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TECHNICAL MEMORANDUM

No. 1105

PLANE AND THREE-DIMENSIONAL FLOW AT HIGH SUBSONIC SPEEDS

By B. Gothert

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Unterschallgeschwindigkeiten

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PLANE AND THREE-DIMENSIONAL FLOW AT HIGH SUBSONIC SPEEDS*

(Extension of the Prandtl Rule)

By B. Göthert, Berlin-Adlershof

Abstract: For two- and three-dimensional flow in a compressible medium, a simple relation is given by which, to a first approximation, the quantitative influence of compressibility upon the velocities and pressures can be understood in a clear manner. In the application of this relation the distinct behaviors of two-dimensional and axially symmetric three-dimensional flow with increasing Mach number are brought out. For slender elliptic cylinders and ellipsoids of revolution, calculations are made of the critical Mach number; that is, the Mach number at which local sonic velocity is achieved on the body. As a further example, the lifting wing of finite span is considered, and it is shown that the increase of wing lift with Mach number at a given angle of attack is greatly dependent upon the aspect ratio b^2/F.

OUTLINE

I. Approximate Equation for Compressible Flow (Prandtl Rule)

II. Distortion of the Perturbation Potential and the Streamline Field in Equal Sense and in Equal Strength (Extension of the Prandtl Rule)

III. Plane and Three-Dimensional Flow without Lift

1. Wing of Finite Thickness without Lift in Two-Dimensional Flow

2. Axially Symmetric Bodies without Lift

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*"Ebene und räumliche Strömung bei hohen Unterschallgeschwindigkeiten" (Erweiterung der Prandtlschen Regel) from report 127, Lilienthal Gesellschaft. pp 97 - 101.

3. Comparison of Two-Dimensional and Axially Symmetric Flow without Lift

4. Determination of Critical Free-Stream Velocity for Two-Dimensional and Axially Symmetric Flow

IV. Wings of Finite Span

V. Summary

I. APPROXIMATE EQUATION FOR COMPRESSIBLE FLOW

(PRANDTL RULE)

If, in a flow which has only small variations of velocity and inclination from parallel flow, the potential is separated into the potential Φ_0 of the parallel stream and the potential Φ of a superimposed perturbation flow, then it can be shown by a development given by Prandtl that, to a first approximation, the equation of continuity for compressible flow assumes the following form:[1])

$$\frac{\partial^2 \Phi_c}{\partial x_c{}^2} + \frac{\partial^2 \Phi_c}{(\sqrt{1 - M^2} \times \partial y_c)^2} + \frac{\partial^2 \Phi_c}{(\sqrt{1 - M^2} \times \partial z_c)^2} = 0 \qquad (1)$$

This differential equation for compressible flow can be simply transformed formally into the known equation for incompressible flow if the following transformations of coordinates are chosen:

$$x_c = x_{ic}; \quad \sqrt{1 - M^2}\, y_c = y_{ic}; \quad \sqrt{1 - M^2}\, z_c = z_{ic} \qquad (2)$$

These transformations mean that each potential field of incompressible flow can be transformed into the potential field of a compressible flow, while in the compressible

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[1]In what follows, the subscript "c" refers to compressible flow, and the subscript "ic" likewise to incompressible flow. Furthermore,

$$M = \frac{V_0}{a_0} = \frac{\text{Free stream velocity}}{\text{Speed of sound}} = \text{Mach number}$$

flow pattern the perturbation potential is the same as in the incompressible flow at the corresponding points determined from equation (2). In this transformation the perturbation velocities Δv_y and Δv_z become smaller at the corresponding points of the compressible flow by the ratio $\sqrt{1 - M^2}$, while the perturbation velocities Δv_x remain unchanged.

If a parallel flow with the velocity $v_{o_c} = v_{o_{ic}}$ is now superimposed upon the perturbation velocities determined in this manner, the inclination of the streamlines will be reduced at the corresponding points in the compressible flow, while the pressures will, to a first approximation, remain unchanged (on the assumption of slight deviations from parallel flow, the pressures are, to a first approximation, only a function of the perturbation velocity v_x). In the transformation to the compressible flow the streamline pattern will, as a result of the smaller inclinations of streamlines in the compressible medium, no longer be compressed according to a simple rule; that is, it will be deformed in an opposite sense from the potential field to a degree not immediately apparent.

