PROBLEM 2—CASCADE-GUST INTERACTION

Geometry

The two-dimensional geometry, shown in Fig. 1, is the unrolled section of a realistic three-dimensional fan outlet guide vane stator. The cascade has a gap-to-chord ratio of $d/c = 2/3$ with the inflow and outflow planes located at $x_{\mp} = \mp 3/2c$. The airfoil definition is given in the accompanying ASCII file and reproduced at the end of this note.

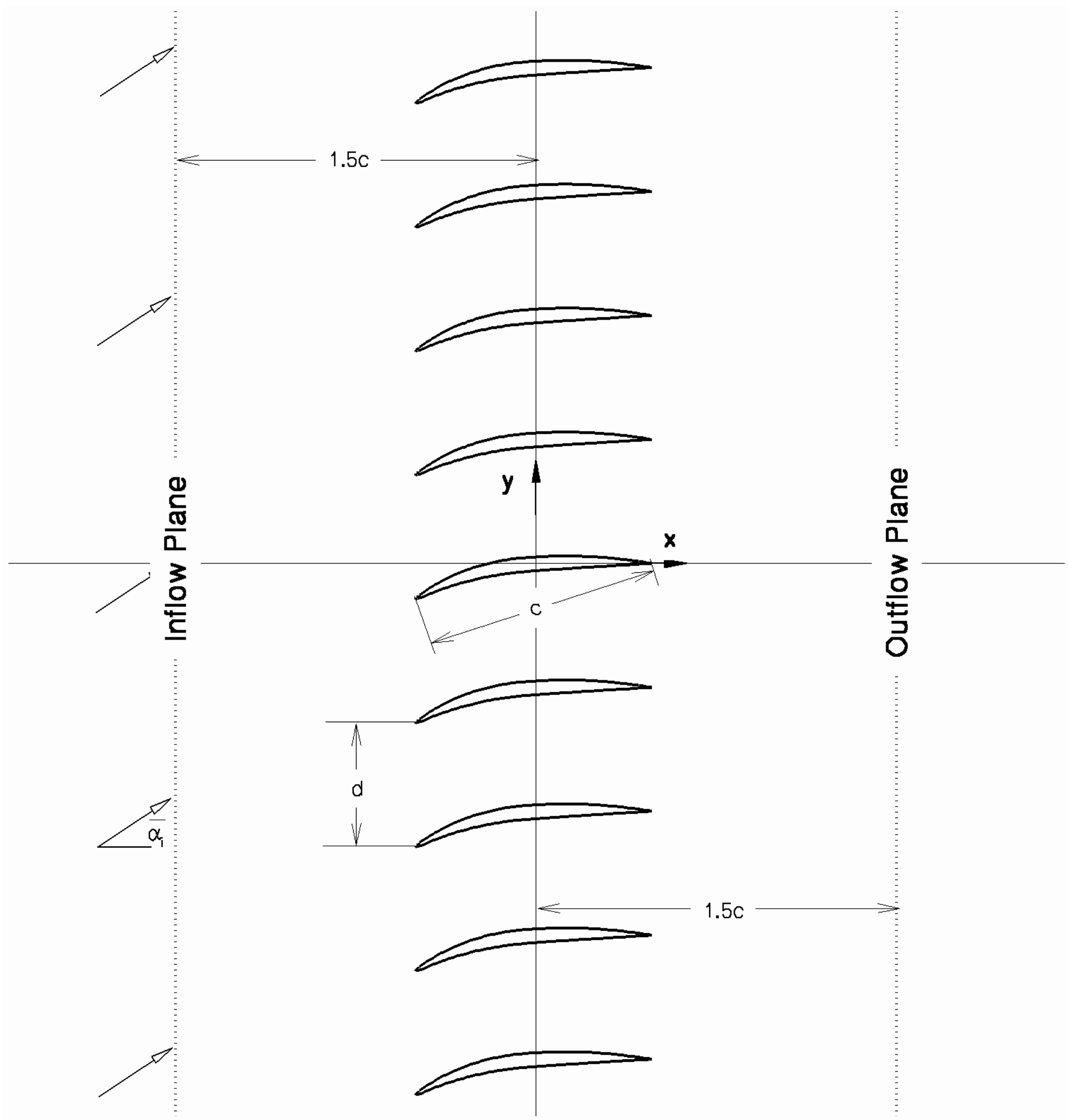

Fig. 1 - Stator Cascade Geometry

Inflow/Outflow Conditions and Gust Input

The mean (i.e., time-averaged) inflow/outflow conditions are:

$$\text{inflow conditions:}\quad \begin{aligned} \bar{P}_i &= 1 \\ \bar{T}_i &= 1 \\ \bar{\alpha}_i &= 36^\circ \end{aligned} \quad , \qquad \text{outflow condition:}\quad \bar{p}_o / \bar{P}_i = 0.92$$

where $\bar{P}_i$ and $\bar{T}_i$ are the normalized inflow plane mean stagnation pressure and mean stagnation temperature. $\bar{\alpha}_i$ is the mean flow angle and $\bar{p}_o$ the normalized outflow plane mean static pressure. Assume the flow to be inviscid and isentropic throughout the domain and that the reference conditions used for normalization are $P_{\text{ref}} = 2116.8 \text{ lb}_\text{f}/\text{ft}^2$, $T_{\text{ref}} = 519\ ^\circ\text{R}$.

