PROBLEM 1—SINGLE AIRFOIL GUST RESPONSE

James R. Scott
NASA Glenn Research Center

The purpose of this problem is to test the ability of a CFD/CAA code to accurately predict the unsteady aerodynamic and aeroacoustic response of a single airfoil to a two-dimensional, periodic vortical gust.

Consider the airfoil configuration shown in Figure 1. The airfoil has chord length c and angle of attack α. The upstream velocity is

$$\vec{U} \;=\; U_\infty \,\vec{\mathrm{i}} \;+\; \vec{a}\, \cos[\vec{k} \cdot (\vec{x} - \vec{\mathrm{i}}\, U_\infty t)] \tag{1}$$

where $\vec{x} = (x_1, x_2)$ denotes the spatial coordinates, $\vec{a} = (a_1, a_2)$ is the gust amplitude vector with $a_1 = -\,\epsilon\, U_\infty\, k_2/|\vec{k}|$, $a_2 = \epsilon\, U_\infty\, k_1/|\vec{k}|$, $\vec{k}$ is the wave number vector, and ϵ is a small parameter satisfying $\epsilon \ll 1$.

The governing equations are the 2-D Euler equations

$$\frac{\partial \rho}{\partial t} \;+\; \frac{\partial}{\partial x}(\rho\, u) \;+\; \frac{\partial}{\partial y}(\rho\, v) \;=\; 0 \tag{2}$$

$$\frac{\partial}{\partial t}(\rho u) \;+\; \frac{\partial}{\partial x}(\rho\, u^2 + p) \;+\; \frac{\partial}{\partial y}(\rho\, u\, v) \;=\; 0 \tag{3}$$

$$\frac{\partial}{\partial t}(\rho v) \;+\; \frac{\partial}{\partial x}(\rho\, u\, v) \;+\; \frac{\partial}{\partial y}(\rho\, v^2 + p) \;=\; 0 \tag{4}$$

$$\frac{\partial E_t}{\partial t} \;+\; \frac{\partial}{\partial x}[(E_t + p)\, u] \;+\; \frac{\partial}{\partial y}[(E_t + p)\, v] \;=\; 0 \tag{5}$$

where ρ, u, v, p and E_t denote the fluid density, velocity, pressure, and internal energy per unit volume.

Since the gust amplitude $\vec{a}$ satisfies $|\vec{a}| \ll U_\infty$, one can alternatively solve the linearized unsteady Euler equations

$$\frac{D_0 \rho'}{Dt} + \rho' \vec{\nabla} \cdot \vec{U}_0 + \vec{\nabla} \cdot (\rho_0 \vec{u}) = 0 \tag{6}$$

$$\rho_0 (\frac{D_0 \vec{u}}{Dt} + \vec{u} \cdot \vec{\nabla} \vec{U}_0) + \rho' \vec{U}_0 \cdot \vec{\nabla} \vec{U}_0 = -\vec{\nabla} p' \tag{7}$$

$$\frac{D_0 s'}{Dt} = 0, \tag{8}$$

where $\frac{D_0}{Dt} = \frac{\partial}{\partial t} + \vec{U}_0 \cdot \vec{\nabla}$ is the material derivative associated with the mean flow, $\vec{u} = (u', v')$, primed quantities are the unknown perturbation variables, and "$_0$" subscripts denote steady mean flow quantities which must be independently solved for and are assumed to be known.

Nondimensionalize the Euler equations as follows:

x_1, x_2	by	$\frac{c}{2}$
$\vec{U} = (u, v)$	by	U_∞
c_0 (sound speed)	by	U_∞
ρ	by	ρ_∞
p	by	$\rho_\infty U_\infty^2$
T	by	T_∞
t	by	$\frac{c}{2U_\infty}$
$\omega = k_1 U_\infty$	by	$\frac{2U_\infty}{c}$
k_1, k_2	by	$\frac{2}{c}$

If solving the linearized unsteady Euler equations, nondimensionalize the mean flow variables as above, and the perturbation variables as follows:

$\vec{u} = (u', v')$	by	U_∞
ρ'	by	ρ_∞
p'	by	$\rho_\infty U_\infty^2$
T'	by	T_∞
$\vec{a}$	by	U_∞

For the following two cases, solve the gust response problem for a Joukowski airfoil in a two-dimensional gust with $k_2 = k_1$ for reduced frequencies $k_1 = 0.1$, 1.0, and 2.0. The nondimensional upstream velocity is $\vec{U} = \vec{\mathrm{i}} + \epsilon\, \vec{a}\, \cos(\vec{k} \cdot \vec{x} - k_1\, t)$, where $\vec{a} = (a_1, a_2) = (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$. Take $\epsilon = .02$.

