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## Summary

The lift on a wind-tunnel model must be accurately known if useful pressure signature data is to be measured. At the same time, the model must be adequately supported in the test section. One method that meets these two requirements consists of bending the support sting to a desired angle of attack and using supersonic wing performance theory to predict the lift. The method is simple, but difficult to accurately apply. A second method consists of machining two annuli in the wind-tunnel test-section support end of the sting which extents from the aft end of the model's fuselage. Two pairs of strain gages are set in these annuli and covered with epoxy which is then contoured to match the sting's surface. This changes the sting into a sting/balance for measuring normal force and pitching moment. Now, the model lift can be accurately set and monitored with a motor-driven angle-of-attack mechanism that holds the sting/balance in the test-section support strut. In this report, a method for designing an integral fuselage/sting/balance is described. A computer code is given for calculating sectional stresses and safety factors as well the sting divergence factor. A sample sting design is outlined to demonstrate the method.

## Introduction

Early sonic-boom research often employed wind-tunnel models with fuselages that merged with their support stings. These stings had to be long enough to prevent support disturbances from interfering with the model's pressure signatures, but short enough to keep bending and shearing stresses within prescribed safety factor limits. Non-lifting models with a cylindrical aft sections, such as those described in references 1 to 5 , had this kind of simplified sting.

When the overpressures were measured on wing-fuselage models at lifting conditions, references 6 to 12 , the design of the sting was considerably more complicated. The sting had to hold the model at a desired lifting attitude and meet Sting Divergence Factor limits as well. Moreover, the lift had to be accurately measured so volume and lift effects, references 13 and 14, would be properly represented when comparisons with theory were made. Adding engine nacelles to the model, references 15 to 18 , added even more complications to the design. Now, the sting had to be long enough to isolate the nacelleinduced effects and the wing-lift effects from potential sting volume and lift interference.

These requirements were met, to a large measure, with a redesign of the sting. By modifying the aft end during its construction, the sting could be transformed into a lift-pitching moment sting/balance. Used with a larger angle-of-attack mechanism, similar to the one described in reference 19, the angle of attack and model lift could be set, measured, and monitored during the wind-tunnel measurements of pressure signatures.

A practical model/sting/balance design is presented and described in this report. Also included is a computer code for predicting the sting divergence factor of the integrated fuselage/sting/balance. Used during the design of the wind-tunnel model, both safety factor limits and Sting Divergence Factor values can be calculated and checked to insure a model that will be acceptable for wind-tunnel testing.

## Nomenclature

$\mathrm{C}_{\mathrm{L}} \quad$ wing lift coefficient
$\mathrm{C}_{\mathrm{L}, 0} \quad$ lift coefficient at zero degree angle of attack
$C_{L \alpha} \quad d C_{L} / d \alpha$ of the wing at zero degree angle of attack, per radian
$d, d(x)$ diameter of the sting at a distance $x$ from start of the cylindrical section, in or ft
$d_{c} \quad$ diameter of the cylindrical section, in or ft
$\mathrm{d}_{\mathrm{s}} \quad$ diameter of the mounting stub, in or ft
$d_{t} \quad$ maximum diameter of the tapered section, in or ft
$E \quad$ modulus of elasticity of the fuselage and sting material, $\mathrm{lb} / \mathrm{ft}^{2}$
$H$ bending moment in the sting due to lift, in- lb or $\mathrm{ft}-\mathrm{lb}$
I moment of inertia of the sting cross section, in ${ }^{4}$ or $\mathrm{ft}^{4}$
$k \quad\left(d_{t}-d_{c}\right) / d_{c}$, equation (6) in the text, and equation (A.13) in Appendix A
$l_{c} \quad$ length of cylindrical section, in or ft
$l_{\text {cent }} \quad$ distance from nose to the centroid of projected planform area, $\mathrm{S}_{\mathrm{p}}$, in or ft
$l_{C L} \quad$ distance from nose to the center of lift, in or ft
$l_{c p} \quad$ distance from centroid of area to start of sting cylinder (used to calculate $l_{c}$ ), or distance from cruise lift center to start of sting cylinder (used to calculate $S D F$ )
$l_{M}$ length of the model from nose to start of cylindrical section, in or ft
$l_{t} \quad$ length of the tapered section of the sting, in or ft
M Mach number
$p \quad$ free-stream reference pressure, psf
$\Delta p \quad$ incremental flow-field pressure, psi
$P_{s t} \quad$ pressure on the model wing due to wind-tunnel "unstart", 288 to 576 psf
$q \quad$ wind-tunnel dynamic pressure, psf
S wing area, $\mathrm{in}^{2}$ or $\mathrm{ft}^{2}$
$\mathrm{S}_{\mathrm{p}} \quad$ projected planform area of the wing, fuselage, and nacelles, $\mathrm{in}^{2}$ or $\mathrm{ft}^{2}$
SCF ratio of the full-scale span to the model span
SF safety factor, ratio of the ultimate stress to model stress, $\sigma_{u l t} / \sigma$
SDF sting divergence factor
W weight of the model and sting, lb
$x \quad$ longitudinal distance, in or ft
$\alpha \quad$ angle of attack, deg
$\beta \quad$ Mach number parameter
$\gamma \quad$ ratio of specific heats, 1.4 for air
$\theta$ sting deflection angle, deg
$\xi \quad$ distance between the center of lift and the end of sting, in or ft
$\sigma \quad$ maximum stress at a station along the sting, psf
$\sigma_{u l t} \quad$ ultimate stress of the sting material, psf

## Previous Sonic-Boom Models

Early sonic-boom wind-tunnel research was performed with wing-body models that were as small as 0.25 to 1.0 inches in length. Angle of attack was often set by bending the sting to put the model at the desired lift attitude. The method was simple, but difficult to accurately apply.

Larger wing-fuselage models were built with tiny prisms mounted in the fuselage or in the angle-ofattack mechanism that supported the model sting, reference 12. A surveying instrument outside the test section was used to monitor and set the angle of attack. A light beam was reflected off the prism and the location of the return was measured. Wind tunnel turbulence, local flow angularity, and model-sting vibration made it difficult, but not impossible, to use this method for setting the model at the desired lift attitude. Moreover, it depended on having an accurate theoretical methods for predicting the lift curve slope, $C_{L \alpha}$, and the zero-angle-of-attack lift coefficient, $\mathrm{C}_{\mathrm{L}, 0}$; methods not always available during the early years of sonic boom research.

