CATEGORY 3, PROBLEM 1

SINGLE AIRFOIL GUST RESPONSE PROBLEM

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The solution to this problem can be obtained by solving the linearized unsteady Euler equations. Let the unsteady flow field be given by

$$\vec{U}(\vec{x},t) = \vec{U}_0(\vec{x}) + \vec{u}(\vec{x},t) \tag{1}$$

$$p(\vec{x},t) = p_0(\vec{x}) + p'(\vec{x},t) \tag{2}$$

$$\rho(\vec{x},t) = \rho_0(\vec{x}) + \rho'(\vec{x},t) \tag{3}$$

$$s(\vec{x},t) = s_0 + s'(\vec{x},t) \tag{4}$$

where the entropy s_0 is constant, and $\vec{u}$, p', ρ', and s' are the unsteady perturbation velocity, pressure, density and entropy, respectively. Zero subscripts denote mean flow quantities which are assumed to be known.

Substituting (1) – (4) into the nonlinear Euler equations and neglecting products of small quantities, one obtains the linearized equations

$$\frac{D_0\rho'}{Dt} + \rho'\vec{\nabla}\cdot\vec{U}_0 + \vec{\nabla}\cdot(\rho_0\vec{u}) \;=\; 0 \tag{5}$$

$$\rho_0(\frac{D_0\vec{u}}{Dt} + \vec{u}\cdot\vec{\nabla}\vec{U}_0) + \rho'\vec{U}_0\cdot\vec{\nabla}\vec{U}_0 \;=\; -\vec{\nabla}p' \tag{6}$$

$$\frac{D_0 s'}{Dt} \;=\; 0, \tag{7}$$

where $\frac{D_0}{Dt} = \frac{\partial}{\partial t} + \vec{U}_0\cdot\vec{\nabla}$ is the convective derivative associated with the mean flow.

If the mean velocity $\vec{U}_0$ can be expressed as the gradient of a potential Φ_0, then equations (5) - (7) can be reduced to a single, non-constant coefficient, inhomogeneous convective wave equation (refs. 1,2)

$$\frac{D_0}{Dt}(\frac{1}{{c_0}^2}\frac{D_0\phi}{Dt}) - \frac{1}{\rho_0}\vec{\nabla}\cdot(\rho_0\vec{\nabla}\phi) \;=\; \frac{1}{\rho_0}\vec{\nabla}\cdot(\rho_0\vec{u}^{(R)}), \tag{8}$$

where the unsteady velocity is decomposed into a known vortical component $\vec{u}^{(R)}$ and an unknown potential component $\vec{\nabla}\phi$,

$$\vec{u}(\vec{x},t) \;=\; \vec{u}^{(R)} + \vec{\nabla}\phi. \tag{9}$$

The unsteady pressure is given by

$$p' \;=\; -\rho_0(\vec{x})\frac{D_0\phi}{Dt}. \tag{10}$$

An unsteady aerodynamic code, called GUST3D (ref. 3), has been developed to solve equation (8) for flows with periodic vortical disturbances. The code uses a frequency-domain approach with second-order central differences and a pressure radiation condition in the far field. GUST3D requires as input certain mean flow quantities which are calculated separately by a potential flow solver. This solver calculates the mean flow using a Gothert's Rule approximation (ref. 3). On the airfoil surface, it uses the solution calculated by the potential code FLO36 (ref. 4). Figures 1-2 show the mean pressure along the airfoil surface for the two airfoil geometries.

In Figures 3 - 8, we present the RMS pressure on the airfoil surface. Each figure shows three GUST3D solutions (calculated on grids with different far-field boundary locations). Three solutions are shown to provide some indication of the numerical uncertainty in the results.

Figures 9 - 13 present the acoustic intensity. We again show three solutions per case. Note that no results are presented for the $k_1 = k_2 = 2.0$ loaded airfoil case, as an acceptable solution could not be obtained.

A few comments need to be made about the results shown.

First, since the last Workshop, the GUST3D code has been substantially upgraded. This includes implementing a more accurate far-field boundary condition (ref. 5) and developing improved gridding capabilities. This is the reason for any differences that may exist between the present results and results from the last Workshop.

Second, the intensity results on the circle $R = 4\,C$ were obtained using a Kirchoff method (ref. 6). The Kirchoff surface was the circle $R = 2\,C$.

Finally, the GUST3D code is most accurate for low reduced frequencies. A new domain decomposition approach (ref. 7) has been developed to improve accuracy. Both the single domain and domain decomposition approaches were used in generating the present results.

REFERENCES

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3. J.R. Scott and H.M. Atassi, "A Finite-Difference, Frequency-Domain Numerical Scheme for the Solution of the Gust Response Problem," *J. Comp. Phys.*, vol. 119, 1995, pp. 75-93.
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5. J.R. Scott, K.L. Kreider, and J.A. Heminger, "Evaluation of Radiation Boundary Conditions for the Gust Response Problem," Accepted for publication in *AIAA Journal*.
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7. J.R. Scott, H.M. Atassi, and R.F. Susan-Resiga, "A New Domain Decomposition Approach for the Gust Response Problem," AIAA Paper 2003-0883, Jan., 2003.