For Case 1, the airfoil has a 12% thickness ratio, free stream Mach number $M_\infty = 0.5$, angle of attack $\alpha = 0^o$, and a camber ratio of zero.

For Case 2, change α to 2^o and the camber ratio to .02.

The airfoil geometries can be generated as follows. Set

$$\zeta_1 = r_0\, e^{i\,\theta} \;+\; \zeta_{0'} \tag{9}$$

where

$$\zeta_{0'} \;=\; -\,\epsilon_1 \;+\; i\,\epsilon_2 \tag{10}$$

is a complex constant. Letting $z = x + i\,y$ denote the airfoil coordinates in the complex z plane, the transformation

$$z = \left(\zeta_1 \;+\; \frac{d^2}{\zeta_1}\right) e^{-\,i\alpha} \tag{11}$$

transforms the ζ_1 circle defined by equation (9) into the desired airfoil shape.

For Case 1, use $r_0 = 0.54632753$, $\epsilon_1 = 0.05062004$, $\epsilon_2 = 0$, $d^2 = 0.24572591$, $\alpha = 0$. Discretize the ζ_1 circle in θ, starting from 0 and going to $2\,\pi$, and then apply equation (11) to get the airfoil coordinates. The values $\theta = 0$ and $\theta = 2\,\pi$ map into the trailing edge point.

For Case 2, use $r_0 = 0.54676443$, $\epsilon_1 = 0.05062004$, $\epsilon_2 = 0.02185310$, $d^2 = 0.24572591$, $\alpha = 0.034906585$. Discretize the ζ_1 circle in θ, starting from $\theta = -\beta$ and going to $\theta = 2\pi - \beta$, where $\beta = 0.039978687$, and then apply equation (11) to get the airfoil coordinates. The values $\theta = -\beta$ and $\theta = 2\pi - \beta$, map into the trailing edge point.

The above procedure for generating the airfoil geometries will generate a Joukowski airfoil of chord length 2, situated very nearly between $x = -1$ and $x = 1$. The airfoil geometries are shown in Figure 2.

For both Case 1 and Case 2, march the discrete equations in time until the solution becomes periodic. On the airfoil surface, calculate the RMS pressure $\sqrt{\overline{(p')^2}}$. In the far field, calculate the intensity $\overline{(p')^2}$ at the following three locations: (i) on a circle of radius $R = 2$ (one chord length) centered about the airfoil center; (ii) on a circle of radius $R = 4$ (two chord lengths); (iii) on a circle of radius $R = 8$ (four chord lengths).

State whether the solution is from the Euler equations or linearized Euler equations. Also state the grid dimensions for each calculation, the number of complete periods computed, the CPU time per period, and the type of machine the calculations were run on.