It now frequently becomes exceedingly difficult to determine the new contour in the distorted potential field. Consider, for example, the very simple case of flow about a plate of infinite span at a small angle of attack. If the potential of the flow about the plate (incompressible) is distorted in the directions of the Y- and Z-axes by the factor $1/\sqrt{1 - M^2}$, the points of the plate, together with the positions of the bound vortices producing the lift, move during the distortion as if the angle of attack of the plate had been increased. On the other hand, however, the slopes of the streamlines become flatter in the ratio $\sqrt{1 - M^2}$. It is evident that after the distortion a flow takes place between the bound vortices. In this simple case, however, the proper relationship between the positions of the bound vortices and the streamline slopes can be achieved easily by a corresponding intensification of the potential in the compressible flow. The relationships become considerably more difficult, even, for example, in the case of inclined flow about a plate of finite span, for which three-dimensional flow results, so that after the potential distortion the flow becomes quite confusing.

II. DISTORTION OF THE PERTURBATION POTENTIAL AND THE STREAMLINE FIELD IN EQUAL SENSE AND IN EQUAL STRENGTH (EXTENSION OF THE PRANDTL RULE)

The difficulties indicated in the preceding section can be in principle lessened if it is possible to distort the streamline pattern in the same sense and to the same extent as the potential field of the perturbation flow. This aim can in fact be achieved by means of a stratagem.

Equation (1) is completely independent of the velocity v_o of the parallel flow upon which the perturbation potential Φ is superimposed; it must only be observed that the perturbation velocities remain small in comparison with the velocity of the parallel flow. It therefore makes no difference whether the velocity $v_{o_{ic}}$ in the incompressible medium is of the same or a different magnitude compared with the velocity v_{o_c} in the compressible medium. If the following relationship is now chosen between the velocities of the parallel stream in the compressible and in the incompressible mediums:

$$v_{o_c} = (1 - M^2) \times v_{o_{ic}}$$

then in corresponding points of the compressible flow the streamline inclination becomes:

$$\frac{dy_c}{dx_c} \approx \frac{\Delta v_{y_c}}{v_{o_c}} = \frac{\sqrt{1 - M^2} \times \Delta v_{y_{ic}}}{(1 - M^2) \times v_{o_{ic}}} \approx \frac{1}{\sqrt{1 - M^2}} \times \frac{dy_{ic}}{dx_{ic}}$$

that is, at corresponding points in the compressible medium the streamline inclination is increased in the ratio $1/\sqrt{1 - M^2}$. The same relation can also be found for the z-direction. This means, however, that the streamline pattern of the compressible flow results simply from distortion of the equivalent incompressible flow in the directions of the y and z axes by the factor $1/\sqrt{1 - M^2}$. The streamline pattern and the perturbation potential field are thus actually distorted in the same way.

The following relationships can be found for the perturbation velocity Δv_x and hence also for the local pressure Δp in the flow:

$$\frac{\Delta p_c}{\frac{\rho}{2} \times {v_o}_c^{\,2}} \approx 2 \times \frac{\Delta {v_x}_c}{{v_o}_c} = 2 \times \frac{\Delta {v_x}_{ic}}{(1 - M^2) \times {v_o}_{ic}}$$

that is, in the compressible flow the pressure ratio $\frac{\Delta p_c}{\rho/2 \times {v_o}_c^{\,2}}$ and the velocity ratio $\Delta {v_x}_c/{v_o}_c$ are larger by the factor $1/(1 - M^2)$ than at corresponding points of the equivalent incompressible flow.

From the basic facts derived above, the following simple proposition can be demonstrated:

The streamline pattern of a compressible flow to be calculated can be compared with the streamline pattern of an incompressible flow which results from contraction along the y and z axes of the profile contour by the factor $\sqrt{1 - M^2}$ (x-axis in the direction of the free stream). In the resulting compressible flow the pressures $\frac{\Delta p}{\rho/2 \times v_o^{\,2}}$ as well as the perturbation velocities $\Delta v_x/v_o$ are greater in the ratio $1/(1 - M^2)$ and the streamline slopes greater in the ratio $1/\sqrt{1 - M^2}$ than at the corresponding points of the equivalent incompressible flow.

