Pseudo-Linear Attitude Determination of Spinning Spacecraft

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This paper presents the overall mathematical model and results from pseudo linear recursive estimators of attitude and rate for a spinning spacecraft. The measurements considered are vector measurements obtained by sun-sensors, fixed head star trackers, horizon sensors, and three axis magnetometers. Two filters are proposed for estimating the attitude as well as the angular rate vector. One filter, called the *q-Filter*, yields the attitude estimate as a quaternion estimate, and the other filter, called the *D-Filter*, yields the estimated direction cosine matrix. Because the spacecraft is gyro-less, Euler's equation of angular motion of rigid bodies is used to enable the estimation of the angular velocity. A simpler Markov model is suggested as a replacement for Euler's equation in the case where the vector measurements are obtained at high rates relative to the spacecraft angular rate. The performance of the two filters is examined using simulated data.

I. Introduction

The advantages of inducing a constant spin rate on a spacecraft are well known. A variety of science missions have used this technique as a relatively low cost method for conducting science. Starting in the later part of the 1970s, NASA pushed toward building spacecraft using 3-axis control as opposed to the single-axis control mentioned above. Considerable effort was expended toward sensor and control system development as well as the development of ground systems to independently process the data. As a result, spinning spacecraft development and their resulting ground system development stagnated. In the 1990s, shrinking budgets made spinning spacecraft a more attractive option for science though most NASA missions continued to be three-axis stabilized. The attitude requirements for spinning spacecraft have become more stringent and the ground systems must be enhanced in order to provide the necessary attitude estimation accuracy.

The current ground attitude determination system at NASA Goddard Space Flight Center consists of a least squares estimator that assumes constant dynamics. Since spinning spacecraft (SC) have no gyroscopes for measuring attitude rate, any new estimator would need to rely on the spacecraft dynamics equations. One estimation technique that utilized the SC dynamics and has been used successfully on 3-axis spacecraft ground systems is the pseudo-linear algorithm. Therefore in this paper, too, SC dynamics as well as spinning spacecraft sensor models are developed to work with the pseudo-linear attitude and rate estimation algorithm. In the rest of this section we present the definition of the coordinate systems used in the filter developments and explain how the important angles are extracted from the estimated variables. In the following section we develop the sensor measurement mathematical models, then, in Section III, we present two filters for attitude

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determination. The filter algorithms are summarized in Section IV, the test that was done is presented in Section V and the conclusions are summarized in the last section.

I.1 Definition of Coordinate Systems

The transformation from the inertial coordinate system to the spacecraft (SC) body system is expressed by a sequence of three Euler angles α, β, and γ in the order $3-2-3$. This sequence is illustrated in Fig. 1

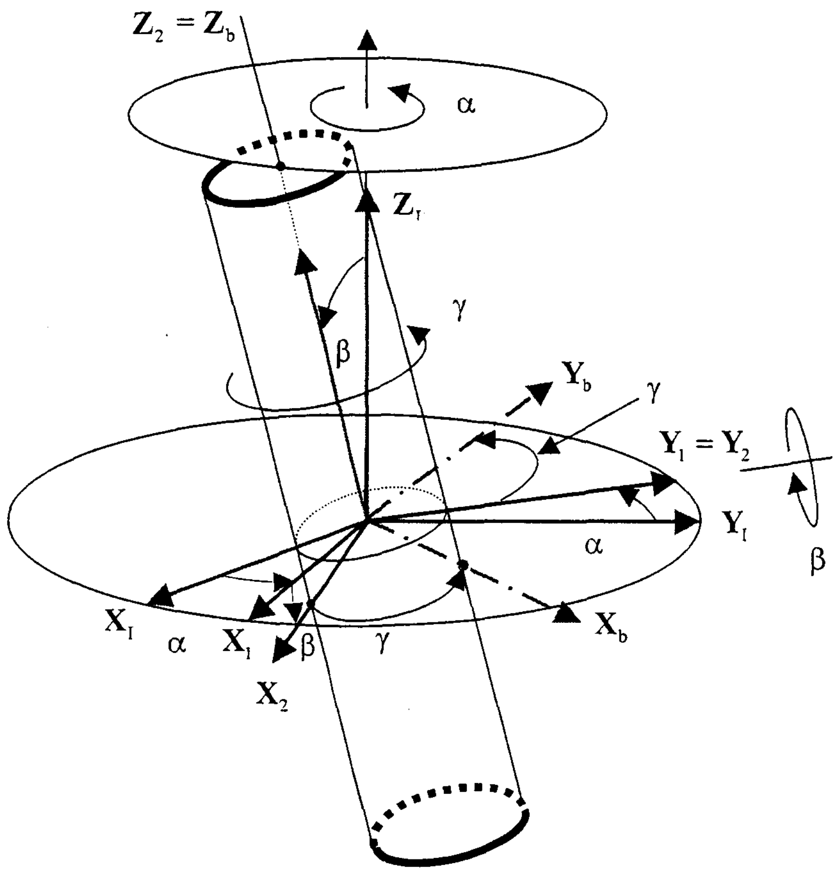

Fig. 1: $3-2-3$ transformation by α, β, γ from inertial to body coordinates.

where the superscript *I* denotes the inertial coordinates. The first rotation by α about the $\mathbf{Z}_I$ axis yields a coordinate system denoted by 1. The next rotation about $\mathbf{Y}_1$ by the angle β results in the intermediate system denoted by 2. Note that $\mathbf{Z}_2$ is identical to the body axis, $\mathbf{Z}_b$, which is the spin-axis. One more rotation by γ about this spin axis yields the body coordinate system denoted by *b*. Actually γ is the spin-angle, which is also known as phase-angle. For this sequence of Euler angles the direction cosine matrix (DCM) which transforms vectors from inertial to body coordinates is:

SC $\mathbf{Z}_b$ - axis. From the geometry it is clear that the three components of the unit vector to the sun expressed in body coordinates are as follows[1]

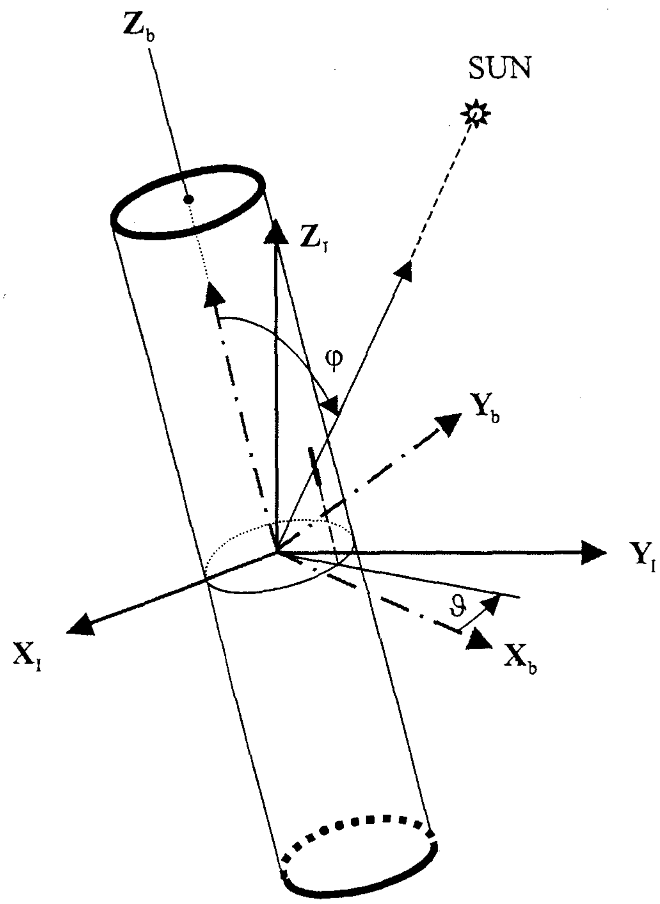

Fig. 2: Sun vector geometry in body coordinates

$$
\begin{aligned}
s_{bx} &= s\varphi \cdot c\vartheta \\
s_{by} &= s\varphi \cdot s\vartheta \\
s_{bz} &= c\varphi
\end{aligned}
\tag{3.a}
$$

where s_{bx}, s_{by} and s_{bz} are the three components of the unit vector to the sun, $\mathbf{1}_{sb}$ expressed in body coordinates. Note that ϑ is a fixed angle and only φ is changing.

If the sun sensor coordinates are not identical to the body coordinates then the measured vector is given in the sun sensor coordinates and it has to be transformed to the SC body coordinates as follows.

[1] Although the sun sensor may measure functions of the angles and not the angles themselves, for simplicity we assume that the angles are the measured quantities. It is indeed easy to express the sine and cosine functions of the respective angles in term of the measured quantities.

where I is the SC inertia tensor, $\boldsymbol{\omega}$ is the angular velocity vector, $\mathbf{h}$ is the angular momentum of the momentum wheels, and $\mathbf{T}$ is the external torque acting on the SC. The symbol $[\mathbf{a}\times]$ denotes the cross product matrix of the general vector $\mathbf{a}$. Attitude is represented by the attitude quaternion whose dynamics equation is [Ref. 1, p. 512]

$$\dot{\mathbf{q}} = \frac{1}{2} Q \boldsymbol{\omega} \tag{8}$$

where

$$Q = \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix} \tag{9}$$

In the q-filter we augment Eqs. (7) and (8) to form the following dynamics equation, which includes the noise terms $\mathbf{w}_\omega$ and $\mathbf{w}_q$.

$$\begin{bmatrix} \dot{\omega} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} I^{-1}[(I\omega+\mathbf{h})\times] & 0 \\ \frac{1}{2}Q & 0 \end{bmatrix} \begin{bmatrix} \omega \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} I^{-1}(\mathbf{T}-\dot{\mathbf{h}}) \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_\omega \\ \mathbf{w}_q \end{bmatrix} \tag{10.a}$$

The unbiased white-noise vector $\mathbf{w}_\omega$ accounts for the inaccuracies in the modeling of the SC angular dynamics, and $\mathbf{w}_q$ is an unbiased white-noise vector that accounts for modeling errors in the quaternion dynamics.

When the measurements come in at a relatively high frequency we may be able to replace the SC angular dynamics model in Eq. (10.a) with a simpler Markov model[7]. Consequently Eq. (10.a) is replaced by the model

$$\begin{bmatrix} \dot{\omega} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} -[\tau] & 0 \\ \frac{1}{2}Q & 0 \end{bmatrix} \begin{bmatrix} \omega \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_\omega \\ \mathbf{w}_q \end{bmatrix} \tag{10.b}$$

where $[\tau]$ is a diagonal matrix whose elements are the inverse of suitable time constants.