The inflow gust (produced, say, by the wake of an upstream blade row) is given, <u>at the inflow plane</u>, by

$$\vec{u}_g(y,t) = \left\{ a_1 \cos\left(k_y y - \omega t\right) + a_2 \cos\left(2\left(k_y y - \omega t\right)\right) + a_3 \cos\left(3\left(k_y y - \omega t\right)\right) \right\} \hat{e}_\beta$$

$$\rho_g(y,t) = 0, \qquad p_g(y,t) = 0$$

$$\hat{e}_\beta = \cos(\beta)\,\hat{e}_x - \sin(\beta)\,\hat{e}_y, \qquad \beta = 44^\circ$$

$$\omega = 3\pi/4, \qquad k_y = 11\pi/9, \qquad \begin{aligned} a_1 &= 5 \times 10^{-3} \\ a_2 &= 3 \times 10^{-3} \\ a_3 &= 7 \times 10^{-4} \end{aligned}$$

where ω is the fundamental reduced frequency[1], k_y is the transverse wavenumber[2], and a_i's are the gust harmonic amplitudes[3].

[1] Frequency is normalized by the chord divided by the ambient speed of sound.
[2] Wavenumber is normalized by the vane chord.
[3] Gust harmonic amplitudes are normalized by the ambient speed of sound.

Requirements

Solve the time-dependent inviscid flow equations for this geometry subject to the specified inflow/outflow mean conditions and the fluctuating inflow velocity distortion.

(1) Compute the unsteady solution until periodicity in pressure is achieved by showing that at least two successive periods are identical[4]. Periodicity must be achieved on both the airfoil surface and the inflow/outflow boundaries.

(2) Once periodicity is achieved, compute the pressure frequency spectra on the reference airfoil on both the upper and lower surfaces at $x = (\ \ 0.25c,\ 0.00,\ +0.25c)$, on the inflow boundary at $(x, y) = \{(\ \ 1.5c,\ \text{-}0.3c), (\ \ 1.5c,\ 0.0), (\ \ 1.5c,\ 0.3c)\}$, and on the outflow boundary at $(x, y) = \{(1.5c,\ \text{-}0.3c), (1.5c,\ 0.0), (1.5c,\ 0.3c)\}$. Express the spectral results in dB using the standard definition $20\ \log(p_{\text{r.m.s.}} / p_{\text{ref.}})$, where $p_{\text{ref.}} = 20\ m\text{Pa}$.

(3) Extract the harmonic pressure distributions on the inflow and outflow boundaries (i.e., on $x = \mp 1.5c$ lines) at the fundamental frequency w and apply a Fourier transform in y direction to identify the spatial (i.e., mode order) structure of the pressure perturbations. Express the result in dB for each mode order. Repeat the process for the frequencies $2w$ and $3w$.

Note: The benchmark solution to this problem will be computed using a frequency-domain linearized Euler code called LINFLUX which has been extensively tested at United Technology Research Center and NASA Glenn Research Center.

Contributed by Ed Envia, Edmane.Envia-1@nasa.gov.

[4] The maximum difference between the spectra of two successive periods must be less than 1% at any of the three input frequencies.

Airfoil Section Data[5]