With this principle, approximate solutions can be obtained in all cases of compressible flow, as long as the corresponding solutions for incompressible flow are known. Thus the problem, mentioned in the beginning, of an infinitely broad plate placed at a small angle of attack, can be solved in a simple manner, as well as the flow about wings of finite span, swept-back wings, axially symmetric bodies, and the like. In this manner, also, approximate solutions are found quite easily for three-dimensional flows which are not axially symmetric, of which an example will be given in the following section IV.

III. PLANE AND THREE-DIMENSIONAL FLOW WITHOUT LIFT

1. Wing of Finite Thickness without Lift in Two-Dimensional Flow

Velocity Along the Surface of the Body

If, for the sake of simplicity, the wing is represented by an elliptic cylinder of large span, then it is known from incompressible flow that the greatest velocity increase appears at the position of maximum thickness, and has the magnitude $\Delta v_{max}/v_o = d/t$.

According to the rule presented previously the perturbation velocities produced by a body in incompressible flow are greater by the factor $1/(1 - M^2)$ than those for a body in incompressible flow thinner by $\sqrt{1 - M^2}$, that is:

$$\left(\frac{\Delta v_{max}}{v_o}\right)_c = \frac{1}{1 - M^2} \times \left(\frac{\Delta v_{max}}{v_o}\right)^*_{ic} = \frac{1}{1 - M^2} \times \left(\frac{d}{t}\right)^*_{ic}$$

With $(d/t)^*_{ic} = \sqrt{1 - M^2}\,(d/t)_c$ there then results for the greatest excess velocity on the wing in the compressible flow:

$$\left(\frac{\Delta v_{max}}{v_o}\right)_c = \frac{1}{\sqrt{1 - M^2}} \times \left(\frac{d}{t}\right)_c$$

The greatest excess velocities, and therefore also the greatest negative pressures acting on the wing, therefore correspond in plane flow to the familiar Prandtl Rule with $1/\sqrt{1 - M^2}$. Since the velocity distribution over the airfoil in compressible flow corresponds to that over a thinner section in incompressible flow, the curve of velocity distribution becomes somewhat more full with increasing Mach number.

Velocity Disturbance at a Great Distance from the Airfoil

According to a calculation carried out in FB 1165[2], in incompressible flow the velocity perturbation $\Delta v_x/v_o$ at a great distance from a slender wing of large span is:

$$\frac{\Delta v_x}{v_o} \sim \frac{d}{t} \times \frac{y^2 - x^2}{(y^2 + x^2)^2} \times t^2$$

According to section II this equation takes the following form for compressible flow:

$$\left(\frac{\Delta v_x}{v_o}\right)_c \sim \frac{1}{1 - M^2} \times \left(\sqrt{1 - M^2} \times \frac{d}{t}\right)_c \times \frac{(1 - M^2) \times y_c^2 - x_c^2}{\left[(1 - M^2) \times y_c^2 + x_c^2\right]^2} \times t_c^2$$

The following relationship therefore results for the increase of the perturbation velocity $\Delta v_x/v_o$ with increase of Mach number at a given point a great distance from a body:

$$\frac{(\Delta v_x/v_o)_c}{(\Delta v_x/v_o)_{ic}} = \frac{1}{\sqrt{1 - M^2}} \times \frac{(1 - M^2) \times y_c^2 - x_c^2}{\left[(1 - M^2) \times y_c^2 + x_c^2\right]^2} \times \frac{(y_c^2 + x_c^2)^2}{y_c^2 - x_c^2}$$

If it is now assumed that x_c is small compared with y_c, that is, if a point a large distance sideways from the body is considered, then according to the above equation the additional velocities and therefore also the pressures increase by $1/(1 - M^2)^{3/2}$ that is, by the third power of the Prandtl factor. If, however, y_c is small compared with x_c, that is, for instance, points far ahead or behind the airfoil are considered, the additional velocities and pressures increase by $1/\sqrt{1 - M^2}$, that is, only with the first power of the Prandtl factor.

2. Axially Symmetric Bodies without Lift

Velocity Distribution over the Surface of the Body

For ellipsoids of revolution the greatest velocity increments, appearing at the position of maximum thickness,

[2] B. Göthert: Einige Bemerkungen zur Prandtlschen Regel in bezug auf ebene und räumliche Strömung (ohne Auftrieb). FB 1165.

are shown in figure 1 for incompressible flow according to a calculation by Weinig[3].

