Minimum-time and vibration avoidance attitude maneuver for spacecraft with torque and momentum limit constraints in redundant reaction wheel configuration

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ABSTRACT

In this paper, we present an optimal open-loop slew trajectory algorithm developed at GSFC for the so-called "Yardstick design" [1] of the James Webb Space Telescope (JWST). JWST is an orbiting infrared observatory featuring a lightweight, segmented primary mirror approximately 6 meters in diameter and a sunshield approximately the size of a tennis court. This large, flexible structure will have significant number of lightly damped, dominant flexible modes. With very stringent requirements on pointing accuracy and image quality, it is important that slewing be done within the required time constraint and with minimal induced vibration in order to maximize observing efficiency. With reaction wheels as control actuators, initial wheel speeds as well as individual wheel torque and momentum limits become dominant constraints in slew performance. These constraints must be taken into account when performing slews to ensure that unexpected reaction wheel saturation does not occur, since such saturation leads to control failure in accurately tracking commanded motion and produces high frequency torque components capable of exciting structural modes. A minimum-time constraint is also included and coupled with reaction wheel limit constraints in the optimization to minimize both the effect of the control torque on the flexible body motion and the maneuver time. The optimization is on slew command parameters, such as maximum slew velocity and acceleration, for a given redundant reaction wheel configuration and is based on the dynamic interaction between the spacecraft and reaction wheel motion. Analytical development of the slew algorithm to generate desired slew position, rate, and acceleration profiles to command a feedback/feed forward control system is described. High-fidelity simulation and experimental results are presented to show that the developed slew law achieves the objectives.

Keywords: Minimum-time slew maneuver, flexible body dynamics, reaction wheel momentum and torque limits, attitude control.

INTRODUCTION

JWST is an orbiting infrared observatory scheduled to be launched in 2011. Its deployed structure consists of a large flexible sunshield, on one side of which is a telescope with a lightweight 6.5-meter deployed primary mirror of 18 segments and a suite of four science instruments, and on the other side is a spacecraft bus where the reaction wheels are mounted. For more details of the JWST configuration see [1].

One of the key performance metrics is the image quality that is driven by wavefront error (WFE) and image motion. The current allocation for WFE and image motion due to structural dynamics is 14 nanometers (RMS) and 4 milli-arc seconds (RMS), respectively. These stringent requirements dictate that dynamic disturbances induced internally be minimized to prevent any significant vibratory motion due to lightly damped system flexible modes that can exceed the allocation for required image quality. For JWST, the primary source of internal disturbance is from the reaction wheel assemblies. In addition to static and dynamic imbalances that continuously produce unwanted forces and torques proportional to the square of the wheel spin speed, [2,3], the reaction wheel torque and momentum limits when encountered with large torque commands will result in discontinuity in the produced torque profile and its derivative, and in turn, create significant vibratory motion in the system. This situation potentially can happen during slew when

reaction wheels undergo a large swing in momentum to affect the motion of the spacecraft whose mass is much greater than that of the reaction wheel. Avoiding reaction wheel limits during slews not only maintains the internal disturbances within the image performance requirements, but also allows for rapid retargeting and better tracking capabilities by preventing control failure in tracking commanded motion during slews.

This paper describes a slew trajectory algorithm developed to minimize both the effect of the control torque produced by reaction wheels on the flexible body motion and the maneuver time. With reaction wheels providing control actuation, this algorithm produces a desired slew trajectory in terms of position, rate, and acceleration profiles as functions of reaction wheel parameters, which are initial wheel speeds, and wheel torque and momentum limits of all the reaction wheels in the system. These reaction wheel parameters are taken into account in the algorithm to ensure that reaction wheel limits can only be tangentially reached during the slew so that no discontinuity in actual torque will occur. These kinematics profiles are used to provide commands to any stable feedback/feedforward control system in order to perform the maneuver. Note that this paper describes the slew trajectory algorithm only as an open-loop system and does not consider it in a closed-loop control design.

DERIVATION

There are two steps in the development of the algorithm. The first step involves selecting the basic slew trajectory, which defines and parameterizes the functional form of the desired slew position, rate, and acceleration profiles. This step, while using an approach similar to that described in [4], extends it by taking into account the reaction wheel limits in obtaining the desired motion profiles. The second step optimizes the slew command parameters, such as maximum slew rate and acceleration defined in the first step, for a given redundant reaction wheel configuration with torque and momentum limits, based on the dynamic interaction between the spacecraft and reaction wheel motion.

In [4], a near optimal maneuver trajectory algorithm is given for the case with no reaction wheel constraints. This algorithm specifies, in a parametric form, a smooth function that basically approximates the sign function of a bang-bang law, which is a well-known form for minimum-time controllers. The trajectory motion in this case is continuously maintained at maximum acceleration over a long portion of the time during the slew period. The duration of this acceleration time portion is selected in a trade-off between minimum maneuver time and the degree of smoothness in the trajectory.

In the presence of reaction wheel momentum and torque limits, arbitrarily long and continuously accelerating motions cannot be supported, implying that bang-bang controllers and the approach given in [4] are not applicable. With limits imposed on the reaction wheel momentum, the best that can be achieved in terms of minimizing maneuver time is to maintain spacecraft motion at its maximum rate allowable by these limits, over as long a portion of the slew time as possible.

