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Pappu L.N. Murthy and Noel N. Nemeth
Glenn Research Center, Cleveland, Ohio

David N. Brewer
U.S. Army Research Laboratory, Glenn Research Center, Cleveland, Ohio

Subodh Mital
University of Toledo, Toledo, Ohio

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University of Toledo, Toledo, Ohio

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Pappu L.N. Murthy and Noel N. Nemeth
National Aeronautics and Space Administration
Glenn Research Center
Cleveland, Ohio 44135

David N. Brewer
U.S. Army Research Laboratory
National Aeronautics and Space Administration
Glenn Research Center
Cleveland, Ohio 44135

Subodh Mital
University of Toledo
Toledo, Ohio 43606

Summary

To demonstrate the advanced composite materials technology under development within the Ultra-Efficient Engine Technology (UEET) Program, it was planned to fabricate, test, and analyze a turbine vane made entirely of silicon carbide-fiber-reinforced silicon carbide matrix composite (SiC/SiC CMC) material. The objective was to utilize a five-harness satin weave melt-infiltrated (MI) SiC/SiC composite material developed under this program to design and fabricate a stator vane that can endure 1000 hours of engine service conditions. The vane was designed to withstand a maximum temperature of 1315 °C (2400 °F) within the substrate and the hot surface temperature of 1482 °C (2700 °F) with the aid of an environmental/thermal barrier coating (EBC/TBC) system. Furthermore, the vane was designed such that the expected maximum stresses to be encountered were kept within the proportional limit strength of the material. Any violation of this design requirement was considered as the failure. This report presents results of a probabilistic analysis and reliability assessment of the vane. Probability of failure to meet the design requirements was computed using the probabilistic analysis methods embedded in the NESSUS software [NESSUS (Numerical Evaluation of Stochastic Structures Under Stress). Final NASA Code, New Manuals, version 6.2, 1995.]. In the analysis, material properties, strength, and pressure loading were considered as random variables. The variations in properties and strength were based on the actual experimental data generated in house. In the present analysis, the pressure loads were considered normally distributed with a nominal variation. A temperature profile on the vane was obtained by performing a computational fluid dynamics (CFD) analysis and was assumed to be deterministic. The results suggest that for the current vane design, the chance of not meeting design requirements is about 1.6 percent.

Introduction

Advanced high-temperature ceramic matrix composites (CMCs) have been recognized as viable candidate materials for propulsion system components. The development of these materials was pursued under the NASA's High-Speed Research (HSR) program and subsequently in the current NASA Ultra-Efficient Engine Technology (UEET) Program. The primary objectives of these programs are to increase

thermal efficiency and to reduce NO_x emissions (www.ueet.nasa.gov). These objectives can be accomplished mainly by raising the turbine inlet temperature and eliminating cooling of the turbine blades, vanes, and combustors. Conventional materials require a large amount of cooling, which reduces the turbine inlet temperatures, thereby reducing the thermal efficiency. CMCs have desirable properties such as lighter weight and higher thermal stability compared to the conventional metallic materials. Hence one can surmise that CMCs can perform well at much higher temperatures thereby increasing the engine efficiency. Furthermore, higher combustion temperatures have the beneficial effect of lowering the NO_x emissions.

Research under NASA's High-Speed Research Enabling Propulsion Materials (HSR/EPM) program led to the emergence of the Sylramic (Dow Corning Corporation, Midland, MI) SiC fiber with chemical-vapor-infiltrated (CVI)-SiC/melt-infiltrated (MI)-SiC matrix 5-harness satin weave CMC material as one of the most promising candidates for propulsion system components (refs. 1 and 2). Successful demonstration of the new CMC technology for propulsion system components is one of the goals of NASA's UEET Program. Under this program, the thermal capability of the material has been raised to 1315 °C (2400 °F). This material is sought for combustor liners and turbine vanes, which see gas temperatures in excess of 1650 °C (3000 °F). Furthermore, the hot side is coated with an environmental/thermal barrier coating (EBC/TBC) system that is stable up to about 1482 °C (2700 °F) (ref. 3). To demonstrate the new CMC technology, it was planned to fabricate, test, and analyze a turbine stator vane made entirely of the MI SiC/SiC composite material developed under NASA's UEET Program. The turbine stator vane was to be fabricated utilizing this CMC material and tested in a high-pressure burner rig at NASA Glenn Research Center. This rig is capable of simulating the engine service environment. The test was to demonstrate that the vane can successfully withstand the harsh engine environment conditions for up to 1000 h. In order to make sure that the vane lasts the duration successfully, it was designed such that the maximum expected stresses are within the proportional limit stress of the CMC material. It should be noted that the life of SiC/SiC materials subjected to stresses that exceed the proportional limit is often severely limited due to the harsh environmental attack of the fibers via matrix cracks.

Due to the brittle nature of the CMC constituents, the properties of CMCs show considerable scatter. Reproducibility is a major issue and a concern. Examination of the MI SiC/SiC stress-strain behavior indicated a substantial amount of scatter in the first matrix cracking strength (proportional limit) as well as the ultimate strength (ref. 4). Furthermore, variations and uncertainties are usually present in geometry, thermal properties, and loading conditions as well. Vane designs based solely upon the mean values for the material properties, geometrical variables, and loads may be unconservative and may lead to unexpected premature failures. These failures are due to a violation of the design constraint: the maximum stresses in critical locations have exceeded the proportional limit. Thus the uncertainties, which add concerns regarding the reliability of the vane performance under the service conditions, need to be quantified.

Current research effort is primarily directed towards assessment of the reliability of an all-CMC turbine stator vane subjected to engine service conditions. Given the scatter in material properties, uncertain loading conditions, and geometrical variations, a probabilistic analysis of the vane is performed in order to quantify the risk of not meeting the design requirements. Since the stresses in the vane depend upon the pressure as well as the temperature gradients through-the-thickness, material thickness variations will have significant effect on the stresses. Variations in the temperature profile (caused by the variation in the gas temperature) and in material thickness and their effect on the vane reliability should be considered in the risk assessment. In addition, other all-CMC vane configurations (e.g., one with a web) will also be assessed for risk and reliability of the design. For the purpose of the present study, it is considered that any violation of a design requirement is a failure. In addition, cumulative probability distribution functions of critical stresses and the sensitivities of various random variables are computed. The proportional limit and strength distributions are computed from the experimental coupon data and Weibull statistics. Measured distributions of thermal properties and pressure loads are not available at this point. These variables are considered normally distributed with nominal coefficients of variation (COVs)

in the current analysis. The vane thickness is considered deterministic in these analyses. It is worth noting that Federal Aviation Administration regulations require that for commercial aircrafts, the probability of failure should range from a high value of 10^{-3} for minor failure conditions to an extremely low value of 10^{-9} for catastrophic failure conditions. Additionally, NASA space missions are striving for a catastrophic failure rate of 10^{-6} .