email: James.R.Scott@nasa.gov

Problem Author: James R. Scott

Problem Submitted By: Milo D. Dahl

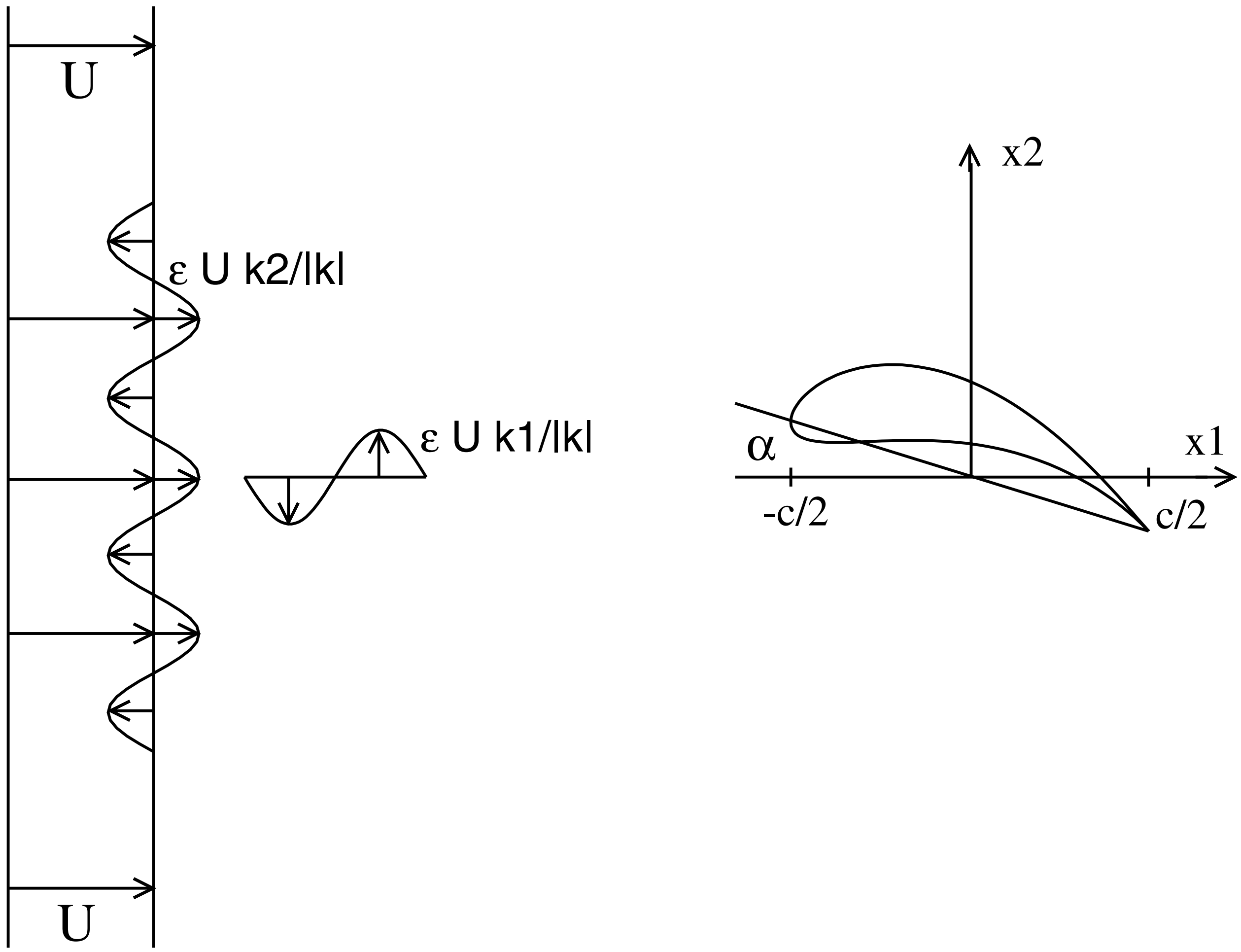

Figure 1   Airfoil in a two-dimensional, periodic gust.