Maximizing this coasting time period means minimizing the time it takes to reach the maximum rate at the start of the slew, and the time it takes to go back to zero rate at the end of the slew. This requires that the maximum allowable acceleration and deceleration, which are bounded by the reaction wheel torque limits and initial wheel momentum, be used in getting to and from the maximum rate.

Thus, the general acceleration profile appropriate for this case starts with an impulse whose amplitude is derived from reaction wheel torque limits, and whose direction is the direction of the desired motion. Following the impulse, the profile remains at zero over the entire slew duration, and ends with an impulse equal to the one applied at the start but in the opposite direction. The resulting rate profile is simply a rectangular curve, whose amplitude is derived from the reaction wheel momentum limits.

In order to minimize the structural mode excitation, a smooth version of the acceleration impulsive profile described above is selected, and is chosen so that it and its first derivative are continuous and expressible in low order polynomials. The basic polynomials used in [4] are employed to provide smooth slew motion including at the two end points.

Derivation of Slew Profile

To achieve a smoothly varying torque, the desired acceleration profile generated by the slew trajectory algorithm has the following functional form:

E.1
$$\alpha(t) = \alpha_d \, \hat{e} \frac{d}{dt} f\left(t, \Delta t_a, \Delta t_c\right)$$

Where α_d is the maximum acceleration amplitude, $\hat{e}$ is a constant vector specifying the slew rotation axis, and $f(\cdot)$ is a positive scalar time-function representing the desired spacecraft rate, parameterized by $(\Delta t_a \Delta t_c)$, and whose first order derivative is given as:

E.2
$$\frac{d}{dt} f(t, \Delta t_a, \Delta t_c) = \begin{cases} \left(\dfrac{t-t_0}{\Delta t_a}\right)^2 \left(3 - 2\left(\dfrac{t-t_0}{\Delta t_a}\right)\right) & t_0 < t < t_1 = t_0 + \Delta t_a \\[2em] \left(\dfrac{t_2-t}{\Delta t_a}\right)^2 \left(3 - 2\left(\dfrac{t_2-t}{\Delta t_a}\right)\right) & t_1 < t < t_2 = t_1 + \Delta t_a \\[2em] 0 & t_2 < t < t_3 = t_2 + \Delta t_c \\[2em] -\left(\dfrac{t-t_3}{\Delta t_a}\right)^2 \left(3 - 2\left(\dfrac{t-t_3}{\Delta t_a}\right)\right) & t_3 < t < t_4 = t_3 + \Delta t_a \\[2em] -\left(\dfrac{t_5-t}{\Delta t_a}\right)^2 \left(3 - 2\left(\dfrac{t_5-t}{\Delta t_a}\right)\right) & t_4 < t < t_5 = t_4 + \Delta t_a \\[2em] 0 & t \geq t_5 \end{cases}$$

The time periods Δt_a and Δt_c can be expressed in term of the maximum acceleration magnitude α_d, the maximum rate magnitude ω_d, and the slew angle $\Delta\theta$ as:

E.3
$$\Delta t_a = \frac{\omega_d}{\alpha_d}$$

$$\Delta t_c = \frac{\Delta\theta}{\omega_d} - 2\frac{\omega_d}{\alpha_d}$$

and the total slew period is:

E.4
$$\Delta T_s = \Delta t_c + 4\Delta t_a$$

It follows that the desired rate profile has the following form:

E.5
$$\omega(t) = \omega_d \, \hat{e} f\left(t, \Delta t_a, \Delta t_c\right)$$

The profile of the slew angle $\Delta\theta$ can be directly obtained by integrating E.5 to give:

E.6
$$\Delta\theta(T) = \omega_d \, \hat{e} \int_0^T f\left(t', \Delta t_a, \Delta t_c\right) dt'$$

