NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM

No. 1099

AIRSCREW GYROSCOPIC MOMENTS

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Luftwissen, Vol. 8, No. 3, March 1941

Reprint of R.T.P. Translation No. 1333
Issued by the Ministry of Aircraft Production
London, England

Washington
September 1946

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AIRSCREW GYROSCOPIC MOMENTS[1]

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When flying in a turn or pulling out of a dive, the airscrew exerts a gyroscopic moment on the aircraft. In the case of airscrews with three or more blades, arranged symmetrically, the value of the gyroscopic moment is $J_x \omega_x \omega_y$, (reference 1) where J_x denotes the axial moment of inertia about the axis of rotation of the airscrew, ω_x the angular speed of the airscrew about its axis, and ω_y the rotary speed of the whole aircraft about an axis parallel to the plane of the airscrew (e.g., when pulling up, the transverse axis of the aircraft). The gyroscopic moment then tends to rotate the aircraft about an axis perpendicular to those of the two angular speeds and, in the case of airscrews with three or more blades, is constant during a revolution of the airscrew. With two-bladed airscrews, on the contrary, although the calculater gyroscopic moment represents the mean value in time, it fluctuates about this value with a frequency equal to twice the revolutions per minute (reference 2). In addition, pulsating moments likewise occur about the other two axes. This fact is known from the theory of the asymmetrical gyro (reference 3); the calculations that have been carried out for the determination of the various gyroscopic moments, however, mostly require an exact knowledge of the gyro theory. The problem will therefore be approached in another manner based on quite elementary considerations.

The considerations are of importance, not only in connection with the gyroscopic moments exerted by the two-bladed airscrew on the aircraft, but also with the stressing of the blades of airscrews with an arbitrary number of blades.

The procedure devised for the calculation of the two gyroscopic moments, the axes of which are perpendicular to the axis of the primary rotation ω_x, is as follows: Consider an element of mass of the screw and calculate its speed in the x-direction due to the two angular speeds ω_x and ω_y (fig.1). By differentiation of the speed with respect to time there is obtained the acceleration and thence the inertial resistance of the element of mass. This inertial resistance may be looked upon as an external load, and thus is found the stresses produced in the blades and also the gyroscopic moments themselves.

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[1]Luftwissen, vol. 8, no. 3, March 1941, pp. 96-97.

NOTE: Reprint of R.T.P. Translation No. 1333; issued by the Ministry of Aircraft Production, London, England.

To simplify the calculation, assume that the elements of mass of the airscrew are combined along the axis of gravity of the blades and that this axis is rectilinear. This assumption holds with sufficient accuracy for airscrews in general use at the present time.

The plane of rotation of the airscrew at the beginning of the calculation is assumed as being turned by the angle χ out of the y-z plane about the y-axis. After calculation of the accelerations the angle χ is again assumed as equal to zero. The airscrew blade is assumed as being turned by the angle φ in the plane of rotation. The element of mass of the screw under consideration is situated at a distance of radius r from the center O. The angular speed in the plane of the airscrew is denoted by ω; it appears first in place of ω_x.

Consider first the velocities and accelerations in the x-direction due to the rotations ω and ω_y.

$$v_x = r \cos \varphi \cos \chi \omega_y - r \sin \varphi \sin \chi \omega$$

Differentiation of this equation gives:

$$\frac{dv_x}{dt} = b_x = -r \cos \varphi \sin \chi \omega_y \frac{d\chi}{dt} - r \sin \varphi \cos \chi \omega_y \frac{d\varphi}{dt}$$

$$-r \sin \varphi \cos \chi \omega \frac{d\chi}{dt} - r \cos \varphi \sin \chi \cdot \omega \frac{d\varphi}{dt}$$

If $\chi = 0$, thus making the plane of rotation coincide with the y-z plane, there is obtained

$$\frac{d\chi}{dt} = \omega_y; \qquad \frac{d\varphi}{dt} = \omega = \omega_x$$

$$b_x = -2 r \sin \varphi \omega_x \omega_y$$

This acceleration gives rise to an inertial resistance which corresponds to an external loading dp_x of the blade in the x-direction.

$$dp_x = b_x dm = 2 r \sin \varphi \omega_x \omega_y dm \tag{1}$$

The bending moment M_b due to this load, which tends to bend the blade forward, that is, in the same direction as the thrust, for the radius r_1, then acquires the value:

$$M_b = \int_{r_1}^{R} d\, p_x\, (r - r_1) = 2\, \omega_x\, \omega_y\, (J_{r_1} - r_1\, S_{r_1}) \sin \varphi \qquad (2)$$

Here $J_{r_1} = \int_{r_1}^{R} r^2\, dm$ is the moment of inertia and $S_{r_1} = \int_{r_1}^{R} r dm$ the static moment of the outer portion of the blade as far as the cross section under consideration relative to the axis of rotation.