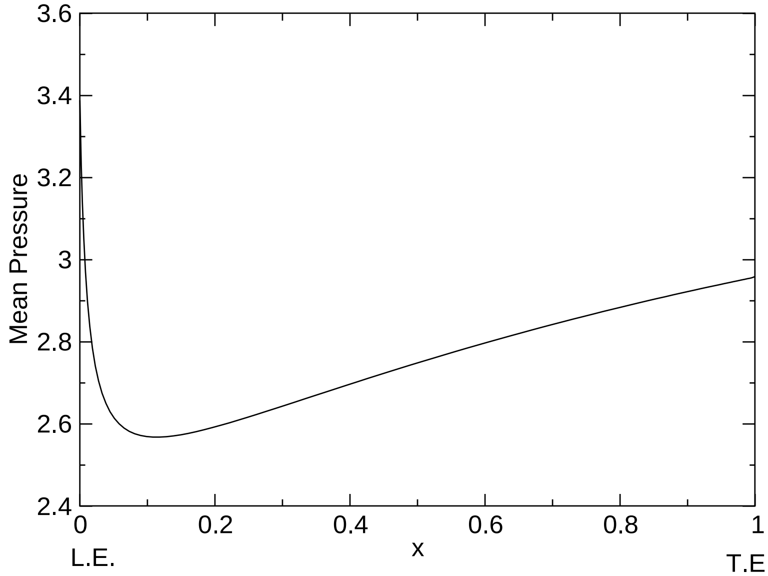

Figure 1 Mean pressure on airfoil surface - Case 1.

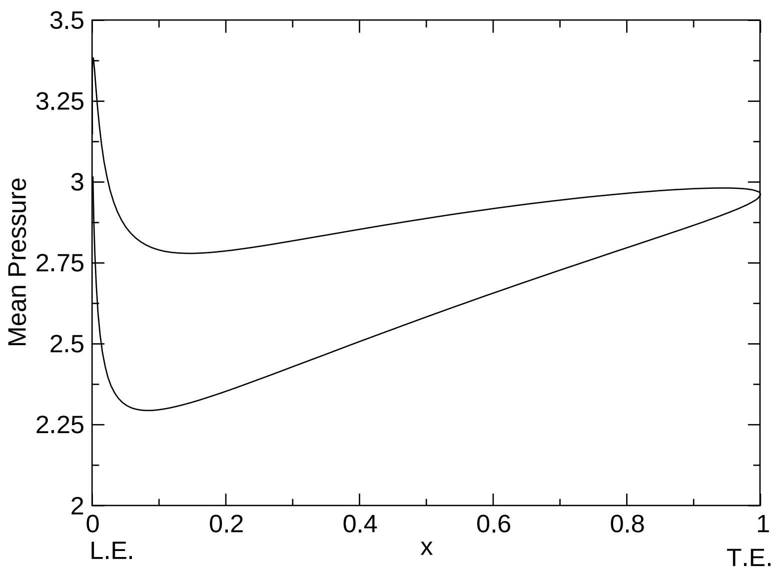

Figure 2 Mean pressure on airfoil surface - Case 2.