According to this, the velocity increments no longer rise linearly, as in two-dimensional flow, but more nearly follow a quadratic law. For small thickness ratios the curve for the velocity increment can be represented in an approximate manner in the following way:

$$\frac{\Delta v_{max}}{v_o} = \frac{1}{2} \times \left(\frac{d}{t}\right)^2 \times ln\left(\frac{1}{d/t}\right)^2$$

If the general rule is applied once more according to section II, one obtains for the greatest velocity increment in compressible flow:

$$\left(\frac{\Delta v_{max}}{v_o}\right)_c = \frac{1}{2} \times \frac{1}{1 - M^2} \times \left[\sqrt{1 - M^2} \times \left(\frac{d}{t}\right)_c\right]^2 \times ln\frac{1}{(1 - M^2) \times (d/t)_c{}^2}$$

and for the ratio of the greatest excess velocities in compressible and incompressible flow for axially symmetric bodies:

$$\frac{(\Delta v_{max}/v_o)_c}{(\Delta v_{max}/v_o)_{ic}} = 1 + \frac{ln(1 - M^2)}{ln(d/t)^2}$$

For small thickness ratios, it is evident that in spite of increasing Mach number the greatest velocity increment on ellipsoids of revolution does not rise. Only for greater thickness ratios does an increase of the velocity increment appear, which is roughly proportional to the square of the thickness ratio. It must be noted, however, that in the approximate calculation terms of the order of magnitude of $\Delta v_x{}^2$ as well as $\Delta v_y{}^2$ have been neglected, which are likewise of the order of magnitude of $(d/t)^2$. It should be consequently realized that the above approximate equation must be corroborated by experimental results. Since the velocity distribution over a body of revolution corresponds in compressible flow to that over a more slender body in incompressible flow, the velocity distribution will become somewhat fuller with increasing Mach number.

[3] F. Weinig: Vergleich der ebenen und der achsensymmetrischen Stromung um Widerstandskorper. Schiffbau 1930 S. 15.

Velocity Disturbance at a Great Distance from Bodies of Revolution

According to a calculation carried out in FB 1165, the velocity perturbation $\Delta v_x/v_o$ at a large distance from a slender body of revolution is, in incompressible flow:

$$\frac{\Delta v_x}{v_o} \sim \left(\frac{d}{t}\right)^2 \times \frac{y^2 - 2x^2}{(y^2 + x^2)^{5/2}} \times t^3$$

For the compressible flow it is then found once more from the corresponding conversion of the equivalent incompressible flow that the velocity disturbances at points a great distance from bodies increase in the following manner:

$$\frac{(\Delta v_x/v_o)_c}{(\Delta v_x/v_o)_{ic}} = \frac{(1 - M^2)y_c^2 - 2x_c^2}{\left[(1 - M^2)y_c^2 + x_c^2\right]^{5/2}} \times \frac{(y_c^2 + x_c^2)^{5/2}}{y_c^2 - 2x_c^2}$$

For points a large distance sideways from the body, that is, y_c large compared with x_c, the velocity perturbations and pressures increase by $1/(1 - M^2)^{3/2}$ that is, with the third power of the Prandtl factor, just as in plane flow. For points far ahead of, or far behind, the body the velocity disturbances and pressures remain constant in spite of increasing Mach number.

3. Comparison of Two-Dimensional and Axially-Symmetric Flow without Lift

In figure 2, as an example, is shown the increase of velocity due to compressibility in the plane of symmetry $x = 0$ for two-dimensional and three-dimensional flow. It is seen from this example that in the immediate vicinity of the body the velocity increment in three-dimensional flow is increased only slightly by compressibility, but that with increase of distance from the body the velocity ratios for plane and three-dimensional flow very quickly approach the common asymptote $1/(1 - M^2)^{3/2}$.

A qualitative comparison with the velocity increments on bodies in incompressible flow (cf. fig. 1) will serve to give an insight into the distinctive behaviors of plane and three-dimensional flow[4]. If one imagines, for example, that the effect of compressibility in the vicinity of the body is somewhat equivalent to an increase in thickness, since as a result of the velocity increment the air expands, then the character of the rise in velocity increment with transition to higher Mach numbers must correspond somewhat to that from increase of thickness ratio. It is evident, in fact, for elliptic cylinders (two dimensions), that the velocity increment rises in proportion to the thickness ratio; that therefore slight changes in thickness ratio, as, for example, also by compressibility, produce quite noticeable variations in velocity increment; this relation is independent of thickness ratio and appears, for example, even with vanishingly small thickness ratios. For ellipsoids of revolution (three dimensions), the velocity increment increases approximately with the second power of the thickness ratio, so that small variations of thickness, as, for example, also from compressibility, produce no discernible change in velocity increment for very slender bodies. Only for very great thickness ratios are the effects of an increase of thickness ratio equal for elliptic cylinders and ellipsoids of revolutions, just as in compressible flow at a large distance from the body the same velocity perturbations are encountered with plane and three-dimensional flow.