Figure 1 shows the normalized slew acceleration, rate, and angle profiles described by E1, E5, and E6, when both α_d and ω_d are set to 1. Also, in these plots, the time periods are selected to be $\Delta t_a = 1$ second and Δt_c, =2 seconds.
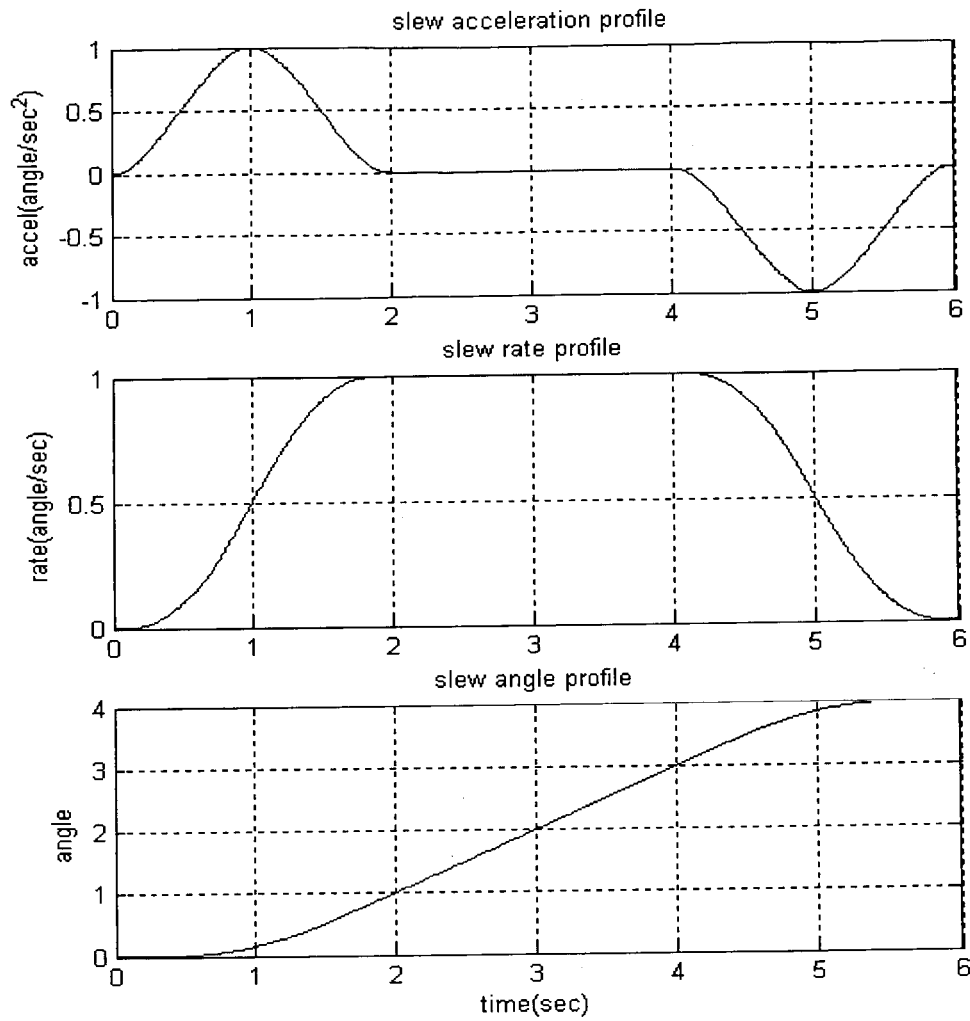
Figure 1 – Idealized Trajectory Profile

Since the dead band time Δt_a cannot be negative, the maximum rate magnitude ω_d must also satisfy the following condition:

E.7
$$\omega_d \leq \sqrt{\frac{\alpha_d \, \Delta\theta}{2}}$$

which can be given in terms of Δt_a by using the first relation in E.3 for α_d, i.e.:

$$2\Delta t_a \leq \frac{\Delta\theta}{\omega_d}$$

For later reference in the derivation of the maximum acceleration magnitude, given here are the relations between α_d, ω_d, Δt_a, and the slew angle $\Delta\theta(\Delta t_a)$, at time Δt_a after the start of the slew, when the desired acceleration profile reaches its positive maximum amplitude α_d, and the desired rate profile reaches half of its maximum value ω_d:

$$\alpha_d = \tfrac{3}{20}\frac{\omega_d^2}{\Delta\theta(\Delta t_a)}$$

E.8

$$\Delta t_a = \tfrac{20}{3}\frac{\Delta\theta(\Delta t_a)}{\omega_d}$$

Derivation of the maximum rate magnitude

The basic equation describing the system momentum of the spacecraft and reaction wheels in the inertial frame is:

E.9
$$H_0 = R^T(t)\left(A_w^b h_w(t) + I_s\omega(t)\right)$$

Where h_w is the wheel momentum vector in wheel frame, and A_w^b represents the constant transformation from the wheel frame to the spacecraft frame. In a four-wheel configuration, h_w is a 4-component vector, and A_w^b is a 3×4 matrix whose columns are reaction wheel rotational axes given in the spacecraft frame and can be obtained directly from the reaction wheel configuration. The left inverse of this matrix, denoted by $\hat{A}_b^w$, is a 4×3 matrix and typically given to minimize the wheel speed magnitude. H_0 and I_s are, respectively, the initial system momentum in the inertial frame, and the system inertia matrix in the spacecraft frame. H_0 is assumed here to be a constant vector, i.e. there is no external torque, and consists only of wheel momentum. $R(t)$ is the direction cosine matrix representing the transformation from the inertial frame to the spacecraft frame corresponding to the desired rate profile $\omega(t)$.

Using the minimum mean squared solution, $\hat{A}_b^w$, for the inverse of A_w^b, the wheel momentum vector of minimum magnitude given in the reaction wheel frame, as a function of the initial system momentum and the desired motion profile can be written as:

E.10
$$h_w = \hat{A}_b^w\left(R(t)H_0 - I_s\omega(t)\right)$$

For a redundant wheel configuration, where there are more than 3 wheels in operation for attitude control, if the null vector in the wheel frame is non-zero initially, then it must be added to E.10 and must be carried in all the subsequent steps in computing the maximum rate magnitude.