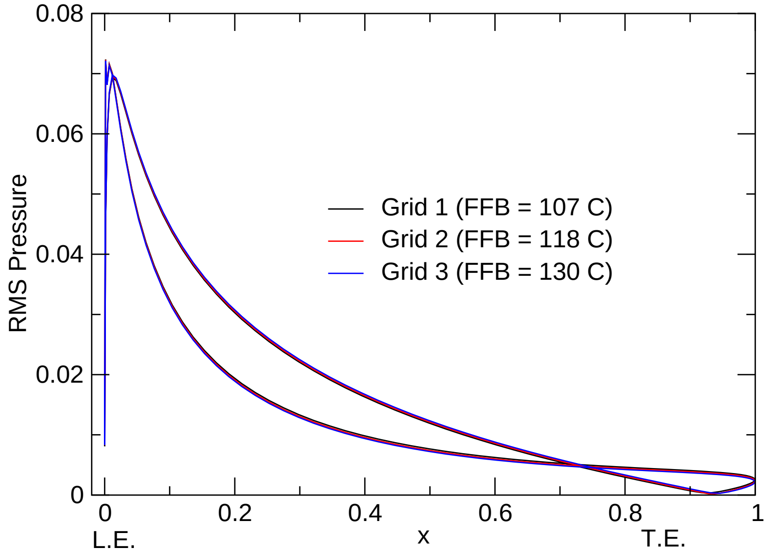

Figure 3 RMS pressure on airfoil surface, Case 1, k1=k2=0.1

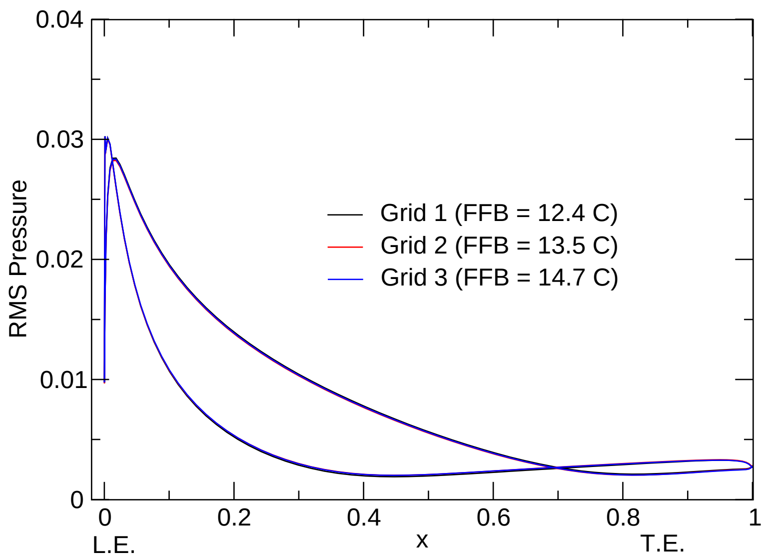

Figure 4 RMS pressure on airfoil surface, Case 1, k1=k2=1.0

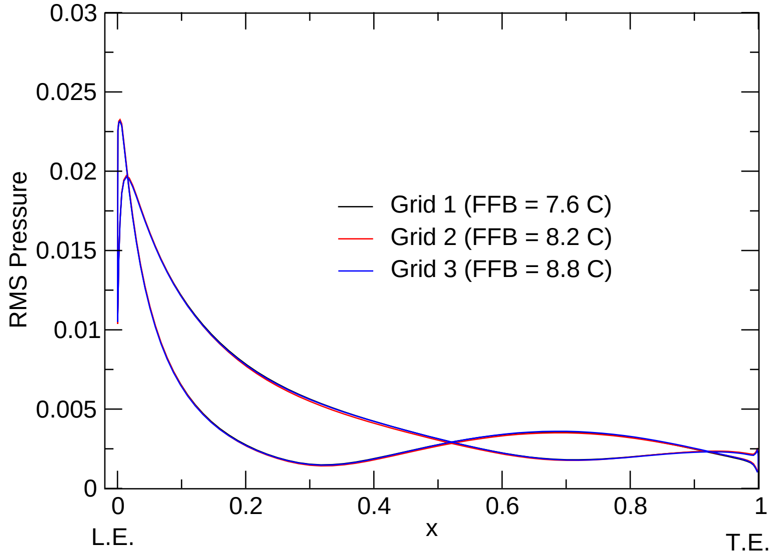

Figure 5 RMS pressure on airfoil surface, Case 1, k1=k2=2.0

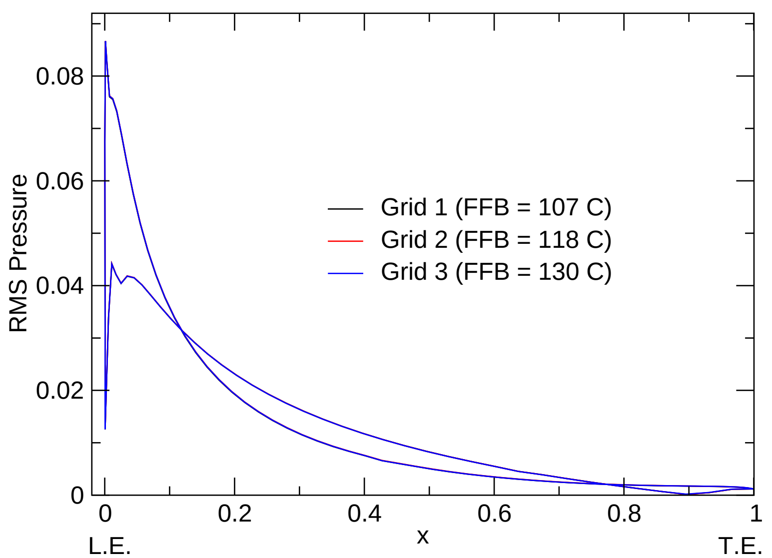

Figure 6 RMS pressure on airfoil surface, Case 2, k1=k2=0.1