## Wind-Tunnel Model Design Problem

A lifting wing/fuselage/fin(s)/nacelle(s) sonic-boom wind-tunnel model and its support sting is required to meet several safety criteria before it is accepted for testing in a wind-tunnel facility. Minimum safety factors must be met to withstand the shear forces and bending moments on both the model and the model support. If the model is mounted on a sting extending from the aft fuselage, the sting must have a Sting Divergence Factor ( $S D F$ ) value that meets specified test criteria. This $S D F$ is a measure of the inertial vibration resistance of the material and the geometry of the sting that supports the lift-generating
model as it responds to airflow turbulence. Also required are assurances that pressure disturbances from the sting or the angle-of-attack mechanism will not impinge on the survey probe when it measures the model's pressure signature at the maximum separation distance.

The sketch in figure 1 shows one, but not the only, model and sting configuration that could be used to measure sonic-boom pressure signatures generated by the model in the wind-tunnel test section at supersonic Mach number.


Figure 1. Sketch of a typical sonic-boom wing-fuselage model with support sting.
The sting on this model has joined cylinder and tapered-cylinder sections. It is secured to the angle-of-attack mechanism or the test-section strut mechanism by a cylindrical stub at the end of the tapered section. Though, simple in design, the sting shown in figure 1 can be readily modified to meet special test requirements.

## Test-Section Apparatus Options

During early sonic-boom research efforts, the flow-field pressures generated by the model were measured with conical probes or with a multi-orificed splitter-plate, reference 1. If conical flow-field probes are selected, there were at least two ways they could be deployed. The first way was to fix the model and move the probes longitudinally. The second method was to fix the probes and move the model longitudinally. Two probes were usually employed: a survey probe which measured the flow-field overpressures from the model; and a reference probe which monitored the test-section static pressure as the pressure signature was recorded. The difference in pressures sensed by the two probes divided by the reference pressure provided a pressure ratio for recording the strength of the disturbances.

When models were small, no hard and fast rules existed for preferring one method over the other. It was the limitations imposed by the wind tunnel and its test section dimensions that made one method more attractive than the other. The desire for larger sonic-boom wind-tunnel models steered the choice
toward the moving model/fixed probes method. This latter method has been used for measuring sonicboom pressure signatures at both the Ames and Langley Research Centers with the model supported by a sting mounted on a traversing angle-of-attack mechanism. If the model was small enough and data was required only at one angle of attack, the desired lifting attitude could be built into the sting, and the angle of attack mechanism would not be required. However, the latest sonic-boom models have increased in length to 12 inches or longer, are tested at more than one angle of attack, i.e. lift condition, and are required to have a more rugged mount for the measurement of pressure signatures in the test section. So, the integrated model/sting/balance on a movable strut with fixed probes has become the apparatus of choice.

## Integral Sting/Balance

The model/sting arrangement shown in figure 1 can be made as large as necessary to adequately support a sonic-boom model that is from 6.0 to 16.0 inches in length. Two pairs of strain gages mounted near the end of the sting form a lift/pitching moment balance. These strain gages are set in over-and-under positions in annuli cut into the aft section of the sting body. Wires from the strain gages run along a groove cut between the annuli, along the outside of the barrel, and out the back end of the sting. After the strain gages and the wires are in place, epoxy is pored into the groove and the annuli to cover the strain gauges and connecting wires. After the epoxy has hardened, the material's surfaces are shaped to match the slopes and radii of the adjoining metal. So, the sting not only supports the model during the windtunnel measurements of pressure signatures, but it contains the instrumentation to measure the lift on the model. When model/sting /balance is calibrated, mounted on the angle-of-attack mechanism, and secured to the wind-tunnel strut, the angle of attack, model lift, the longitudinal location, and the radial position of the model's nose can be accurately known during the measurement of the pressure signature.

## Equations for Calculating Sting Segments Lengths

Aircraft volume and lift are the dominant sources of sonic-boom disturbances from a High-Speed Civil Transport (HSCT) aircraft. If the engine nacelles are mounted under the wings, then nacelle volume and nacelle-wing interference lift are additional sources of flow-field disturbances. Of these four sources, the equivalent area due to lift is larger than the equivalent areas due to volume and nacelle-wing interference lift by a factor of from 3 to 5 . Therefore, the wind-tunnel model sting must be sized to withstand stresses from lift-induced bending moments as well as shearing forces. Since wind-tunnel models are scaled-down versions of their HSCT counterpart, it would seem logical to expect that lift at cruise lift coefficient would generated the critical loads on the model during the pressure signature measuring sessions in the test section. However, the real critical loads come from possible wind-tunnel "unstart" or unexpected blockage of air flow. Pressures as high as 288 to 576 psf , which can be from 2 to 5 times larger than nominal cruise lift coefficient loads on the model, are often specified as "unstart" pressures. So it is the "unstart" pressures, $\mathrm{P}_{\text {st }}$, acting at the centroid of projected planform area that are used to calculate critical stresses on the model and its support sting.

The model/sting/balance design that will be discussed is shown in figure 2 , and is repeated as figure A2 in Appendix A.


Figure 2. Typical sonic-boom model with simple cylinder/tapered-cycle sting.
In addition to the previously-mentioned limits, there is also the maximum length limit on the model/sting. The total length of the model and sting, $l_{\max }$, will permit a complete pressure signature to be measured only out to a maximum model-probe separation distance dictated by the test section length, the test Mach number, and the boundary layer depth among the walls. Assuming this condition is met, the desired model-sting length consists of three sections:

$$
\begin{equation*}
l_{\text {max }}=l_{M}+l_{c}+l_{t} \tag{1}
\end{equation*}
$$

The stresses at the end of the model and cylindrical section, $l_{M}+l_{c}$, are calculated from:

$$
\begin{equation*}
\sigma=\frac{H d_{c}}{2 I}=\frac{32 P_{s t} S\left(l_{c p}+l_{c}\right)}{\pi d_{c}^{3}} \tag{2}
\end{equation*}
$$

Note that in equation (2),

$$
\begin{equation*}
l_{c p}=l_{M}-l_{c e n t} \tag{2a}
\end{equation*}
$$

because these stresses are due to unstart conditions rather than wind-tunnel running loads. A safety factor, SF, defined by:

$$
\begin{equation*}
S F=\frac{\sigma_{u l t}}{\sigma} \geq 4 \tag{3}
\end{equation*}
$$

must be met along the entire length of the model and sting. Tables of ultimate stress, $\sigma_{u l t}$, for different metals can be found in Strength And Materials textbooks, in Physics textbooks, or in Machine-Shop manuals.