Vane Subelement Fabrication and Testing

Vane subelements were fabricated from a silicon carbide-fiber-reinforced SiC/SiC composite and were coated with an environmental barrier coating (EBC). In order to address realistic critical design features of a turbine airfoil, the vane subelement cross section was derived from an existing production aircraft engine vane. A unique woven cloth configuration was used to provide a sharp trailing edge with continuous fiber reinforcement (ref. 5). Fabrication of vanes with a sharp trailing edge was considered to be one of the more challenging features for fabricating a ceramic composite vane. The vanes were densified through the CVI/slurry cast/silicon MI process. Both nondestructive and metallographic examinations revealed that the quality of the final as-fabricated composite vanes was consistent with that typically obtained for the same composite material fabricated into flat panels. One consisted of a thin-walled (1.5-mm) shell with a continuously reinforced sharp trailing edge. A vane subelement manufactured in this study is shown in figure 1. Each vane had a constant cross section over a height of 50 mm, with a trailing edge radius of 0.26 mm, a leading edge radius of 3.1 mm, and a cord length of 50 mm, as shown in figure 1. All vanes were manufactured with the CVI/slurry-cast/MI SiC/SiC material system using Sylramic SiC fiber-reinforcing cloth.

A fiber architecture was developed to address the fabrication challenges presented at the vane trailing edge as well as provide a fiber architecture in the remaining regions of the vane that had been well characterized and successfully demonstrated in other CMC turbine engine components. The fiber tows forming the trailing edge section are interlocked, thereby enhancing the through-the-thickness strength capability of the composite material. The sharp trailing edge is then naturally formed within the vertex of the Y-shaped cloth. This avoids sharply bent fiber filaments and its associated strength reduction. The vane test program with test rig description, test conditions, and vane test configuration is described in reference 5. An interesting point to note is that the CMC vane was surrounded by a metallic vane on either side to help establish close to realistic flow around the SiC/SiC test specimen (fig. 2).

Analyses of Vane During Rig Testing

Prior to testing, computational fluid dynamics (CFD) and finite element analyses were performed to predict the temperature and stress conditions present in the vane during rig testing (ref. 6). Analyses were performed for a preliminary vane design that did not include trailing edge cooling. An updated model including cooling holes is currently being analyzed. The CFD analysis for a cascade of blades was performed using a two-dimensional Euler (i.e., inviscid flow) equation solver. Local pressure and velocity results were used to determine heat transfer coefficients for the vane exterior surface. Calculated in-plane tensile stress values ranged from 27 MPa in the axial direction to a maximum transverse or “hoop” stress of 105 MPa. The predicted interlaminar tensile (ILT) stresses were found to be rather high, although in a very small area. This vane has some through-the-thickness reinforcement (because of the unique geometry at the critical location as explained earlier) that is likely to provide a higher ILT strength than found in a two-dimensional flat specimen. Therefore, even though ILT stresses were high, because of the small region and the additional reinforcement provided, they were not considered to be a major design issue and are not addressed in this study. The finite element model of the vane with the boundary conditions is shown in figure 3. In this study, analysis is focused primarily on the hoop stress since

measured data is readily available for the in-plane modulus and proportional limit strength. Based upon this data, the statistics pertaining to strength and modulus could be established. Such statistical data is not available for ILT tensile strength.

Probabilistic Analysis

As mentioned earlier, the measured MI SiC/SiC material strength and modulus showed a substantial amount of scatter. The deterministic analyses, specifically near the trailing edge region, indicated that the most critical stresses are in-plane hoop stresses. These stresses are affected primarily by the in-plane stiffness of the material as well as by the loading conditions and need to be compared to the stress allowables (i.e., proportional limit in this case). The in-plane Young's modulus and proportional limit strength of the UEET material data taken for 24 samples is shown in the form of histograms in figures 4 and 5, respectively. It is evident that these two measured material properties show considerable scatter. Design/analysis solely based upon mean values for these properties, therefore, might lead to unexpected failures during the rig testing due to the wide scatter range. Consequently, it was decided to perform a probabilistic (risk) analysis to quantify the probability of vane performance not meeting the design requirement, which is referred to as failure (i.e., the hoop stress exceeding the proportional limit). Two cases for probabilistic analysis were evaluated:

Case I: Only the material Young's modulus, Poisson's ratio, coefficient of thermal expansion and proportional limit are considered as random variables. Among these variables the strength and modulus statistics are based upon the measured data. Probabilistic-distribution-related parameters are assumed for the remaining variables. All other pertinent parameters (e.g., material thickness, gas temperature, or other loading parameters) are considered deterministic in this evaluation.

Case II: In addition to the variables considered in case I, two other parameters related to the loading conditions—internal pressure of the cooling air and the external aerodynamic pressure on the vane—were considered as random variables with assumed distributions and nominal values for the means and COVs.

Estimation of Weibull Parameters

The stochastic behavior of the MI SiC/SiC in-plane Young's modulus and proportional limit at 1204 °C (2200 °F) were characterized from experimental data using the two-parameter Weibull distribution (ref. 7). This information was subsequently included in the probabilistic analysis. The two-parameter Weibull distribution is expressed as

$$P = 1 - \exp \left[- \left(\frac{\alpha}{\beta} \right)^\gamma \right] \quad (1)$$

where P is the probability of occurrence, α is the particular value of data for which probability is to be calculated, β is the Weibull characteristic value—the value where the probability of occurrence is 63.21 percent, and γ is the Weibull modulus, which measures the degree of dispersion or scatter in the data. For the composite proportional limit, which could be regarded as a strength measurement, α is a value of strength while β is the characteristic strength. Both α and β have units of stress. The Weibull modulus γ is dimensionless. As γ increases, the amount of dispersion decreases. Typical values describing monolithic ceramic strength dispersion range from about 5 to more than 30. Ceramic composites likely fall within this same range. When the distribution is used to describe strength, P in equation (1) is

interpreted as a probability of failure. Likewise, when characterizing the in-plane Young's modulus using the Weibull distribution, α is the value of Young's modulus, β is the characteristic value, and α and β both have units of stress.

