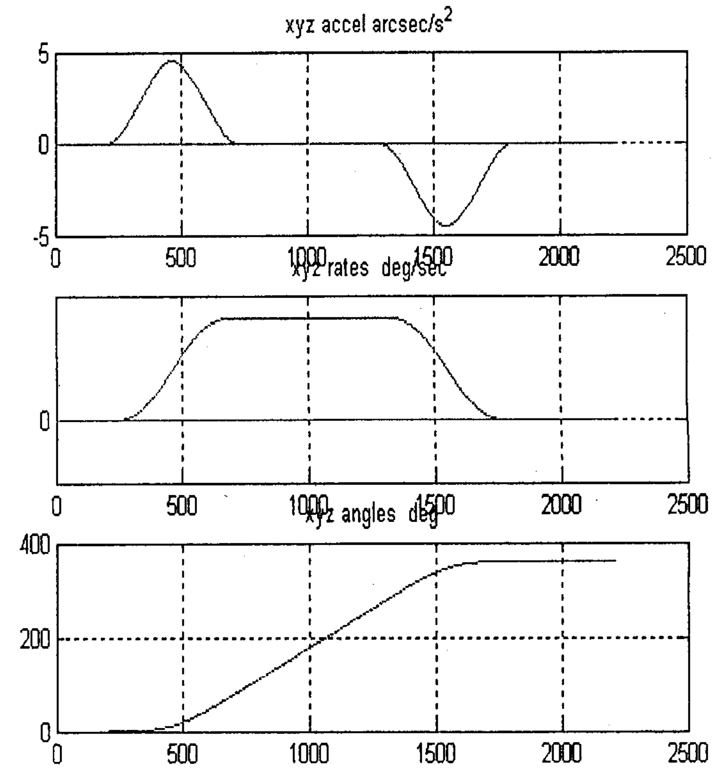
To illustrate the operation of the slew algorithm in a closed-loop system to handle reaction wheel limits, results from a simple rigid body dynamic simulation developed to provide a preliminary test of the algorithm are shown.

Simulation Setup:

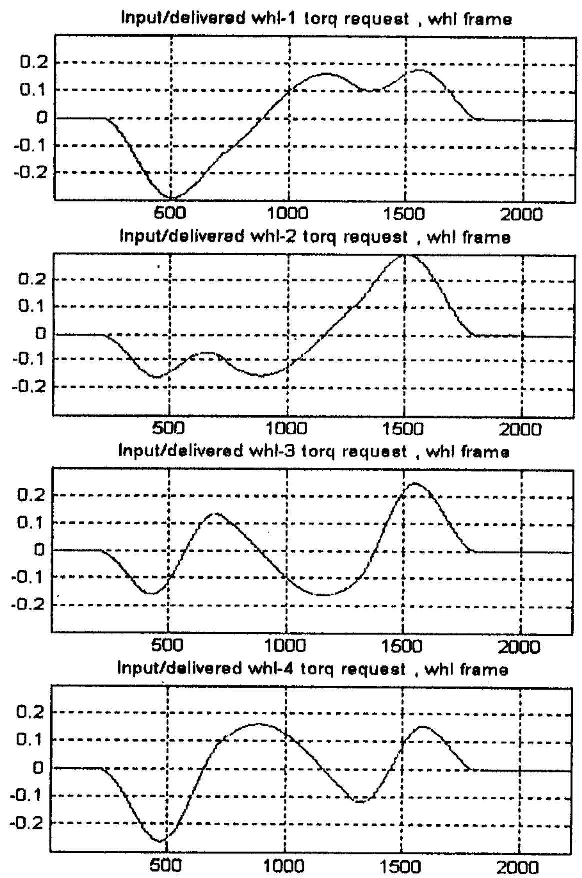
- Rectangular integration with dt = 1 sec, with only rigid body dynamic included.
- A feedback/feedforward controller: analog PD feedback, w=1Hz, acceleration and gyroscopic term (wxh) feedforward.
- Full body inertia matrix $\begin{pmatrix} 11730 & 142 & -7206 \\ 142 & 27476 & -113 \\ -7206 & -113 & 20268 \end{pmatrix}$ kg*m^2
- 4-wheel pyramid configuration, a rectangular speed-torque curve with momentum limit = 60Nms, torque limit = .3Nm, inertia = .0948 kg*m^2. For this simulation, torque limit used in the algorithm is set at 99% of the true limit, if not, commanded torque grows slightly above the true torque limit.
- 359-degree slew along the body y-axis.
- Two cases: Initial wheel momentum vector is set: case 1= (53.608, 41.695, 11.913, 5.956)Nms
  case 2= (-23.826, -41.695,11.913,5.956)Nms

