Close out and Final report for
NASA Glenn Cooperative Agreement NCC3-1021

Development in Source Modeling and Sound Propagation for Jet Noise Predictions

Summary of Research During Tenure

The development of an accurate, physics-based, prediction method for turbulent jet mixing noise that can be reliably used over a wide range of operating conditions is a key element in efforts aimed at reducing the overall noise emitted by commercial jet aircraft. The ability to predict the effects of relatively small flow changes on the radiated sound field is critical to the economical development of successful noise-reduction technologies.

The most commonly used jet noise prediction schemes are based on analogies between the fluctuations in a real turbulent flow with those produced by nominal source terms and propagating linearly through a prescribed base flow. Perhaps the most well developed of these is the JeNo code (Khavaran & Bridges 2004) which is based on the Lilley (1974) form of acoustic analogy. In its current version, the method has incorporated solutions of the Reynolds-averaged Navier-Stokes (RANS) equations and anisotropic source models into the computational procedure for the prediction of jet noise. This method is capable of predicting results which are generally in good agreement with experimental data for axisymmetric jets at polar angles around ninety degrees to the jet axis and angles near the peak noise levels for subsonic jets. A number of issues remain unresolved however, particularly at supersonic Mach numbers and for heated jets near the peak noise direction. As increasingly stringent noise regulations are imposed on commercial aircraft, greater accuracy will be required of jet noise prediction methods and they will be required to perform over a broader range of operating conditions.

The purpose of the research carried out under this cooperative agreement was to develop tools that could be used to improve upon the current state of the art in the prediction of noise emitted by turbulent exhaust jets. Both the source modeling and sound propagation aspects of the prediction of jet noise by acoustic analogy were examined with a view toward the development of methods which yield improved predictions over a wider range of operating conditions.

The starting point for the work was the generalized acoustic analogy formulation of Goldstein (2002) who showed that the Navier-Stokes equations can be written as the linearized Navier-Stokes equations, but with different dependent variables and with the viscous stress replaced by a generalized Reynolds stress and the heat flux vector replace by a generalized enthalpy flux. The overall goal of the work was to determine whether there are any potential advantages of basing a noise prediction method on this generalized acoustic analogy formulation which could result in improved predictions of the noise emitted by turbulent jets.

October 1, 2002 – Sept. 30, 2003

The Generalized Acoustic Analogy for a Parallel Mean Flow

(A) General Formulation and Source Modeling

During the first year of this cooperative agreement, work was focused on the use of the generalized acoustic analogy for jet noise prediction in the case when the base flow is taken to a locally parallel shear flow. A formal solution for the fluctuating pressure was derived in terms of a vector Green's function for the system of equations governing the propagation of sound and the nominal source terms of the generalized acoustic analogy formulation. This solution was then used to derive a general expression for the far-field sound spectral density. The latter, which involves integration of products of the Green's function with the fourth-order, two-point, time-delayed, density-weighted fluctuating velocity/enthalpy correlation tensor, is quite complicated, and a considerable amount of simplification is required to reduce it to a form that can be used to make practical noise predictions. A significant amount of effort during the first year of this cooperative agreement was devoted to a careful and systematic approach to modeling the source terms resulting from the generalized acoustic analogy formulation. Most of the approximations and assumptions that were made are similar to those typically made in jet noise work, but the emphasis here was on a full and rigorous application of the mathematical implications of these approximations.

It was first assumed that the source is compact in a reference frame moving with the nominal convection speed of a typical turbulent eddy. This results in the spectral density being dependent only on the integral of the correlation tensor over the correlation volume, which makes the results much less sensitive to the details of any empirical models which are eventually invoked. It was further assumed that the density was uniform in the source region so that the required correlations reduce to the more usual fourth-order velocity/enthalpy correlations.

To obtain any additional simplification, assumptions must be made about the statistical properties of the turbulence in the jet. A commonly made assumption in jet noise source modeling is that the properties of normal (Gaussian) statistics may be used to express the fourth-order velocity correlations in terms of second order ones (Millionschikov, 1941 ; G. K. Batchelor, 1953). It has been demonstrated (Freund, 2003) that this `quasi-normal' approximation is quite good when used in this context. For the source terms appearing in the generalized acoustic analogy, a generalization of this idea was employed, and it was assumed that the properties of jointly normal statistics could be used to express the fourth-order velocity/enthalpy correlations in terms of second-order ones.

In order to reduce the number, and possible forms, of the required, second-order, correlation functions, the properties of axisymmetric turbulence (Batchelor, 1946; Chandrasekhar, 1950; Goldstein & Rosenbaum 1973; Lindborg, 1995) were used to further simplify the formulation. Using the symmetry properties of axisymmetric vectors and second-order tensors, the integrals appearing in the formula for the sound spectral density were reduced to a form involving a minimum number of independent correlation functions.

When the base flow in the generalized acoustic analogy is taken to a locally parallel shear flow, the governing system of equations can be written as a single equation for the fluctuating pressure. The left hand side of the resulting equation is the same as the form of the linearized Lilley operator which is implemented in the JeNo scheme for axisymmetric mean flows. The main difference between the generalized analogy considered in this work and formulation used in the JeNo approach is in the specification of the source terms. A qualitative comparison of the source terms in the two formulations was made to try to see under what conditions their differences might result in significant differences in noise predictions.

Source terms in the Lilley analogy resulting from linearization about the base flow and those due to non-uniform density are generally neglected in the JeNo scheme. In the present formulation no such linearization is required, but there are two additional source terms in the convective wave equation which do not appear in the JeNo formulation. The form of these additional source terms suggests that the differences in sound source definition between the present formulation and JeNo might be expected to be most significant for heated jets, since one of the additional source terms involves enthalpy (or temperature) fluctuations in addition to velocity fluctuations. However, even in the case of unheated jets, there are differences in the source terms.