Results from the measurements of the in-plane elastic modulus for 24 specimens are shown in figure 6 in the form of a Weibull plot. Table I lists the values of the Weibull parameters estimated from this data set. This information was obtained using the CARES/LIFE code, and these procedures are described in reference 8. The Weibull line that was best-fit to the data and corresponds to the parameters in table I is also shown in figure 6. The parameters were obtained using the maximum likelihood estimation method (ref. 8). From figure 6 it can be seen that there is significant scatter in Young's modulus where the data have a mean of 181.5 GPa, a standard deviation of 13.8 GPa, and a COV of 7.6 percent. For the Weibull distribution, the scatter is described with the Weibull modulus γ , which has a value of 14.1. The data visually shows a good fit to the two-parameter Weibull distribution, and this is confirmed with Kolmogorov-Smirnov (K-S) (ref. 8) and Anderson-Darling (A-D) (ref. 8) goodness-of-fit significance levels of 49 and 82 percent, respectively. The A-D test is more sensitive to the tails of the distribution, thus the interpretation of the percentages is that a better fit is achieved towards the tails than the central portions. Fitting a three-parameter Weibull distribution to the data would yield improved goodness-of-fit scores, however there are too few data points to conclude with any reasonable certainty that the underlying distribution has a three-parameter Weibull behavior (ref. 7). Also provided in table I are the 90 percent confidence bounds (the 5 and 95 percent values) on the Weibull parameters. The relative spread in the values is a function of the number of data points used in the estimation and the Weibull modulus γ . Note that there is always more relative spread in γ than in β . In this case 24 specimens were used in the estimation, which yielded reasonably narrow confidence bounds. A standard rule-of-thumb is that 30 or more specimens are desirable to obtain sufficiently narrow confidence bounds.

TABLE I.—WEIBULL PARAMETERS FOR IN-PLANE ELASTIC MODULUS
AND PROPORTIONAL LIMIT STRENGTH
[Also shown are confidence bounds on parameters and goodness-of-fit statistics.]

Property	Sample size	Weibull modulus, γ	90% confidence bounds on γ	Weibull characteristic value, β , MPa	90% confidence bounds on β	K-S goodness-of-fit statistic (and significance level, %)	A-D goodness-of-fit statistic (and significance level, %)
Modulus	24	14.1	17.7 9.9	189.1×10^3	194.6×10^3 183.8×10^3	0.17 (49 %)	0.43 (82 %)
Strength	24	7.4	9.2 5.3	177.5	187.0 168.7	0.13 (87 %)	0.51 (74 %)

Results from the measurements of the in-plane proportional limit for 24 specimens and the best-fit Weibull line obtained from maximum likelihood analysis are shown in figure 7. Table I lists the values of these estimated parameters. There is a significant scatter in the data, which have a mean of 166.0 MPa, a standard deviation of 27.2 MPa, and a COV of 16.4 percent. The Weibull modulus γ has an estimated value of 7.4, which indicates considerably more scatter than the in-plane modulus data. The K-S and A-D goodness-of-fit significance levels were 87 and 74 percent, respectively, which indicates a good fit across the entire range of data to the estimated parameters. The 90 percent confidence bounds on the estimated parameters show the uncertainty range as to the true values of the parameters, which in this case is between 9.2 and 5.3 for γ and 187.0 and 168.7 for β . The in-plane proportional limit Weibull parameter estimates were also based on 24 measurements which, as noted before, yielded reasonably narrow confidence bounds. In summary, both the in-plane modulus and proportional limit data fit the two-parameter Weibull distribution reasonably well. The degree of scatter was considerably larger in the proportional limit data than the in-plane modulus data. The sample size of 24 for each measured quantity provided an adequate degree of confidence that the estimated parameters were representative of their true

values. The degree of scatter in the data in both figures 6 and 7 highlight the necessity of using probabilistic methodology in any risk assessment of a component to meet its design requirements.

Probabilistic Analysis Approach

The need to account for uncertainties in engineering design has long been recognized. No structure can be guaranteed to be absolutely safe because of the unpredictability of the loads, uncertainties in the in situ material properties, the use of simplified assumptions in the analysis (which include limitations of the numerical methods used), and human factors (errors and omissions). Nevertheless, the probability of failure is usually required to be shown to be within a specified acceptable range for each specific application. Therefore, the estimation of failure probability and risk assessment of a structure becomes an important task for the design/analysis engineers.

Design requirements dictate that a structure satisfies various criteria of safety, serviceability and durability under the action of anticipated loads during its useful lifetime. That is, the structural strength or resistance should be greater than the effects caused by the action of loads. A simplified model (referred to as R–S model) consists of two variables (one relating to strength or resistance, R , and the other to the loads, S , of the structure) as shown in figure 8. Both R and S are random in nature and that randomness is characterized by their respective probability density functions. The nominal (deterministic) values, R_N and S_N , used in a safety-factor-based approach, are also shown in figure 8. In a deterministic approach, the design safety is assured by requiring that the R_N be greater than S_N with a specified safety margin. A safe design requires that R_N is greater than S_N at all times. The intent of this approach and other similar deterministic approaches can be understood by considering the area of the overlap of two probability density functions (shaded region), which provides a measure of the probability of failure. This overlap depends upon

1. The relative positions of the curves represented by the mean values (μ_S and μ_R) of the two variables S and R . As the distance between the two curves increase, the area of the overlap (probability of failure) decreases.
2. The dispersion of the two curves is characterized by the standard deviations (σ_S and σ_R) of the two variables S and R . For narrower curves, the area of overlap and thus the probabilities of failure are smaller.
3. The shape of the curves that are represented by the probability density functions.

To achieve a safe design, the design variables must be chosen such that the area of overlap is minimized. The basic concept of the classical theory of structural reliability and risk-based design starts with the identification of relevant load and resistance parameters, called basic variables or sometimes referred to as random variables X_i (such as loads, material properties, and so forth) and the functional relationship between the response variable Z (e.g., stress at a point, deflection, frequency, etc.) and the basic random variables. Mathematically, it can be described as

$$Z = Z(X_1, X_2, X_3, \dots, X_n) \quad (2)$$

A limit state function g (sometimes referred to as performance function) is defined as

$$g = Z(X) - Z_0 \quad (3)$$

where Z_0 is a particular value of Z . A limit state function can be an implicit or explicit function of random variables and is divided in such a way that $g(X) = 0$ is a boundary between the failure region $g \leq 0$ and

safe region $g > 0$. It should be noted that since the cumulative distribution function (CDF) of Z at Z_0 equals the probability that $g \leq 0$, the CDF can be computed by varying the Z_0 and computing the point probability.