4. Determination of Critical Free-Stream Velocities for Two-Dimensional and Axially Symmetric Flow

It is known from wind-tunnel experiments that flow over slender bodies goes smoothly so long as the velocity of sound is not reached or exceeded at any point in the flow field. If the free-stream velocity is raised so high that on the body at the point of maximum velocity increase sonic velocity is just reached, the flow begins to deteriorate more or less rapidly. This free-stream velocity shall accordingly be termed the critical free-stream velocity, and the corresponding Mach number the critical Mach number M^*. If the greatest velocity increment for incompressible flow $\Delta v_{ic}/v_o$ is known for a body,

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[4]I thank Prof. Bock, DVL, for the basic idea of this comparison.

the basic principles of the Prandtl rule presented previously permit calculation of the critical Mach number for slender bodies.

Two-dimensional flow:

$$\frac{\Delta v_{ic}}{v_o} = \sqrt{1 - M*^2}\left[\sqrt{\frac{\gamma - 1}{\gamma + 1} + \frac{2}{\gamma + 1} \times \frac{1}{M*^2}} - 1\right]$$

Three-dimensional flow (with $d/t = f\left[\Delta v_{ic}/v_o\right]$ according to figure 1):

$$\left[1 + \frac{ln\ (1 - M*^2)}{ln\ (d/t)^2}\right] \times \frac{\Delta v_{ic}}{v_o} = \sqrt{\frac{\gamma - 1}{\gamma + 1} + \frac{2}{\gamma + 1} \times \frac{1}{M*^2}} - 1$$

For elliptic cylinders and ellipsoids of revolution the critical Mach numbers calculated from the above equations are shown in figure 3.

From the figure it is seen that the three-dimensional flow affords considerably more favorable results than the plane flow, since on the one hand for equal thickness ratios the maximum velocity increment is considerably lower at $M = 0$, and on the other hand upon going to higher Mach numbers the velocity increments rise only insignificantly. For example, an elliptic cylinder airfoil at 15-percent thickness ratio possesses according to this a critical Mach number of $M* = 0.78$, while an elliptical fuselage of the same thickness ratio reaches the critical region only at a Mach number of $M* = 0.93$.

This result indicates that for aircraft the achievement of the greatest possible slenderness should be concentrated in particular upon the wing, since the fuselage and engine nacelles of favorable shape will reach local sonic velocity only at considerably higher velocities than correspond to the critical speed of the wing.

This result is naturally valid only for smooth bodies without protuberances.

IV. WING OF FINITE SPAN

If one considers next an idealized wing, represented by a flat plate of aspect ratio $\Lambda = b^2/F$ which moves at a small angle of attack α and Mach number M, then this plate has in incompressible flow at the same angle of attack the following lift coefficient according to the lifting-line theory[5]:

$$C_{L_{ic}} = \frac{\partial C_L}{\partial \alpha}\alpha \approx \frac{5.65}{1 + 1.8/\Lambda} \times \alpha$$

In compressible flow the pressures and hence also the lift coefficient are greater in the ratio $1/(1 - M^2)$ than for an equivalent plate in incompressible flow, whose span and angle of attack are smaller in the ratio $\sqrt{1 - M^2}$, that is:

$$C_{L_c} = \frac{1}{1 - M^2} \times \frac{5.65}{1 + \frac{1.8}{\sqrt{1 - M^2} \times \Lambda_c}} \sqrt{1 - M^2} \times \alpha_c = \frac{5.65 \times \Lambda_c}{1.8 + \sqrt{1 - M^2} \times \Lambda_c} \times \alpha_c$$

For the fractional increase of lift coefficient with rise in Mach number one finds therefore:

$$\frac{C_{L_c}}{C_{L_{ic}}} = \frac{1.8 + \Lambda_c}{1.8 + \sqrt{1 - M^2}\Lambda_c}$$

For the limiting case $\Lambda \rightarrow \infty$ the above equation gives the familiar prediction of the Prandtl rule, that the lift coefficients rise in the ratio $1/\sqrt{1 - M^2}$. For the limiting case of the plate with very small aspect ratio $\Lambda \rightarrow 0$ it is found, however, that the lift coefficients remain constant despite increase of Mach number.