The derivation of the $S D F$ is outlined in Appendix A and its value, determined from equations (A.10), (A12), and (A.13), is:

$$
\begin{equation*}
S D F=\frac{1.0}{d \theta / d \alpha} \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
\frac{d \theta}{d \alpha}=\frac{32 C_{L a} q S}{3 \pi d_{c}^{4}(1+k)^{3}}\left[l_{t}^{2}(3+k)+2 l_{t}\left(l_{c p}+l_{c}\right)\left(3+3 k+k^{2}\right)+3\left(l_{c p}+l_{c}\right)^{2}(1+k)^{3}\right] \tag{5}
\end{equation*}
$$

and the parameter, $k$, is defined as:

$$
\begin{equation*}
k=\frac{d_{t}-d_{c}}{d_{c}} \tag{6}
\end{equation*}
$$

In equation (5), $l_{c p}$ is:

$$
\begin{equation*}
l_{c p}=l_{M}-l_{C L} \tag{6a}
\end{equation*}
$$

because the $S D F$ is based on model running loads rather than wind-tunnel unstart conditions.
After the value of the $S D F$ is determined to be greater than 2 (A value of 2.0 is a minimum. Preferable is a $S D F$ that is greater than 3 or 4 ). The stress at the sting stub is calculated from:

$$
\begin{equation*}
\sigma=\frac{32 P_{s t}\left(l_{c p}+l_{c}+l_{t}\right)}{\pi d_{s}^{3}} \tag{7}
\end{equation*}
$$

and compared with the load criteria in equation (3) to make sure that all safety conditions are satisfied. Then, the angle along the tapered section of the sting is found from:

$$
\begin{equation*}
\text { tangent }(\text { Taper Angle })=\frac{d_{t}-d_{c}}{2 l_{t}} \tag{8}
\end{equation*}
$$

The sting taper angle should be much less than the Mach angle, preferably no more than 1.5 to 2.0 degrees for a Mach number of 2.0 . This insures that an isentropic compression rather than a shock forms at the smoothly-rounded junction of the constant-area cylinder and the tapered section. A shock starting in this location would move forward during radial propagation from the model through the flow field to the probe, and might intrude on the aft portion of the model's pressure signature. Eventually, if the separation distance is large enough, a shock will form from the coalescence and merging of isentropic pressure waves, move forward, and interact with the model's tail shock. This shock coalescence distance can be calculated and compared with the maximum separation distance desired during the measurement of pressure signatures.

## Design of the Sting-Balance

Using the equations presented in the previous section, a sting-balance can be designed for a sonicboom model once the model length and maximum model-sting length, $l_{\max }$, have been calculated. The following steps should be followed to determine the rest of the dimensions of the candidate sting-balance:

1. Select a station along the aft fuselage to start the constant-area cylindrical section, and determine the diameter, $d_{c}$, at this station.
2. Calculate the centroid of planform area which includes the wing and fuselage projected area up to the station selected on step 1. Calculate the shearing stress at the fuselage station selected in step 1, and determine the safety factor, SF. If SF is 4.0 or less, a larger diameter must be used or a station further forward must be chosen. If SF is greater than 4.0, use equations (2) and (3) to calculate the cylinder length, $l_{c}$. This cylinder has, ideally, a length larger than several $\beta d_{c}$ to keep the compression that forms at the cylinder-tapered section junction from encroaching on the tail shock of the model's pressure signature.
3. Calculate the length of tapered section, $l_{t}$, from equation (1) with the model length, $l_{M}$, and the value of $l_{c}$ obtained from step 2.
4. Use equations (4) to (6) to obtain an initial value of $S D F$. Model and sting values of, $C_{L \alpha}, \mathrm{~S}, d_{c}$, $\mathrm{E}, d_{t}$, and $l_{t}$ are required. The stub diameter, $d_{s}$, can be used as an initial value of $d_{t}$ to obtain this initial result.
5. If the $S D F$ is 2.0 or less, decrease $l_{c p}$ or increase $d_{c}$. If fuselage has a boat-tail, decrease $l_{M}$. Return to step 3.
If $S D F$ is greater than 2.0 but less than 3.0 and the ratio of $l_{c} d_{c}$ is less than $\beta d_{c}$, reduce $l_{c p}$ and/or increase $d_{c}$. Return to step 2.
If $S D F$ is greater than 3.0 but less than 4.0, decrease $l_{c p}$, increase $d_{c}$, or try increasing $d_{t}$. Return to step 2. (Check the possibility that a different, more suitable material can be used. If this choice is made, return to step 2.)
If $S D F$ is greater than 4.0 , move on to step 6 .
6. Calculate the stress and $S F$ in the stub using equations (3) and (7).
7. Calculate the taper angle with equation (8). Ideally, it should be between 1 and 1.5 degrees and no more than 2.0 degrees at a Mach number of 2.0 to minimize sting-induced disturbances. If so, move on to step 8 . If the taper angle is too large, consider increasing $d_{c}$ and returning to step 1.
8. Increase the calculated diameter, $d_{t}$, by 4 times the depth of a strain gage to account for their presence on the sting. The strain gages will be fastened to the top and bottom of annuli cut into the aft end of the sting. They will be covered with epoxy, and the surface contours smoothed to match radii and slopes of adjacent sections.

To illustrate the application of this iterative procedure, a design of a model-sting-balance for the measurement of pressure signatures in the wind tunnel will be outlined in the next section of this report. The required calculations are presented, and the details that were considered during the design and analysis procedure are discussed.

## Sample Sting-Balance Design

The procedure for obtaining a sonic-boom model-sting-balance was applied to the HSCT-10B concept described in reference 17, and shown in figure 3.


Figure 3. Three view of the HSCT-10B concept.
A sonic-boom wind-tunnel model of the HSCT-10B concept was constructed at 1:300 scale. The span was 6.0 inches, and the length was 12.4 inches from nose to wing-tip trailing edge. The HSCT-10B concept data needed to design the wind-tunnel model-sting are listed below.

| Wing area, S | $10,465.5 \mathrm{ft}^{2}$ |
| :--- | :--- |
| Planform area, $\mathrm{S}_{\mathrm{p}}$ | $11,381.3 \mathrm{ft}^{2}$ |
| Distance to start of constant-area cylinder, $l_{M}$ | 300.0 ft |
| Distance to centroid of planform area, $l_{\text {cent }}$ | 206.5 ft |
| Distance to center of cruise lift, $l_{C L}$ | 226.0 ft |
| Diameter of fuselage, $d_{c}$, at fuselage-cylinder junction | 8.4 ft |
| Wind-tunnel "unstart" pressure, $P_{s t}$ | 2.0 psi |
| Wind-tunnel dynamic pressure, $q$ | 455.5 psf |
| Ultimate stress of $15-5$ Ph 925 steel, $\sigma_{u l t}$ | $170,000.0 \mathrm{psi}$ |
| Modulus of elasticity of $15-5 \mathrm{Ph} 925$ steel, $E$ | $28.5 \times 10^{6} \mathrm{psi}$ |
| Total length of model and sting, $l_{\text {max }}=l_{M}+l_{c}+l_{t}$ | 800.0 ft |
| Diameter of sting stub, $d_{s}$, and maximum diameter of sting, $d_{t}$ | 18.75 ft |
| Slope of lift curve near $\alpha=0.0$ degree, $C_{L \alpha}$ | 1.828 per radian |
| Mach number, M | 1.8 |

For convenience and ease of calculation in this example, all model and sting dimensions were scaled to concept size. This could be done because the equivalent areas along with the concept dimensions were scaled to design the wind-tunnel model. However, in the code which is given in Appendix B, the diameter of the sting stub was input in inches.