With Equation E.3, E.10 can be expressed in terms of slew parameters as:

E.11
$$h_w = \hat{A}_b^w\left(R(t)H_0 - I_s\hat{e}\omega_d f(t)\right)$$

Let $\pm h_{lim}$ and $\pm\tau_{lim}$, $h_{lim} > 0$ and $\tau_{lim} > 0$, denote the momentum limits and torque limits, respectively, of each reaction wheel. These limits correspond to the four corners of a rectangular torque-speed curve. Although a typical torque-speed curve may have regions of decreasing torque as the wheel approaches saturation speeds, only the rectangular torque-speed curve is considered here.

The momentum limits when applied to a reaction wheel leads to the following inequality:

E.12
$$-h_{\text{lim}} \leq h_{wi} = \left(\hat{A}_b^w \right)_i \left(R(t)H_0 - I_s \hat{e} \omega_d f(t) \right) \leq h_{\text{lim}}$$

Where the index i indicates the ith wheel, and $(\hat{A}_b^w)_i$ denotes the ith row of the body-to-wheel transformation matrix. E.12 leads to the following upper and lower bounds of the spacecraft rate component along the ith wheel axis, due to the momentum limits of the ith wheel:

E.13
$$-h_{\text{lim}} + \left(\hat{A}_b^w \right)_i R(t)H_0 \leq \left(\hat{A}_b^w \right)_i I_s \hat{e} \omega_d f(t) \leq h_{\text{lim}} + \left(\hat{A}_b^w \right)_i R(t)H_0$$

Global bounds over the slew period can be obtained by taking the appropriate maximum and minimum values of the time-dependent terms over the period, i.e.

E.14
$$-h_{\text{lim}} + \underset{t \in \Delta T_s}{\text{Max}} \left\{ \left(\hat{A}_b^w \right)_i R(t)H_0 \right\} \leq \left(\hat{A}_b^w \right)_i I_s \hat{e} \omega_d f(t) \leq h_{\text{lim}} + \underset{t \in \Delta T_s}{\text{Min}} \left\{ \left(\hat{A}_b^w \right)_i R(t)H_0 \right\}$$

The expression inside the above **Min** and **Max** operators taking over the slew time period represents the component of the system momentum that the ith wheel must be able to carry during the slew. As the spacecraft rotates, there are for each reaction wheel, two attitudes at which the system momentum on the wheel are maximum and minimum, which occur when the total momentum vector in the spacecraft frame is closest and farthest from the wheel's rotational axis. In addition the initial and final slew attitudes may represent extreme points in the case where there is no extreme point during the slew. For the four wheel case, there are 10 of these rotation angles, and the exact solutions of these angles can be obtained since $R(\cdot)$ can be expressed in terms of position without being explicitly dependent on time.

The computation of these attitudes is simplified by considering a coordinate frame, referred to as the slew frame, in which the z-axis coincides with the slew axis, $\hat{e}$, and the x axis is defined such that the initial system momentum H_o is in the x-z plane. In this frame, motion of the vector H_o describes a cone about the z-axis as the slew progresses, and for a given wheel rotational axis vector, the minimum and maximum functions of E.14 can be computed straightforwardly. They can be shown to correspond to having H_o at either the end points or lying in the plane spanned by the z-axis and the ith wheel's rotational axis.

Since $f(\cdot)$ is a positive function with the two end points at zero value, the upper and lower bounds on the body rate given in E.14 must have one bound positive and the other negative. Thus, the maximum rate amplitude, denoted by ω_{di}, based on the ith wheel's momentum limit is simply the larger of the two bounds (i.e. the positive bound). That is:

E.15
$$\omega_{di} = \text{Max} \left\{ \frac{-h_{\text{lim}} + \underset{t \in \Delta T_s}{\text{Max}} \left\{ \left(\hat{A}_b^w \right)_i R(t)H_0 \right\}}{\left(\hat{A}_b^w \right)_i I_s \hat{e}}, \quad \frac{h_{\text{lim}} + \underset{t \in \Delta T_s}{\text{Min}} \left\{ \left(\hat{A}_b^w \right)_i R(t)H_0 \right\}}{\left(\hat{A}_b^w \right)_i I_s \hat{e}} \right\}$$

Also, the condition for a valid command slew is that the upper and lower bounds defined in E.14 must have opposite signs. This assures that the component of the system momentum on a reaction wheel cannot exceed h_{lim} during the slew.

The sign of the denominator in E.15 plays a very important role in determining which of the two bounds is the maximum rate amplitude, as it indicates the direction in which that wheel must be rotated to achieve the desired spacecraft motion in the absence of the initial momentum. In general, it has the same sign as that of the wheel momentum limit that will be reached during the slew. In the case where this quantity is identically zero for the ith wheel,

which means that there is no contribution from this reaction wheel to the slew and thus, it can be removed from the computation of the slew maximum rate and acceleration magnitudes.

