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CALIBRATION AND MEASUREMENT IN TURBULENCE RESEARCH

BY THE HOT-WIRE METHOD

By Laszlo Kovasznay

Translation

"HITELESÍTÉSI ÉS MÉRÉSI RENDSZER
A HÖDRÓTOS TURBULENCIA
KUTATÁSBAN"
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BY THE HOT-WIRE METHOD*

Laszlo Kovasznay

INTRODUCTION

The problem of turbulence in aerodynamics is at present being attacked both theoretically and experimentally. In view of the fact however that purely theoretical considerations have not thus far led to satisfactory results the experimental treatment of the problem is of great importance. Among the different measuring procedures the hot wire methods are so far recognized as the most suitable for investigating the turbulence structure. The several disadvantages of these methods however, in particular those arising from the temperature lag of the wire can greatly impair the measurements and may easily render questionable the entire value of the experiment.

The name turbulence is applied to that flow condition in which at any point of the stream the magnitude and direction of the velocity fluctuate arbitrarily about a well definable mean value. This fluctuation imparts a certain whirling characteristic to the flow.

At any point in the flow the velocity can be decomposed into a mean velocity and a so-called turbulence component. The time-average value of the latter is by definition equal to zero. The turbulent velocity fluctuations according to investigations thus far made show no regularity in their details and hence it is evident that they obey only statistical laws. Between the velocity fluctuations measured at different points a certain correlation is observed which serves to define the turbulence dimensions in space.

The order of magnitude of the turbulent fluctuations in actual flows is relatively small, being altogether 0.25 to 2.5 percent of

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*"Hitelesítési És Mérési Rendszer A Hödrótos Turbulencia Kutatásban." A Müegyetem Aerodinamikai Intézetében Készült Munka Budapest, Hungary, 1943.

the time-averaged value, and only in extreme cases will it attain
5 percent of that value. The frequency range of the turbulence
fluctuations however is much greater, the frequency numbers lying
between 10-3000/second.

On the basis of what has been said above an instrument for
measuring the velocity fluctuations must satisfy the following
requirements:

1. It must be able to measure up to about a 1 percent effective
velocity fluctuation and up to about 5 percent of the latter or,
referred to the time-averaged value, to about 0.05 percent accuracy.

2. The size of the measuring device must be small so that it
should not be larger than the characteristic dimensions of the tur-
bulence and therefore in many cases it must not exceed 1 millimeter.

3. It must faithfully reproduce the sine components for the
different frequencies as far as possible within the range of
10-3000/second.

4. If the connection (correlation number) between the velocity
fluctuations measured at two or more points is investigated, the
velocity fluctuations must be reproduced with the greatest possible
faithfulness. This uniform reproduction of the frequency range in
addition to the phase relations however imposes severe requirements
on the instrument.

Hot wire measurements go back to about 30 years ago when for the
first time its possibilities for measuring stream velocities were
investigated. In 1914 L. V. King (reference 1) lowered a heated wire
into a heat dissipating air stream to investigate the air flow laws.
At the same time as King, Dr. Istavan Schrodt (reference 8) was occupied
with the development of a zero inertia velocity measuring instrument
using for this purpose slender heated filaments. Unfortunately however
he was unable to give any relation expressed by equations.

Following King's lead Huguenard, Magnan and Planiol (refer-
ence 6) concerned themselves with hot wire velocity fluctuation
measurements, investigating in particular the velocity fluctuations in
the free atmosphere.

H. L. Dryden (reference 2) was the first to determine theoret-
ically the thermal lag effect of the wire and its numerical value and
to design a compensated amplifier. This amplifier system was further
developed by W. C. Mock (references 3 and 5) and his coworkers.

Independently of the above, Doetsch (reference 9) in Germany conducted hot-wire velocity fluctuation measurements without however compensating for the thermal lag.

In Holland, Ziegler (reference 11) independently of Dryden designed a condenser compensated apparatus and (reference 12) measured the thermal lag of the wire using a variable current bridge.

In England, Simmons and Salter conducted several tests with their hot-wire apparatus (reference 7) especially their well-known turbulence spectrum measurements (reference 10).

Important turbulence measurements without using the hot-wire method were conducted by Schubauer (reference 4) on the basis of the increased heat diffusion due to turbulence.

