NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM

No. 1147

GENERAL CHARACTERISTICS OF THE FLOW THROUGH NOZZLES AT NEAR CRITICAL SPEEDS

By R. Sauer

Translation

"Allegemeine Eigenschaften der Strömung durch Düsen in der Nähe der kritischen Geschwindigkeit"

Deutsche Luftfahrtforschung, Forschungsbericht Nr. 1992

Washington
June 1947

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SUMMARY

The characteristics of the position and form of the transition surface through the critical velocity are computed for flow through flat and round nozzles from subsonic to supersonic velocity. Corresponding considerations were carried out for the flow about profiles in the vicinity of sonic velocity.

I. INTRODUCTION

While useful methods of calculation exist for pure subsonic and for pure supersonic flows of compressible gases difficulties arise in the mathematical treatment of mixed flows with subsonic and supersonic ranges. Such mixed flows exist: (1) in the transition from subsonic to supersonic speeds in Laval nozzles, (2) about profiles in a flow at high subsonic speed on the appearance of local supersonic ranges, (3) in front of blunt bodies in a flow at supersonic velocity with a local subsonic region behind the compression shock in the vicinity of the stagnation point.

The present report takes up the first as the simplest of the three problems named. It is a fact that for this case a rough view of flow conditions sufficient for many problems can be obtained by the methods of hydraulics, that is, the nozzle is considered a flow tube and the magnitude of the velocity is considered

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*"Allgemeine Eigenschaften der Strömung durch Düsen in der Nähe der kritischen Geschwindigkeit," FB 1992, Zentrale für wissenschaftliches Berichtswesen der Luftfahrtforschung des Generalluftzeugmeisters (ZWB) Berlin - Adlershof, Sept. 25, 1944.

constant in every cross-sectional plane. The critical velocity (flow velocity = sonic velocity) is then reached exactly in the minimum cross section of the Laval nozzle. In the strict two-dimensional treatment, however, a curved line (critical curve) is obtained for the passage of the velocity through the critical value in the plane of the longitudinal section of the nozzle; it begins at the nozzle wall in front of the minimum cross section and cuts the nozzle axis downstream of the minimum cross section. (See fig. 1.)

In various reports, especially those of Th. Meyer (1), G. J. Taylor (2), H. Görtler (3), and Kl. Oswatitsch and H. Rothstein (4), expansions in power series for the determination of the critical curves are given. In what follows, general statements on the position and curvature of the critical curve for a given nozzle are obtained from such expansions in power series by breaking off the series after the first two terms. The investigations are concerned with flat and round nozzles and provide sufficiently accurate results for practical purposes for nozzle curvatures that are not too sharp.

Corresponding expansions in power series are applied, in conclusion, to the flows through flat nozzles with curved axes. In this way, information is obtained on the variation of the critical curve for profile flows with local supersonic regions.

II. POTENTIAL EQUATION

The nozzle axis is selected as the x-axis and the origin of the coordinate system is placed at the point on the axis at which the critical velocity is first reached. (See fig. 1.) With the hypothesis of non-vortical flow and perfect flow of a perfect gas with constant specific heats c_p, c_v $(k = c_p/c_v)$ the potential equation

$$\frac{\partial \tilde{u}}{\partial x}\left(a^2 - \tilde{u}^2\right) + \frac{\partial \tilde{v}}{\partial y}\left(a^2 - \tilde{v}^2\right) - 2\tilde{u}\tilde{v}\frac{\partial \tilde{u}}{\partial y} + \sigma a^2 \frac{\tilde{v}}{y} = 0 \qquad (1)$$

holds. In this $\tilde{u}$, $\tilde{v}$ are the x- and y-components of the flow velocity and a, the local sonic velocity, which is related to $\tilde{u}$, $\tilde{v}$ and the critical velocity a^* through

$$a^2 = \frac{k+1}{2} a^{*2} - \frac{k-1}{2} \left(\tilde{u}^2 + \tilde{v}^2\right) \tag{2}$$

$\sigma = 0$ characterizes the flat or two-dimensional flow, $\sigma = 1$ the three-dimensional flow through stream tubes of of circular cross section; x, y are cartesian for $\sigma = 0$, cylindrical coordinates for $\sigma = 1$.