In addition to the bending moment due to the thrust there is thus a pulsating bending moment due to the gyroscopic forces, the frequency of which is equal to the angular speed ω_x. To obtain an idea of its magnitude, take an example. For an engine developing 1000 horsepower with 1300 rpm at 4-kilometer altitude, the thrust of the airscrew at a flight speed of 400 kilometers per hour is about 500 kilograms. The metal airscrew has three blades and is 4 meters in diameter. The bending moment, due to the thrust, to which a blade cross section at a distance of 0.25 meter from the hub is subjected, is then about 200 meters per kilogram. Assuming, now, that the aeroplane flying at this speed is pulled up with an acceleration of 6g, an angular speed $\omega_y = 0.53\ \text{sec}^{-1}$ is produced. With a value $J_{r_1} - r_1\, S_{r_1}$ = 2.0 meters per kilogram per second square, a bending moment of about 300 meters per kilogram due to the gyroscopic forces will result, which is thus 150 percent of the bending moment due to the thrust and, as opposed to the latter, gives rise to an alternating stress. Pass on to the gyroscopic moment of the whole airscrew. Consider the moment about the z-axis, and the effect of a blade with an angle of rotation φ (fig. 1). By using equation (1) there is obtained

$$M'_z = \int_0^R d\, p_x\, r \sin \varphi = 2\, \omega_x\, \omega_y\, J'_x \sin^2 \varphi$$

where $J'_x = \int_0^R r^2\, dm$ is the moment of inertia of a blade relative to the x-axis. This may also be written:

$$M_z' = \omega_x \, \omega_y \, J_x' \quad (1 - \cos 2\,\varphi) \tag{3}$$

For the two-bladed airscrew, therefore, the gyroscopic moment composed of the effect of the two blades is:

$$M_z = \omega_x \, \omega_y \, J_x' \left[1 - \cos 2\,\varphi + 1 - \cos 2\,(\varphi + \pi)\right]$$

or

$$M_z = \omega_x \, \omega_y \, J_x \,(1 - \cos 2\,\varphi) \tag{4}$$

Where $J_x = 2 \cdot J_x'$ is the axial moment of inertia of the whole screw, relative to the x-axis.

The gyroscopic moment of the two-bladed screw thus fluctuates about a mean value of the quantity $\omega_x \omega_y J_x$ with the period of twice the revolutions per minute; its minimum value of zero is attained with the blades in the perpendicular position and its maximum which is equal to twice the mean value, with the blades in the horizontal position.

As may be easily seen, for airscrews with three or more blades arranged symmetrically, as is the usual practice, the term $\cos 2\,\varphi$ in equation (4) is ruled out, and the gyroscopic moment has the known constant value of $\omega_x \, \omega_y \, J_x$.

The gyroscopic moment about the y-axis is obtained by appropriate consideration from equation (1). For a two-bladed screw:

$$M_y = \omega_x \, \omega_y \, J_x \sin 2\,\varphi \tag{5}$$

The gyroscopic moment of the two-bladed screw about the y-axis thus pulsates by the same amount and at the same frequency as the gyroscopic moment about the z-axis; its mean value, however, is zero. Its maximum values, which occur with the 45° positions of the screw, are equal to the mean value of the gyroscopic moment about the z-axis. In the case of airscrews with three or more blades, the gyroscopic moment about the y-axis becomes zero.

The course of the gyroscopic moments about the z- and y-axes is plotted in figure 2 against the angle of rotation φ. The blade positions are shown by the sketch.

For a 400 horsepower engine running at 1800 rpm driving a two-bladed metal airscrew cf 2.8-meter diameter with an axial moment of inertia $J_x = 1.2$ meters per kilogram per second square assuming an angular speed $\omega_y = 0.53 \text{ sec}^{-1}$ (which was also taken as a basis earlier), the amplitude

$\omega_x \omega_y J_x$ of the gyroscopic moments is 120 meters per kilogram

Owing to the rotation about the y-axis (fig. 1), centrifugal forces are also produced in the blade; at an element of mass d m situated at a distance r from the center, its value is

$$d\ Z = \omega_y^2\ r \cos \varphi\ d\ m$$

As a result, in a cross section at radius r_1 there is produced, first, an additional centrifugal force in the direction of the blade which is of the value:

$$\int_{r_1}^{R} d\ Z \cos \varphi = \frac{\omega_y^2}{2} (1 + \cos 2\ \varphi)\ S_{r_1}$$

and is supplementary to the centrifugal force due to the angular speed ω_x, and secondly, a bending moment in the plane of rotation of the value

$$M_{bx} = - \int_{1}^{R} d\ Z\ r \sin \varphi = - \frac{\omega_y^2}{2} (J_{r_1} - r_1\ S_{r_1}) \quad \sin 2\ \varphi \qquad (6)$$

The notations here are the same as in equation (2).