The procedure for conducting the hot-wire measurements consists generally in placing a very thin platinum wire (prepared by the Wollaston method) in the flow, the wire being heated by an electric current. The air stream cools the wire to a degree depending on the stream velocity. The wire temperature thus fluctuates in correspondence with the velocity fluctuations and with it the wire resistance and therefore the voltage drop across it. The voltage fluctuations of the wire are therefore proportional to the air velocity fluctuations. This however is true only if the thermal lag of the wire is small for those frequencies which are under investigation.

Unfortunately however it follows from theory that the effect of the thermal lag cannot be compensated electrically up to any frequency limit. At higher frequencies the wire responds with a smaller amplitude because it cannot heat up and cool down with sufficient rapidity. A compensating device is precisely a means for giving a more faithful amplification of the fluctuations at the higher frequencies. There is a limit however to the amplification as the higher frequencies are continuously increased and therefore we require only of the compensating apparatus that it restore a 1 percent drop in amplitude due to the inertia effect to approximately its initial value.

In addition to the above primary compensating amplifier a further amplifier is naturally required for the cases where the fluctuations are led to an oscillograph or the effective value of the fluctuations is measured with a thermocouple voltmeter.

With the apparatus that has thus far been developed abroad (reference 5) the compensator frequency characteristic shows a drop of about 10 percent for the frequency range of 25-1000/second and about 30 percent for the frequency range 1000-3000/second.

The design of the amplifying apparatus still leaves much to be desired. There are three fundamental conditions encountered in the hot-wire method of measuring the velocity fluctuations:

2   1. The known velocity fluctuation amplitude should be capable of being fixed in advance especially at the higher frequencies.

2. The thermal lag of the wire should be compensated in the desired frequency range and within the latter the response should be faithful.

3. The quantity characterizing the thermal lag of the wire (the time constant) should be known accurately under the actual conditions. The compensation for the desired measurements is determined on the basis of these conditions.

The object of the present paper is to describe a calibration and measuring apparatus and a method by which

1. The wire constants can be determined by simple measurements; the scatter of the latter can be readily smoothed out, and on this basis the amplitude of the fluctuations determined.

2. The thermal lag (time constant) is determined by a separate method as under actual conditions and the required compensation and faithful response of the entire apparatus can be readily controlled.

3. Because of the above-mentioned possibility of control of the compensation and thus the measuring accuracy and entire response an essentially better new amplifying apparatus is obtained.

On the basis of known principles it was possible in this paper to design a reliable hot-wire turbulence measuring apparatus. To give a detailed account of it would however exceed the scope of the present paper.

The theoretical and experimental scientific work underlying the present investigation is part of the research program of the Aero-dynamics Institute. I here wish to express my sincere and grateful

thanks to Dr. Elod Abody, the director of the Aerodynamic Institute, for his careful guidance and far reaching support. It was he who pointed out that without turbulence tests fruitful aerodynamic research cannot be conducted. In the autumn of 1939, I was entrusted with the design of a turbulence measuring apparatus. In designing the apparatus and in working out the principles of the present investigation I have received much useful advice and much material aid without which this experimental investigation could not have been undertaken.

TRANSFORMATION OF THE KING FORMULA FOR THE

HEAT LOSS OF A HOT WIRE

According to the relation obtained by L. V. King (reference 1) the heat loss of a thin metal wire in an air stream can be expressed in the following form:

$$H = (\theta_1 - \theta_0)(\beta \sqrt{v} + \gamma) \tag{1}$$

where

H   heat loss per unit length of wire

v   velocity of air

θ_1 temperature of wire

θ_0 temperature of air

β and γ are constants which depend only on the wire diameter and other data (specific heat, density, conductivity). The validity of the equation extends down to such small velocities for which the free convection velocity of the heated air may be neglected in comparison with the blower velocity. The formula moreover can be used as long as the radiation loss of the wire is not too large or until the wire is visibly glowing. At certain values of the Reynolds number the flow about a cylinder undergoes a change and according to test results the constants in the equation undergo a discontinuous change but the validity of the equation still holds over separate intervals.