Substituting (2) in equation (1) and introducing the dimensionless velocity components

$$U = \tilde{u}/a^*, \qquad V = \tilde{v}/a^* \tag{3}$$

the potential equation (1) becomes:

$$\frac{\partial U}{\partial x}\left(1 - U^2 - \frac{k-1}{k+1} V^2\right) + \frac{\partial V}{\partial y}\left(1 - \frac{k-1}{k+1} U^2 - V^2\right)$$
$$- \frac{4}{k+1}\frac{\partial U}{\partial y} UV + \sigma\left[1 - \frac{k-1}{k+1}\left(U^2 + V^2\right)\right]\frac{V}{y} = 0 \tag{4}$$

Since the present investigation was limited to the vicinity of the critical curve,

$$U = 1 + u, \qquad V = v \tag{5}$$

in which u and v are small quantities, and equation (4) becomes

$$\frac{\partial u}{\partial x}\left(2u + u^2 + \frac{k-1}{k+1} v^2\right) - \frac{\partial v}{\partial y}\left[\frac{2}{k+1} - 2\frac{(k-1)}{(k+1)} u\right.$$
$$\left. - \frac{k-1}{k+1} u^2 - v^2\right] + \frac{4}{k+1}\frac{\partial u}{\partial y}(1 + u)\, v - \sigma\left[\frac{2}{k+1}\right.$$
$$\left. - 2\frac{(k-1)}{k+1} u - \frac{k-1}{k+1}\left(u^2 + v^2\right)\right]\frac{v}{y} = 0 \tag{6}$$

As $x \to 0$, $y \to 0$, u, and v approach zero and also on the basis of (6) $\partial v/\partial y$ and, consequently, the quotient v/y approach zero, while the velocity rise $\partial u/\partial x$ along the axis ought not vanish at the origin. Then if the small quantities u, v, $\partial v/\partial y$, and v/y are considered linear only, the following approximate relation is obtained from (6)

$$(k + 1)\, u \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} + 2v \frac{\partial u}{\partial y} - \sigma \frac{v}{y} = 0 \qquad (7)$$

Since the nozzle flow is symmetrical with respect to the x-axis, $\partial u/\partial y$ also approaches zero for $x \to 0$, $y \to 0$. Consequently, the term $2v\ \partial u/\partial y$ may be ignored, by means of which (7) becomes further simplified to

$$(k + 1)\, u \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} - \sigma \frac{v}{y} = 0 \qquad (8)$$

With $\sigma = 0$, (8) was already applied in another connection by Kl. Oswatitsch and K. Wieghardt (5).

III. FLAT NOZZLES

Equation (8) furnishes general relations, for $\sigma = 0$, for the flow through flat nozzles in the vicinity of the critical curve. Like Kl. Oswatitsch and W. Rothstein (4), taking into account the symmetry around the x-axis, setting

$$\left.\begin{aligned} \varphi &= f_0(x) + y^2 f_2(x) + y^4 f_4(x) + \ldots \\ u &= \frac{\partial \varphi}{\partial x} = f'_0(x) + y^2 f'_2(x) + y^4 f'_4(x) \ldots \\ v &= \frac{\partial \varphi}{\partial y} = 2y f_2(x) + 4y^3 f_4(x) + \ldots \end{aligned}\right\} \qquad (9)$$

in which the primes signify derivatives with respect to x, and, obtain from (8).

$$(k+1)\,(f'_0 + y^2 f'_2 + \dots)\,(f''_0 + y^2 f''_2 + \dots)$$

$$= 2f_2 + 12y^2 f_4 + \dots + 2\sigma(f_2 + 2y^2 f_4 + \dots)$$

By arranging in order of powers of y and equating coefficients of the individual powers to zero this system of equations is obtained

$$\begin{aligned} 2(1+\sigma)\,f_2 &= (k+1)\,f'_0 f''_0 \\ 2(6+2\sigma^*)f_4 &= (k+1)\,\left(f'_0 f''_2 + f''_0 f'_2\right) \\ &\dots\dots\dots\dots \end{aligned} \qquad (10)$$

through which the functions $f_2(x)$, $f_4(x)\dots$ in order can be expressed by the function $f'_0(x)$ and its derivatives. The function

$$f'_0 = u_0(x)$$

which characterizes the velocity distribution along the nozzle axis, remains undetermined. Given $u_0(x)$ the flow is completely established through (10) and can be understood to be nozzle flow, if a pair of streamlines that are mirror images of one another with respect to the x-axis are assumed as nozzle walls. For further discussion, setting

$$f'_0 = u_0(x) = \alpha\, x \qquad (11)$$

from which

$$f_2 = \frac{k+1}{2}\,\alpha^2 x, \qquad f_4 = \frac{(k+1)^2}{24}\,\alpha^3, \dots \qquad (12(a))$$