These inputs were repeated in Appendix C, and the output of the final value of the calculated design diameter, the $S D F$ value, the sting dimensions, and the surface slope of the sting's tapered section were given in Appendix D. All the calculations required to arrive at this final estimate of the $S D F$ value were provided in Appendix E. A shortened summary of these calculations is given in the text to provide an overview of the procedure and some typical values of the important parameters encountered in the design process.

The sting lengths and diameters were calculated as follows:
Step 1. From the list of data:

$$
l_{M}=300.0, d_{c}=8.4, \text { and } l_{c p}=l_{M}-l_{\text {cent }}=300.0-206.5=93.5 \mathrm{ft}
$$

Step 2. Using equations (2) and (3) with $S F=4.0$ :

$$
l_{c}=15.14 \mathrm{ft} \text {, which was rounded off to: } l_{c}=15.0 \mathrm{ft}
$$

The length of the cylindrical section, $l_{\mathrm{c}}$ was only a bit longer than $\beta d_{c}=12.57 \mathrm{ft}$, but it was well behind the wing center-section trailing edge and the engine inlets. In this example, the design process continued with the proviso that if the $S D F$ was too small, the design would be restarted with a decreased $l_{M}$, which would provide a larger value of $\beta d_{\mathrm{c}}$, or a larger value of $\mathrm{d}_{\mathrm{t}}$. Either of these values would increase $l_{c}$.

Step 3. Combining the result of Step 2, the length of the model, and the desired total length:

$$
l_{t}=800.0-l_{M}-l_{c}=800.0-300.0-15.0=485.0 \mathrm{ft}
$$

Step 4. Using the results of the previous steps as input to equations (4) and (5), gave:

$$
S D F=3.90
$$

NOTE: in equation (5),

$$
l_{c p}=l_{M}-l_{C L}=300.0-226.0=74.0 \mathrm{ft}
$$

because $S D F$ is calculated with the lift at the supersonic-cruise center of lift, $l_{C L}$.
Step 5. This value of $S D F$ was a bit small. While the SDF could be improved by decreasing $l_{M}$ and $l_{c p}$ to increase $\mathrm{d}_{\mathrm{c}}$, there was also the possibility that a larger $\mathrm{d}_{\mathrm{t}}$ would significantly increase the $S D F$. Returning to Step 4 with:

$$
d_{t}=21.875 \mathrm{ft}
$$

gave:

$$
S D F=5.08
$$

which was a marked improvement. However, the value of $l_{\mathrm{c}}=15.0 \mathrm{ft}$ was still low.

The 15-5 Ph 925 steel was replaced with a steel like VascoMax C-300, which had these properties:

$$
\sigma_{u l t}=360 \times 10^{5} \mathrm{psf} \text {, and } E=396 \times 10^{7} \mathrm{psf}
$$

Then, the calculation returned to Step 2.
Step 2. With the VascoMax C-300 properties inserted in equation (3),

$$
l_{c}=66.0 \mathrm{ft} \text { was obtained, }
$$

which was 5.25 times larger than $\beta d_{\mathrm{c}}=12.57 \mathrm{ft}$. The new length $1_{\mathrm{t}}$ was:

$$
l_{t}=800.0-l_{M}-l_{c}=800.0-300.0-66.0=434.0 \mathrm{ft}
$$

Step 5. Using equation (5) with these new values gave:

$$
S D F=4.90
$$

which was a satisfactory value, and very close to $S D F=5.08$ that was obtained previously.
Step 6. $d_{\mathrm{t}}$ had been increased, and VascoMax C-300 had been substituted for $15-5 \mathrm{Ph} 925$ steel.
These changes resulted in a satisfactory $S F$ at the sting's mounting stub of:

$$
S F=13.7
$$

Step 7. These new lengths, along with the previous diameter $d_{\mathrm{c}}=8.4 \mathrm{ft}$, gave:

$$
\text { Taper Angle }=1.30 \mathrm{deg}
$$

which was larger than before, but still acceptable. Thus, satisfactory results could be obtained by changing dimensions, by changing materials, or by both.

Step 8. Since $d_{\mathrm{t}}$ had already been increased in the previous steps to increase a marginal $S D F$ value of 3.9 to a more acceptable value of 4.90 , the additional increment in $\mathrm{d}_{\mathrm{t}}$ could be omitted because the real $S D F$ value will most likely be well above 4.0.

These preliminary calculations have demonstrated that a satisfactory wind-tunnel model-sting design can be obtained by solving a relatively simple set of equations. If these equations are encoded on a programmable desk calculator or in a digital computer code, the iterations can be done quickly, and the complex calculations can be performed error-free. When these calculations are performed simultaneously with the preliminary design of the concept, there can be fewer problems in transferring key component features from full-scale concept to the reduced-scale wind-tunnel model.

## Wing Downwash Effects

There is one other consideration that can influence the design of the fuselage/sting/balance even though it is not crucial to its configuration. It is the lift induced on the sting by the downwash of the wing which is usually just ahead of the fuselage-sting junction. This downwash will generate a negative lift on the sting and be sensed by the balance at the end of the sting. By having the sting at a small angle of attack when the model is at its cruise lift coefficient, the sting lift can partially, or even completely, nullify the induced wing downwash lift. If this effect can be achieved with smoothly contoured surfaces from fuselage through to the straight center line of the sting, the setting and monitoring of model lift during the test can be made considerably easier.

## Concluding Remarks

Credible comparisons of measured and predicted pressure signatures from lifting sonic-boom windtunnel models depend on a well-designed sting support. On small sonic-boom wind-tunnel models, the sting can be built with a bend in it to provide the necessary angle of attack. This simplified approach will be sufficient if only one angle-of-attack attitude is required for the pressure signature measurements. This method of positioning a wind-tunnel model at an angle of attack has been done, but it is difficult to apply so as to provide an accurate level of lift. Sting flexure with its attendant increase in attitude can be estimated when the model is at angle of attack, but these predictions are based on average text-book or shop manual values of elasticity.

For the larger sonic-boom wind-tunnel models, especially those of HSCT concepts, this simple approach fails because of large aerodynamic forces on the models and/or because of a requirement to use several lifting attitudes during the tests. To meet the need for accurate and multiple-attitude positioning of the wind-tunnel model, an integrated model/sting/balance was devised for use in conjunction with an angle-of-attack positioning mechanism. This model/sting/balance had two pairs of strain gages built into the aft section of the sting to permit lift to be measured and set during the recording of the pressure signatures.