The probability of failure, P_f , is given by the integral

$$P_f = \int \dots \int_{\Omega} f_X(X_1, X_2, \dots, X_n) dX_1 dX_2 \dots dX_n \quad (4)$$

in which $f_X(X_1, X_2, \dots, X_n)$ is the joint probability density function for the random variables X_1, X_2, \dots, X_n , and the integration is performed over the failure region Ω where $g \leq 0$ (ref. 9). If the random variables are statistically independent, then the joint probability density function can be replaced by individual density functions. This integral can be computed by the standard Monte Carlo procedure which is rather straightforward (ref. 9). However, depending upon the number of random variables involved and the level of P_f (usually very small), this must be repeated thousands of times to accurately build the response variable's stochastic characteristics. Although the method is inherently simple, the large number of output sets that must be generated to build an accurate cumulative distribution function of the output variable becomes its obvious disadvantage. Furthermore, if the deterministic computation of the response is complicated (e.g., need for a large nonlinear finite element analysis), the computational costs could become prohibitive. Thus the need for more efficient approaches to perform such tedious tasks in a timely and cost-effective manner to routinely assess the reliability of a design cannot be overemphasized.

For over two decades, NASA Glenn Research Center has been involved in developing efficient probabilistic analysis tools for aerospace applications. As a result of this effort, a collection of methods called fast probability integration (NESSUS/FPI) techniques were developed to solve a large class of engineering problems (ref. 10). In the current study the FPI approach is implemented as illustrated in the following procedure:

- (1) A set of input random variables are identified, and the corresponding probabilistic distributions are selected. For a given set of random variables, a deterministic finite element analysis is run using the ANSYS finite element code (ref. 11). The response results are collected from the finite element analysis output.
- (2) The above process is repeated a number of times to generate a table of response variable values that correspond to the perturbed values of the selected input random variables.
- (3) The FPI analysis then uses the previously generated table to compute the CDF and corresponding sensitivities of the response.

In addition to the CDF of the response, the FPI technique provides additional information regarding the sensitivity of the response to the random variables as shown schematically in the flow diagram of figure 9. The magnitude of the sensitivity factor provides a way to rank the random variables that have the most influence on the uncertainty of the response variable. This helps the user to prioritize the data collection resources. Also, by controlling scatter in the more significant variables, the reliability can be improved.

Results and Discussion

As mentioned in the Probabilistic Analysis section, two cases were evaluated for the risk assessment of the vane design as described below:

Case I: Young's modulus, Poisson's ratio, the coefficient of thermal expansion for computing the

variability in the stress, and the proportional limit of the material were considered random variables. Other parameters were assumed to be deterministic for this analysis. Table II shows the probabilistic characteristics of these random variables. The modulus and proportional limit statistics are based on observed scatter (measured data) and the Weibull parameter analysis described earlier. The probabilistic analysis, using the advanced first-order reliability method (AFORM), is done in two steps. The first step involves the computation of the probabilistic characteristics (mean value and standard deviation) of the hoop stress at the critical location. Figure 10(a) shows such a CDF with its predicted statistical characteristics. Sensitivity factors for the hoop stress are shown in figure 10(b). The Young's modulus has the highest effect on the variability of hoop stress among the random variables selected. The second step involves risk quantification by employing the basic R–S reliability model, where S represents the computed stress at the critical location, and R is the resistance (strength/proportional limit) based on the observed scatter and the Weibull parameter analysis described earlier. By using the advanced first-order reliability methods, the probability of failure is computed as 0.00994 (approximately 10 failures out of 1000 trials). Sensitivity information from this analysis indicates that scatter in strength virtually controls the reliability of the vane design. It is worth mentioning again that since the vanes were originally designed to assure that under the high-pressure burner rig test conditions, the maximum stresses will be below the proportional limit stress, a failure would mean that the vane design fails to meet this design criterion.

TABLE II.—PARAMETERS FOR PROBABILISTIC ANALYSES

Property	Mean value	Standard deviation	Distribution type
In-plane modulus, GPa	181.5	13.8	Weibull
Poisson's ratio	0.17	0.009	Normal
Coefficient of thermal expansion (in-plane), $10^{-6}/^{\circ}\text{C}$	4.5	0.23	Normal
Proportional limit, MPa	166.0	27.2	Weibull

Case II: In the second case, two additional variables—namely, internal cooling air pressure and external aerodynamic pressure on the vane—were added as random variables. Since at this time, no measured data on pressures are available, a hypothetical study was conducted to evaluate the effect of certain loading parameters. The internal cooling air pressure is assumed to be normally distributed with a mean value of 862 kPa (125 psi) and a coefficient of variation (COV) of 0.04 (i.e., a standard deviation of 35 kPa (5 psi)). The external aerodynamic pressure is also assumed to be normally distributed with a mean value of 552 kPa (80 psi) and a COV of 0.08. Figure 11(a) shows a CDF of hoop stress at the critical location. Figure 11(b) shows the ranking of the sensitivity factors (at a probability level of 0.999). As expected, the loading parameters, namely the internal and external pressures, are the dominant ones that control the scatter in hoop stress. The stress distribution is used to quantify risk by using the standard R–S model. Accordingly, the probability of failure is computed as 0.0162 (~1.6 percent or 16 failures out of 1000 trials). In this case, stress uncertainty also contributes to this failure probability. The higher failure probability is due to an increase in uncertainty of hoop stress, which, in turn, occurs because of the inclusion of loading parameters as random variables. A summary of the risk assessment for both cases is shown in figure 12.

Concluding Remarks

The ANSYS finite element code and the in-house probabilistic analysis methods in the FPI (fast probability integration) technique were used to perform a formal reliability assessment of an all-ceramic matrix composite (CMC) turbine stator vane. Two cases were evaluated—one with three random variables for stress (all related to material properties) and the other with five random variables for stress

(with the addition of load-related parameters). Results show that the failure probability is 10/1000 and 16/1000 for the two cases, respectively, for not meeting the design requirements under the high-pressure burner rig test conditions. Results also showed that load-related parameters have a more significant effect on the uncertainty in hoop stress than material- or geometry-related parameters. Reliability of the vane design can be controlled primarily by reducing the scatter in the proportional limit of the vane material. However, as the CMC materials continue to develop and fabrication techniques for complex CMC structural shapes are perfected, these failure rates will reduce. In any case, such methodologies provide a quantifiable way to assess the risk and provide a quantifiable tool to tailor specific designs for a given reliability.