*Translator's note: The number 2 was omitted from the German report. The correction here changes several subsequent equations and several values in the table.

follows immediately from (10) with $\sigma = 0$. According to (9)

$$\boxed{\begin{aligned} u &= \alpha x + \frac{k+1}{2}\alpha^2 y^2 + \ldots \\ v &= (k+1)\,\alpha^2 xy + \frac{(k+1)^2}{6}\alpha^3 y^3 + \ldots \end{aligned}} \qquad (13(a))$$

is obtained for the components of the flow velocity. The dots represent terms of higher order of y. If (11) is taken as the first term of a power series expansion for $u_o(x)$, the expressions for f_2 and f_4 in (12(a)) are also the first terms of power series expansions.

From (13(a)) the critical curve is obtained from the requirement

$$(1 + u)^2 + v^2 = 1$$

from which in (13(a)) the parabola

$$x_K = -\frac{k+1}{2}\alpha {y_K}^2 \qquad (14(a))$$

is obtained, therefore, with $u = 0$ as a first approximation. It cuts the nozzle axis, normally, at the point $x = y = 0$ and its curvature there is

$$1/\rho_K = (k + 1)\,\alpha \qquad (15(a))$$

The vertices, therefore the points, of the streamlines, adjacent to the nozzle axis, are given by $v = 0$ and, according to (13(a)), lie on the curve

$$x_S = -\frac{k+1}{6}\alpha {y_S}^2 \qquad (16(a))$$

Considering, now, the streamlines at a given vertex distance y_S (fig. 1) from the nozzle wall, then

$$\epsilon = -x_S = \frac{k+1}{6}\alpha {y_S}^2 \qquad (17(a))$$

is the distance from the center point of the narrowest cross section downstream to the intersection point of the nozzle axis and the critical curve.

The junction point of the critical curve with the nozzle wall is upstream and is obtained, as a first approximation from (14(a)) with $y_K = y_S$, which yields

$$\eta = -\left(x_K - x_S\right) = \frac{k + 1}{3} \alpha y_S{}^2 = 2\epsilon \qquad (18(a))$$

for the distance η from the junction point to the narrowest cross section.

Finally the calculation of the curvature at the vertex of the nozzle wall is to be made. In moving outward from the vertex to the wall by an element of curve $ds = dx$, the tangent turns through the angle

$$d\omega = \frac{dv}{1 + u} = \frac{1}{1 + u} \frac{\partial v}{\partial x} dx = \frac{1}{1 + u} \frac{\partial v}{\partial x} ds$$

Therefore the curvature at the vertex is

$$\frac{1}{\rho_S} = \frac{d\omega}{ds} = \frac{1}{1 + u} \frac{\partial v}{\partial x}$$

and taking into account (13(a)) and (15(a))

$$1/\rho_S = (k + 1)\ \alpha^2 y_S = 1/\rho_K \alpha y_S \qquad (19(a))$$

Up to now the velocity distribution $u_o(x)$ had been considered as given by equation (11) and the nozzle walls computed for it. Practically, the reverse is the problem, namely to ascertain for a particular nozzle, that is for assigned values of y_S, ρ_S, the flow in the narrowest cross section, therefore the values α, ϵ, η, and ρ_K. On the basis of the equations that have preceded it is possible to write down the solution to the problem, immediately, namely

$$\alpha = \sqrt{\frac{1}{(k+1)\rho_S y_S}}$$

$$\rho_K = \sqrt{\frac{\rho_S y_S}{k+1}} \qquad (20(a))$$

$$\eta = 2\epsilon = \frac{y_S}{3}\sqrt{(k+1)\frac{y_S}{\rho_S}}.$$

If the flow is in the positive direction of the x-axis, as in figure 1, then the subsonic region is to the left and the supersonic region to the right and, accordingly, $\alpha > 0$.

In practice it is recommended that all distances be expressed, dimensionless, in terms of y_S, and accordingly set $y_S = 1$ throughout (20(a)).

