A Direct Approach For Minimum Fuel Maneuvers of Distributed Spacecraft In Multiple Flight Regimes

Abstract Only: Not for Publication

Steven P. Hughes*

D.S. Cooley†

Jose J. Guzman‡

In this work we present a method to solve the impulsive minimum fuel maneuver problem for a distributed set of spacecraft. We develop the method assuming a fully non-linear dynamics model and parameterize the problem to allow the method to be applicable to any flight regime. Furthermore, the approach is not limited by the inter-spacecraft separation distances and is applicable to both small formations as well as constellations.

We assume that the desired relative motion is driven by mission requirements and has been determined a-priori. The goal of this work is to develop a technique to achieve the desired relative motion in a minimum fuel manner. We define the minimum fuel problem for a distributed set of m spacecraft, where the trajectory of the k^{th} spacecraft has n_k total maneuvers, as

$$\min(J) \tag{1}$$

where

$$J = \sum_{k=1}^{m} \Delta V_k \tag{2}$$

and

$$\Delta V_k = \sum_{j=1}^{n_k} f_s(\Delta v_{jk}) \Delta v_{jk} \tag{3}$$

where Δv_{jk} is the j^{th} maneuver performed by the k^{th} spacecraft. The function $f_s(\Delta v_{jk})$ in Eq. (3) is included to remove a singularity in the derivative of J for small Δv_{jk}. This function will be discussed in detail in a later section. For now it suffices to say that $f_s(\Delta v_{jk}) = 1$ for values of Δv_{jk} large enough to avoid numerical difficulties. To equalize the fuel expenditure among the spacecraft, we define a set of constraints as follows:

$$(\Delta V_\ell - \Delta V_n)^2 \leq c_{\ell n} \qquad \begin{array}{c} 1 \leq \ell \leq k \\ 1 \leq n \leq k \\ \ell \neq n \end{array} \tag{4}$$

where $c_{\ell n}$ is a vector of tolerances.

To permit applicability to multiple flight regimes, we have chosen to parameterize the cost function in terms of the maneuver times expressed in a useful time system and the maneuver locations

*Aerospace Engineer, Flight Dynamics Analysis Branch, NASA Goddard Space Flight Center, (301) 286-0145, (301) 286-0369 (FAX), email: Steven.P.Hughes@nasa.gov

†Aerospace Engineer, Flight Dynamics Analysis Branch, NASA Goddard Space Flight Center, (301) 286-0145, (301) 286-6653 (FAX), email: D.S.Cooely@nasa.gov

‡Mission Analyst

expressed in their cartesian vector representations. We also include as an independent variable the initial reference orbit to solve for the optimal injection orbit to minimize and equalize the fuel expenditure of distributed sets of spacecraft with large interspacecraft separations. In this work we derive the derivatives of the cost and constraints with respect to all of the independent variables (*Deriving the gradients is a non-trivial effort that has already been completed. However, it is not included in the extended abstract due to space limitations*). We assume a dynamics model given by the second order differential equation

$$\ddot{\mathbf{r}} = f(\mathbf{r}, t) \tag{5}$$

To demonstrate the method we have applied it to several test problems. The reference orbit for the test problem discussed here has a semimajor axis of 43000 km and an eccentricity of .83. The formation is composed of four spacecraft and is designed to form a regular tetrahedron with side lengths around 1000 km at apoapsis. Each spacecraft undergoes three maneuvers in the maneuver sequence. In the analysis shown below, we have included the gravitational affects of the Sun, Moon, Earth and J_2. In the first case, we have not included the launch injection orbit as an independent variable or attempted to equalize the ΔV among the spacecraft. Results for test case one are shown in Table 1. In the second test case, we have included the initial reference orbit as an independent variable in order to find the optimal launch injection orbit. Results for test case two are shown in Table 2. As expected, by varying the initial injection orbit, the total fuel expenditure for the maneuver sequence is lower than that for case one. In the third test case, we have included the initial reference orbit as an independent variable and we have and enforced the constraints shown in Eq. (5) to equalize the fuel use among the spacecraft. The results for case three are found in Table 3. Here we see that the total ΔV has increased. However, the spacecraft all expend the same amount of fuel. *Note: The final version of this paper will include a Libration Point example not included here.*

Table 1: Test Case One

Property	S/C 1	S/C 2	S/C 3	S/C 4
Δv_1 (m/s)	0.044	0.042	24.8	24.7
Δv_2 (m/s)	44.2	8.06	4.18	0.030
Δv_3 (m/s)	4.93	26.75	3.60	11.0
$\sum \Delta v_j$ (m/s)	49.2	34.849	32.6	35.7

Total ΔV = 152.3 m/s

Table 2: Test Case Two

Property	S/C 1	S/C 2	S/C 3	S/C 4
Δv_1 (m/s)	16.2	16.1	9.21	.049
Δv_2 (m/s)	32.1	9.26	1.90	.032
Δv_3 (m/s)	11.5	14.3	8.54	5.04
$\sum \Delta v_j$ (m/s)	59.9	39.7	19.7	5.1154

Total ΔV = 124.4 m/s

Table 3: Test Case Three

Property	S/C 1	S/C 2	S/C 3	S/C 4
Δv_1 (m/s)	3.48	0.867	27.9	19.6
Δv_2 (m/s)	39.2	11.6	8.50	12.3
Δv_3 (m/s)	0.20	31.0	6.75	11.0
$\sum \Delta v_j$ (m/s)	42.87	43.5	43.2	42.8

Total ΔV = 172.3 m/s