III.1.2 *The Measurement Model*

Let $\mathbf{r}_j$ denote the column vector measured by the $j^{\underline{th}}$ sensor and expressed in the reference inertial coordinate system (this vector is taken from the almanac). The relationship between this column vector and $\mathbf{b}_j$, which is that vector resolved in body coordinates, is expressed by

$$\mathbf{b}_j = D_b^I \mathbf{r}_j \tag{11}$$

The measured $\mathbf{b}_j$ vector, which we denote by $\mathbf{b}_{jm}$, contains an additive zero-mean white-noise vector, $\mathbf{v}_{j,b}$, thus

$$\mathbf{b}_{jm} = D_b^I \mathbf{r}_j + \mathbf{v}_{jb} \tag{12}$$

Using the expression for D_b^I given in Eq. (2.b), and after some algebraic manipulation, the last equation can be written as

$$\mathbf{b}_{jm} = H_j(\mathbf{r}_j, \mathbf{q})\mathbf{q} + \mathbf{v}_{jb} \tag{13.a}$$

where

$$H_j(\mathbf{r}_j,\mathbf{q}) = \begin{bmatrix} q_1r_1 + q_2r_2 + q_3r_3 & -q_2r_1 + q_1r_2 - q_4r_3 & -q_3r_1 + q_4r_2 + q_1r_3 & q_4r_1 + q_3r_2 - q_2r_3 \\ q_2r_1 - q_1r_2 + q_4r_3 & q_1r_1 + q_2r_2 + q_3r_3 & -q_4r_1 - q_3r_2 + q_2r_3 & -q_3r_1 + q_4r_2 + q_1r_3 \\ q_3r_1 - q_4r_2 - q_1r_3 & q_4r_1 + q_3r_2 - q_2r_3 & q_1r_1 + q_2r_2 + q_3r_3 & q_2r_1 - q_1r_2 + q_4r_3 \end{bmatrix}_j \tag{13. b}$$

It should be noted that H is not unique; that is, $D_b^I(\mathbf{q})\mathbf{r}_j$ can be written in the shape $H_j(\mathbf{r}_j,\mathbf{q})\mathbf{q}$ where $H_j(\mathbf{r}_j,\mathbf{q})$ can take several forms. However, the form of $H_j(\mathbf{r}_j,\mathbf{q})$ shown in Eq. (13.b) is the most symmetrical one and it is exactly half the sensitivity matrix, $\delta[D_b^I(\mathbf{q})\mathbf{r}_j]/\delta\mathbf{q}$, obtained when developing the corresponding Extended Kalman Filter measurement equation. Finally, Eq. (13.a) can be written also as

$$\mathbf{b}_{jm} = \begin{bmatrix} 0 & H_j(\mathbf{r}_j,\mathbf{q}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \mathbf{v}_{jb} \tag{13.c}$$

The last equation is the measurement equation associated with any vector measurement.

III.1.3 *Vector Propagation*

As mentioned before, it was found in the past[6] that in non-spinning SC a similar filter performed well only when the different vector measurements were taken at short time intervals from one another. Therefore we propose to propagate forward each measurement after it has been used, and use the propagated vector together with the newly acquired vector measurement. The projection is done as follows. Suppose that at the time when $\mathbf{b}_1$ is measured, the quaternion of the transformation from inertial to body coordinates is $\mathbf{q}_{Ib1}$ and at the time when $\mathbf{b}_2$ is measured, that quaternion is $\mathbf{q}_{Ib2}$, then $\mathbf{q}_{Ib2}$ satisfies the following relation

$$\mathbf{q}_{Ib2} = \mathbf{q}_{Ib1} \otimes \delta\mathbf{q} \tag{14.a}$$

where $\delta\mathbf{q}$ is the quaternion that transforms from the body coordinates at the time $\mathbf{b}_1$ is measured to the body coordinates at the time $\mathbf{b}_2$ is measured. From Eq. (14.a) we obtain

$$\delta\mathbf{q} = \mathbf{q}_{Ib1}^{-1} \otimes \mathbf{q}_{Ib2} \tag{14.b}$$

The corresponding DCM, which we denote by δD_2^1, is computed using

$$\delta D_2^1 = \begin{bmatrix} \delta q_1^2\text{-}\delta q_2^2\text{-}\delta q_3^2+\delta q_4^2 & 2(\delta q_1\delta q_2+\delta q_3\delta q_4) & 2(\delta q_1\delta q_3\text{-}\delta q_2\delta q_4) \\ 2(\delta q_1\delta q_2\text{-}\delta q_3\delta q_4) & \text{-}\delta q_1^2+\delta q_2^2\text{-}\delta q_3^2+\delta q_4^2 & 2(\delta q_2\delta q_3+\delta q_1\delta q_4) \\ 2(\delta q_1\delta q_3+\delta q_2\delta q_4) & 2(\delta q_2\delta q_3\text{-}\delta q_1\delta q_4) & \text{-}\delta q_1^2\text{-}\delta q_2^2+\delta q_3^2+\delta q_4^2 \end{bmatrix} \tag{14.c}$$

hence the projected $\mathbf{b}_1$, denoted by $\mathbf{b}_1^p$ is computed as

$$\mathbf{b}_1^p = \delta D_2^1 \mathbf{b}_1 \tag{14.d}$$

Both $\mathbf{b}_1^p$ and $\mathbf{b}_2$ are used in the filter update at the time $\mathbf{b}_2$ is measured. If $\mathbf{b}_1$ is a sun sensor or a star tracker measurement, then $\mathbf{r}_1^p$, the vector corresponding to $\mathbf{b}_1^p$, does not change between the two measurement time points; that is, $\mathbf{r}_1^p = \mathbf{r}_1$. However, when the measurement is of the magnetic field or of the nadir, $\mathbf{r}_1$ does change, but the new vector is easily computable independently of the attitude.