IV. ROUND NOZZLES

Corresponding relations for round nozzles come from (9) and (10) with $\sigma = 1$. Retaining (11) for the velocity distribution $u_o(x)$ along the nozzle axis, the following relations are obtained in place of (12(a)) through (20(a)):

$$f_2 = \frac{k+1}{4}\alpha^2 x, \quad f_4 = \frac{(k+1)^2}{64}\alpha^3 \qquad (12(b))$$

$$u = \alpha x + \frac{k+1}{4}\alpha^2 y^2 + \ldots$$

$$v = \frac{k+1}{2}\alpha^2 xy + \frac{(k+1)^2}{16}\alpha^3 y^3 + \ldots \tag{13(b)}$$

$$x_K = -\frac{k+1}{4}\alpha y_K{}^2 \tag{14(b)}$$

$$\frac{1}{\rho_K} = \frac{k+1}{2}\alpha \tag{15(b)}$$

$$x_S = -\frac{k+1}{8}\alpha y_S{}^2 \tag{16(b)}$$

$$\epsilon = -x_S = \frac{k+1}{8}\alpha y_S{}^2 \tag{17(b)}$$

$$\eta = -(x_K - x_S) = \frac{1}{8}(k+1)\alpha y_S{}^2 = \epsilon \tag{18(b)}$$

$$\frac{1}{\rho_S} = \frac{k+1}{2}\alpha^2 y_S = \frac{1}{\rho_K}\alpha y_S \tag{19}$$

$$\alpha = \sqrt{\frac{2}{(k+1)\rho_S y_S}}$$

$$\rho_K = \sqrt{\frac{2\rho_S y_S}{k+1}} \qquad (20(b))$$

$$\eta = \epsilon = \frac{y_S}{8}\sqrt{2(k+1)\frac{y_S}{\rho_S}}$$

To compare a flat nozzle with a round one of the same longitudinal section, formulas (20(a)) and (20(b)) are compared. (See fig. 2.)

For the flat nozzle $\eta = 2\epsilon$, for the round nozzle $\eta = \epsilon$.

Moreover

$$\frac{\epsilon \text{ round}}{\epsilon \text{ flat}} = \frac{3}{4}\sqrt{2} \approx 1.06$$

$$\frac{\eta \text{ round}}{\eta \text{ flat}} = \frac{3}{8}\sqrt{2} \approx 0.53$$

$$\frac{\alpha \text{ round}}{\alpha \text{ flat}} = \sqrt{2} \approx 1.41$$

The velocity increase is, therefore, around 40 percent larger for the round nozzle than for the flat nozzle. All of these results are fully confirmed by the nozzle flows calculated by Kl. Oswatitsch and W. Rothstein, as the enclosed table on page 18 shows.

V. COMPARISON WITH THE FLOW-TUBE APPROXIMATION

The author's results are now to be compared with the flow-tube approximation by computing the increase in velocity α by formulas (20(a)) and (20(b)), and by the flow-tube theory. For this purpose the flow density along the nozzle axis in the vicinity of the narrowest cross section is expanded, as follows

$$\theta = \frac{\rho}{\rho^*} U$$

$$= \text{const. } U\left(\frac{k+1}{k-1} - U^2\right)^{\frac{1}{k-1}}$$

$$= \text{const. } (1+u)\left(\frac{k+1}{k-1} - 1 - 2u - u^2\right)^{\frac{1}{k-1}}$$

$$\approx \text{const. } (1+u)\left(\frac{2}{k-1} - 2u\right)^{\frac{1}{k-1}}$$

$$\approx \text{const. } (1-u^2)$$

Taking the velocity distribution (11) as a basis, for the nozzle walls in the neighborhood of the narrowest cross section the following is obtained in the case of a flat nozzle

$$y\left(1-u^2\right) = \text{const.} = y_S,\ y = y\left(1-u^2\right)^{-1} = y_S\left(1+\alpha^2x^2\right)$$

and in the case of a round nozzle

$$y^2\left(1-u^2\right) = \text{const.} = y_S^{\,2},\ y = y_S\left(1-u^2\right)^{-\frac{1}{2}} = y_S\left(1+\frac{\alpha^2}{2}x^2\right)$$

From this, the curvature at the vortex is

$$1/\rho_S = 2\alpha^2 y_S$$

and

$$1/\rho_S = \alpha^2 y_S$$

respectively,

and in place of the first equations of (20(a)) and (20(b))

$$\alpha = \sqrt{\frac{1}{2\rho_S y_S}} \qquad (21(a))$$

$$\alpha = \sqrt{\frac{1}{\rho_S y_S}} \qquad (21(b))$$

The comparison of (20(a),20(b)) and (21(a),21(b)) shows that the velocity increase α in the flow-tube approximation for both flat and round nozzles is too large by a factor $\sqrt{\frac{k+1}{2}}$, that is for $k = 1.4$, about 10 percent. The values for the flow-tube approximation are added to the table, page 18, in square brackets.