The description of this model-sting-balance design method, presented and discussed in this report, has been successfully used with the conceptual design procedures of a full-scale aircraft concept. In this mode of simultaneous concept/wind-tunnel model design, there is a high degree of assurance that a wind-tunnel model can be built that will faithfully generate most of the important sonic-boom characteristics for measurement with conventional apparatus.

Note that there are no electrical wiring diagrams for the strain gauges given in this report. These details are handled by competent electricians and/or electrical engineers once the structural and aerodynamic features of the fuselage/sting/balance are supplied by the model designer.

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## Appendix A

## Derivation of the Sting Divergence Factor, SDF

The structural members of a lifting sonic-boom model and sting can be represented by the components shown in figure A1.


Figure A1. Schematic of a beam representing a lifting model and its sting.
The deflection and slope of a beam under a concentrated load can be found from:

$$
\begin{equation*}
\frac{d}{d x}\left(\frac{d z}{d x}\right)=\frac{H}{E I} \tag{A.1}
\end{equation*}
$$

which is based on the assumption that the deflection is small relative to its length and that the slope is so small that

$$
\begin{equation*}
\frac{d z}{d x}=\tan \theta \approx \theta \tag{A.2}
\end{equation*}
$$

Variable $z$ is in the lift direction and normal to the longitudinal axis $x$. Between the center of lift and the start of a constant-diameter cylinder, the fuselage is represented by a cylinder with the diameter, $d_{c}$. A tapered cylinder with diameters $d_{c}$ and $d_{t}$ follows these two cylindrical sections. At each station between the lift and the end of the sting, the circular cross section has a moment of inertia expressed as

$$
\begin{equation*}
I=\frac{\pi}{64} d^{4} \tag{A.3}
\end{equation*}
$$

where the diameter, $d$, is a function $d(x)$, between the load, $L$, and the support end of the sting. With the
mentioned assumptions, equation (A.1) can be written as

$$
\begin{equation*}
\frac{d \theta}{d x}=\left(\frac{64}{\pi E}\right)\left(\frac{H}{d^{4}}\right) \tag{A.4}
\end{equation*}
$$

From this, it is possible to write

$$
\begin{equation*}
\theta=\left(\frac{64}{\pi E}\right) \int_{0}^{x} \frac{H}{d^{4}} d x \tag{A.5}
\end{equation*}
$$

The modulus $E$ is assumed to be constant, the bending moment, $H$, due to the load, $L$, acts along the sting over a distance where

$$
\begin{equation*}
\xi=l_{c p}+l_{c}+l_{t} \tag{A.6}
\end{equation*}
$$

The quantity $H$ is defined by

$$
\begin{equation*}
H=L x \tag{A.7}
\end{equation*}
$$

and the lift load, $L$, is

$$
\begin{equation*}
L=C_{L} q S \tag{A.8}
\end{equation*}
$$

For angles of attack close to zero

$$
\begin{equation*}
C_{L}=C_{L, 0}+C_{L \alpha} \alpha \tag{A.9}
\end{equation*}
$$

The sting divergence factor, SDF, is defined by

$$
\begin{equation*}
S D F=\frac{1}{\frac{d \theta}{d \alpha}} \tag{A.10}
\end{equation*}
$$

The model weight, W, and load, $L$, act normal to the beam, but only the load, $L$, is dependent on angle of attack. Taking the derivative of equation (A.5) with respect to $\alpha$, and substituting equations (A.7) to (A.9) gives

$$
\begin{equation*}
\frac{d \theta}{d \alpha}=\left(\frac{64 C_{L q} q S}{\pi E}\right) \int_{0}^{x} \frac{x}{d^{4}} d x \tag{A.11}
\end{equation*}
$$

which depends only on the lift characteristics of the wing and the geometry of the sting for the sting on the model shown in figure A2.


Figure A2. Typical model with a simple cylinder/tapered-cylinder sting.
In figure A2, the model and sting are linearly aligned. Often, the model is set at cruise angle of attack to keep the sting aligned with the wind-tunnel flow so it generates no lift. Should this not be possible at all the lift attitudes required during the measurements of pressure signatures, the sting lift must be accounted for in setting the model angle of attack.

For the model sting shown in figure A2, equation (A.11) can be analytically integrated, section by section, to provide the equation:

$$
\begin{equation*}
\frac{d \theta}{d \alpha}=\frac{32 C_{L a} q S}{3 \pi E d_{c}^{4}(1+k)^{3}}\left[l_{t}^{2}(3+k)+2 l_{t}\left(l_{c p}+l_{c}\right)\left(3+3 k+k^{2}\right)+3\left(l_{c p}+l_{c}\right)^{2}\left(1+k^{3}\right)\right] \tag{A.12}
\end{equation*}
$$

where, $k$, is defined as:

$$
\begin{equation*}
k=\frac{d_{t}-d_{c}}{d_{c}} \tag{A.13}
\end{equation*}
$$

Equations (A12) and (A.13) do not account for the annuli and longitudinal groove cut into the sting for the mounting of strain gages and wires to form the lift-pitching moment balance. Since these annuli and groove are cut into the thickest section of the sting, their presence will cause a negligible decrease in the Sting Divergence Factor value. Once the strain gages are in place, the annuli and groove are filled with epoxy and machined so that the original surface contour of the sting is restored. However, equations (A12) and (A13) can be used to obtain a preliminary value of the $S D F$.

After an acceptable preliminary sting design is obtained, three options for a more accurate estimate of $S D F$ are possible. With option one, the diameter along the sting, $d$, is represented as a function $d(x)$. Then, equation (A.11) is integrated numerically to obtain a more accurate value of the SDF for a sting with annuli machined in the aft six or seven inches of its length. Using option two, the annular grooves cut for the strain gauges are assumed to be machined to a constant depth below the sting surface. Using option
three, the annular grooves cut for the strain gauges are assumed to be machined to a constant radii. Thus, the annuli contours have the same taper angle as the sting or have constant diameter along their length. With either choice, this would permit equation (A.11) to be solved analytically or numerically for a very accurate value of the $S D F$ if a closed form solution were desired.