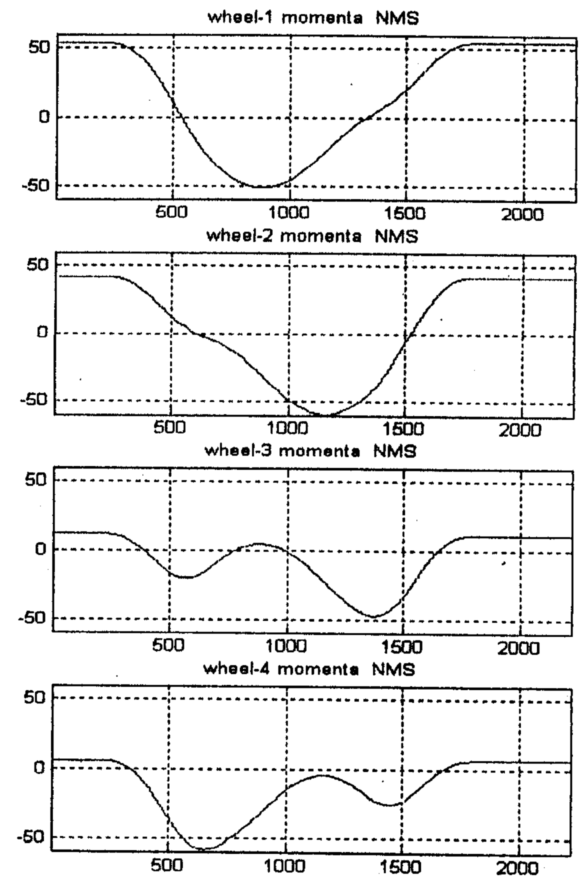
Case 1: Body Motion Simulation Results



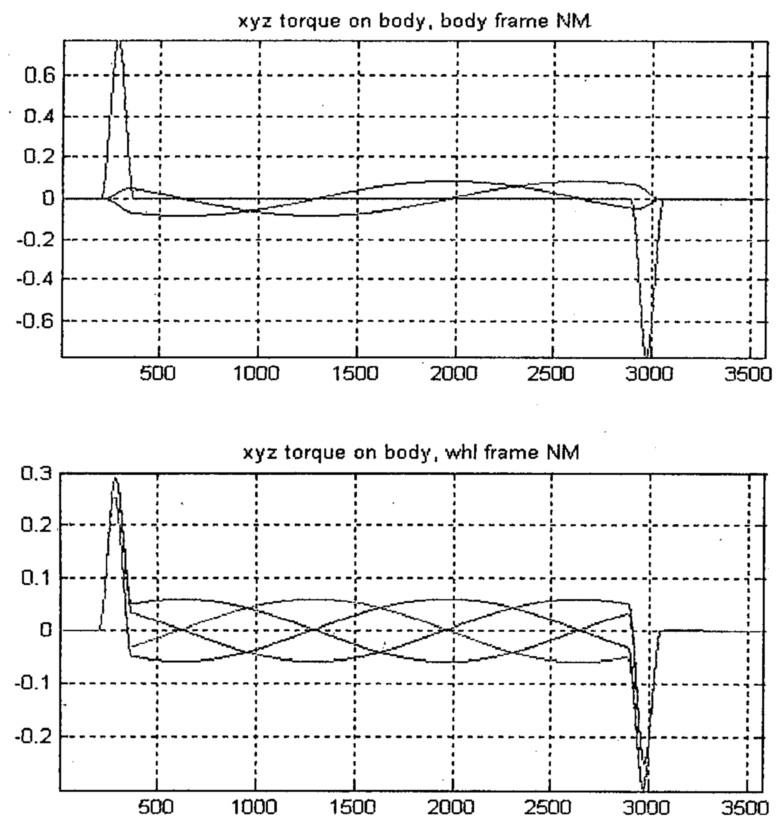


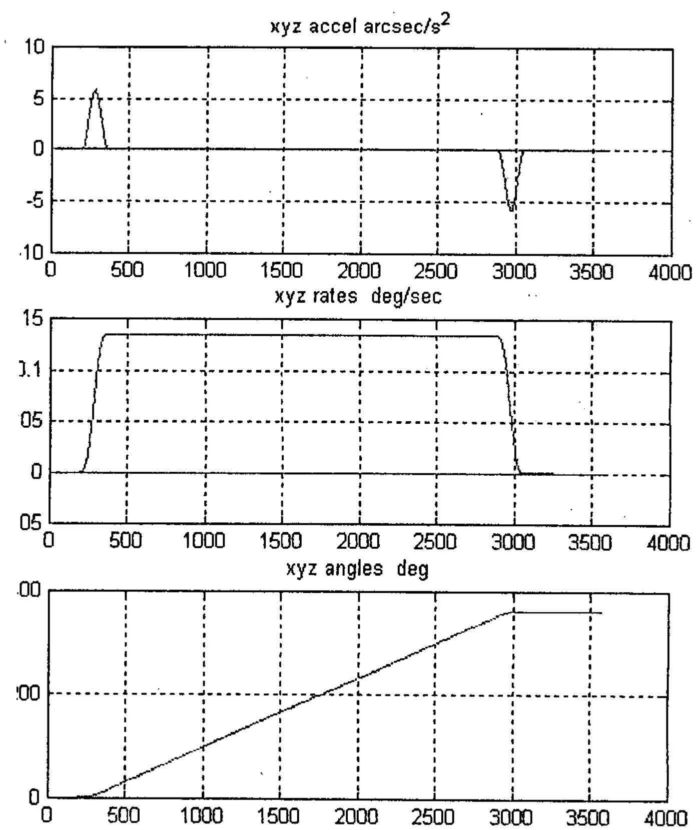
Case 1: Reaction Wheel Simulation Results