VI. FLOW ABOUT PROFILES

Through similar power-series expansions the flow past a profile in the vicinity of transit through the critical velocity can be discussed. If a streamline adjacent to the profile is thought of a rigid wall, the flow may also be explained as flow through a nozzle with a curved axis. The origin of the coordinate system is located at that point of the profile at which the critical velocity is just reached, the x-axis downstream on the profile tangent and the y-axis normal to the profile, outward. (See fig. 3.)

Since the flow is no longer symmetrical to the x-axis, instead of (8) the rather complicated equation (7) which must be specified with $\sigma = 0$, by all means, is taken as a basis and in place of (9)

$$\left.\begin{aligned}
\varphi &= f_0(x) + yf_1(x) + y^2f_2(x) + y^3f_3(x) + \ldots \\
u &= \frac{\partial\varphi}{\partial x} = f'_0(x) + yf'_1(x) + y^2f'_2(x) + y^3f'_3(x) + \ldots \\
v &= \frac{\partial\varphi}{\partial y} = f_1(x) + 2yf_2(x) + 3y^2f_3(x) + \ldots
\end{aligned}\right\} \qquad (22)$$

By putting (22) in (7)

$$(k+1)\left(f'_0 + yf'_1 + \ldots\right)\left(f''_0 + yf''_1 + \ldots\right) = \left(2f_2 + 6yf_3 + \ldots\right) - \left(2f_1 + 4yf_2 + \ldots\right)\left(f'_1 + 2yf'_2 + \ldots\right)$$

and by comparing coefficients

$$\left.\begin{aligned} f_2 &= \frac{k+1}{2} f'_0 \, f''_0 + f_1 \, f'_1 \\ 6f_3 &= (k+1)\left(f'_0 f''_1 + f''_0 f'_1\right) + 4\left(f_1 f'_2 + f'_1 f_2\right) \end{aligned}\right\} \qquad (23)$$

while only the function $f'_0(x)$ had remained arbitrary in the nozzle flow, here there are two arbitrary functions $f'_0(x)$, $f_1(x)$. With $f'_0(x)$ and $f_1(x)$ the remaining coefficient functions $f_2(x)$, $f_3(x)$,... in order may be computed from the system of equations (23), by which the flow is completely established.

Similar to (11) set

$$f'_0 = \alpha x, \qquad f_1 = -\beta x \qquad (24)$$

and obtain from (23)

$$f_2 = \left(\frac{k+1}{2} \alpha^2 + \beta^2\right) x$$

$$f_3 = -\frac{k+1}{6} \alpha\beta - \frac{4\beta}{3}\left(\frac{k+1}{2} \alpha^2 + \beta^2\right) x$$

and from (22)

$$
\left.\begin{aligned}
u &= \alpha x - \beta y + \left(\frac{k+1}{2}\alpha^2 + \beta^2\right) y^2 + \ldots \\
v &= -\beta x + \left[(k+1)\alpha^2 + 2\beta^2\right] xy \\
&- \left\{\frac{k+1}{2}\alpha\beta + 4\beta\left[\frac{(k+1)}{2^*}\alpha^2 + \beta^2\right] x\right\} y^2 + \ldots
\end{aligned}\right\} \qquad (25)
$$

Here α signifies the velocity increase $\frac{\partial u}{\partial x}$ along the profile on transit through the critical velocity; it is therefore positive at the starting point A and negative at the terminal point B (fig. 4) of a local supersonic region.

The profile curvature is given with $\beta > 0$ at the point A or B as

$$\frac{1}{\rho_p} = -\left(\frac{\partial v}{\partial x}\right)_{x=y=0} = \beta \qquad (26)$$

For the slope of the critical curve with respect to the profile tangent with

$$(1 + u)^2 + v^2 \approx 2u = 0$$

from (25) the relation

$$\tan \vartheta = \frac{y}{x} \approx \frac{\alpha}{\beta} \approx \alpha\rho_p \qquad (27)$$

is obtained.

*Translator's Note: This 2 was omitted through error in the original German report. The correction applied here changes values of the constant in the succeeding equations and some values in the table.