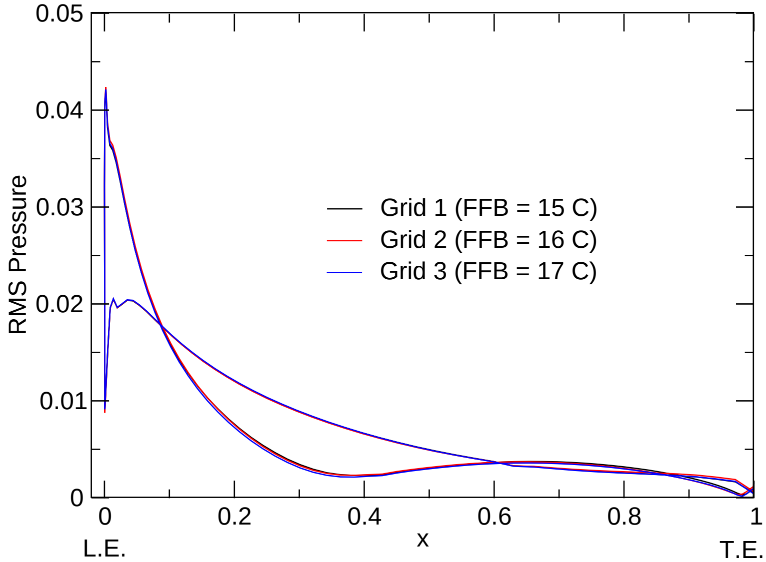

Figure 7   RMS pressure on airfoil surface, Case 2, k1=k2=1.0

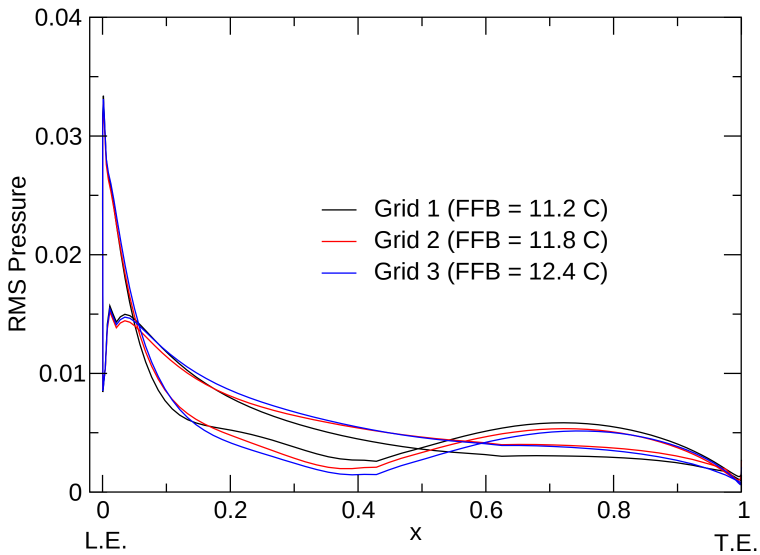

Figure 8   RMS pressure on airfoil surface, Case 2, k1=k2=2.0

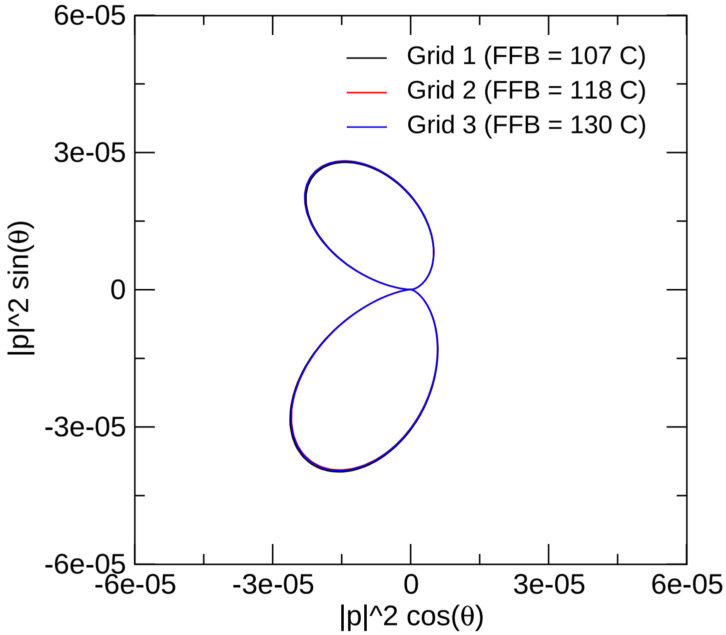

Figure 9.a Acoustic intensity on circle R = 1 C,
Case 1, k1 = k2 = 0.1.

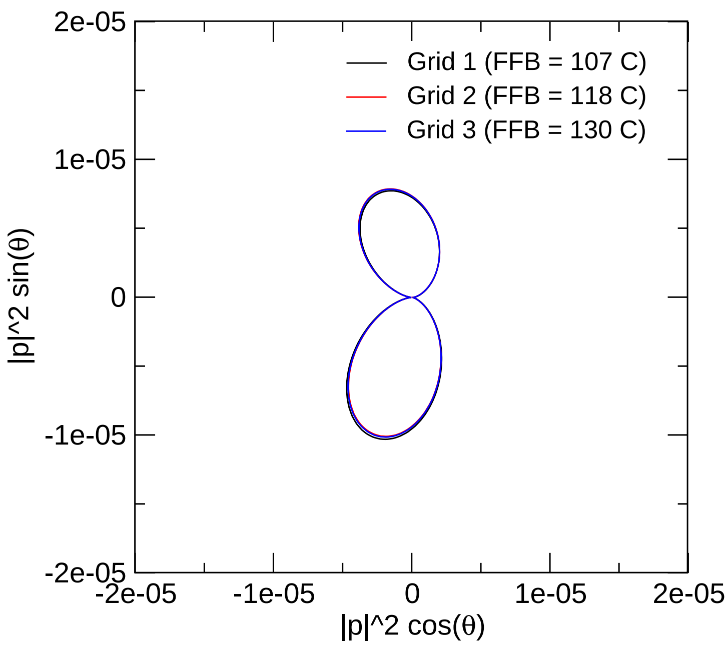

Figure 9.b Acoustic intensity on circle R = 2 C,
Case 1, k1 = k2 = 0.1.