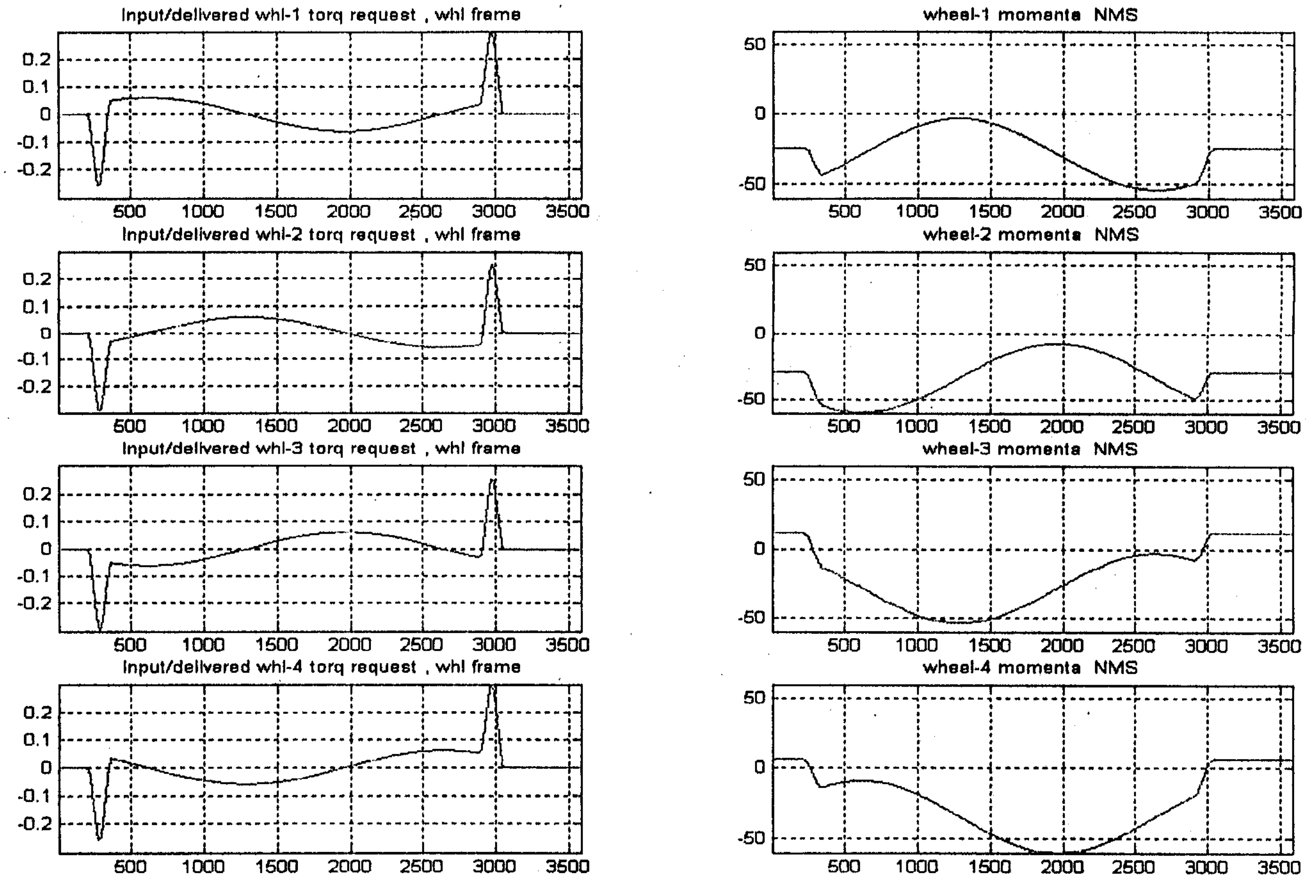


Case 2: Body Motion Simulation Results





Case 2: Reaction Wheel Simulation Results



Conclusion

- We have presented an algorithm for trajectory control of a spacecraft that minimizes the time to perform slews, including settling, by avoiding reaction wheel torque and momentum limits that would excite flexible structural modes
- This algorithm was validated by simulation during the design of the NGST "Yardstick" (pre-cursor to JWST)
- Performance verification of a reduced form for single-axis slews was carried out using the MIT Origins Testbed
- It is currently baselined for use by TPF-Coronagraph