The maximum rate amplitude ω_d that satisfies the momentum limits of all the reaction wheels in the system, is the smallest of all the ω_{di} :

E.16
$$\omega_d = \underset{i}{\text{Min}}\left\{\omega_{di}\right\}$$

In the special case, where the initial system momentum H_o is along the slew axis $\hat{e}$, Equation E.13 for all the reaction wheels, reduces to:

$$-h_{\text{lim}} + \left(\hat{A}_b^w\right)_i H_0 \leq \left(\hat{A}_b^w\right)_i I_s \hat{e}\omega_d f(t) \leq h_{\text{lim}} + \left(\hat{A}_b^w\right)_i H_0$$

Which has constant bounds so that the maximum rate magnitude is easily obtained in this case.

Derivation of the maximum acceleration magnitude

The wheel torque required to carry out the desired motion is obtained by taking the derivative of E.10, which gives:

E.17
$$\frac{d}{dt}h_w = \hat{A}_b^w\left(\left[\omega(t)\right]R(t)H_0 - I_s\frac{d}{dt}\omega(t)\right)$$

Where the square brackets denote the matrix corresponding to the cross product operator, i.e. $\left[a\right]b = a \times b$.

With Equations E.1 and E.3, E.17 can be rewritten in terms of slew parameters as:

E.18
$$\frac{d}{dt}h_w = \omega_d \hat{A}_b^w\left[\hat{e}\right]R(t)H_0 f(t) - \alpha_d \hat{A}_b^w I_s \hat{e}\frac{d}{dt}f(t)$$

Using the same convention as in the above section in expressing components related to a reaction wheel, the following inequality is obtained when applying the torque limits to the i$^{\text{th}}$ wheel:

E.19
$$-\tau_{\text{lim}} \leq \frac{d}{dt}h_{wi} = \omega_d\left(\hat{A}_b^w\right)_i\left[\hat{e}\right]R(t)H_0 f(t) - \alpha_{di}\left(\hat{A}_b^w\right)_i I_s \hat{e}\frac{d}{dt}f(t) \leq \tau_{\text{lim}}$$

The index i appears in α_{di} to indicate that the derived parameter in this case is the maximum allowable acceleration due to the i$^{\text{th}}$ wheel. Rearranging E.19 leads to the following upper and lower bounds on the component of the spacecraft acceleration along the i$^{\text{th}}$ wheel axis, due to the torque limits on that wheel:

E.20
$$-\tau_{\text{lim}} + \omega_d\left(\hat{A}_b^w\right)_i\left[\hat{e}\right]R(t)H_0 f(t) \leq \alpha_{di}\left(\hat{A}_b^w\right)_i I_s \hat{e}\frac{d}{dt}f(t) \leq \tau_{\text{lim}} + \omega_d\left(\hat{A}_b^w\right)_i\left[\hat{e}\right]R(t)H_0 f(t)$$

At this point, an approach similar to what was used previously to derive the maximum rate magnitude can be used to obtain a legitimate maximum value for the acceleration profile. This involves obtaining the global bounds for the time-dependent term in E.20, which is the gyroscopic effect term, over the slew period, and taking the minimum of the norm of the two bounds as the maximum allowable acceleration by the i$^{\text{th}}$ wheel. The desired acceleration magnitude is simply the minimum of these maximum allowable acceleration values over all wheels, i.e. by simply setting f(t) and its time-derivative respectively to ½ and ±1, which are the values when the acceleration is at its maximum values, Equation E.20 can be manipulated to yield the following maximum wheel accelerations:

$$\alpha_{di} = Min \left\{ \left| \frac{-\tau_{\lim} + \frac{1}{2}\omega_d \underset{t\in\Delta T_s}{Max}\left\{\left(\hat{A}_b^w\right)_i [\hat{e}]R(t)H_0\right\}}{\left(\hat{A}_b^w\right)_i I_s\hat{e}} \right|, \left| \frac{\tau_{\lim} - \frac{1}{2}\omega_d \underset{t\in\Delta T_s}{Min}\left\{\left(\hat{A}_b^w\right)_i [\hat{e}]R(t)H_0\right\}}{\left(\hat{A}_b^w\right)_i I_s\hat{e}} \right| \right\}$$

so that:
$$\alpha_d = \underset{i}{Min}\left\{\alpha_{di}\right\}.$$

This is an admissible solution because as the acceleration profile, which is a function of the derivative of $f(t)$ given in Equation E.2, changes sign from positive to negative in the course of the slew, the norm of the smaller bound will ensure that the acceleration profile will not exceed torque limit of the ith wheel. By taking the minimum over these smaller bounds of all the reaction wheels, the acceleration profile will be within all the wheel torque limits. Although it produces a legitimate solution for the maximum acceleration magnitude, and is actually quite easy to implement, this approach in general results in sub-optimum acceleration profile.

An approach, which yields an optimum solution for the maximum acceleration amplitude, derives the maximum acceleration values for the positive and negative regions of the acceleration profile separately, and selects the minimum of the two as the solution. This approach turns out to be more complicated to implement since it requires an iterative method for finding roots of an equation involving transcendental functions.