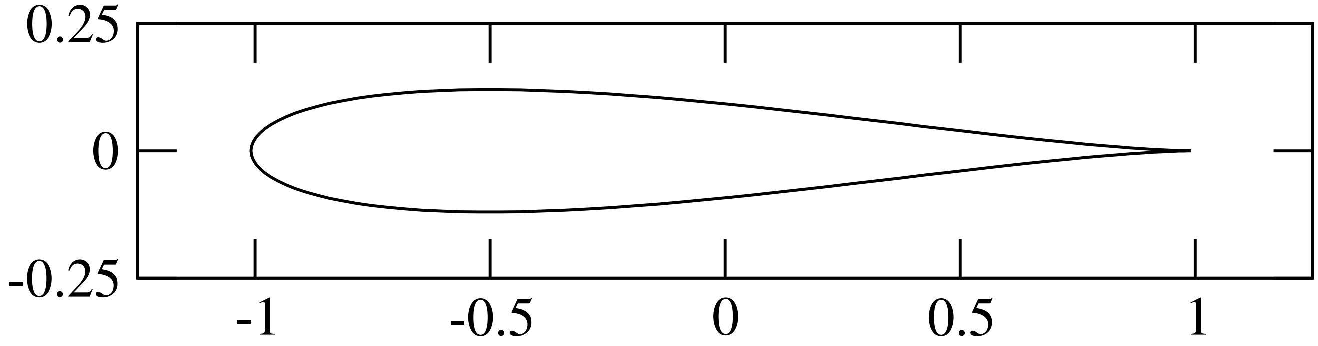

Figure 2a   Airfoil geometry for Case 1.

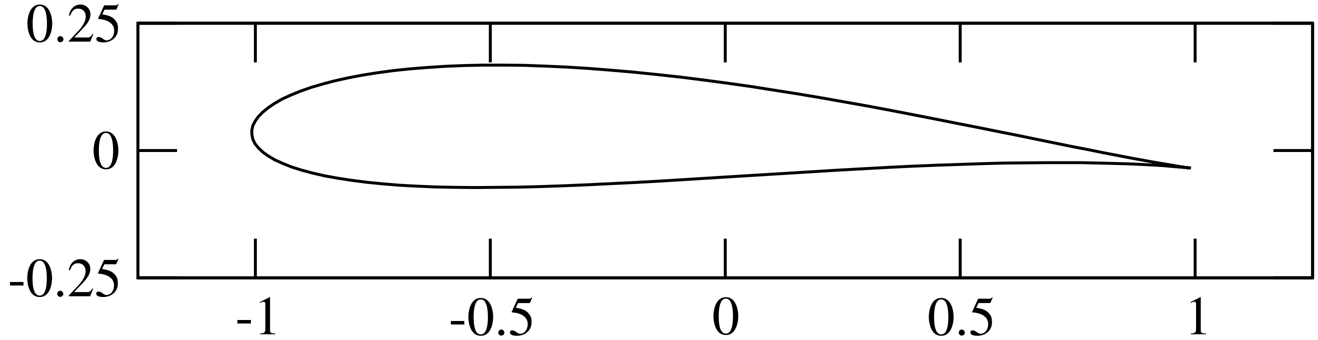

Figure 2b   Airfoil geometry for Case 2.

PROBLEM 2—CASCADE-GUST INTERACTION

Geometry

The two-dimensional geometry, shown in Fig. 1, is the unrolled section of a realistic three-dimensional fan outlet guide vane stator. The cascade has a gap-to-chord ratio of $d/c = 2/3$ with the inflow and outflow planes located at $x_{\mp} = \mp 3/2c$. The airfoil definition is given in the accompanying ASCII file and reproduced at the end of this note.