L. V. King derived this equation theoretically. For the purposes of our investigation we shall only make use of it when the above

equation is valid or the heat loss of the heated wire is proportional
to the temperature difference and to the square root of the velocity
while the constants remain such within a certain range.

The second law on which our considerations are based is that the
resistance of the platinum wire varies linearly with the temperature
coefficient. The deviations from the linear law for the range of
measurements considered lie within small enough limits so that they
can be neglected.

Since the temperature of the wire cannot be easily measured
directly the heat equilibrium equation referring to the steady state
must be written in such a form that only measurable quantities remain
in the equation. We introduce the following notation:

W — heat in joules produced per unit length of wire per unit time

r_0 — resistance per unit length of wire at air temperature θ_0

r — resistance per unit length of air at test temperature θ_1

l — length of wire

$rl = R$ — total resistance of wire under test conditions

$r_0 l = R_0$ — so-called cold resistance of wire, or resistance at air temperature

J — strength of wire heating current

α — temperature coefficient of wire resistance

In the steady state there is equilibrium between the heat produced
and heat conducted away or

$$J^2 r_0 \left[1 + \alpha (\theta_1 - \theta_0) \right] = (\theta_1 - \theta_0)(\beta \sqrt{v} + \gamma)$$

We introduce the following quantities

$$a = \alpha(\theta_1 - \theta_0) = \frac{r - r_0}{r_0} = \frac{R - R_0}{R_0}, \quad q = J^2 \; ; \; y = \sqrt{v}$$

instead of the variables, and the constants

$$c = \frac{\beta}{\alpha\, r_o} \qquad y_o = \frac{\gamma}{\beta} \qquad q = J^2$$

and moreover let $y' = y + y_o$. The equation written with the new variables and constants is

$$qr_o(1 + a) = \frac{a}{\alpha}\, \beta(y + y_o)$$

$$q(1 + a) = acy' \tag{2}$$

Solving the above equation for the variables we obtain the following simple formulas:

$$a = \frac{q}{cy' - q} \tag{3}$$

$$q = \frac{a}{a + 1}\, cy' \qquad = \quad \frac{cy'}{1 + \frac{1}{a}} \tag{4}$$

$$y = \frac{1 + a}{a}\, \frac{q}{c} - y_o \tag{5}$$

Three variables occur in the equation: the temperature coefficient, the heating current strength (squared) and the air velocity (square root). The simple connection between them is expressed by the above formulas. For greater ease of manipulation we shall keep some variable as constant treating it as a parameter and thus discuss only the relation between the other two. We cannot however leave out of consideration the fact that during the actual measurements the changes in the wire resistance react on the heating current and therefore we must discuss the so-called dynamic characteristic.

THE THREE SETS OF STATIC CHARACTERISTIC CURVES

We shall first investigate the case where the wire temperature and therefore its resistance is kept constant and shall find the current strength corresponding to which the given velocity produces the arising heat quantity:

$$q = \frac{a}{a + 1}\, cy'$$

(according to equation (4) of preceding section). This means that
q is proportional to y' from which it follows that we obtain
straight lines which start from the point y' = o, y = - y_O. The
negative values of y naturally have no physical meaning. We can
now very easily obtain the wire calibration or the determination of
the constants c and y_O (fig. 1). The wire is calibrated by con-
necting it in a resistance (Wheatstone or Thompson) bridge and
balancing the desired resistance and therefore the temperature. For
each wind velocity we obtain the current given by the balanced bridge
and in this way the wire temperature. Through the points we draw the
characteristic curve. The segment intercepted by the abscissa axis
gives the value of y_O while the value of c is obtained from the
slope. The slope is

$$\left(\frac{\partial q}{\partial y}\right)_a = \frac{a}{a + 1} \quad c = \frac{q}{y'} \tag{6}$$

From this we can see that for a = 1 the slope is 1/2 of c and
only for a = ∞ is the entire value of c obtained. The case
a = ∞ is indicated by the dotted line of figure 1.