For the slope of the streamline at a distance y_S from the profile

$$\frac{1}{\rho_S} = -\left(\frac{1}{1+u}\,\frac{\partial v}{\partial x}\right)_{x=0} \approx \frac{1}{1-\beta y_S}\left\{\beta - \left[(k+1)\,\alpha^2 + 2\beta^2\right] y_S\right\}$$

$$\approx \beta - \left[(k+1)\,\alpha^2 + \beta^2\right] y_S$$

is obtained and taking (26) into consideration

$$\boxed{\begin{aligned} \frac{1}{\rho_S} &= \frac{1}{\rho_p} - \left[(k+1)\,\alpha^2 + \frac{1}{{\rho_p}^2}\right] y_S \\ \rho_S &\approx \rho_p + y_S\left[1 + (k+1)\,\alpha^2 {\rho_p}^2\right] \end{aligned}} \qquad (28)$$

according to (27) the critical curve at A ($\alpha > 0$) and B ($\alpha < 0$) is steeper the larger the velocity increase or decrease and the less the profile is curved. By (28) in the limiting case of $\alpha = 0$ the circle of curvature at S is concentric with the circle of curvature of the profile and, as is already known from the flow-tube approximation, becomes still flatter with increasing α.

VII. SUMMARY

Approximation formulas were developed for the position and curvature of the critical curve for transition through the critical velocity in the neighborhood of the narrowest cross section of flat and round Laval nozzles. In comparison with nozzle flows calculated by Kl. Oswatitsch and W. Rothstein (4) they showed satisfactory agreement. In addition, corresponding approximation formulas were deduced for the flow at profiles with local supersonic regions.

Translated by Dave Feingold
National Advisory Committee
for Aeronautics

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TABLE

COMPARISON OF APPROXIMATION FORMULAS (20(a)) AND (20(b))

WITH THE NOZZLE FLOWS COMPUTED BY KL. OSWATITSCH

AND W. ROTHSTEIN

	10	$\rho_S/y_S = 5$	10/3
α flat	0.20 (0.20)	0.29 (0.27)	0.35 (0.32)
[α flat Tube]	[0.22]	[0.32]	[0.39]
α round	0.29 (0.28)	0.41 (0.37)	0.50 (0.42)
[α round Tube]	[0.32]	[0.45]	[0.55]
ε flat	0.08 (0.08)	0.12 (0.12)	0.14 (0.16)
ε round	0.09 (0.10)	0.12 (0.14)	0.14 (0.18)
η flat	0.16 (0.16)	0.23 (0.23)	0.28 (0.25)
η round	0.09 (0.08)	0.12 (0.12)	0.14 (0.14)

The unbracketed numbers have been computed with approximation formulas (20(a)) and (20(b)), the numbers in curved brackets taken from figures 7 and 10 of the paper by Kl. Oswatitsch and W. Rothstein (4). The square bracketed numbers are from formulas (21(a)) and (21(b)) for the flow-tube approximation.