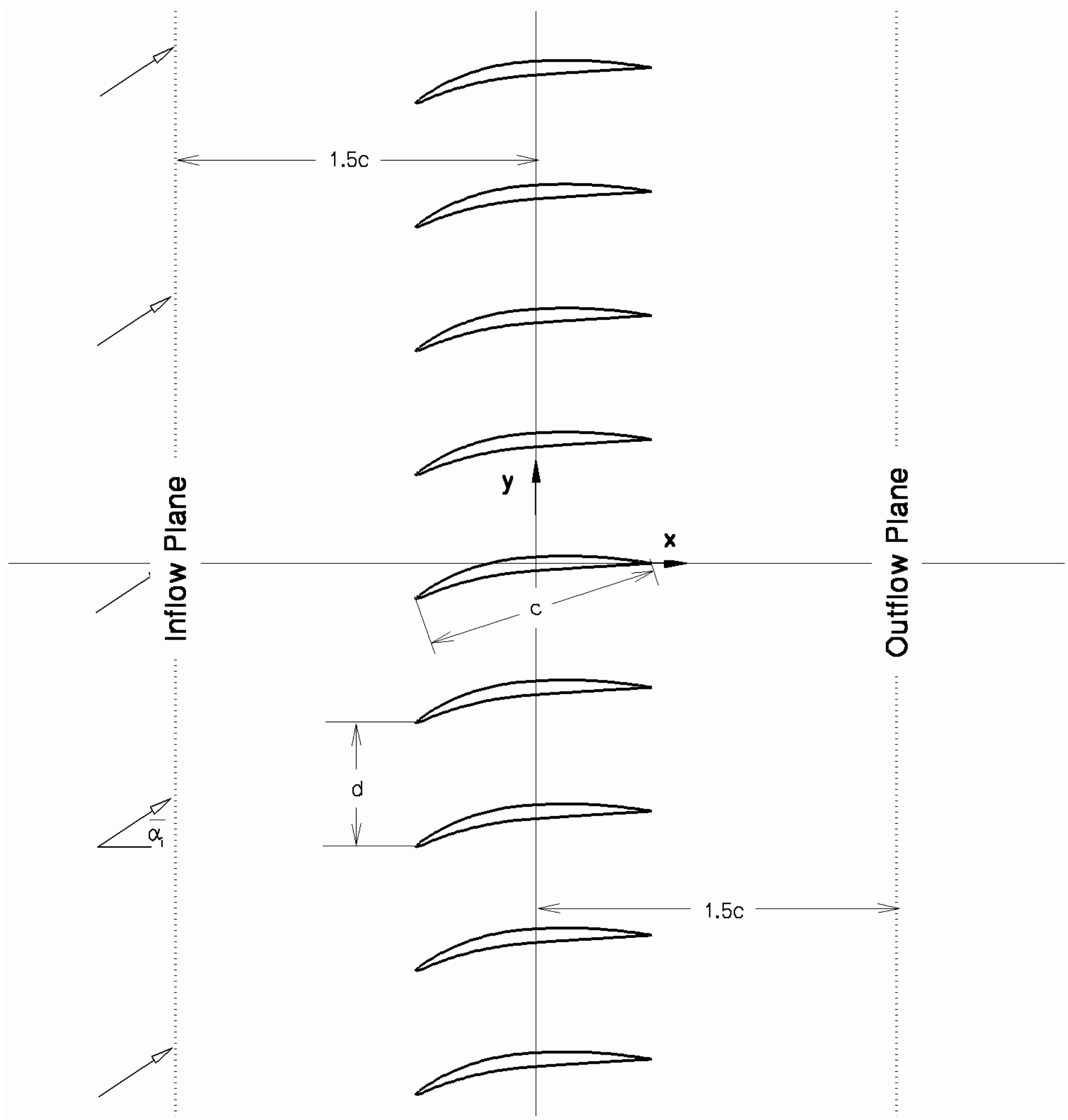

Fig. 1 - Stator Cascade Geometry

Inflow/Outflow Conditions and Gust Input

The mean (i.e., time-averaged) inflow/outflow conditions are:

$$\text{inflow conditions:}\quad \begin{array}{l} \bar{P}_i = 1 \\ \bar{T}_i = 1 \\ \bar{\alpha}_i = 36^\circ \end{array}\quad , \qquad \text{outflow condition:}\quad \bar{p}_o / \bar{P}_i = 0.92$$

where $\bar{P}_i$ and $\bar{T}_i$ are the normalized inflow plane mean stagnation pressure and mean stagnation temperature. $\bar{\alpha}_i$ is the mean flow angle and $\bar{p}_o$ the normalized outflow plane mean static pressure. Assume the flow to be inviscid and isentropic throughout the domain and that the reference conditions used for normalization are $P_{\text{ref}} = 2116.8\ \text{lb}_\text{f}/\text{ft}^2$, $T_{\text{ref}} = 519\ ^\circ\text{R}$.

The inflow gust (produced, say, by the wake of an upstream blade row) is given, <u>at the inflow plane</u>, by

$$\vec{u}_g(y,t) = \left\{ a_1 \cos\left(k_y y - \omega t\right) + a_2 \cos\left(2\left(k_y y - \omega t\right)\right) + a_3 \cos\left(3\left(k_y y - \omega t\right)\right) \right\} \hat{e}_\beta$$

$$\rho_g(y,t) = 0, \qquad p_g(y,t) = 0$$

$$\hat{e}_\beta = \cos(\beta)\,\hat{e}_x - \sin(\beta)\,\hat{e}_y, \qquad \beta = 44^\circ$$

$$\omega = 3\pi/4, \qquad k_y = 11\pi/9, \qquad \begin{array}{l} a_1 = 5 \times 10^{-3} \\ a_2 = 3 \times 10^{-3} \\ a_3 = 7 \times 10^{-4} \end{array}$$

where ω is the fundamental reduced frequency[1], k_y is the transverse wavenumber[2], and a_i's are the gust harmonic amplitudes[3].