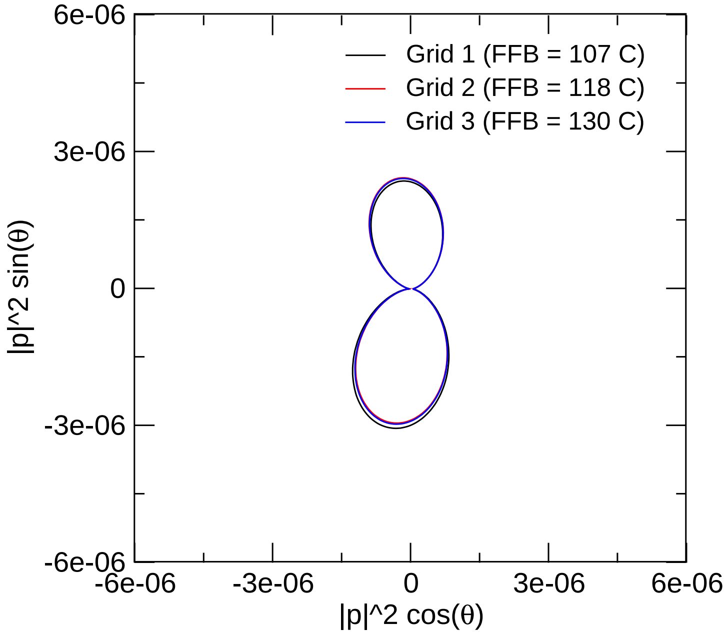

Figure 9.c Acoustic intensity on circle R = 4 C,
Case 1, k1 = k2 = 0.1.

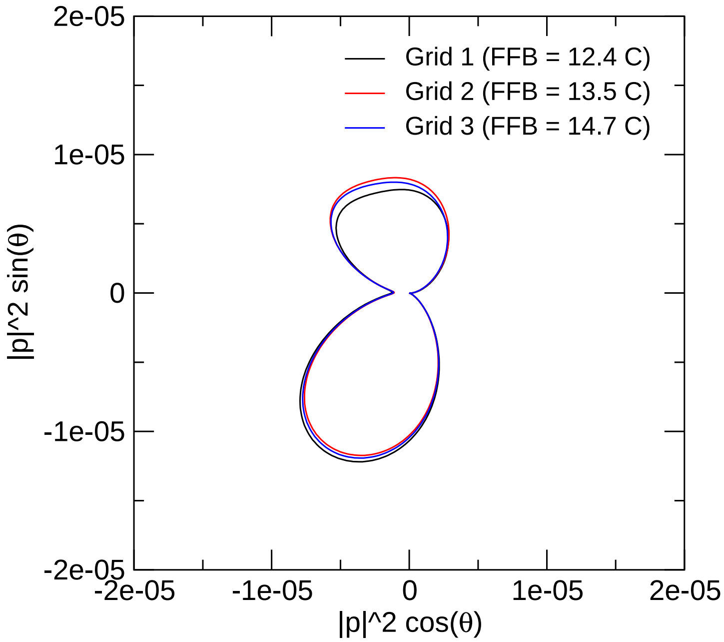

Figure 10.a Acoustic intensity on circle R = 1 C,
Case 1, k1 = k2 = 1.0.

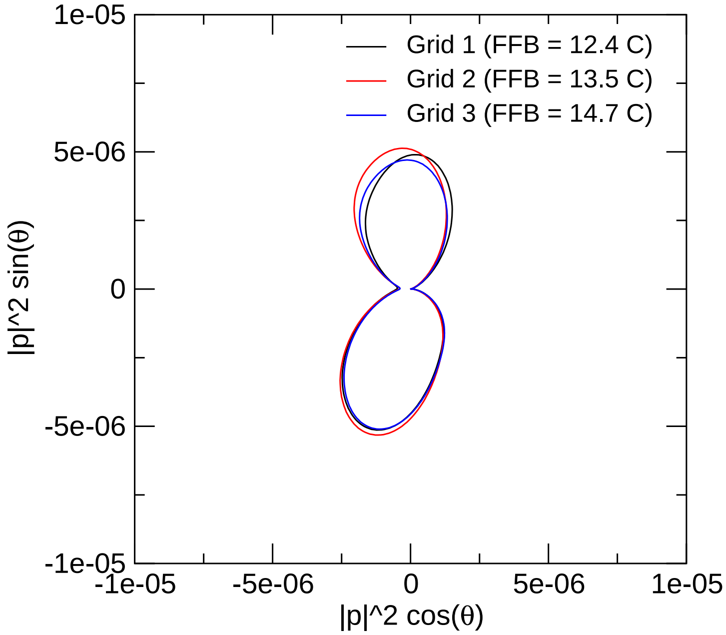

Figure 10.b Acoustic intensity on circle R = 2 C,
Case 1, k1 = k2 = 1.0.