The inequality given by Equation E.20 when applied to the positive region of the acceleration profile reduces to:

E.21
$$0 \le \alpha_{di}(+)\frac{d}{dt}f(t) \le \left| \frac{\tau_{\lim}}{\left(\hat{A}_b^w\right)_i I_s\hat{e}} \right| + \frac{\omega_d\left(\hat{A}_b^w\right)_i [\hat{e}]R(t)H_0 f(t)}{\left(\hat{A}_b^w\right)_i I_s\hat{e}}$$

Since Equation E.21 is true for time between the starting time t_0 and $t_0 + 2\Delta t_a$, evaluating the above inequality at time Δt_a when the acceleration profile reaches its positive maximum magnitude and the derivative of $f(t)$ equals to 1, leads to the following inequality:

E.22
$$\alpha_{di}(+) \le \left| \frac{\tau_{\lim}}{\left(\hat{A}_b^w\right)_i I_s\hat{e}} \right| + \frac{\frac{1}{2}\omega_d\left(\hat{A}_b^w\right)_i [\hat{e}]R(\Delta t_a)H_0}{\left(\hat{A}_b^w\right)_i I_s\hat{e}}$$

This inequality contains two unknown parameters, $\alpha_{di}(+)$ and Δt_a, which, when obtained by solving the equality of E.22, are the optimum allowable acceleration and the shortest time to reach the maximum rate magnitude, satisfying the system torque constraints and relations in E.5 for the ith wheel in the positive acceleration region. Although it is possible for Equation E.22 to have more than one set of solutions, the set which is the desired solution has the largest $\alpha_{di}(+)$ and smallest Δt_a.

With Equation E.8, the equality of E.22 can be rewritten in terms of just one variable, the slew angle $\Delta\theta_i$ evaluated at Δt_a, for the ith wheel:

E.23
$$\frac{3}{20}\frac{\omega_d^2}{\Delta\theta_i(\Delta t_a)} = \left| \frac{\tau_{\lim}}{\left(\hat{A}_b^w\right)_i I_s\hat{e}} \right| + \frac{\frac{1}{2}\omega_d\left(\hat{A}_b^w\right)_i [\hat{e}]R(\Delta\theta_i(\Delta t_a))H_0}{\left(\hat{A}_b^w\right)_i I_s\hat{e}}$$

By using the slew coordinate frame defined in the previous section, E.23 can be simplified to the following form:

E.24
$$\frac{A}{\Delta\theta_a} = B + C\sin(\Delta\theta_a + \phi)$$

Where $\Delta\theta_a \equiv \Delta\theta_i(\Delta t_a)$ is the slew angle at which the acceleration profile attains its positive maximum magnitude α_d and the rate profile attains half of its maximum magnitude ω_d, and A, B, C, and ϕ are constants corresponding to terms in Equation E.23, i.e.

$$A = \tfrac{3}{20}\omega_d^2, \quad B = \left|\frac{\tau_{\lim}}{\left(\hat{A}_b^w\right)_i I_s\hat{e}}\right|, \quad C = \frac{\tfrac{1}{2}\omega_d\|H_0\| S_1\left(\left(\hat{A}_b^w\right)_i, H_0, \hat{e}\right)}{\left(\hat{A}_b^w\right)_i I_s\hat{e}}, \quad \phi = \operatorname{atan}\left(S_2\left(\left(\hat{A}_b^w\right)_i\right)\right)$$

Where $S_1(\cdot)$ and $S_2(\cdot)$ are scalar functions given in terms of specified parameters, with S_1 assumed to have values between $(-1,1)$, which limits the possible size of C. The terms of E.24 can be visualized as follows: B is the maximum possible acceleration that the ith wheel can provide along the slew rotation axis $\hat{e}$. The term $C\sin(\Delta\theta_a + \phi)$ is the acceleration required to commutate the system momentum through the wheels as the slew progresses. The left-hand side of Equation E.24 is the remaining acceleration available to slew the body.

Although Equation E.24 can not be solved analytically for $\Delta\theta_a$, some simple iterative search methods can be devised for finding the smallest root, which corresponds to the desired (minimum slew time) solution.

One adequate search technique to find the first root is to start at the top left of the region, which corresponds to the zero angle and acceleration $(B+|C|)$. The angle corresponding to this point using the left-hand side of E.24, i.e.

$$\theta_0 = \frac{A}{B+|C|},$$ should be within the bounded region and on the left of the first root. Starting with this angle, by

stepping in an adequately small increment along the curve of the left-hand side of E.24, the interval, which contains the solution, can be determined. The following iteration method can be shown to yield the desired solution within this interval:

E.26
$$\left\{\theta_k = \frac{A}{\hat{\alpha}_{k-1}} < \theta_{\max} \implies \hat{\alpha}_k = B + C\sin(\theta_k + \phi)\right\} \implies \hat{\alpha}_i \to \alpha_{di}(+)$$

However, at any point in the above process, if an angle $\theta_i > \theta_{\max}$, where $\theta_{\max}$ is the angle defines the boundary of the first quadrant of the total slew angle, the maximum rate magnitude must be decreased, according to E.7, to give a smaller A that will permit the above process to continue. This rate reduction is simply to adjust the desired motion profile so that available reaction wheel torque in the system can allow the motion to accelerate to the maximum rate within the time constraint to have a dead band in the acceleration profile.