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[5] Fuchs, Hopf, Seewald: Aerodynamik, Bd. 2 (1935) S. 137, Verlag Springer, Berlin 1935.

Between these two limiting cases lies a complete intermediate region, which is represented in figure 4. Thus for a wing of aspect ratio $\Lambda = 1$ the increase due to compressibility in the lift from Mach numbers in the range $M = 0$ to 0.4 up to a Mach number of $M = 0.9$ amounts to about 35 percent, still only approximately 20 percent of the value found for infinite aspect ratio.

For small aspect ratios, to which the linear airfoil theory is no longer applicable, Bollay[6] has presented investigations for incompressible flow in which, for example, for a plate of vanishingly small aspect ratio there results: $C_L = 2\pi \sin^2\alpha$. From this equation it is likewise found that the lift coefficient is not increased by compressibility, since the angle of attack α of the equivalent plate must be chosen smaller by the ratio $\sqrt{1 - M^2}$, while the pressure is subsequently increased by $1/(1 - M^2)$.

The influence of aspect ratio upon increase of lift coefficient which was found by this method will be of importance in the stability of aircraft. If, for example, the empennage has a smaller aspect ratio than the wing, the $\partial C_L/\partial\alpha$ of the wing rises more rapidly than the $\partial C_L/\partial\alpha$ of the tail, so that the airplane becomes unstable with increasing Mach number. If the moment of the fuselage is to be considered in the stability of the airplane, it must be realized that the fuselage moment remains practically constant, and a further postponement is accordingly to be anticipated.

V. SUMMARY

1. From the continuity equation for three-dimensional flow in a compressible medium the following rule, valid for slender bodies, is found by means of a transformation similar to that employed in the derivation of the Prandtl rule:

The streamline pattern of a compressible flow to be calculated can be compared with the streamline pattern of

[6]Bollay: A Nonlinear Wing Theory and its Application to Rectangular Wings of Small Aspect Ratio. Z.A.M.M., Bd. 19 (1939) pp. 21 to 35.

an incompressible flow which results from contraction along the y and z axes of the profile contour by the factor $\sqrt{1 - M^2}$ (x-axis in the direction of the free stream). In the resulting compressible flow the pressures $\frac{\Delta p}{\rho/2 \times v_0^2}$ as well as the perturbation velocities $\Delta v_x/v_0$ are greater in the ratio $1/(1 - M^2)$ and the streamline slopes greater in the ratio $1/\sqrt{1 - M^2}$ than those at the corresponding points of the equivalent incompressible flow.

2. It is shown for slender bodies in two-dimensional flow the velocity ratios $\Delta v_x/v_0$ and the pressure ratios $\frac{\Delta p}{\rho/2 \times v_0^2}$ along the surface of the body increase with increasing free-stream velocity by the ratio $1/\sqrt{1 - M^2}$, while for axially symmetric flow these ratios remain constant in spite of increasing Mach number.

3. At a great distance to one side of the body $(x = 0)$ the perturbation velocities Δv_x and the pressures increase to the same degree in both two- and three-dimensional flow in the ratio $1/(1 - M^2)^{3/2}$. At a great distance ahead of and behind the body the velocity Δv_x and pressure rise in the ratio $1/\sqrt{1 - M^2}$ in two-dimensional flow, but remain constant in three-dimensional flow.

4. For elliptic cylinders and ellipsoids of revolution the Mach numbers are calculated for which sonic velocity is just reached locally on the body (fig. 3). Accordingly, for example, an elliptic cylinder of 0.15 thickness ratio possesses a critical Mach number of 0.78, while an ellipsoid of revolution of equal thickness ratio shows a critical Mach number of 0.93.

5. It is demonstrated the lift coefficient of airfoils rises according to the Prandtl rule with $1/\sqrt{1 - M^2}$ only with infinitely large aspect ratio; for finite aspect ratio the increase is somewhat smaller, until in the limiting case of vanishingly small aspect ratio it completely disappears, that is, for $\Lambda \rightarrow 0$ an increase of Mach number

at constant angle of attack produces no increase in lift coefficient. This influence of aspect ratio is presented in a graph (fig. 4).