When this approach is used, situations may arise where the attitude is unobservable even though attitude is completely defined from two pairs of vectors [see Ref. 6 for such a case]. To avoid such cases it is proposed to use the technique that was used successfully in[6]. Generate a pseudo-measurement pair, $\mathbf{r}_3$ and $\mathbf{b}_3$ where

$$\mathbf{r}_3 = \mathbf{r}_1^p \times \mathbf{r}_2 \tag{15.a}$$

and

$$\mathbf{b}_3 = \mathbf{b}_1^p \times \mathbf{b}_2 \tag{15.b}$$

Using Eq. (12) the noise vector, $\mathbf{v}_3$, associated with the pseudo-measurement $\mathbf{b}_3$ is given by

$$\mathbf{v}_3 = (\delta D_2^1 \mathbf{r}_1) \times \mathbf{v}_2 + \mathbf{v}_1 \times (\delta D_2^1 \mathbf{r}_2) + \mathbf{v}_1 \times \mathbf{v}_2 \tag{16}$$

Note that $\mathbf{v}_3$ is correlated with both $\mathbf{v}_1$ and $\mathbf{v}_2$. Since $\mathbf{v}_1$ as well as $\mathbf{v}_2$ consist of components that are zero mean sequences which are uncorrelated with themselves at different time points and with each other at all time points, it is not too difficult to compute the covariance matrix of R_3; however, since the suitable measurement covariance matrix is normally determined experimentally by tuning[8], the exact computation of R_3 is, in practice, unnecessary. The corresponding measurement model when using vector projection is

$$\begin{bmatrix} \mathbf{b}_{1m}^p \\ \mathbf{b}_{2m} \\ \mathbf{b}_3 \end{bmatrix} = \begin{bmatrix} 0 & H_1(\mathbf{r}_1^p, \mathbf{q}) \\ 0 & H_2(\mathbf{r}_2, \mathbf{q}) \\ 0 & H_3(\mathbf{r}_3, \mathbf{q}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} \delta D_2^1 \mathbf{v}_1 \\ \mathbf{v}_2 \\ \mathbf{v}_3 \end{bmatrix} \tag{17}$$

where the subscript *m* denotes measured quantity. The covariance matrix that corresponds to $\delta D_2^1 \mathbf{v}_1$ is $\delta D_2^1 R_1 (\delta D_2^1)^T$ where R_1 is the covariance matrix corresponding to $\mathbf{v}_1$. Because of the relationship, expressed in Eq. (16.a), between $\mathbf{v}_3$ and $\mathbf{v}_1$, and between $\mathbf{v}_3$ and $\mathbf{v}_2$, it is obvious that the covariance matrix of the noise vector in Eq. (17) has off-diagonal elements. The simplest way to avoid the computation of these elements is to disregard these elements and compensate for their elimination by proper tuning.

III.2 The D-Filter

III.2.1 *The Dynamics Model*

We start with the well-known first order DCM differential equation[9]

$$\dot{D} = -[\boldsymbol{\omega} \times] D \tag{18}$$

where $[\boldsymbol{\omega} \times]$ is the cross product matrix of $\boldsymbol{\omega}$ defined as follows.

$$[\omega\times]=\begin{bmatrix}0 & -\omega_z & \omega_y\\ \omega_z & 0 & -\omega_x\\ -\omega_y & \omega_x & 0\end{bmatrix} \tag{19}$$

Define the columns of D as 3 vectors; that is,

$$\mathrm{D}=\left[\mathbf{d}_1 \mid \mathbf{d}_2 \mid \mathbf{d}_3\right] \tag{20}$$

then Eq. (18) can be written as

$$\left[\dot{\mathbf{d}}_1 | \dot{\mathbf{d}}_2 | \dot{\mathbf{d}}_3\right]=-[\omega\times]\left[\mathbf{d}_1 | \mathbf{d}_2 | \mathbf{d}_3\right] \tag{21}$$

It is easy to see that

$$\left[\dot{\mathbf{d}}_1 \mid \dot{\mathbf{d}}_2 \mid \dot{\mathbf{d}}_3\right]=\left[\ [\mathbf{d}_1\times] \mid [\mathbf{d}_2\times] \mid [\mathbf{d}_3\times]\ \right]\omega \tag{22}$$

or

$$\left[\dot{\mathbf{d}}_i\right]=\left[\mathbf{d}_i\times\right]\omega \quad i=1,2,3 \tag{23}$$

Define the matrix $\mathcal{D}$ as follows

$$\mathcal{D}=\begin{bmatrix}[\mathbf{d}_1\times]\\ [\mathbf{d}_2\times]\\ [\mathbf{d}_3\times]\end{bmatrix} \tag{24}$$

where, obviously, $\mathcal{D}\in\Re^{9\times 3}$. Also define the vector $\boldsymbol{d}$ as follows

$$\boldsymbol{d}^T=\left[\mathbf{d}_1^T \quad \mathbf{d}_2^T \quad \mathbf{d}_3^T\right] \tag{25}$$

then Eqs. (22) can be written in one vector differential equation as

$$\dot{\boldsymbol{d}}=\mathcal{D}\omega \tag{26}$$

Grouping Eqs. (7) and (26), yields

$$\begin{bmatrix}\dot{\omega}\\ \dot{\boldsymbol{d}}\end{bmatrix}=\begin{bmatrix}I^{-1}[(I\omega+\mathbf{h})\times] & 0\\ \mathcal{D} & 0\end{bmatrix}\begin{bmatrix}\omega\\ \boldsymbol{d}\end{bmatrix}+\begin{bmatrix}I^{-1}(\mathbf{T}-\dot{\mathbf{h}})\\ 0\end{bmatrix} \tag{27}$$