(B) Sound Propagation in non-axisymmetric Mean Flows

Advanced nozzle designs for jet noise reduction often produce complicated exhaust flows and accurate prediction of the noise emitted by such nozzles requires the capability to calculate the propagation of sound through more general, non-axisymmetric, base flows. Another part of the work carried out during the first year of this cooperative agreement was the development of a computer code to calculate the fundamental solution (Green's function) for the Euler equations linearized about a general, transversely sheared, mean flow. A computational procedure, based on the adjoint formulation for the linearized Euler equations and the principle of reciprocity (Tam & Auriault 1998), was formulated for this problem.

The numerical method consisted of second-order central differences in space along with a Runge Kutta method in time to compute the time-harmonic solution (reduced Green's function) to this problem. The physical domain was surrounded by perfectly matched layers to absorb all waves encountering an artificial boundary and prevent their reflection back into the physical domain. A computer code based on this scheme was written (in cooperation with Professor Thomas Hagstrom of the University of New Mexico), and a preliminary set of calculations was run for testing using an analytic mean flow profile in a super-elliptic jet. Solutions were obtained for the non-axisymmetric Green's function, but it was determined that the CPU time required to obtain solutions was much too long to be acceptable for use in a practical jet noise prediction scheme. Another scheme, based on an iterative Generalized Minimum Residual (GMRES) method, suffered from a similar difficulty. Further work is required to develop an accurate and efficient solver for the non-axisymmetric Green's function. An alternate approach might be to try to develop an approximate representation of the non-axisymmetric Green's function.

Dec. 1, 2003 – April 30, 2004

The Role of Instability Waves in the Prediction of Jet Noise

During the second year, work was begun on a study of the role of linear instability waves in the prediction of jet noise within an acoustic analogy formulation. The existing prediction methodologies are generally based on linearized, inhomogeneous equations whose complete solution consists, in general, of a particular solution and an appropriate combination of homogeneous solutions as determined by the boundary and initial conditions. When the base flow is taken to be a locally parallel mean flow, the system of governing equations admits spatially growing instability waves as homogeneous solutions. If these solutions are included as part of the general solution they would make the far-field sound exponentially large. For this reason, the contribution of linear instability waves is neglected in jet noise calculations based on the parallel flow approximation. This, however, precludes the use of the causal solution in the noise calculations.

The complete solution to the generalized acoustic analogy equations has been constructed in terms of a vector Green's function, assuming only that the mean flow varies slowly in the streamwise direction, relative to its variation in transverse directions (Goldstein & Leib 2004). The analysis shows that, in order to satisfy causality requirements, spatially growing linear instability waves must be included as part of the Green's function. These waves initially grow and then decay on the long streamwise length scales over which the flow evolves, and the final result is a uniformly valid, leading- order approximation to the exact Green's function in the limit of small mean flow spread rate.

Using the causal Green's function, along with the source terms from the acoustic analogy formulation, an expression for the far-field sound spectrum was derived. Upon making a number of simplifying approximations to the general formula, it was shown that the sound spectrum can be expressed as the sum of two terms: one of which is analogous to the non-causal, locally parallel mean flow solution considered during the first year of this work (and similar to that used in the JeNo method), and a second term arising from the linear instability waves.

Numerical calculations were carried out based on this simplified formula for the case of an axisymmetric jet with an analytically prescribed mean flow and an isotropic source model. Results obtained for Mach numbers (based on the ambient speed of sound) of between 1.1 and 1.5 for cold jets show that the instability wave component makes a negligible contribution to the predicted far-field sound at polar angles near 90 degrees from the jet axis, but that its relative importance increases as the angle from the jet axis decreases, with its effects being manifest at larger angles for higher Mach numbers. Even at the smallest Mach number considered (M=1.1), the instability wave component has a significant effect on the predicted sound levels at low frequencies for angles less than around 50 degrees. Results obtained at Mach 1.5 were chosen for qualitative comparison with experimental data and it was found that the computed results exhibited the general characteristics of the data: the spectral peak at thirty degrees to the jet axis is about a factor of ten larger, and occurs at about half the frequency of that at ninety degrees.

The computed results were also found to exhibit the experimentally observed Strouhal number scaling of the peak frequency at ninety degrees, and Helmholtz number scaling at small angles.

Conclusions and Future Work

Stricter noise regulations imposed on commercial aircraft will require continued improvement of the existing noise prediction tools to guide the development of innovative noise-reduction technologies. In the work carried out under this cooperative agreement, we have examined the potential benefits of basing a prediction scheme for one of the most significant components of aircraft noise, jet noise, on the generalized acoustic analogy put forward by Goldstein (2002).

The source terms appearing in the generalized acoustic analogy were examined and a number of approximations and assumptions commonly made in noise-source modeling were strictly applied to these terms to provide simplified formulas suitable for use in numerical calculations.

The potentially important role of linear instability waves in the prediction of jet noise was highlighted by a preliminary set of numerical calculations based on the causal solution to the generalized acoustic analogy equations. These calculations suggest that the inclusion of linear instability waves, through the use of the causal Green's function, could explain discrepancies between experimental data and current predictions for supersonic jets, and also possibly for the technologically important case of heated (subsonic and supersonic) jets.

Future work will be concerned with developing the computational capability needed to evaluate the causal Green's functions for more realistic turbulent jet flows. In addition, building upon the analysis carried out here, more refined models for the turbulence correlation functions will be required that make use of the latest available experimental data on the statistical flow properties of turbulent jets.

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