Translation by Milton Van Dyke
National Advisory Committee
for Aeronautics

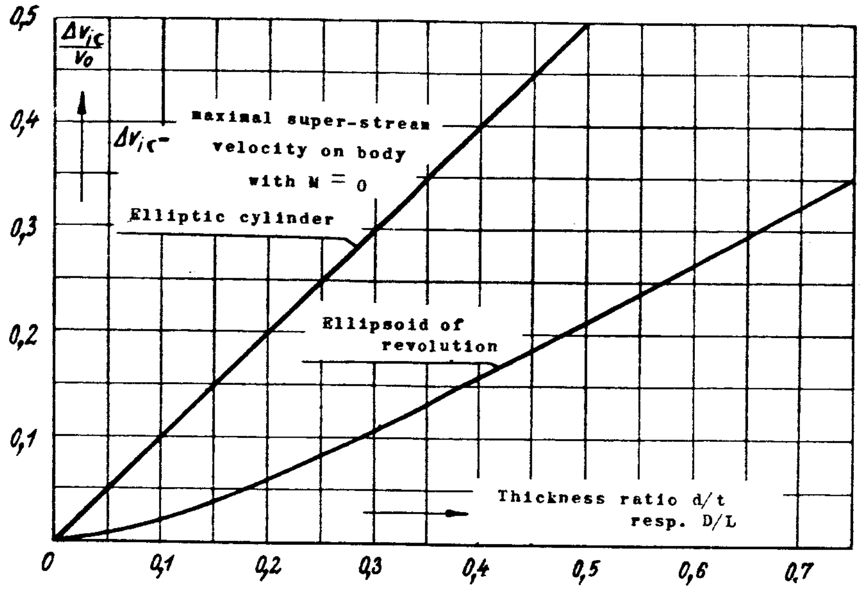

Fig. 1 Maximum super-stream velocity for elliptical cylinders and ellipsoids of revolution.

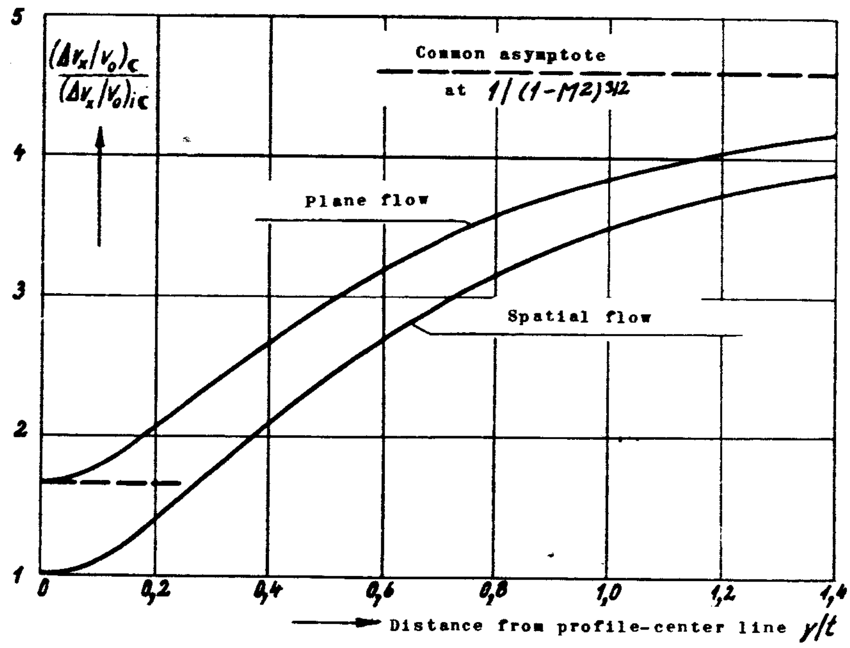

Fig. 2 Perturbation velocities for X=0 through a plane and through rotationally symmetric disposition of source-depressions in incompressible and compressible flow. (Mach number M=0,8).