[1] Frequency is normalized by the chord divided by the ambient speed of sound.
[2] Wavenumber is normalized by the vane chord.
[3] Gust harmonic amplitudes are normalized by the ambient speed of sound.

Requirements

Solve the time-dependent inviscid flow equations for this geometry subject to the specified inflow/outflow mean conditions and the fluctuating inflow velocity distortion.

(1) Compute the unsteady solution until periodicity in pressure is achieved by showing that at least two successive periods are identical[4]. Periodicity must be achieved on both the airfoil surface and the inflow/outflow boundaries.

(2) Once periodicity is achieved, compute the pressure frequency spectra on the reference airfoil on both the upper and lower surfaces at $x = (\ \ 0.25c,\ 0.00,\ +0.25c)$, on the inflow boundary at $(x, y) = \{(\ \ 1.5c,\ \text{-}0.3c), (\ \ 1.5c,\ 0.0), (\ \ 1.5c,\ 0.3c)\}$, and on the outflow boundary at $(x, y) = \{(1.5c,\ \text{-}0.3c), (1.5c,\ 0.0), (1.5c,\ 0.3c)\}$. Express the spectral results in dB using the standard definition $20\ \log(p_{\text{r.m.s.}} / p_{\text{ref.}})$, where $p_{\text{ref.}} = 20\ m\text{Pa}$.

(3) Extract the harmonic pressure distributions on the inflow and outflow boundaries (i.e., on $x = \mp 1.5c$ lines) at the fundamental frequency w and apply a Fourier transform in y direction to identify the spatial (i.e., mode order) structure of the pressure perturbations. Express the result in dB for each mode order. Repeat the process for the frequencies $2w$ and $3w$.

Note: The benchmark solution to this problem will be computed using a frequency-domain linearized Euler code called LINFLUX which has been extensively tested at United Technology Research Center and NASA Glenn Research Center.

Contributed by Ed Envia, Edmane.Envia-1@nasa.gov.

[4] The maximum difference between the spectra of two successive periods must be less than 1% at any of the three input frequencies.

Airfoil Section Data[5]

Suction Side *x*	Suction Side *y*	Pressure Side *x*	Pressure Side *y*
-0.5000000	-0.1901280	-0.5000000	-0.1901280
-0.5002920	-0.1894140	-0.4993730	-0.1905700
-0.5002540	-0.1884780	-0.4984500	-0.1907420
-0.4999420	-0.1873940	-0.4973270	-0.1906820
-0.4994300	-0.1862410	-0.4960960	-0.1904450
-0.4984710	-0.1845490	-0.4942390	-0.1899110
-0.4961960	-0.1812740	-0.4905490	-0.1884920
-0.4915950	-0.1757120	-0.4840870	-0.1854640
-0.4836860	-0.1674000	-0.4741110	-0.1801880
-0.4716500	-0.1560880	-0.4599970	-0.1722870
-0.4550310	-0.1414710	-0.4410590	-0.1619740
-0.4443170	-0.1324970	-0.4290580	-0.1557100
-0.4334350	-0.1237260	-0.4169740	-0.1496080
-0.4201140	-0.1134380	-0.4024270	-0.1425270
-0.4065630	-0.1034570	-0.3877700	-0.1356790
-0.3959670	-0.0959786	-0.3764920	-0.1306010
-0.3852400	-0.0886887	-0.3651540	-0.1256600
-0.3743860	-0.0815891	-0.3537570	-0.1208550
-0.3621360	-0.0738989	-0.3410850	-0.1156980
-0.3497360	-0.0664518	-0.3283470	-0.1107070
-0.3371910	-0.0592516	-0.3155440	-0.1058820
-0.3234920	-0.0517603	-0.3017670	-0.1009010
-0.3096350	-0.0445642	-0.2879220	-0.0961104
-0.2956260	-0.0376667	-0.2740120	-0.0915107
-0.2845230	-0.0324608	-0.2631260	-0.0880571
-0.2733330	-0.0274428	-0.2522040	-0.0847192
-0.2620600	-0.0226143	-0.2412460	-0.0814972
-0.2507070	-0.0179772	-0.2302560	-0.0783912
-0.2390860	-0.0134608	-0.2191170	-0.0753706
-0.2273890	-0.0091454	-0.2079460	-0.0724691
-0.2156200	-0.0050325	-0.1967460	-0.0696870
-0.2037810	-0.0011236	-0.1855160	-0.0670250
-0.1918560	0.0025865	-0.1742920	-0.0644907
-0.1798690	0.0060908	-0.1630430	-0.0620746
-0.1678230	0.0093899	-0.1517680	-0.0597753
-0.1557230	0.0124843	-0.1404710	-0.0575911
-0.1435580	0.0153764	-0.1291750	-0.0555260
-0.1313440	0.0180566	-0.1178570	-0.0535799
-0.1190850	0.0205178	-0.1065200	-0.0517589
-0.1067840	0.0227529	-0.0951624	-0.0500690
-0.0944786	0.0247549	-0.0837561	-0.0485073