In order to use the last equation as a dynamics equation in a KF, we need to add to it zero-mean white-noise to account for modeling inaccuracies. This results in the following dynamics equation.

$$\begin{bmatrix}\dot{\omega}\\ \dot{\boldsymbol{d}}\end{bmatrix}=\begin{bmatrix}I^{-1}[(I\omega+\mathbf{h})\times] & 0\\ \mathcal{D} & 0\end{bmatrix}\begin{bmatrix}\omega\\ \boldsymbol{d}\end{bmatrix}+\begin{bmatrix}I^{-1}(\mathbf{T}-\dot{\mathbf{h}})\\ \mathbf{0}\end{bmatrix}+\begin{bmatrix}\mathbf{w}_\omega\\ \mathbf{w}_\delta\end{bmatrix} \tag{28.a}$$

where, as before, $\mathbf{w}_\omega$ accounts for the inaccuracies in the modeling of the SC angular dynamics and $\mathbf{w}_{\boldsymbol{d}}$ accounts for modeling errors of the DCM dynamics. As with the q-filter, here too, when the measurements are obtained at a relatively high frequency, we can use the simpler dynamics equation. In such case the dynamics equation reduces to

$$\begin{bmatrix}\dot{\omega}\\ \dot{\boldsymbol{d}}\end{bmatrix}=\begin{bmatrix}-[\tau] & 0\\ \mathcal{D} & 0\end{bmatrix}\begin{bmatrix}\omega\\ \boldsymbol{d}\end{bmatrix}+\begin{bmatrix}\mathbf{w}_\omega\\ \mathbf{w}_{\boldsymbol{d}}\end{bmatrix} \tag{28.b}$$

III.2.2 The Measurement Model

Equation (12) can be written as

$$\mathbf{b}_{j,m} = \left[\mathbf{d}_1 r_{j1} + \mathbf{d}_2 r_{j2} + \mathbf{d}_3 r_{j3}\right] + \mathbf{v}_{j,b} \tag{29}$$

which can be written also as

$$\mathbf{b}_{j,m} = R_j \boldsymbol{d} + \mathbf{v}_{j,b} \tag{30}$$

where

$$R_j = \left[r_{j1} I_3 \quad r_{j2} I_3 \quad r_{j3} I_3\right] \tag{31}$$

and $\boldsymbol{d}$ is as defined in Eq. (25). Note that usually $\mathbf{r}_j$ is taken from an almanac and is not measured; therefore, it is very accurate. The measurement model is

$$\mathbf{b}_{jm} = \left[0_3 \quad R_j\right] \begin{bmatrix} \omega \\ \boldsymbol{d} \end{bmatrix} + \mathbf{v}_{jb} \tag{32}$$

III.2.3 *Vector Projection*

As with the q-filter, here too we propose vector projection. Here, however, δD_2^I is obtained as follows

$$D_{b2}^I = \delta D_2^I D_{b1}^I \tag{33.a}$$

where D_{b1}^I is the transformation from inertial to body when the measurement is of $\mathbf{b}_1$ and D_{b2}^I is the transformation from inertial to body when the measurement is of $\mathbf{b}_2$. From Eq. (33.a) we obtain

$$\delta D_2^I = (D_{b1}^I)^T D_{b2}^I \tag{33.b}$$

The handling of the vector measurements in this filter is identical to that of the q-filter. The measurement model is different though. In the D-filter it is as follows.

$$\begin{bmatrix} \mathbf{b}_{1m}^p \\ \mathbf{b}_{2m} \\ \mathbf{b}_3 \end{bmatrix} = \begin{bmatrix} 0 & R_1 \\ 0 & R_2 \\ 0 & R_3 \end{bmatrix} \begin{bmatrix} \omega \\ \boldsymbol{d} \end{bmatrix} + \begin{bmatrix} \delta D_2^I \mathbf{v}_{1b} \\ \mathbf{v}_{2b} \\ \mathbf{v}_{3b} \end{bmatrix} \tag{33.c}$$

IV. SUMMARY

IV.1 Using the q-Filter

For the dynamics equation use either

$$\begin{bmatrix} \dot{\omega} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} I^{-1}[(I\omega+\mathbf{h})\times] & 0 \\ \frac{1}{2}Q & 0 \end{bmatrix} \begin{bmatrix} \omega \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} I^{-1}(\mathbf{T} - \dot{\mathbf{h}}) \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_\omega \\ \mathbf{w}_q \end{bmatrix} \tag{10.a}$$

or

$$\begin{bmatrix} \dot{\omega} \\ \dot{\mathbf{q}} \end{bmatrix} = \begin{bmatrix} -[\tau] & 0 \\ \frac{1}{2}Q & 0 \end{bmatrix} \begin{bmatrix} \omega \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_\omega \\ \mathbf{w}_q \end{bmatrix} \tag{10.b}$$

and for the measurement equation use

$$\begin{bmatrix} \mathbf{b}^p_{1m} \\ \mathbf{b}_{2m} \\ \mathbf{b}_3 \end{bmatrix} = \begin{bmatrix} 0 & H_1(\mathbf{r}^p_1, \mathbf{q}) \\ 0 & H_2(\mathbf{r}_2, \mathbf{q}) \\ 0 & H_3(\mathbf{r}_3, \mathbf{q}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \mathbf{q} \end{bmatrix} + \begin{bmatrix} \delta D^1_2 \mathbf{v}_{1b} \\ \mathbf{v}_{2b} \\ \mathbf{v}_{3b} \end{bmatrix} \tag{17}$$