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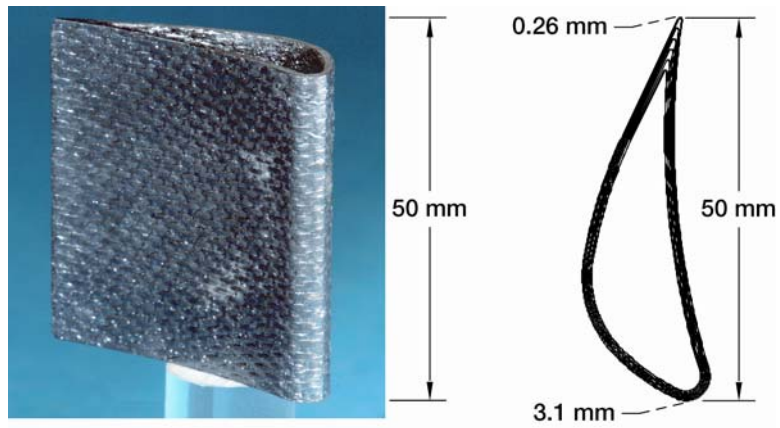


Figure 1.—SiC/SiC CMC turbine stator vane.

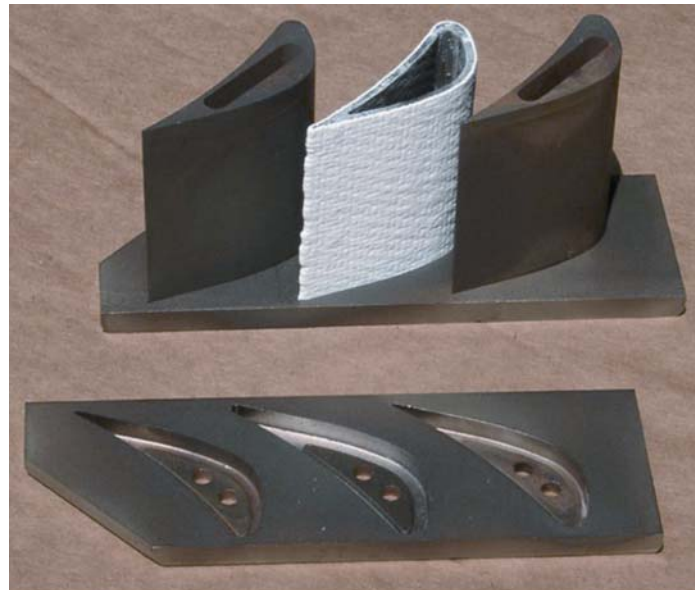


Figure 2.—Vanes in holder prior to testing. SiC/SiC test vane is in center with metallic vanes on either side.

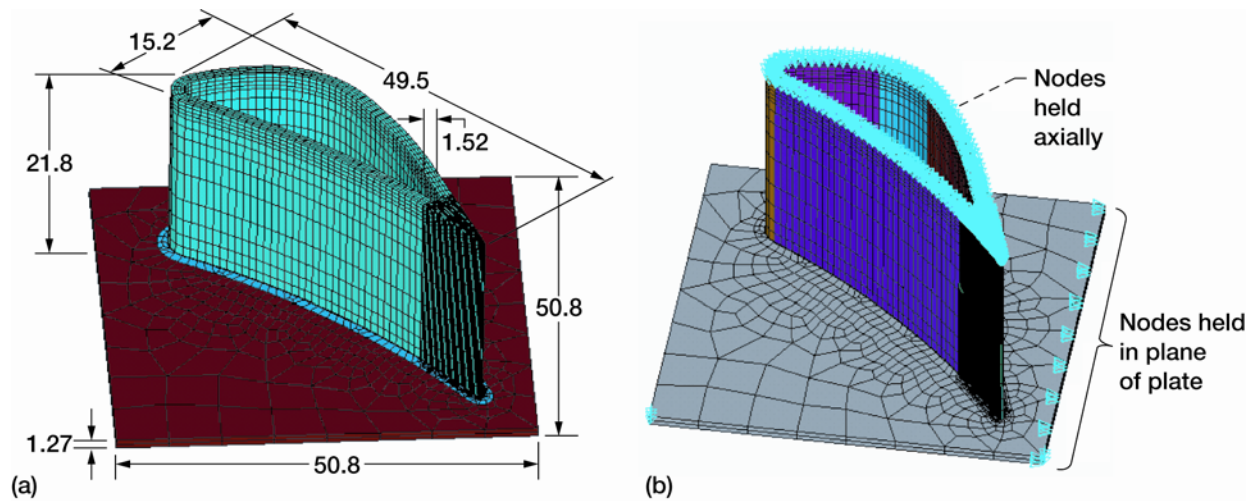


Figure 3.—Finite element model and boundary conditions for vane. (a) Finite element mesh details and vane geometry. All dimensions are in millimeters. (b) Boundary conditions for finite element model.

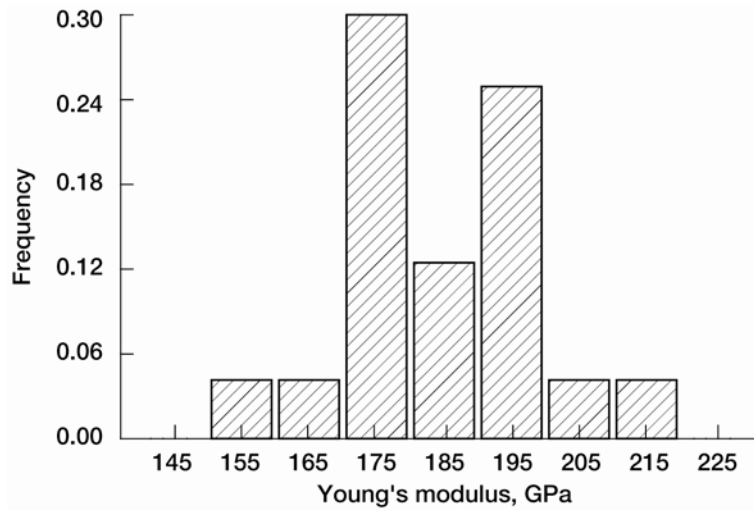


Figure 4.—Frequency histogram of in-plane Young's modulus of MI SiC/SiC for 24 data points. Mean value is 181.5 GPa and standard deviation, 13.8 GPa.