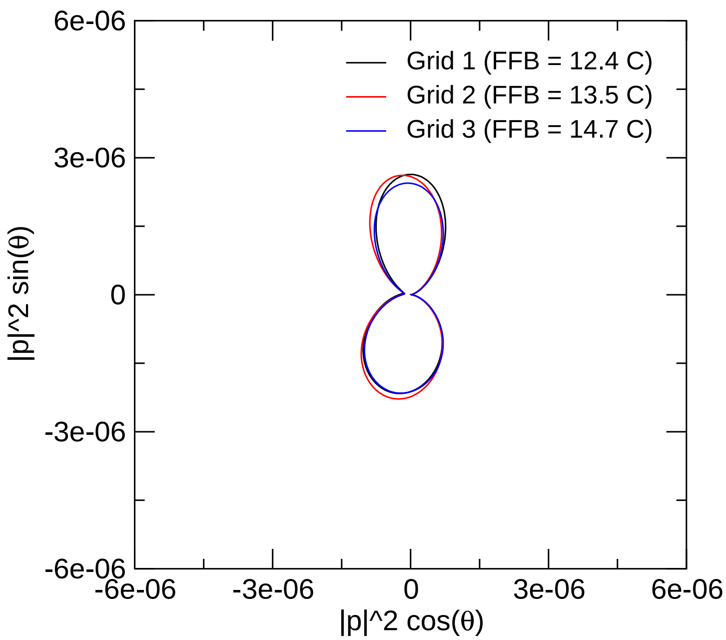

Figure 10.c Acoustic intensity on circle R = 4 C,
Case 1, k1 = k2 = 1.0.

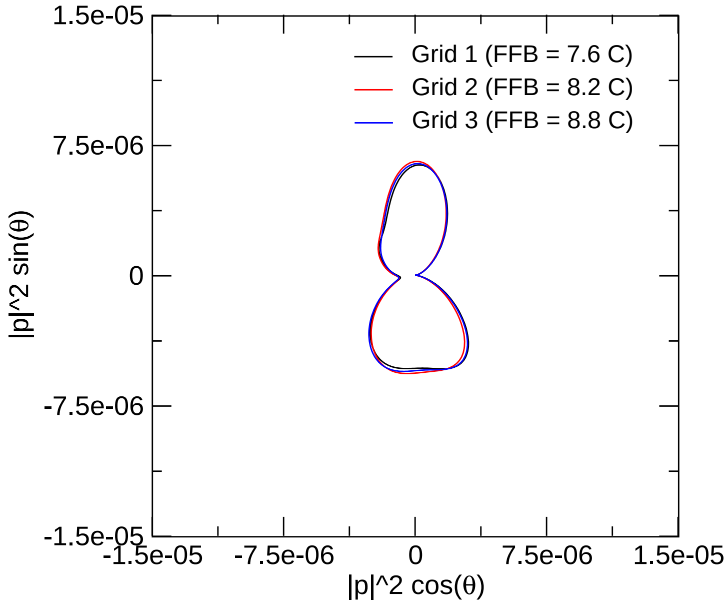

Figure 11.a Acoustic intensity on circle R = 1 C, Case 1, k1 = k2 = 2.0.

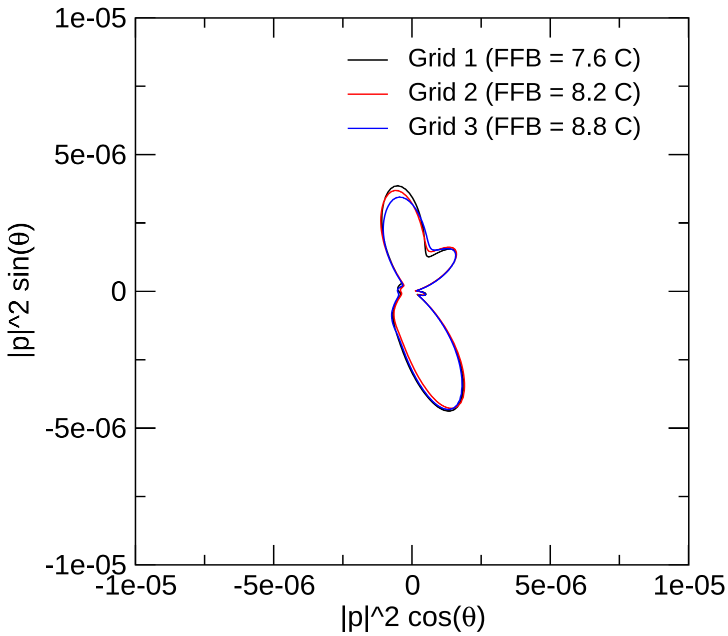

Figure 11.b Acoustic intensity on circle R = 2 C, Case 1, k1 = k2 = 2.0.

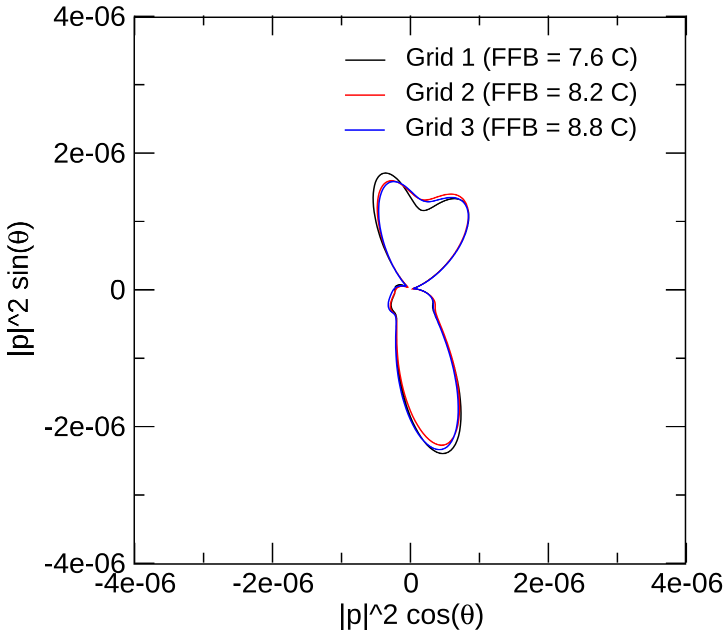

Figure 11.c Acoustic intensity on circle R = 4 C,
Case 1, k1 = k2 = 2.0.