In the characteristic of the fluctuation measurement the current
intensity and therefore q is constant. To the different values of
the velocity and therefore y there correspond different values of
the temperature, that is, a. The latter is simply connected with y
and the parameter q. This idealized state however would be realized
if the wire, whose resistance is assumed very large, is heated from
a very large source of voltage across it and thus the resistance
fluctuations would not react on the wire heating current. The equa-
tion of the curves

$$a = \frac{q}{cy' - q}; \quad q = \text{const.}$$

represents hyperbolas whose asymptotes are parallel to the coordinate
axes, the horizontal asymptote coinciding with the axis of abscissas
(fig. 2), that is, when y = ∞   a = o. The position of the vertical
asymptote is determined as follows: Let y' = y'_a where a = ∞ and
y' = y_1 where a = 1, then

$$a = 1 = \frac{q}{cy'_1 - q} \quad \text{and} \quad a = \infty = \frac{q}{cy'_a - q}$$

from which $cy'_1 = 2q$ and $cy'_a = q$ and therefore $y'_a = y'_1/2$.

If the asymptote cuts the positive y axis, then when the wind velocity is zero the wire is cooled down. It is still necessary however to have a current because otherwise the wire would not be sufficiently sensitive at greater velocities.

The measure of the sensitivity is given by the derivative of the curve:

$$\left(\frac{\partial a}{\partial y}\right)_q = - \frac{qc}{(cy' - q)^2} = - \frac{a(1 + a)}{y'} \tag{7}$$

From this as a basis there are computed the voltage fluctuations which are produced by the velocity fluctuations.

The third class of characteristic curves are the wire heating curves. Here we consider the velocity constant and thus the temperature as a function of the current and therefore the degree of heating. These characteristics are likewise hyperbolas whose asymptotes are parallel to the coordinate axes. The horizontal asymptote is given by $a = -1$, that is,

$$a = \frac{q}{cy' - q}; \quad \lim_{q \to \infty} \frac{q}{cy' - q} = -1$$

Naturally the part of the curve below the q axis has no physical meaning. The position of the vertical asymptote is computed as follows. Let

$$q = q_a \quad \text{for} \quad a = \infty$$

$$q = q_1 \quad \text{for} \quad a = 1$$

$$\text{then} \quad a = 1 = \frac{q_1}{cy' - q_1} \quad \text{and} \quad a = \infty = \frac{q_a}{cy' - q_a}$$

and therefore $cy' = 2q_1$ and $cy' = q_a$ from which $q_a = 2q_1$

If the curve is plotted not with the values of a but with the resistance R itself we obtain a quite similar curve. In this case however we express the reciprocal of the resistance $1/R$ as a function of q and this must give the straight line

$$\frac{R}{R_0} = a + 1 = \frac{cy'}{cy' - q};$$

$$\frac{1}{R} = \frac{1}{R_0}\left(1 - \frac{q}{cy'}\right); \quad \text{if} \quad q \to 0 \quad \frac{1}{R} = \frac{1}{R_0} \tag{8}$$

From the intercept on the vertical axis we thus obtain the resistance for zero heating current condition, that is R_0, the so-called cold resistance which corresponds to the air temperature.

We also require for later purposes the slope $\left(\frac{\partial a}{\partial q}\right)_y$ given by

$$\left(\frac{\partial a}{\partial q}\right)_y = \frac{\partial}{\partial q}\frac{q}{cy' - q} = \frac{(cy' - q) - (-q)}{(cy' - q)^2} = \frac{a(1 + a)}{q} \tag{9}$$

With the aid of this coefficient we can compute the effect of the heating current fluctuations on the thermal state of the wire.

DYNAMIC CHARACTERISTIC AND THE COMPUTATION

OF THE VOLTAGE FLUCTUATIONS

Thus far we have considered the wire by itself. Actually however the wire is connected in a heating current circuit. If the velocity fluctuates, the wire temperature and therefore the entire resistance will fluctuate. This means however a variation of the resistance of the current circuit and therefore of the wire heating current. We consequently can not regard the latter as constant and must thus consider a further characteristic curve of the wire, the so-called dynamic characteristic which is analogous to that of the electron tube (fig. 4).