where

$$\mathbf{r}_3 = \mathbf{r}^p_1 \times \mathbf{r}_2 \tag{15.a}$$

$$\mathbf{b}^p_1 = \delta D^1_2 \mathbf{b}_1 \tag{14.d}$$

$$\mathbf{b}_3 = \mathbf{b}^p_{1m} \times \mathbf{b}_{2m} \tag{15.b}$$

and

$$H_j(\mathbf{r}_j, \mathbf{q}) = \begin{bmatrix} q_1 r_1 + q_2 r_2 + q_3 r_3 & -q_2 r_1 + q_1 r_2 - q_4 r_3 & -q_3 r_1 + q_4 r_2 + q_1 r_3 & q_4 r_1 + q_3 r_2 - q_2 r_3 \\ q_2 r_1 - q_1 r_2 + q_4 r_3 & q_1 r_1 + q_2 r_2 + q_3 r_3 & -q_4 r_1 - q_3 r_2 + q_2 r_3 & -q_3 r_1 + q_4 r_2 + q_1 r_3 \\ q_3 r_1 - q_4 r_2 - q_1 r_3 & q_4 r_1 + q_3 r_2 - q_2 r_3 & q_1 r_1 + q_2 r_2 + q_3 r_3 & q_2 r_1 - q_1 r_2 + q_4 r_3 \end{bmatrix}_j \tag{13. b}$$

IV.2 Using the D-Filter

For the dynamics equation use either

$$\begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \dot{\boldsymbol{d}} \end{bmatrix} = \begin{bmatrix} I^{-1}[(I\boldsymbol{\omega} + \mathbf{h})\times] & 0 \\ \mathcal{D} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{d} \end{bmatrix} + \begin{bmatrix} I^{-1}(\mathbf{T} - \dot{\mathbf{h}}) \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_\omega \\ \mathbf{w}_\delta \end{bmatrix} \tag{28.a}$$

or

$$\begin{bmatrix} \dot{\boldsymbol{\omega}} \\ \dot{\boldsymbol{d}} \end{bmatrix} = \begin{bmatrix} -[\tau] & 0 \\ \mathcal{D} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{d} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} + \begin{bmatrix} \mathbf{w}_\omega \\ \mathbf{w}_d \end{bmatrix} \tag{28.b}$$

where

$$\boldsymbol{d}^T = \begin{bmatrix} \mathbf{d}^T_1 & \mathbf{d}^T_2 & \mathbf{d}^T_3 \end{bmatrix} \tag{25}$$

and

$$\mathcal{D} = \begin{bmatrix} [\mathbf{d}_1 \times] \\ [\mathbf{d}_2 \times] \\ [\mathbf{d}_3 \times] \end{bmatrix} \tag{24}$$

and for the measurement equation use

$$\begin{bmatrix} \mathbf{b}^p_{1m} \\ \mathbf{b}_{2m} \\ \mathbf{b}_3 \end{bmatrix} = \begin{bmatrix} 0 & R_1 \\ 0 & R_2 \\ 0 & R_3 \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{d} \end{bmatrix} + \begin{bmatrix} \delta D^1_2 \mathbf{v}_{1b} \\ \mathbf{v}_{2b} \\ \mathbf{v}_{3b} \end{bmatrix} \tag{33.c}$$

where

$$R_j = \begin{bmatrix} r_{j1} I_3 & r_{j2} I_3 & r_{j3} I_3 \end{bmatrix} \tag{31}$$

V. TEST RESULTS

V.1 The q-Filter Results

In this simulation we considered a SC spinning at a rate of 120 deg/sec. The initial Euler angles describing the attitude were $\alpha = -86.4$, $\beta = 33.3$, $\gamma = 8.2$ degrees. The simulated measurements included Sun Sensor (SS) measurements, Three Axis Magnetometer (TAM) measurements and Sun Pulse time. The latter was used to obtain a rough initial spin rate estimate. For better convergence, the initial SS and TAM measurement were used in the TRIAD algorithm[10] to obtain a rough initial attitude estimate. The attitude and rate errors mean and standard deviation are presented, respectively in Tables I and II. Plots of the attitude and rate errors are shown in Figs. 3 and 4.

Table I: Attitude error mean and standard deviation using the q-filter (deg).

	Component	*Mean error*	*Standard Deviation*
ATTITUDE ERRORS	x	-0.068204	0.036345
	y	-0.003806	0.064319
	z	0.076284	0.002280

Table II: Rate error mean and standard deviation using the q-filter (deg/sec).

	Component	*Mean error*	*Standard Deviation*
RATE ERRORS	x	-0.244370	0.076580
	y	-0.007258	0.011394
	z	0.000006	0.000113