Suction Side		Pressure Side	
x	*y*	*x*	*y*
-0.5000000	-0.1901280	-0.5000000	-0.1901280
-0.5002920	-0.1894140	-0.4993730	-0.1905700
-0.5002540	-0.1884780	-0.4984500	-0.1907420
-0.4999420	-0.1873940	-0.4973270	-0.1906820
-0.4994300	-0.1862410	-0.4960960	-0.1904450
-0.4984710	-0.1845490	-0.4942390	-0.1899110
-0.4961960	-0.1812740	-0.4905490	-0.1884920
-0.4915950	-0.1757120	-0.4840870	-0.1854640
-0.4836860	-0.1674000	-0.4741110	-0.1801880
-0.4716500	-0.1560880	-0.4599970	-0.1722870
-0.4550310	-0.1414710	-0.4410590	-0.1619740
-0.4443170	-0.1324970	-0.4290580	-0.1557100
-0.4334350	-0.1237260	-0.4169740	-0.1496080
-0.4201140	-0.1134380	-0.4024270	-0.1425270
-0.4065630	-0.1034570	-0.3877700	-0.1356790
-0.3959670	-0.0959786	-0.3764920	-0.1306010
-0.3852400	-0.0886887	-0.3651540	-0.1256600
-0.3743860	-0.0815891	-0.3537570	-0.1208550
-0.3621360	-0.0738989	-0.3410850	-0.1156980
-0.3497360	-0.0664518	-0.3283470	-0.1107070
-0.3371910	-0.0592516	-0.3155440	-0.1058820
-0.3234920	-0.0517603	-0.3017670	-0.1009010
-0.3096350	-0.0445642	-0.2879220	-0.0961104
-0.2956260	-0.0376667	-0.2740120	-0.0915107
-0.2845230	-0.0324608	-0.2631260	-0.0880571
-0.2733330	-0.0274428	-0.2522040	-0.0847192
-0.2620600	-0.0226143	-0.2412460	-0.0814972
-0.2507070	-0.0179772	-0.2302560	-0.0783912
-0.2390860	-0.0134608	-0.2191170	-0.0753706
-0.2273890	-0.0091454	-0.2079460	-0.0724691
-0.2156200	-0.0050325	-0.1967460	-0.0696870
-0.2037810	-0.0011236	-0.1855160	-0.0670250
-0.1918560	0.0025865	-0.1742920	-0.0644907
-0.1798690	0.0060908	-0.1630430	-0.0620746
-0.1678230	0.0093899	-0.1517680	-0.0597753
-0.1557230	0.0124843	-0.1404710	-0.0575911
-0.1435580	0.0153764	-0.1291750	-0.0555260
-0.1313440	0.0180566	-0.1178570	-0.0535799
-0.1190850	0.0205178	-0.1065200	-0.0517589
-0.1067840	0.0227529	-0.0951624	-0.0500690
-0.0944786	0.0247549	-0.0837561	-0.0485073

[5] These coordinates are normalized by the vane chord.

-0.0821404	0.0265503	-0.0723342	-0.0470608
-0.0697762	0.0281647	-0.0609005	-0.0457072
-0.0573926	0.0296234	-0.0494585	-0.0444238
-0.0436985	0.0310812	-0.0361405	-0.0429933
-0.0299907	0.0323960	-0.0228173	-0.0416151
-0.0162715	0.0335834	-0.0094903	-0.0402770
-0.0025430	0.0346593	0.0038395	-0.0389665
0.0111930	0.0356362	0.0171762	-0.0376738
0.0249356	0.0365136	0.0305150	-0.0364027
0.0386844	0.0372880	0.0438564	-0.0351594
0.0524387	0.0379556	0.0572009	-0.0339503
0.0661944	0.0385130	0.0705583	-0.0327780
0.0799541	0.0389579	0.0839183	-0.0316348
0.0937171	0.0392878	0.0972799	-0.0305095
0.1074820	0.0395006	0.1106420	-0.0293905
0.1212460	0.0395944	0.1240260	-0.0282670
0.1350090	0.0395694	0.1374110	-0.0271416
0.1487720	0.0394264	0.1507950	-0.0260174
0.1625330	0.0391659	0.1641800	-0.0248975
0.1759700	0.0387987	0.1772820	-0.0238079
0.1894040	0.0383201	0.1903840	-0.0227245
0.2028330	0.0377299	0.2034870	-0.0216466
0.2162570	0.0370281	0.2165910	-0.0205733
0.2287510	0.0362742	0.2288210	-0.0195753
0.2412380	0.0354237	0.2410520	-0.0185808
0.2537190	0.0344771	0.2532830	-0.0175899
0.2661920	0.0334347	0.2655140	-0.0166029
0.2809030	0.0320815	0.2799840	-0.0154403
0.2956010	0.0305956	0.2944550	-0.0142832
0.3102850	0.0289773	0.3089260	-0.0131318
0.3225580	0.0275219	0.3210570	-0.0121710
0.3348200	0.0259744	0.3331870	-0.0112142
0.3470710	0.0243354	0.3453180	-0.0102617
0.3614500	0.0222929	0.3596040	-0.0091454
0.3758110	0.0201246	0.3738900	-0.0080337
0.3965670	0.0167648	0.3945990	-0.0064307
0.4101550	0.0144220	0.4081910	-0.0053863
0.4180750	0.0130020	0.4161300	-0.0047777
0.4253860	0.0116572	0.4234920	-0.0042149
0.4306480	0.0106702	0.4290990	-0.0037887
0.4434420	0.0081752	0.4422550	-0.0027922
0.4562150	0.0055715	0.4554110	-0.0017987
0.4689730	0.0028954	0.4685680	-0.0008073
0.4817240	0.0001831	0.4817240	0.0001831