## Appendix B

## Computer Code For Calculating SDF Values

The program listed below calculates a value of the STING DIVERGENCE FACTOR, SDF, from input listed at the top of the program. This code was written in FORTRAN IV, but is compatible with a FORTRAN 77 compiler. A sample case with input is presented in Appendix C, and the output from that case is given in Appendix D.

```
1 C PROGRAM STING
2 C
3 C PROGRAM TO COMPUTE STRESSES, DIVERGENCE, AND STING
4 C LENGTHS. INPUT IS IN FULL-SCALE AIRCRAFT DIMENSIONS.
5 C MINIMUM ALLOWABLE SAFETY FACTOR OF 4.0 IS ASSUMED.
6 C
7 C THE EFFECT OF STING PLANFORM AREA IS NOT INCLUDED
8C
9C dmin = minimum diameter on sting (usually on aft end
10 C of fuselage in aircraft scale, feet
11 C xac = x-distance from nose to the center of lift in aircraft
1 2 ~ C ~ s c a l e , ~ f e e t
13 C xcent = x-distance from nose to centroid of planform area,
1 4 \mathrm { C } u s e d ~ t o ~ d e t e r m i n e ~ m a x i m u m ~ l e n g t h ~ o f ~ c y l i n d e r , ~ f e e t
15 C xfus = length of fuselage in aircraft scale, feet
16 C xtap = x-distance from nose to end of sting, feet
17 C cla = lift curve slope, per radian
18 C emat = modulus of elasticity, psi
19 C sult = ultimate stress in psi
20 C sref = reference wing area used for Sting Divergence
21 C calculation, square feet
22 C splan = planform area used to determine maximum length of
23 C cylindrical section, square feet
24 C default value = 1.25*sref
25 C sf = model scale factor
26 C qwt = wind-tunnel dynamic pressure, psf
27 C dmax = maximum diameter at end of sting in aircraft scale, feet
28 C pst = wind-tunnel starting load, psi
29 C default value is 2.0 psi
30 C dstub = diameter of support stub on end of sting-balance, inches
31 C delx = incremental length used to calculate cylinder, feet
32 C safact = minimum safety factor, default is 4.0
33 C
34
35 C
36 Character ident(1)*80
37 C
38 NAMELIST/INPUT/dmin,dmax,pst,xac,xfus,xtap,cla,emat,sult,sref,sf,
39
40 C
4 1
    safact=4.0
```

```
    delx=2.0
    pst=2.0
    dstub=.75
65 C Calculation of length of cylinder on end of fuselage
80 sig=sult/safact
81 force=pst*splan
82 xst=sig*(pi/32.0)*(dmin**4)*144.0/(dmin*force)
83 xcylmax=xst+xcent
8 4 ~ w r i t e ( 6 , 9 0 2 ) ~ x c y l m a x ~
85 902 format(/,8x,24hMax. Cylinder Distance =,f10.4,4h ft.)
87 C Sting divergence is calculated for a sting with a cylinder-
88 C tapered cylinder shape. xcyl is computed in program and can
89 C be controlled with the input value of SAFACT as long as
90 C SAFACT is > or = to 4.0
xcyl=float(int(xcylmax))
```

45 C
46
47
48
49
50 C
51
52
53
54
55
56
57
58
59
60
61
62
63
64 C
66 C
67
68
69
86 C
91 C
92
$101 \mathrm{~b}=3.0^{*} \mathrm{pi}^{*}(\mathrm{dmin} * * 4) *$ emat $^{*}((1.0+\mathrm{xlk}) * * 3)$
$102 \mathrm{c}=$ ell*ell*(3.0+xlk)

113 C
$114 \mathrm{k}=1$
115 dfin=dmax
11610 sigma $=.5 *$ pst $^{*}$ sref $^{*}($ xtot-xac $) *$ dfin $/((\mathrm{pi} / 64.0) *($ dfin $* * 4) * 144.0)$
117 factor=sult/sigma
118 write $(6,908)$ dfin,xtot,factor
119908 format(/, $2 \mathrm{x}, 6 \mathrm{hDia} .=, \mathrm{f} 7.3,4 \mathrm{~h} \mathrm{ft} ., 3 \mathrm{x}, 10 \mathrm{hx}($ total $)=, \mathrm{f} 8.3,4 \mathrm{~h} \mathrm{ft} ., 3 \mathrm{x}$,
$120 \quad 115 \mathrm{hSafety}$ Factor $=, \mathrm{f} 7.3$ )
$121 \quad \mathrm{k}=\mathrm{k}+1$
$122 \operatorname{if}(\mathrm{k}-2) 11,11,12$
123 11 dfin=dstub*sf/12.0
124 write $(6,909)$
125909 format(////,2x,45hLoad Factor in the scaled-up sting mount stub,/)
126 go to 10
127 C
128
129
130
131910 format(/,2x,28hAirplane Plus Sting Length =,f9.3,////,5x,25hW.T. M
132 lodel Scale Factor =,f7.1,/)
133 C
134 C Model and sting lengths resized with the scale factor
135 C
136 xfus=xfus*12.0/sf
137 xcyl=xcyl*12.0/sf
138 xtot=xtot*12.0/sf
139 dmin=dmin*12.0/sf
140 dmax=dmax*12.0/sf
141 write $(6,912)$ xfus,xcyl,xtot
142912 format(/,2x,14hModel length $=, 48.4,4 \mathrm{~h}$ in., $3 \mathrm{x}, 21 \mathrm{hModel}$ Plus Cylinde
if(xcyl .lt. xfus) go to 950
write $(6,904)$ xcyl
904 format(/,10x, 15 hXCYL Distance $=, \mathrm{f10.4,4hft)}$.
ell=xtap-xcyl
eta=xfus-xac
el=xcyl-xfus
$\mathrm{xlk}=(\mathrm{dmax}-\mathrm{dmin}) / \mathrm{dmin}$
$\mathrm{a}=32.0$ * qut* ${ }^{\text {sref* }}$ cla
$\mathrm{c} 1=2.0^{*} \mathrm{ell} *(\mathrm{eta}+\mathrm{el}) *\left(3.0+3.0^{*} \mathrm{xlk}+\mathrm{xlk} * * 2\right)$
$\mathrm{c} 2=3.0 *\left((\mathrm{eta}+\mathrm{el})^{* *} 2\right) *((1.0+\mathrm{xlk}) * * 3)$
calc $=a^{*}(c+c 1+c 2) / b$
div=1.0/calc
write $(6,906)$ eta,el,ell
906 format( $/ /, 2 \mathrm{x}, 6 \mathrm{hdLcp}=, \mathrm{f8} .3,4 \mathrm{~h} \mathrm{ft} .3 \mathrm{x}, 4 \mathrm{hLc}=, \mathrm{f8} .3,4 \mathrm{~h} \mathrm{ft} ., 3 \mathrm{x}, 4 \mathrm{hLt}=$
$1,49.3,4 \mathrm{hft}$.
write $(6,907)$ div
907 format(/,2x,18hSTING DIVERGENCE $=, 48.5$ )
xtot=xfus+el+ell
C
$12 \mathrm{dzdx}=.5^{*}(\mathrm{dmax}-\mathrm{dmin}) / \mathrm{ell}$
eps $=180.0 * \operatorname{atan}(\mathrm{dzdx}) / \mathrm{pi}$
write $(6,910)$ xtot,sf
1odel Scale Factor =,f7.1,/)
Model and sting lengths resized with the scale factor
xfus $=x$ fus* $12.0 /$ sf
$\mathrm{xcyl}=\mathrm{xcyl} * 12.0 / \mathrm{sf}$
dmin=dmin*12.0/sf
dmax $=$ dmax* $12.0 / \mathrm{sf}$
912 format(/,2x,14hModel length $=, \mathrm{f} 8.4,4 \mathrm{~h}$ in., $3 \mathrm{x}, 21 \mathrm{hModel}$ Plus Cylinde
$1 \mathrm{r}=, \mathrm{f8} .4,4 \mathrm{~h}$ in., $/ /, 2 \mathrm{x}, 41 \mathrm{hModel}$ Plus Cylinder Plus Tapered Length $=$

```
144 2,f8.4,4h in.)
145 write(6,914) dmin,dmax,dstub
146 914 format(/,5x,6hDmin =,f8.4,4h in.,/,5x,6hDmax =,f8.4,4h in.,/,5x,7h
147 1Dstub =,f7.4,4h in.,/)
148 C
1 4 9 ~ w r i t e ( 6 , 9 1 6 ) ~ e p s ~
150 916 format(/,5x,23hTapered Section Slope =,f9.5,1x,4hdeg.///)
151 go to 100
152 C
153 950 write(6,952)
154 952 format(/,1x,75hLength needed to have safety factor of 4.0 is less
155 1than XFUS, case aborted.,///)
156 100 END
157 C
```