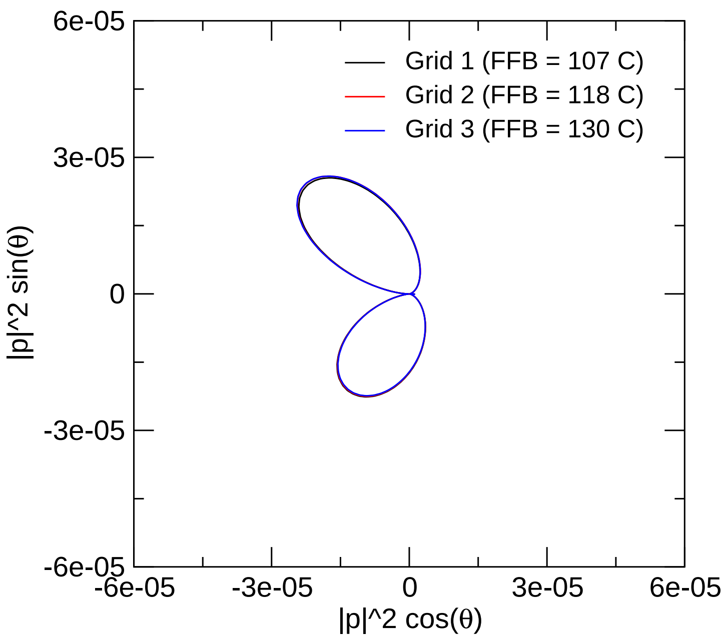

Figure 12.a Acoustic intensity on circle R = 1 C,
Case 2, k1 = k2 = 0.1.

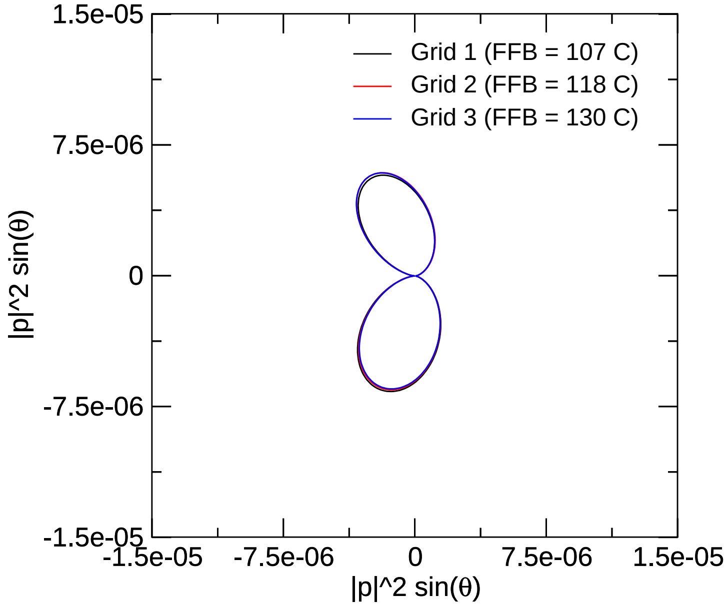

Figure 12.b Acoustic intensity on circle R = 2 C,
Case 2, k1 = k2 = 0.1.

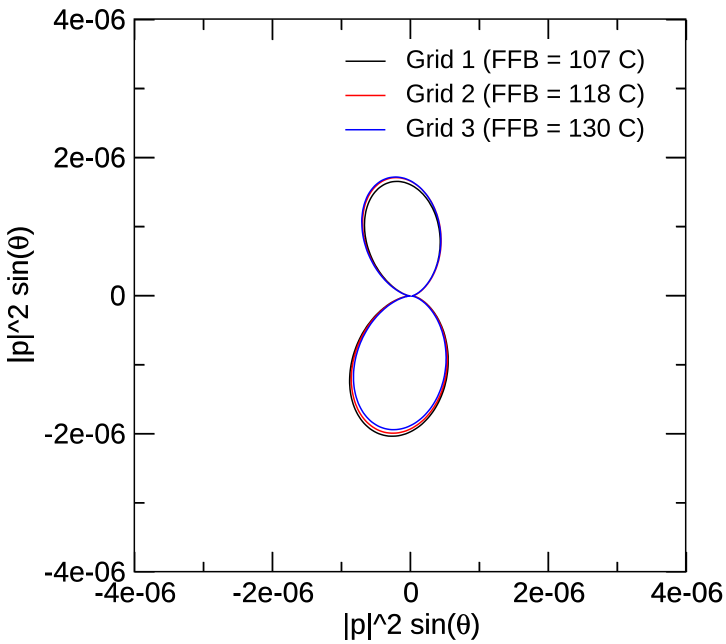

Figure 12.c Acoustic intensity on circle R = 4 C,
Case 2, k1 = k2 = 0.1.