The same method described above can be applied to the negative region of the acceleration profile to obtain the desired solution for the negative region. The equivalent equation to E.23 for the negative region of the acceleration profile is the following:

E.27
$$\tfrac{3}{20}\frac{\omega_d^2}{\Delta\theta(\Delta t_s) - \Delta\theta(\Delta t_s - \Delta t_a)} = \left|\frac{\tau_{\lim}}{\left(\hat{A}_b^w\right)_i I_s\hat{e}}\right| - \frac{\tfrac{1}{2}\omega_d\left(\hat{A}_b^w\right)_i[\hat{e}]R(\Delta\theta(\Delta t_s - \Delta t_a))H_0}{\left(\hat{A}_b^w\right)_i I_s\hat{e}}$$

Having obtained the maximum allowable acceleration values for each of the reaction wheels in the positive, $\alpha_{di}(+)$, and negative, $\alpha_{di}(-)$, regions, the maximum acceleration magnitude is the minimum value taken over all these values, i.e.

$$\alpha_d = \underset{i}{\text{Min}} \left\{ \alpha_{di}(-), \alpha_{di}(+) \right\}$$

In the special case, where the initial system momentum H_o is along the slew axis $\hat{e}$, the properly modified version of E.23 and E.27 for the ith wheel, which has non-zero term $\left(\hat{A}_b^w \right)_i I_s \hat{e}$, leads to the following solution:

$$\alpha_{di} = \tfrac{3}{20} \min \left\{ \frac{\omega_d^2}{\left| \frac{\tau_{\lim}}{\left(\hat{A}_b^w \right)_i I_s \hat{e}} \right| + \frac{\frac{1}{2} \omega_d \left(\hat{A}_b^w \right)_i [\hat{e}] H_0}{\left(\hat{A}_b^w \right)_i I_s \hat{e}}}, \frac{\omega_d^2}{\left| \frac{\tau_{\lim}}{\left(\hat{A}_b^w \right)_i I_s \hat{e}} \right| - \frac{\frac{1}{2} \omega_d \left(\hat{A}_b^w \right)_i [\hat{e}] H_0}{\left(\hat{A}_b^w \right)_i I_s \hat{e}}} \right\}$$

Where ω_d and α_d are computed from the above formulation based on reaction wheel configuration, momentum and torque limits, and slew command, and completely define the desired slew trajectory.

SIMULATION EXAMPLE

A simple time domain simulation was created to demonstrate spacecraft slew performance using this trajectory algorithm. The simulation was implemented using rectangular integration with a 1 second time step. The system model included only rigid body dynamics of the spacecraft, with the following inertia matrix (from an early conceptual design for JWST)

$$I = \begin{bmatrix} 11730 & 142 & -7206 \\ 142 & 27476 & -113 \\ -7206 & -113 & 20268 \end{bmatrix} \quad (kg\ m^2)$$

The system model also included PD feedback control with a 1 Hz bandwidth, and feedforward control ($\omega \times h$) to compensate for gyroscopic effects. The reaction wheel assembly in this model was a 4-wheel configuration. Each wheel was modeled using a rectangular torque-speed curve, with a 60 Nms momentum limit, a 0.3 Nm torque limit, and a rotor inertia of 0.0948 kg m^2. The initial wheel momentum vector was (53.608, 41.695, 11.913, 5.956) Nms. The spacecraft was commanded to slew 359 degrees about the Y axis. The results are shown in figures 2 and 3.