## Appendix C

## Input for Sample Case

```
1 \text { HSCT-10B sting calculation: VascoMax C-300 steel}
2 $input
3 dmin=8.400,
4 dmax=28.125000,
5 xcent=206.5000,
6 xac=226.000,
7 xfus=300.000,
8 xар=800.00,
9 cla=1.828,
10 emat=3960000000.0,
11 sult=250000.0,
12 sref=10465.5,
13 splan=11381.20,
14 sf=300.0,
15 qwt=455.5,
16 pst=2.0,
17 delx=5.0,
18 $
```


## Appendix D

## Output From Sample Case

```
HSCT-10B sting calculation: VascoMax C-300 steel
3
Dia.(min) = 8.400 ft.
Dia.(max) = 28.125 ft.
x(cp) = 226.000 ft.
x(t)=800.000 ft.
Fus. Length = 300.000 ft.
Ref. Area = 10465.50 sq.ft.
Lift Slope = 1.8280 per radian
Young's Mod. = 3960000000.0 psf
Ult. Stress = 250000.0 psi
Scale Factor = 300.00
q}(\mathrm{ wind tunnel })=455.50 ps
"Unstart" pressure = 288.00 psf
Dia.(stub) = 0.750 in.
Planform Area = 11381.20 sq.ft.
x(centroid ) = 206.500 ft.
x=300.00 ft. sigma }=36575.637 psi sig(ult)/sig =6.8352
x}=305.00\textrm{ft.}\mathrm{ sigma }=38531.553 psi sig(ult)/sig = 6.488
x=310.00 ft. sigma = 40487.470 psi sig(ult)/sig =6.1747
x=315.00 ft. sigma = 42443.386 psi sig(ult)/sig = 5.8902
x=320.00 ft. sigma = 44399.303 psi sig(ult)/sig = 5.6307
x=325.00 ft. sigma = 46355.219 psi sig(ult)/sig = 5.3931
x=330.00 ft. sigma =48311.135 psi sig(ult)/sig = 5.1748
x=335.00 ft. sigma = 50267.052 psi sig(ult)/sig=4.9734
x=340.00 ft. sigma = 52222.968 psi sig(ult)/sig = 4.7872
x=345.00 ft. sigma = 54178.885 psi sig(ult)/sig = 4.6143
x=350.00 ft. sigma = 56134.801 psi sig(ult)/sig = 4.4536
x=355.00 ft. sigma = 58090.717 psi sig(ult)/sig = 4.3036
x=360.00 ft. sigma = 60046.634 psi sig(ult)/sig = 4.1634
x=365.00 ft. sigma = 62002.550 psi sig(ult)/sig = 4.0321
x =370.00 ft. sigma = 63958.467 psi sig(ult)/sig = 3.9088
Max. Cylinder Distance = 366.2717 ft.
XCYL Distance = 366.0000 ft.
dLcp}=74.000 ft. Lc = 66.000 ft. Lt = 434.000 ft.
STING DIVERGENCE = 4.89938
```

2

47 Dia. $=28.125 \mathrm{ft} . \mathrm{x}($ total $)=800.000 \mathrm{ft}$. Safety Factor $=45.448$

48
49
50
51
52
53
54
55 Dia. $=18.750 \mathrm{ft} . \mathrm{x}($ total $)=800.000 \mathrm{ft}$. Safety Factor $=13.466$
56
57 Airplane Plus Sting Length $=800.000$
58
59
60
61 62
63
64
65
66 Model Plus Cylinder Plus Tapered Length $=32.0000$ in.
67
$68 \operatorname{Dmin}=0.3360 \mathrm{in}$.
69 Dmax $=1.1250$ in.
70
71
72
73
Load Factor in the scaled-up sting mount stub
W.T. Model Scale Factor $=300.0$

Model length $=12.0000$ in. Model Plus Cylinder $=14.6400$ in.

Dstub $=0.7500 \mathrm{in}$.

Tapered Section Slope $=1.30180 \mathrm{deg}$.

These results are repeated and discussed in the text.

## Appendix E

## Calculations of SDF for the Sample Case

The procedure for designing a sonic-boom model-sting-balance was applied to the HSCT-10B concept, reference 17. It was shown as text figure 3 and repeated as figure E1.


Figure E1. Three view of the HSCT-10B concept.
A 1:300 scale sonic-boom wind-tunnel model was designed and built from the HSCT-10B concept (Figure E2). The span of the model was 6.0 inches, and the length was 12.4 inches from nose to wing-tip trailing edge.


Figure E2. Three view of the HSCT-10B wind-tunnel model. (Only front of sting is shown.)
The full-scale HSCT-10B concept data needed to design the wind-tunnel model-sting-balance are listed in the text and repeated below.

| Wing area, S | $10,465.5 \mathrm{ft}^{2}$ |
| :--- | :--- |
| Planform area, $\mathrm{S}_{\mathrm{p}}$ | $11,381.3 \mathrm{ft}^{2}$ |
| Distance to start of constant-area cylinder, $l_{M}$ | 300.0 ft |
| Distance to centroid of planform area, $l_{\text {cent }}$ | 206.5 ft |
| Distance to center of cruise lift, $l_{C L}$ | 226.0 ft |
| Diameter of fuselage, $d_{c}$, at fuselage-cylinder junction | 8.4 ft |
| Wind-tunnel "unstart" pressure, $P_{s t}$ | 2.0 psi |
| Wind-tunnel dynamic pressure, $q$ | 455.5 psf |
| Ultimate stress of $15-5$ Ph 925 steel, $\sigma_{\text {ult }}$ | $170,000.0 \mathrm{psi}$ |
| Modulus of elasticity of $15-5 \mathrm{Ph} 925$ steel, $E$ | $28.5 \times 10^{6} \mathrm{psi}$ |
| Total length of model and sting, $l_{\text {max }}=l_{M}+l_{c}+l_{t}$ | 800.0 ft |
| Diameter of sting stub, $d_{s}$, and maximum diameter of sting, $d_{t}$ | 18.75 ft |
| Slope of lift curve near $\alpha=0.0$ degree, $C_{L \alpha}$ | 1.828 per radian |
| Mach number, M | 1.8 |

For convenience and ease of calculation in this example, all model and sting dimensions were scaled to concept size. This could be done because the equivalent areas along with the concept dimensions were scaled to design the wind-tunnel model. However, in the code which is given in Appendix B, the diameter of the sting stub was input in inches.