Notation of coordinates:

Perturbation velocity in incompressible flow:

$(\Delta V_x / V_0)_{ic}$

Perturbation velocity in compressible flow: (M=0,8) :

$(\Delta V_x / V_0)_c$

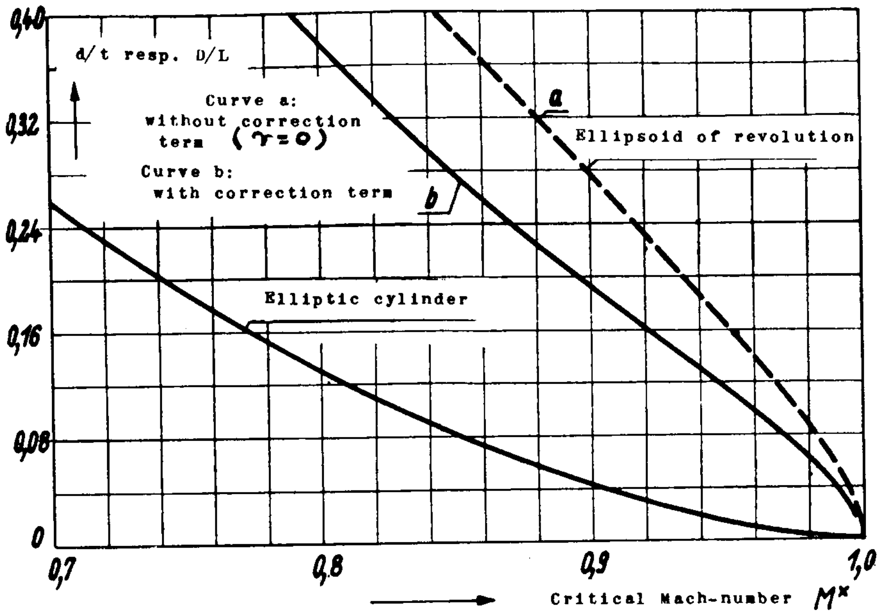

Fig. 3 Maximum super-stream velocity and critical Mach number M^* for elliptical cylinders and ellipsoids of revolution in symmetric flow.
M^*= Mach number of the flow velocity at which the body attains sonic velocity.

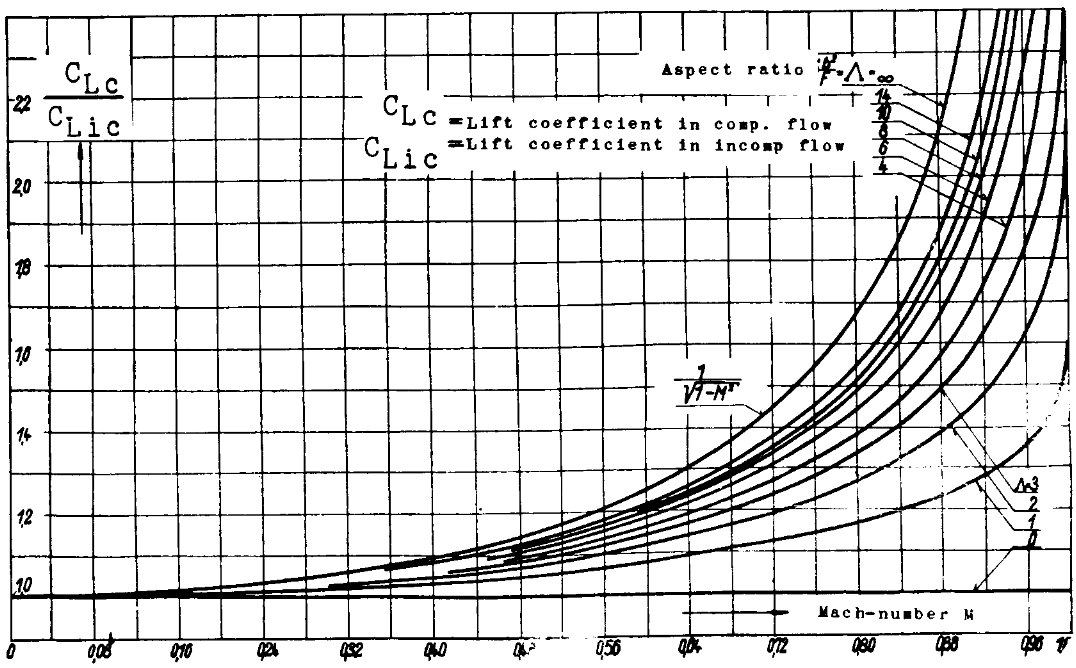

Fig. 4 The influence of aspect ratio upon the increase of the lift coefficient at the same angle of attack by increasing the Mach number M