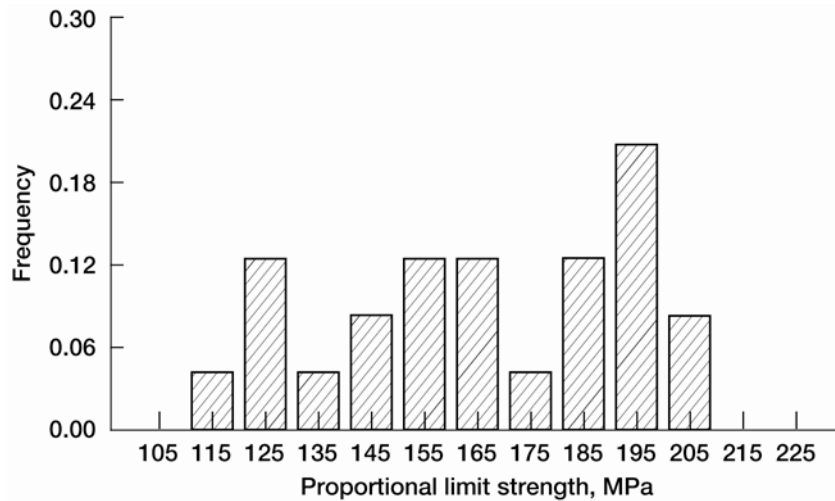


Figure 5.—Frequency histogram of proportional limit strength of MI SiC/SiC for 24 data points. Mean value is 166 MPa and standard deviation, 27.2 MPa.

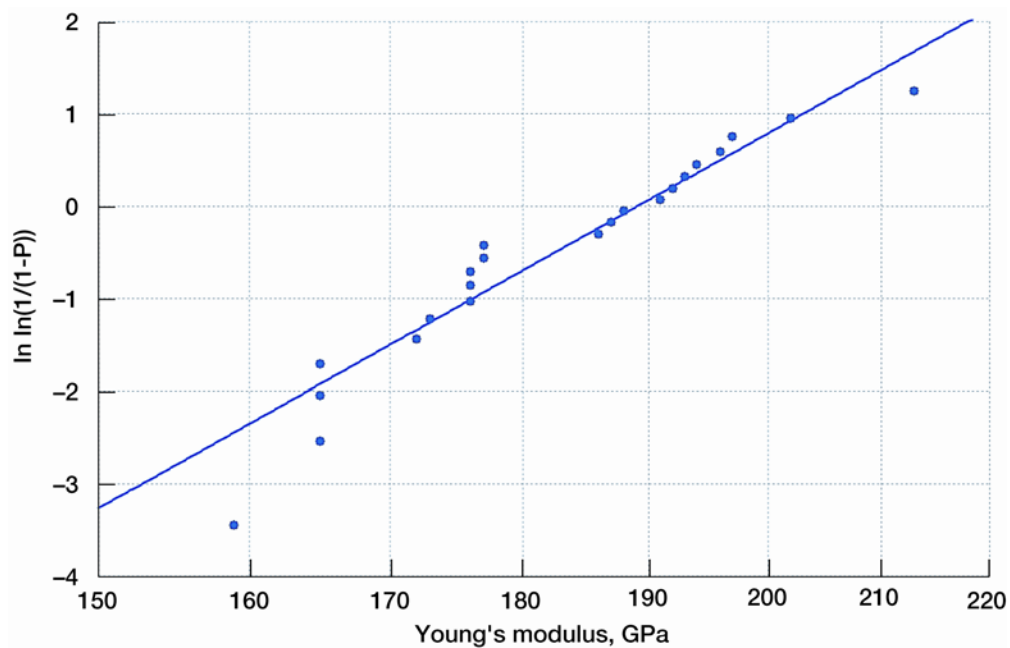


Figure 6.—Weibull plot of CMC in-plane Young's modulus for 24 specimen measurements (also shown is best-fit line through data). For any x , P is the probability that the value is less than or equal to x .

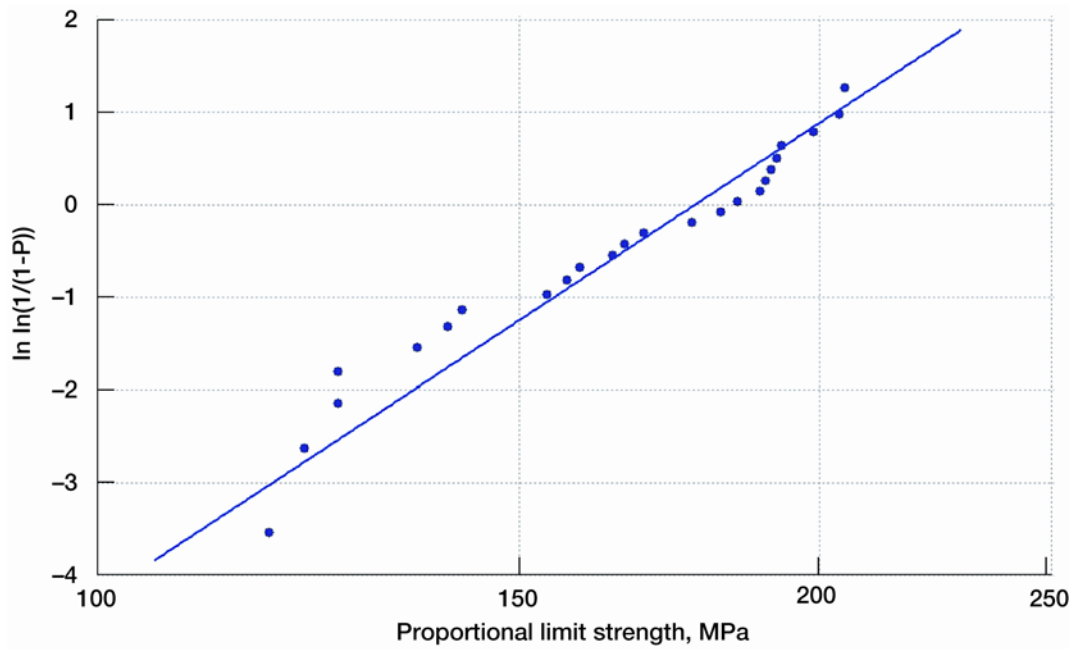


Figure 7.—Weibull plot of CMC in-plane measured proportional limit strength for 24 specimen measurements (also shown is best-fit line through data). For any x , P is the probability that the value is less than or equal to x .