The sting lengths and diameters were calculated as follows:
Step 1: From the list of data:

$$
l_{M}=300.0, \text { and } d_{c}=8.4
$$

Also,

$$
l_{c p}=l_{M}-l_{\text {cent }}=206.5=93.5 \mathrm{ft}
$$

Step 2: Setting $l_{c}=0.0$ in equation (2), the maximum shear stress and the $S F$ at the end of the model, i.e., start of cylinder was:

$$
\sigma=32 * 288.0 * 11,381.3 * 93.5 /\left(\pi * 8.4^{3}\right)=5,266,891.9 \mathrm{psf}
$$

so

$$
S F=244.8 \times 10^{5} / 5,266,891.9=4.65
$$

Using equations (2) and (3) with $\mathrm{SF}=4.0$ :

$$
l_{c}=244.8 \times 10^{5} * \mathrm{p} * 8.4^{3} /\left(4 * 32 * 288.0^{*} 11,381.2\right)-93.5=15.14 \mathrm{ft}
$$

which was rounded off to:

$$
l_{c}=15.0 \mathrm{ft}
$$

The length of the cylindrical section, $l_{c}$ was only a bit larger than $\beta d_{c}=12.57 \mathrm{ft}$, but its position on the aft fuselage was well behind the wing center-section trailing edge, and the engine nacelle
inlets. In this example, the design would be continued with the proviso that if the $S D F$ was too small, the design would be restarted with a decreased $l_{M}$, which would provide a larger value of $d_{c}$, or a larger value of $d_{t}$.

Step 3: Combining the result of Step 2, the length of the model, and the desired total length:

$$
l_{t}=800.0-l_{M}-l_{c}=800.0-300.0-15.0=485.0 \mathrm{ft}
$$

Step 4: Using the results of the previous steps as input to equations (4) and (5) yielded:

$$
S D F=3.90
$$

NOTE: in equation (5), $l_{c p}=l_{M}-l_{C L}=300.0-226.0=74.0 \mathrm{ft}$ because $S D F$ was calculated with the lift at the supersonic-cruise center of lift, $l_{C L}$.

Step 5: The $S D F$ was a bit small. While the $S D F$ would be improved by decreasing $l_{M}$ and $l_{c p}$ to increase $d_{c}$, there was also the option of using a larger $d_{t}$ to significantly increase the $S D F$. Returning to Step 4 with $d_{t}$ increased to:

$$
d_{t}=21.875 \mathrm{ft}
$$

gave:

$$
S D F=5.08
$$

which was a marked improvement. However, the value of $l_{c}=15.0 \mathrm{ft}$ was still on the marginallylow side. Considering this value as provisional, the design was continued so that the end results with all values known could be judged.

Step 6: The maximum stress and $S F$ in the stub, calculated from equation (7) was:

$$
\begin{gathered}
\sigma=32 * 288.0 * 10,465.5 *(74.0+15+485) /\left(\pi * 18.75^{3}\right)=2,673,379.7 \mathrm{psf} \\
S F=244.8 \times 10^{5} / 1,683,527.8=9.15
\end{gathered}
$$

which was no cause for concern.
Step 7: The angle along the tapered section of the sting is determined from equation (8).

$$
\text { tangent }(\text { Taper Angle })=(21.875-8.4) /(2 * 485.0)=0.013892,
$$

from which an acceptable value of:

$$
\text { Taper Angle }=0.796 \text { deg was obtained. }
$$

So, everything seemed to be acceptable except for the cylinder length:

$$
l_{c}=15.0 \mathrm{ft}
$$

Instead of changing sting dimensions, the $15-5 \mathrm{Ph} 925$ steel was replaced with a steel like VascoMax C-300, which had the following properties:

$$
\sigma_{u l t}=360 \times 10^{5} \mathrm{psf}, \text { and } E=396 \times 10^{7} \mathrm{psf}
$$

Then, returning to Setp 2 in the calculation.
Step 2: Using VascoMax C-300 properties in equation (3) yielded:

$$
l_{c}=66.0 \mathrm{ft}
$$

which was 5.25 times larger than $\beta d_{c}=12.757 \mathrm{ft}$, the previous value.

Then, the new length $l_{t}$ was:

$$
l_{t}=800.0-l_{M}-l_{c}=800.0-300.0-66.0=434.0 \mathrm{ft}
$$

Now, Step 5 was repeated with this newly-calculated length.
Step 5: Using equation (5) with these new values gave:

$$
S D F=3.71
$$

which was slightly lower than obtained earlier. However, if $d_{t}$ was increased to:

$$
d_{t}=28.125 \mathrm{ft}
$$

and the previous calculations were repeated, a new value of:

$$
S D F=4.90
$$

was found which was very close to the value of $S D F=5.08$ that was obtained before.

Step 6: Maximum sting diameter, $d_{t}$, had been increased, and VascoMax C-300 had been substituted for 15-5 Ph 925 steel. These changes resulted in a Safety Factor, $S F$, at the sting's mounting stub of:

$$
S F=13.7, \text { which was again, satisfactory. }
$$

Step 7: These new lengths, along with the previous diameter, $d_{c}=8.4 \mathrm{ft}$, gave:

$$
\begin{gathered}
\text { tangent }(\text { Taper Angle })=(28.125-8.4) /(2 * 434.0)=0.022725, \text { and } \\
\text { Taper Angle }=1.30 \mathrm{deg}
\end{gathered}
$$

which was larger than before, but still acceptable. Thus, satisfactory results could be obtained by changing dimensions, by changing materials, or by both. This flexibility should be kept in mind during this entire design procedure.

Step 8: Since $d_{t}$ had already been increased in the previous steps to increase a marginal $S D F$ value of 3.71 to a more acceptable value of 4.90 , the additional increment in $d_{t}$ could be omitted because the real SDF value will most likely be well above 4.0.

This completed the preliminary fuselage/sting/balance design calculations, and the model/sting unit was ready for an independent follow-up evaluation before sending it to the shops to be built as a model/sting/balance.