The total time to complete the slew was approximately 1800 seconds, as can be seen from the acceleration, velocity, and angle trajectories in figure 2. Comparing figure 2 to figure 1 shows that the salient features of the ideal trajectory are present, namely the smooth transitions between the segments of the trajectory, the smooth peak in the acceleration, and the flat coasting at maximum velocity. Figure 3 shows that hard saturation in both wheel torque and wheel momentum is avoided. The torque limit is approached in wheel 1 (at t $\cong$ 500 seconds) and in wheel 2 (at t $\cong$ 1500 seconds). The momentum limit is approached in wheel 2 (at t $\cong$ 1150 seconds) and in wheel 4 (at t $\cong$ 650 seconds). Thus, excitation of flexible modes would be mitigated.
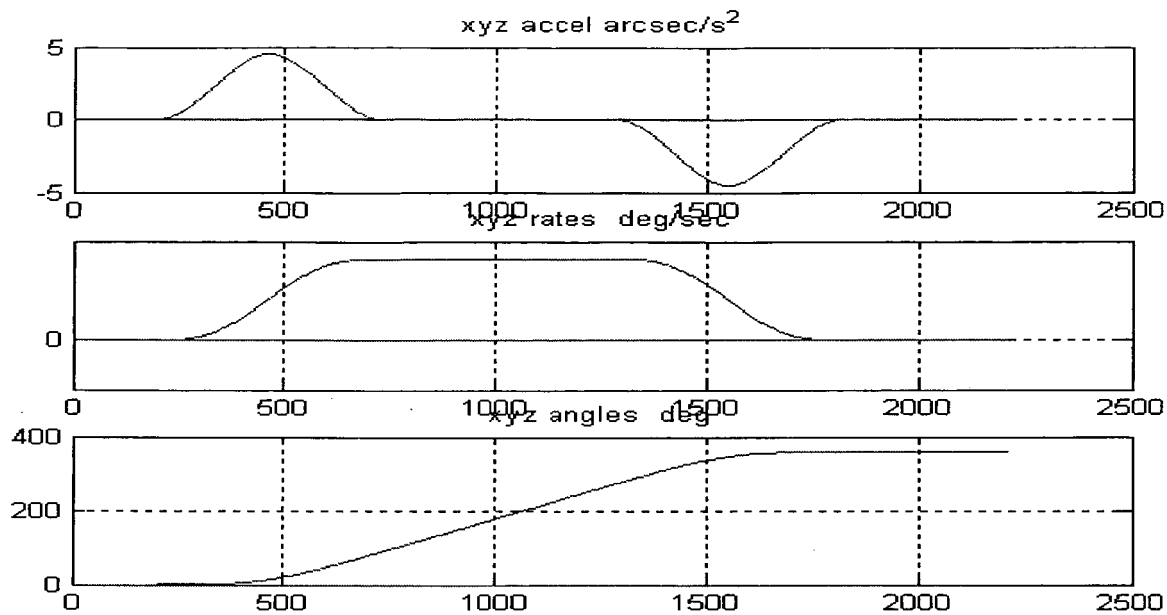
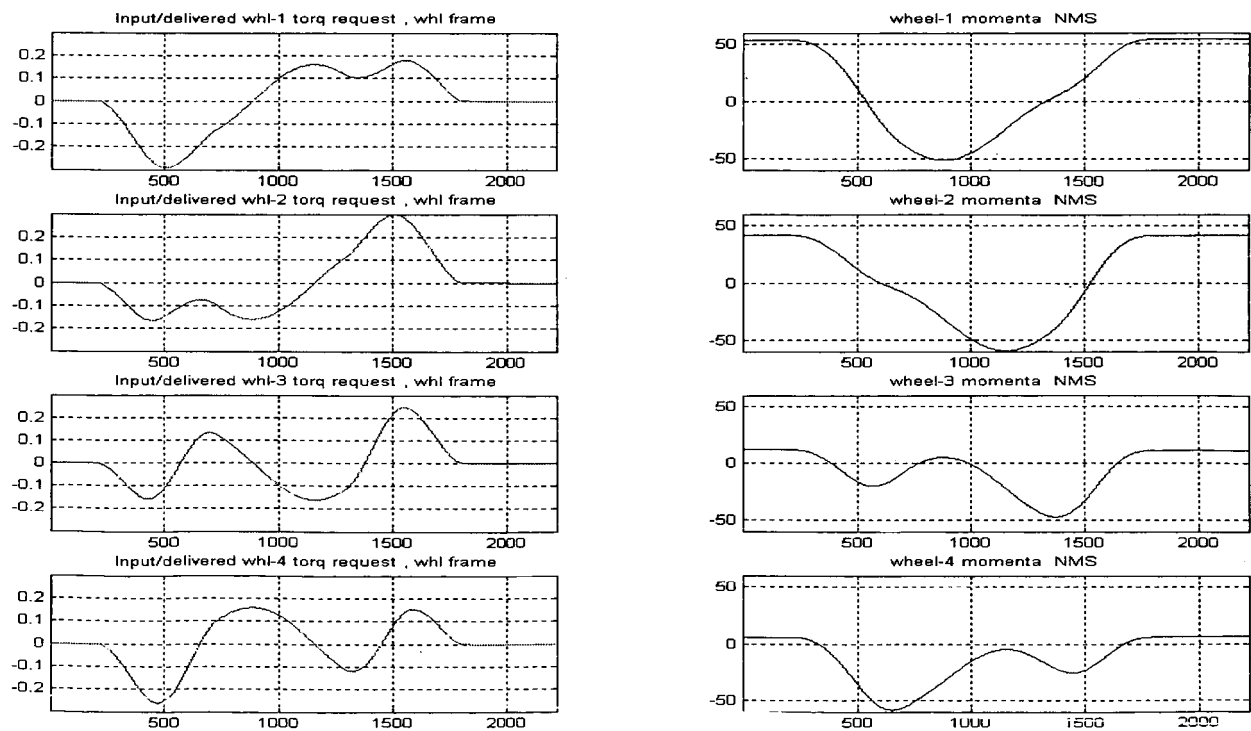**Figures 2 – Slew Trajectories from Simulation**

Figure 3 – Wheel Torque (left) and Wheel Momentum (right) Profiles from Simulation

CONCLUSION

An optimal open-loop slew trajectory algorithm is described. This algorithm enables minimum-time maneuvers of a spacecraft while avoiding the excitation of structural modes that would otherwise impact settling time. Furthermore, the algorithm does this without knowledge, or a model, of the flexible structural dynamics – all that is required is knowledge of the spacecraft rigid body mass properties and the reaction wheel torque and momentum limits. It has been implemented and tested via simulation to support the Yardstick design of the JWST. Additionally, experiments were performed with the MIT Origins Testbed, using a reduced-order form for a single-axis system, which confirmed the ability to track the commanded trajectory and to avoid excitation of unmodeled dynamics.

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