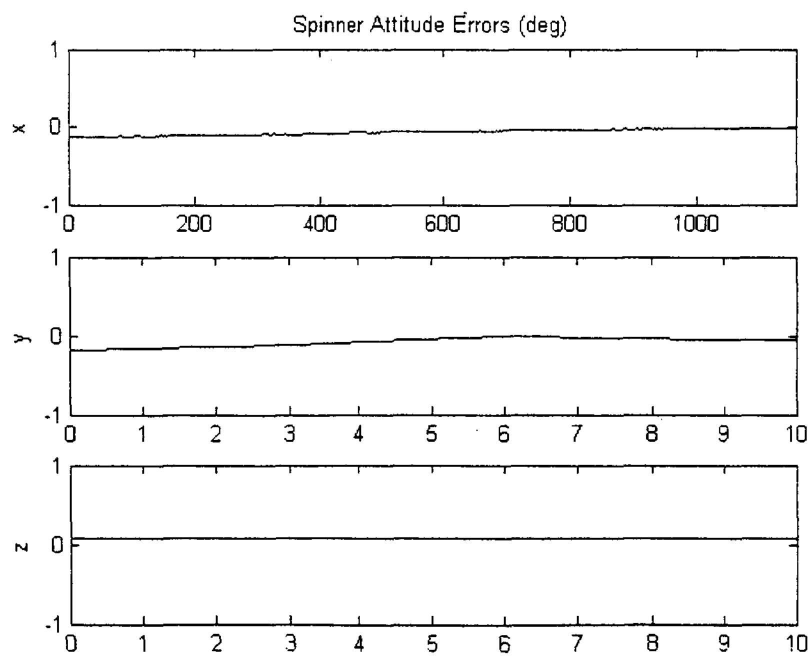

Fig. 3: The spinning SC attitude estimation errors.

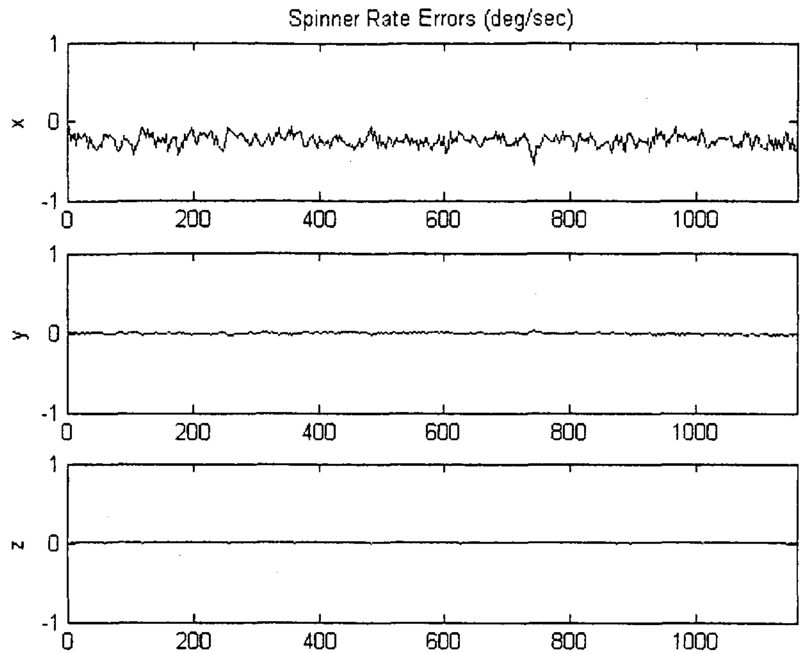

Fig. 3: The spinning SC attitude estimation errors.

V.2 The D-Filter Results

The D-filter was run under the same conditions. The plots of the attitude and rate errors that are plotted on the same scale as the q-filter plots look similar to those shown in Figs. 3 and 4, respectively. The mean and standard deviation of the attitude and the angular rate errors are presented, respectively, in Table III and Table IV.

Table III: Attitude error mean and standard deviation using the D-filter (deg).

	Component	*Mean error*	*Standard Deviation*
ATTITUDE ERRORS	x	-0.094162	0.036783
	y	-0.169427	0.018187
	z	0.032968	0.022750

Table IV: Rate error mean and standard deviation using the D-filter (deg/sec).

	Component	*Mean error*	*Standard Deviation*
RATE ERRORS	x	0.000006	0.000004
	y	0.000014	0.000015
	z	-0.000000	0.000000

VI. CONCLUSIONS

This paper treats the problem of *recursive* attitude and rate estimation of spinning spacecraft from vector measurement. The sensors considered are 3-axis magnetometers, sun sensors, star trackers, and horizon sensors. Error models suitable for Kalman filtering of these sensors were developed. Since usually spinning spacecraft carry no gyros, it was necessary to use Euler's equation of the angular dynamics of rigid bodies to estimate the angular rate. The use of the nonlinear Euler's equation and the fact that quaternions are related to vector measurements in a nonlinear fashion turned the estimation problem into a nonlinear one. Following our good experience with the application of pseudo-linear Kalman filtering to similar problems, we adopted that filter in this work too.

Two pseudo-linear Kalman filters were designed; namely, the q-filter and the D-filter. The former estimates attitude in terms of the attitude quaternion, and the latter does it in terms of the direction cosine matrix. Since usually vector measurements are not acquired simultaneously we used vector projection to project forward past measurements to coincide with present ones and thereby increase the robustness of the filters.

A simpler Markov model is suggested as a replacement for Euler's equation in the case where the vector measurements are obtained at high rates relative to the spacecraft angular rate. The performance of the two filters was examined using simulated data which confirmed the efficiency of the recursive algorithms in estimating attitude and rate of spinning spacecraft.

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Appendix A: Sun Sensor Measurement Equation[2]

The geometry involved in the sun sensor measurement is presented in Fig. A.1[2].