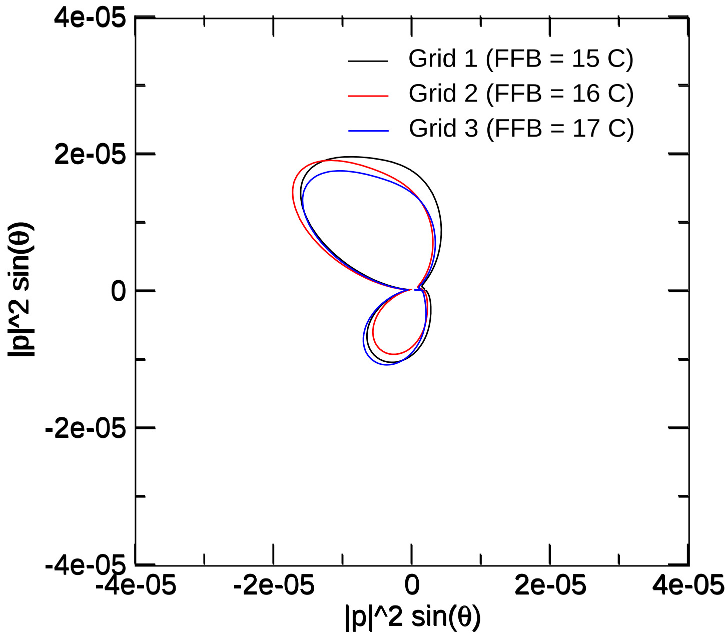

Figure 13.a Acoustic intensity on circle R = 1 C, Case 2, k1 = k2 = 1.0.

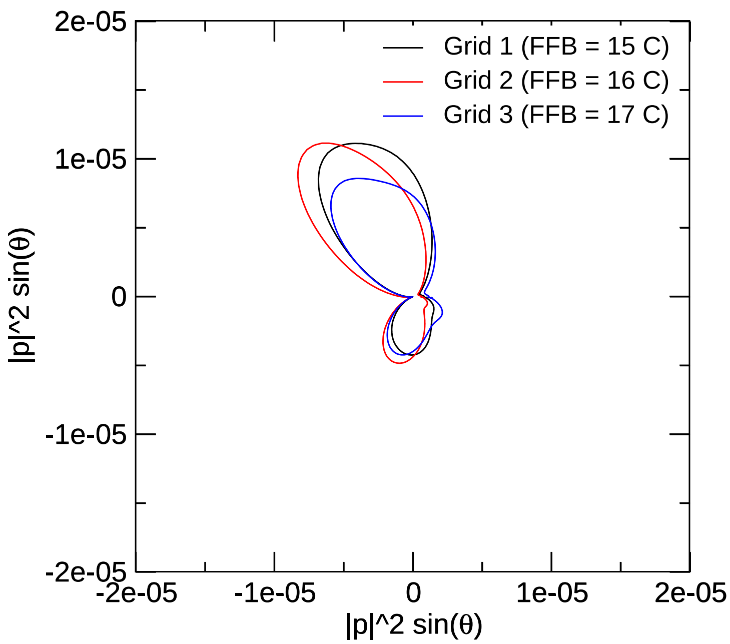

Figure 13.b Acoustic intensity on circle R = 2 C, Case 2, k1 = k2 = 1.0.

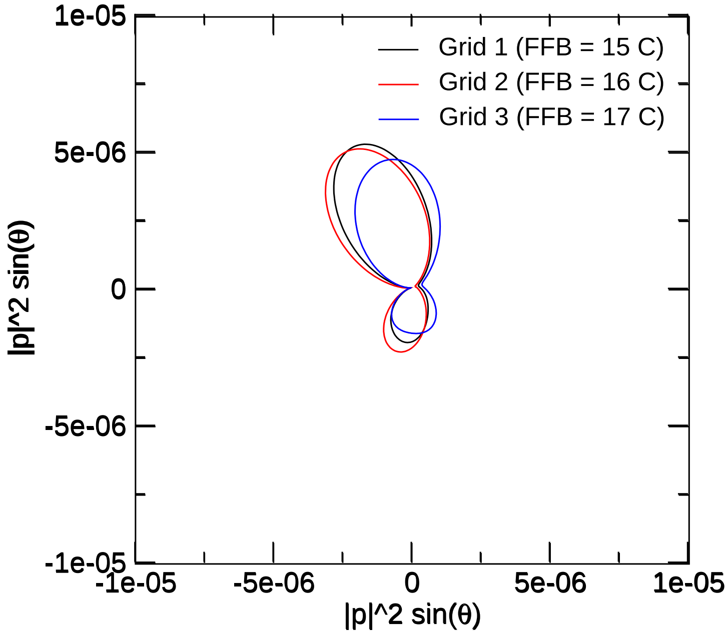

Figure 13.c Acoustic intensity on circle R = 4 C,
Case 2, k1 = k2 = 1.0.