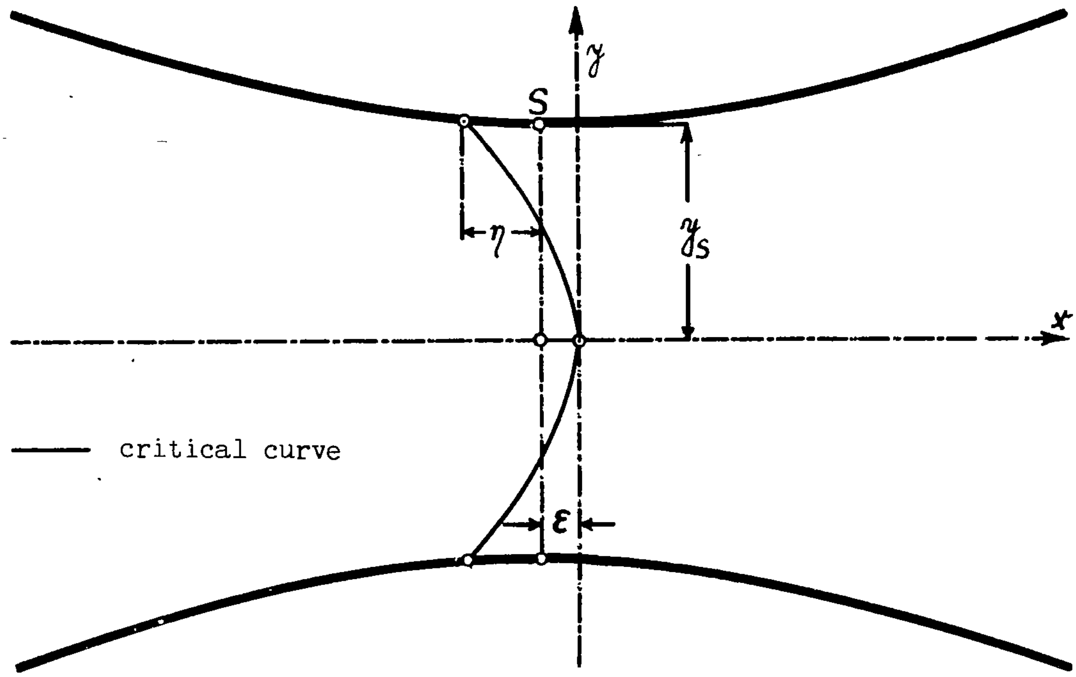

Figure 1. Illustration of terms.

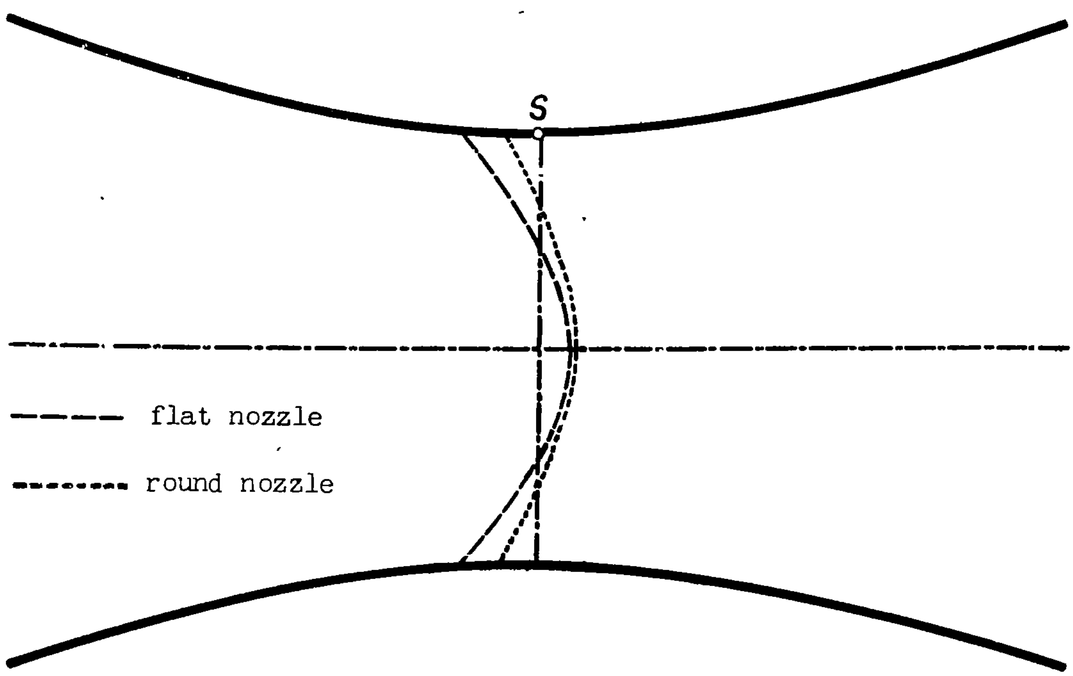

Figure 2. Comparison of a flat and a round nozzle.