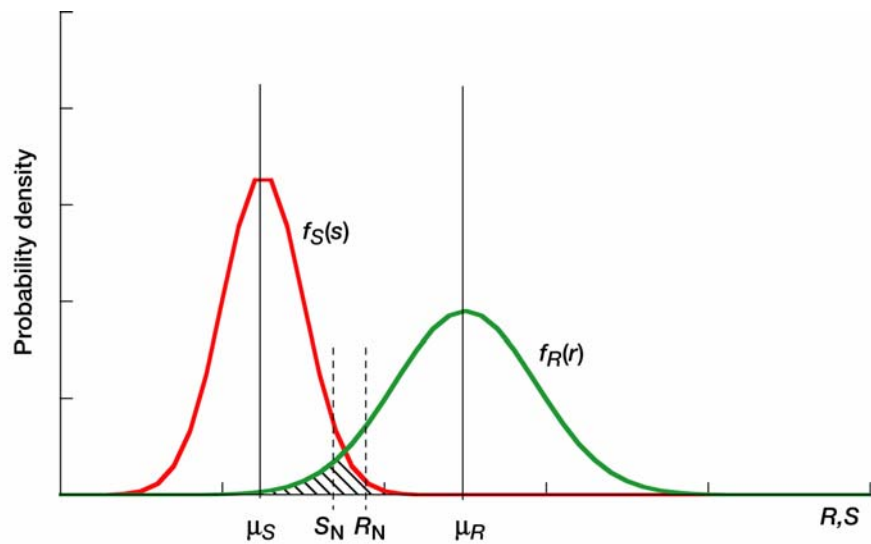


Figure 8.—Failure probability evaluation. S_N and R_N are nominal values of stress and resistance (strength), respectively. The $f_S(s)$ and $f_R(r)$ are corresponding failure probability density functions.

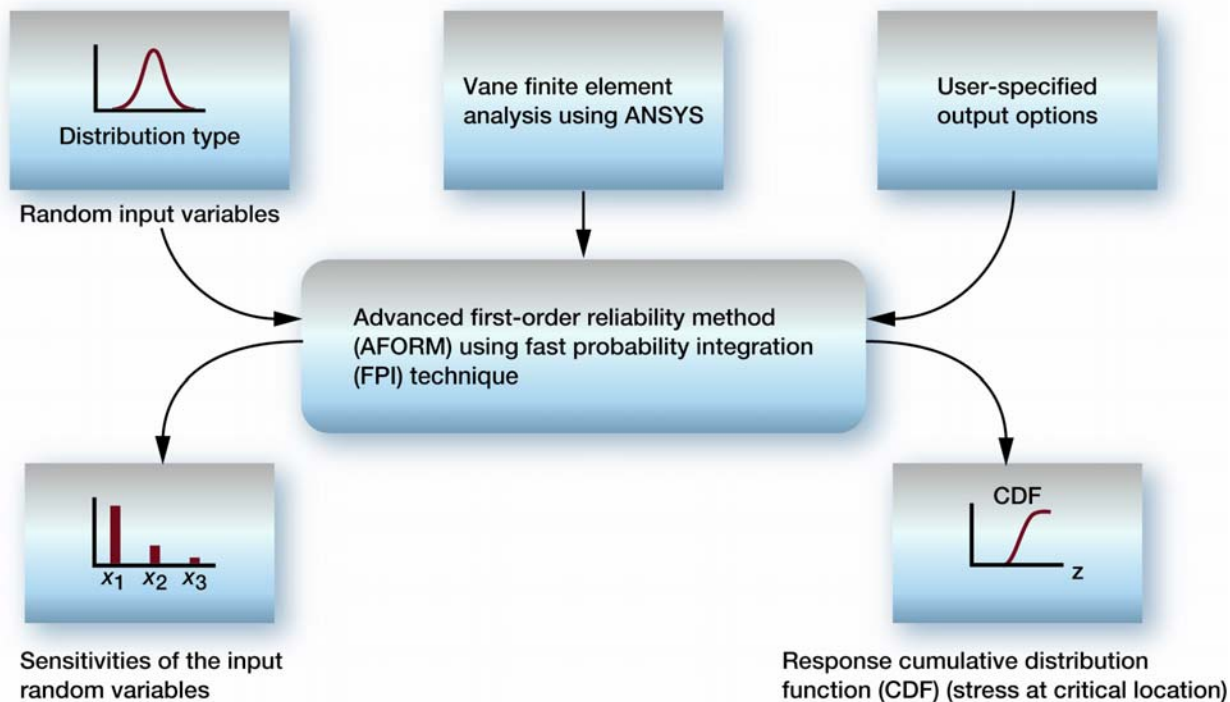


Figure 9.—Probabilistic analysis flow diagram. ANSYS (ANSYS, Inc., Canonsburg, PA) is finite element code.

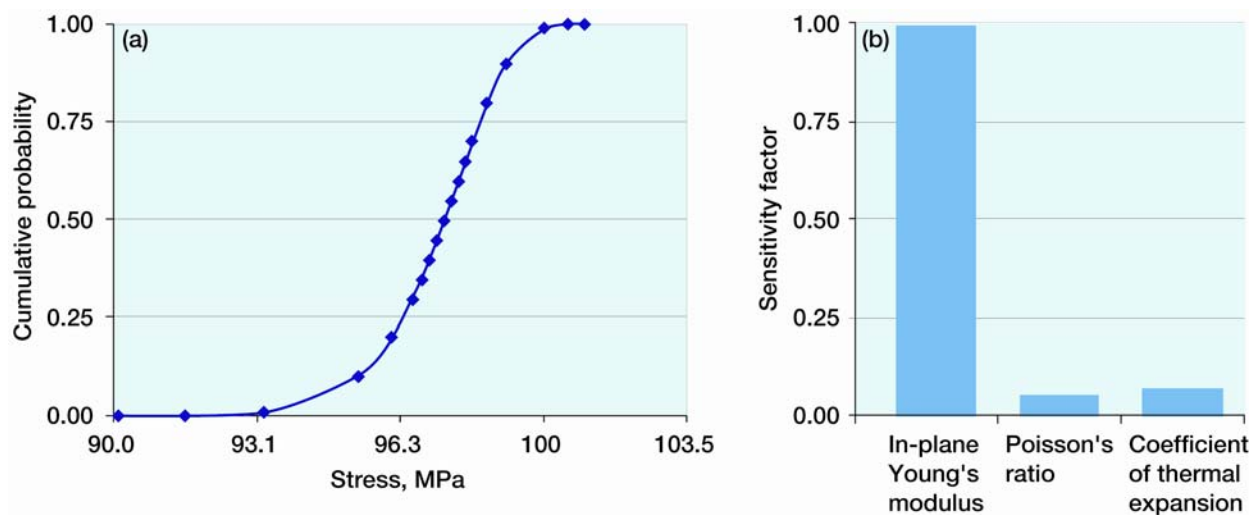


Figure 10.—Probabilistic analysis results for three of the random variables for SiC/SiC turbine stator blade design, case I. Stress mean value is 97.4 MPa and standard deviation, 1.4 MPa. (a) Cumulative distribution function (CDF) of hoop stress. (b) Sensitivity factors.