Correspondingly, for the two-bladed airscrew a pulsating gyroscopic moment about the x-axis of the value

$$M_x = - \frac{\omega_y^2}{2} \quad J_x \sin 2\ \varphi \qquad (7)$$

is produced.

Compared with the airscrew torque this gyroscopic moment is in most cases very small. For example, for the 400-horsepower engine referred to above under the same conditions, as those according to figure 2, the amplitude $\frac{\omega_y^2}{2} J_x$ of the gyroscopic moment is 0.17 meter kilogram; whereas the engine torque, with reference to the airscrew shaft, is 160 meters per kilogram.

In the case of the airscrew with three or more blades, no gyroscopic moment about the x-axis appears.

Summing up, it may therefore be said that:

With a rotation of the whole airplane, as a result of inertia forces, additional pulsating bending moments of the airscrew blades are produced. The bending moment perpendicular to the plane of rotation (equation (2)) is of the frequency of the revolutions per minute and may attain the value of the bending moment due to the thrust. The bending moment in the plane of rotation (equation (6)) has a frequency of twice the revolutions per minute and compared with that due to the airscrew torque is negligibly small.

Of the gyroscopic moments acting on the aircraft, the moment about the axis perpendicular to the axis of rotation has a mean value in time of $\omega_x \omega_y J_x$; in the case of the two-bladed screw the gyroscopic moment pulsates about this mean value between zero and twice this value, at a frequency equal to twice the revolutions per minute (equation (4)), in the case of a screw with three or more blades the gyroscopic moment is constant. Further, in the case of the two-bladed airscrew a gyroscopic moment is produced about the axis of the secondary rotation ω_y, which pulsates with the same amplitude and frequency as that given previously, but has a mean value of zero (equation (5)), and a gyroscopic moment about the axis of the primary rotation (equation (7)), which, however, in most cases is negligibly small in comparison with the other moments. In the case of airscrews with three or more blades, gyroscopic moments about the two axes of rotation are absent.

Translation by M. Flint.

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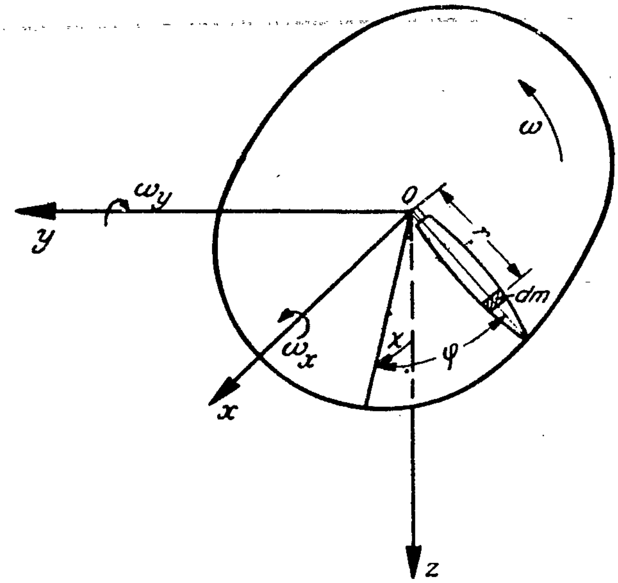

Figure 1.- The motion of the airscrew in a curvilinear flight path.

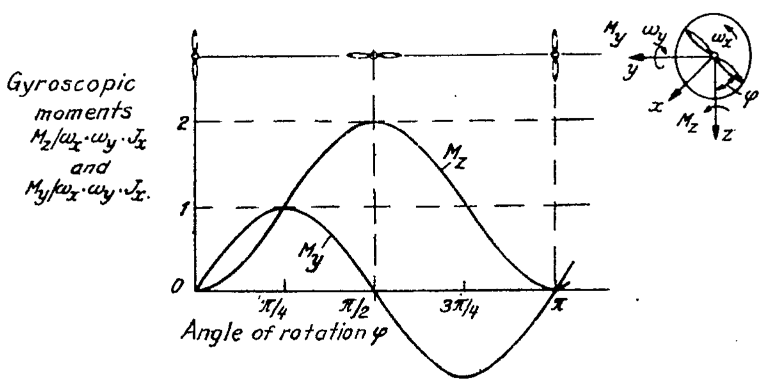

Figure 2.- The gyroscopic moments M_z and M_y of the two bladed airscrew when pulling out of a dive.

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Propellers - Gyroscopic moment
Propeller blades - Stresses