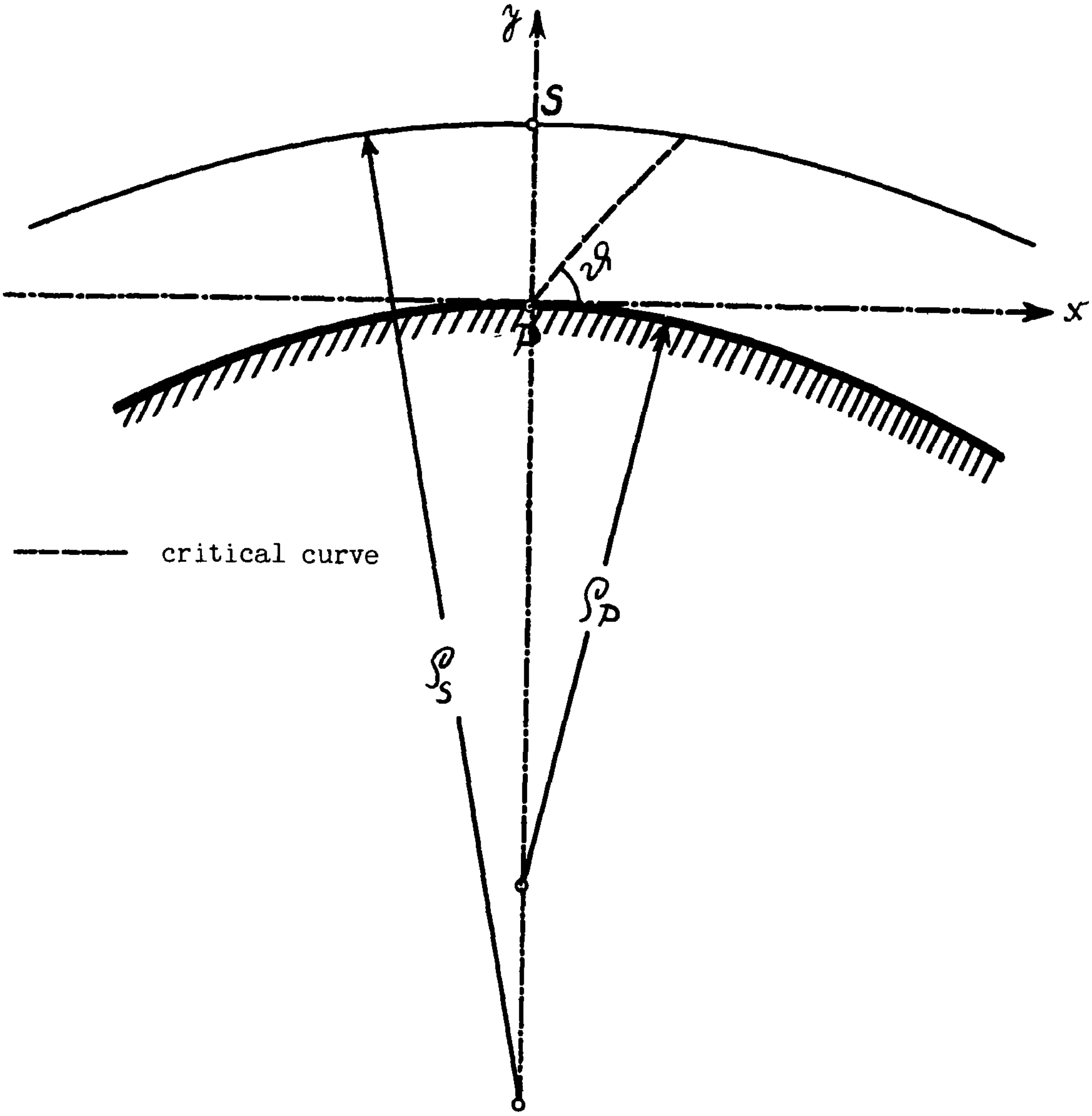

Figure 3. Profile flow at transit through the critical velocity.

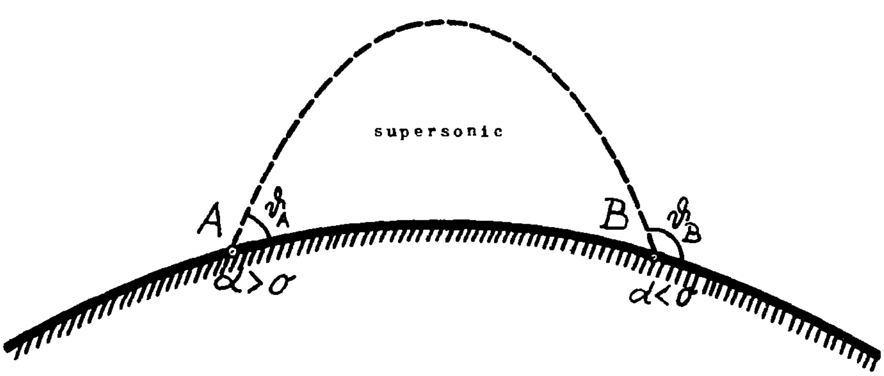

Figure 4. Profile with supersonic region.