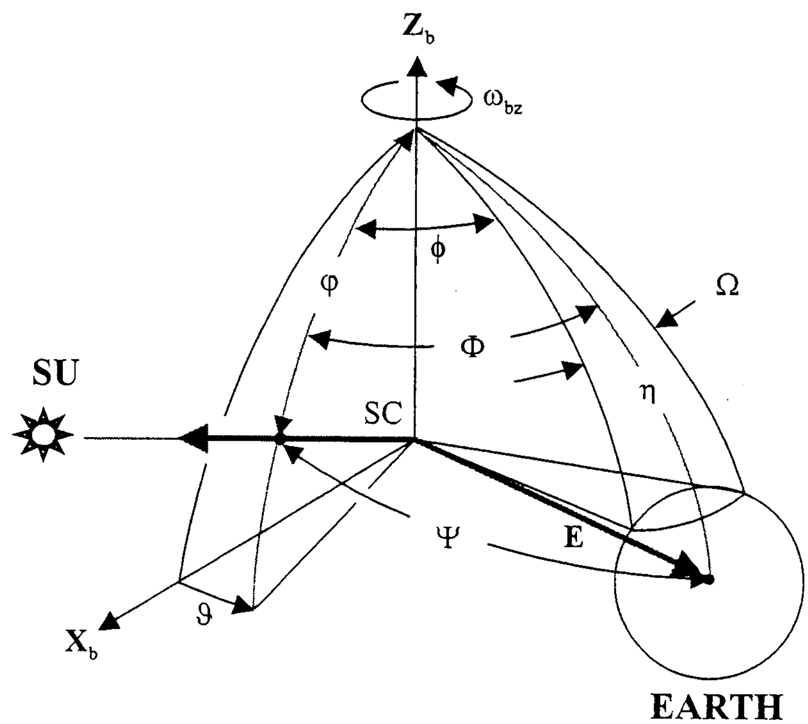

Fig. A.1: Sun Sensor Measurement Geometry.

The known angles are the following:

Measured:

ϕ - dihedral angle between the sun line and earth horizontal crossing.

Ω - earth width (dihedral angle from earth-in to earth-out).

φ - arc length between the sun line and $\mathbf{Z}_b$.

From Ephemeris:

Ψ - arc length between the sun and earth.

The computed unknown angles are:

Φ - dihedral angle between the sun line and the center of earth.

η - Nadir Angle, the arc angle between the $\mathbf{Z}_b$ and the direction to the center of earth.

From Fig. (A.1) it is seen that the computation of Φ is rather trivial because

$$\Phi = \phi + \frac{\Omega}{2} \tag{A.1}$$

The computation of η is more involved. We start by using the following connection [2].

$$\cos\Psi = \cos\varphi \cdot \cos\eta + \sin\varphi \cdot \sin\eta \cdot \cos\Phi \tag{A.2}$$

In order to solve the last equation for η define

$$\begin{aligned} a &= \cos(\varphi) \\ b &= \sin(\varphi)\cos(\Phi) \\ c &= \cos(\Psi) \end{aligned} \tag{A.3}$$

then Eq. (A.2) becomes

$$c = a\cos(\eta) + b\sin(\eta) \tag{A.4}$$

Divide this equation by $\sqrt{a^2+b^2}$ to obtain

$$\frac{c}{\sqrt{a^2+b^2}} = \frac{a}{\sqrt{a^2+b^2}}\cos(\eta) + \frac{b}{\sqrt{a^2+b^2}}\sin(\eta) \tag{A.5}$$

Define an angle λ such that

$$\sin(\lambda) = \frac{a}{\sqrt{a^2+b^2}} \tag{A.6.1}$$

and

$$\cos(\lambda) = \frac{b}{\sqrt{a^2+b^2}} \tag{A.6.2}$$

The definition of this angle is explained in Fig. (A.2).

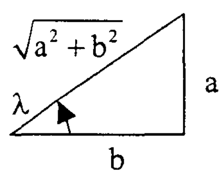

Fig. A.2: Definition of the angle

Also let $A = c\big/\sqrt{a^2+b^2}$ then Eq. (A.4) can be written as

$$A = \sin(\lambda)\cos(\eta) + \cos(\lambda)\sin(\eta) \tag{A.7}$$

Using the formula for the sine of the sum of two angles, the last equation can be written as

$$A = \sin(\lambda + \eta) \tag{A.8}$$

therefore

$$\eta = \arcsin(A) - \lambda \tag{A.9}$$

which can be written as

$$\eta = \arcsin(A) - \arctan\left(\frac{a}{b}\right) = \arcsin\left(\frac{c}{\sqrt{a^2 + b^2}}\right) - \arctan\left(\frac{a}{b}\right) \tag{A.10}$$

Substituting the values for a, b and c given in Eq. (A.3) yields the final solution for η.

$$\eta = \arcsin\left(\frac{\cos\Psi}{\sqrt{\cos^2\varphi + \sin^2\varphi \cdot \cos^2\Phi}}\right) - \arctan\left(\frac{\cos\varphi}{\sin\varphi \cdot \cos\Phi}\right) \tag{A.11}$$

and using Eq. (A.1) the last equation becomes

$$\eta = \arcsin\left(\frac{\cos\Psi}{\sqrt{\cos^2\varphi + \sin^2\varphi \cdot \cos^2(\phi + \Omega/2)}}\right) - \arctan\left(\frac{\cos\varphi}{\sin\varphi \cdot \cos(\phi + \Omega/2)}\right) \tag{A.12}$$

With η on hand we can compute the unit nadir vector in boddy coordinates as follows

$$\mathbf{1}_{hb} = \begin{bmatrix} \sin\eta \cdot \cos(\phi + \Omega/2 + \vartheta) \\ \sin\eta \cdot \sin(\phi + \Omega/2 + \vartheta) \\ \cos\eta \end{bmatrix} \tag{A.13}$$