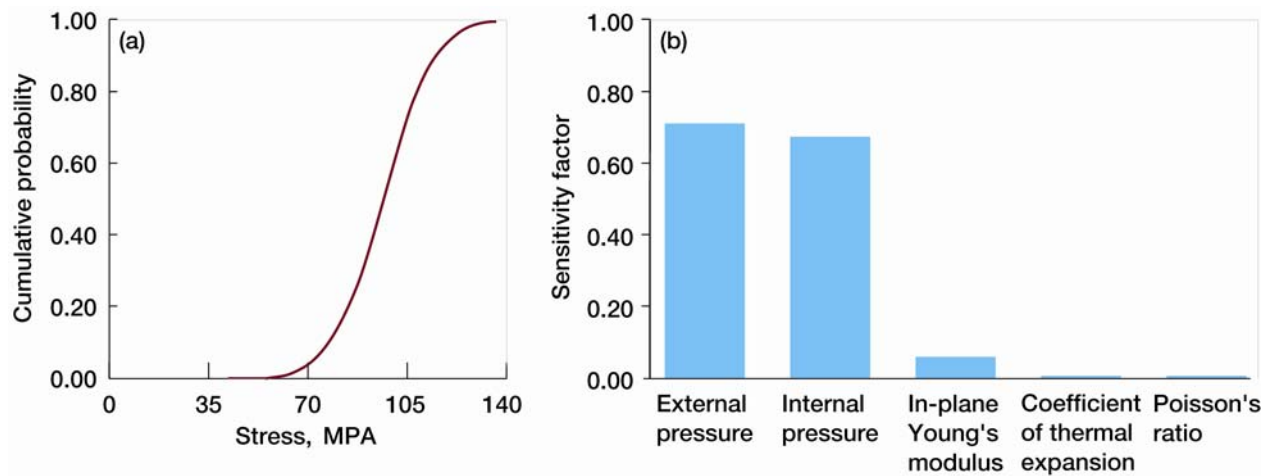


Figure 11.—Probabilistic analysis results for five of the random variable for SiC/SiC turbine stator vane design, case II. (a) Cumulative distribution function (CDF) of hoop stress. Stress mean value is 96.5 MPa and standard deviation, 14.7 MPa. (b) Sensitivity factors.

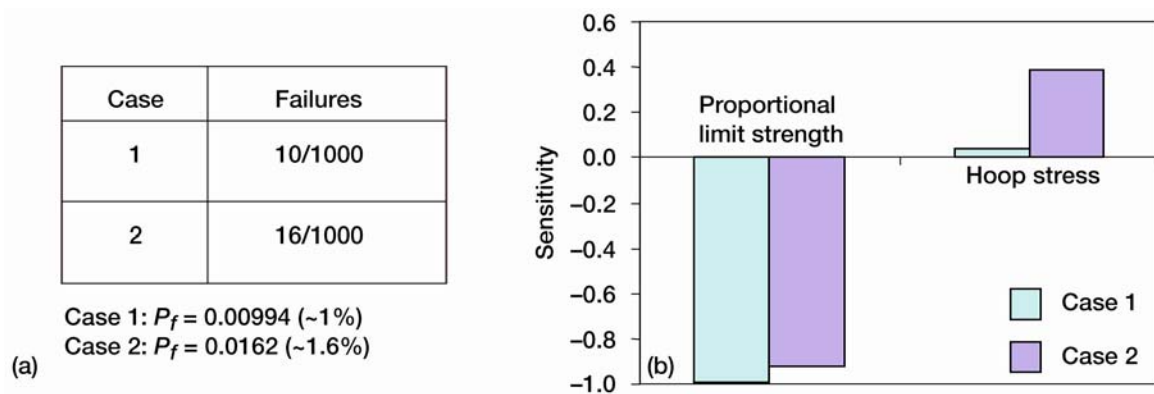


Figure 12.—Risk analysis results summary for SiC/SiC turbine stator vane design. P_f is failure probability.

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE November 2004	3. REPORT TYPE AND DATES COVERED Technical Memorandum		
4. TITLE AND SUBTITLE Probabilistic Analysis of a SiC/SiC Ceramic Matrix Composite Turbine Vane		5. FUNDING NUMBERS WBS-22-714-30-20		
6. AUTHOR(S) Pappu L.N. Murthy, Noel N. Nemeth, David N. Brewer, and Subodh Mital				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration John H. Glenn Research Center at Lewis Field Cleveland, Ohio 44135-3191		8. PERFORMING ORGANIZATION REPORT NUMBER E-14792		
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Washington, DC 20546-0001		10. SPONSORING/MONITORING AGENCY REPORT NUMBER NASA TM-2004-213331		
11. SUPPLEMENTARY NOTES Pappu L.N. Murthy and Noel N. Nemeth, NASA Glenn Research Center; David N. Brewer, U.S. Army Research Laboratory, NASA Glenn Research Center; and Subodh Mital, University of Toledo, Toledo, Ohio 43606 and NASA Resident Research Associate at Glenn Research Center. Responsible person, Pappu L.N. Murthy, organization code 5920, 216-433-3332.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Unclassified - Unlimited Subject Category: 24 Available electronically at http://gltrs.grc.nasa.gov This publication is available from the NASA Center for AeroSpace Information, 301-621-0390.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) To demonstrate the advanced composite materials technology under development within the Ultra-Efficient Engine Technology (UEET) Program, it was planned to fabricate, test, and analyze a turbine vane made entirely of silicon carbide-fiber-reinforced silicon carbide matrix composite (SiC/SiC CMC) material. The objective was to utilize a five-harness satin weave melt-infiltrated (MI) SiC/SiC composite material developed under this program to design and fabricate a stator vane that can endure 1000 hours of engine service conditions. The vane was designed such that the expected maximum stresses were kept within the proportional limit strength of the material. Any violation of this design requirement was considered as the failure. This report presents results of a probabilistic analysis and reliability assessment of the vane. Probability of failure to meet the design requirements was computed. In the analysis, material properties, strength, and pressure loading were considered as random variables. The pressure loads were considered normally distributed with a nominal variation. A temperature profile on the vane was obtained by performing a computational fluid dynamics (CFD) analysis and was assumed to be deterministic. The results suggest that for the current vane design, the chance of not meeting design requirements is about 1.6 percent.				
14. SUBJECT TERMS Probabilistic analysis; CMC vane; Cumulative distribution function; Probability density function; Scatter; Weibull distribution; Strength; Proportional limit; Design requirements			15. NUMBER OF PAGES 21	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT Unclassified	20. LIMITATION OF ABSTRACT	