[5] These coordinates are normalized by the vane chord.

-0.0821404	0.0265503	-0.0723342	-0.0470608
-0.0697762	0.0281647	-0.0609005	-0.0457072
-0.0573926	0.0296234	-0.0494585	-0.0444238
-0.0436985	0.0310812	-0.0361405	-0.0429933
-0.0299907	0.0323960	-0.0228173	-0.0416151
-0.0162715	0.0335834	-0.0094903	-0.0402770
-0.0025430	0.0346593	0.0038395	-0.0389665
0.0111930	0.0356362	0.0171762	-0.0376738
0.0249356	0.0365136	0.0305150	-0.0364027
0.0386844	0.0372880	0.0438564	-0.0351594
0.0524387	0.0379556	0.0572009	-0.0339503
0.0661944	0.0385130	0.0705583	-0.0327780
0.0799541	0.0389579	0.0839183	-0.0316348
0.0937171	0.0392878	0.0972799	-0.0305095
0.1074820	0.0395006	0.1106420	-0.0293905
0.1212460	0.0395944	0.1240260	-0.0282670
0.1350090	0.0395694	0.1374110	-0.0271416
0.1487720	0.0394264	0.1507950	-0.0260174
0.1625330	0.0391659	0.1641800	-0.0248975
0.1759700	0.0387987	0.1772820	-0.0238079
0.1894040	0.0383201	0.1903840	-0.0227245
0.2028330	0.0377299	0.2034870	-0.0216466
0.2162570	0.0370281	0.2165910	-0.0205733
0.2287510	0.0362742	0.2288210	-0.0195753
0.2412380	0.0354237	0.2410520	-0.0185808
0.2537190	0.0344771	0.2532830	-0.0175899
0.2661920	0.0334347	0.2655140	-0.0166029
0.2809030	0.0320815	0.2799840	-0.0154403
0.2956010	0.0305956	0.2944550	-0.0142832
0.3102850	0.0289773	0.3089260	-0.0131318
0.3225580	0.0275219	0.3210570	-0.0121710
0.3348200	0.0259744	0.3331870	-0.0112142
0.3470710	0.0243354	0.3453180	-0.0102617
0.3614500	0.0222929	0.3596040	-0.0091454
0.3758110	0.0201246	0.3738900	-0.0080337
0.3965670	0.0167648	0.3945990	-0.0064307
0.4101550	0.0144220	0.4081910	-0.0053863
0.4180750	0.0130020	0.4161300	-0.0047777
0.4253860	0.0116572	0.4234920	-0.0042149
0.4306480	0.0106702	0.4290990	-0.0037887
0.4434420	0.0081752	0.4422550	-0.0027922
0.4562150	0.0055715	0.4554110	-0.0017987
0.4689730	0.0028954	0.4685680	-0.0008073
0.4817240	0.0001831	0.4817240	0.0001831