All our considerations thus far have referred to the stage of thermal equilibrium hence to phenomena where the thermal lag of the wire is not taken into account. From the preceding discussion we know that the velocity fluctuations are small in comparison with the fundamental velocity. It is thus convenient to consider every variable as made up of two parts namely, the average value with respect to time and the fluctuating component. We denote the average by a bar over the symbol and the fluctuating component by Δ. We thus have

$$v = \bar{v} + \Delta v; \quad J = \bar{J} + \Delta J; \quad R = \bar{R} + \Delta R$$

$$y = \bar{y} + \Delta y; \quad q = \bar{q} + \Delta q; \quad a = \bar{a} + \Delta a$$

(In what follows we shall everywhere use the differential coefficients in place of the different coefficients, which corresponds to neglecting the terms of higher order in the Taylor series.) We compute the constants from the mean values. The error thus committed assuming a ±10 percent velocity fluctuation remains below 0.5 percent.

According to the transformed equation (3) $a = a(y, q)$; hence in the case of small changes, neglecting the terms of higher order,

$$\Delta a = \left(\frac{\partial a}{\partial y}\right)_q \Delta y + \left(\frac{\partial a}{\partial q}\right)_y \Delta q$$

But $q = q(a)$ and $I = I(R)$, hence $\Delta q = \left(\frac{\partial q}{\partial a}\right)_{r_E} \Delta a$

$$\Delta a = \left(\frac{\partial a}{\partial y}\right)_q \Delta y + \left(\frac{\partial a}{\partial q}\right)_y \left(\frac{\partial q}{\partial a}\right)_r \Delta a$$

$$\frac{\Delta a}{\Delta y} = \left(\frac{da}{dy}\right)_{dyn} = \left(\frac{\partial a}{\partial y}\right)_q \frac{1}{1 - \left(\frac{\partial a}{\partial q}\right)_y \left(\frac{\partial q}{\partial a}\right)_r}$$

The coefficient $\left(\frac{da}{dy}\right)_{dyn}$ is the so-called dynamic slope. The coefficient $\left(\frac{\partial q}{\partial a}\right)_r$ gives the variation of the current with the resistance. The subscript r indicates that the current circuit remains unchanged in the differentiation and should not be confused with the quantity $\left(\frac{\partial q}{\partial a}\right)_y$ referring to the wire.

We obtain the voltage fluctuation in the following manner. The wire voltage, in the same manner as above, is decomposed into a mean value and a fluctuating component.

$$P = \bar{P} + \Delta P = (\bar{J} + \Delta J)(\bar{R} + \Delta R)$$

Neglecting the second degree terms

$$\Delta P = R_0(1 + \bar{a}) \Delta J + R_0 \bar{J} \Delta a \qquad\qquad (10)$$

$$\Delta J = \left(\frac{\partial J}{\partial a}\right)_r \Delta a; \quad \Delta a = \left(\frac{da}{dy}\right)_{dyn} \Delta y$$

$$\Delta P = R_o \bar{J} \left[1 + \frac{1 + \bar{a}}{J} \left(\frac{\partial J}{\partial a} \right)_r \right] \Delta a \tag{11}$$

$$y = \sqrt{v} \qquad \Delta y = \frac{\bar{y}}{2\bar{v}} \Delta v$$

$$\Delta P = - \bar{J}\bar{R}_o \frac{1 + \dfrac{1 + \bar{a}}{\bar{J}} \left(\dfrac{\partial J}{\partial a} \right)_r}{1 - \left(\dfrac{\partial a}{\partial q} \right)_y \left(\dfrac{\partial q}{\partial a} \right)_r} \frac{\bar{a}(1 + \bar{a})}{\bar{y'}} \frac{\bar{y}}{2\bar{v}} \Delta v$$

$$\frac{\Delta P}{\bar{P}} = - \frac{\bar{a}}{2} \frac{1 + \dfrac{1 + \bar{a}}{\bar{J}} \left(\dfrac{\partial J}{\partial a} \right)_r}{1 - \left(\dfrac{\partial a}{\partial q} \right)_y \left(\dfrac{\partial q}{\partial a} \right)_r} \frac{\bar{y}}{\bar{y'}} \frac{\Delta v}{\bar{v}}$$

The above can be briefly written as in the following form:

$$\frac{\Delta P}{\bar{P}} = - \frac{\bar{a}}{2} \, \mathrm{Ez} \, \frac{\Delta v}{\bar{v}} \tag{12}$$

This equation holds only for the slow so-called quasi-stationary volt-fluctuations. For rapid changes the thermal lag of the wire modifies the relations. Hence the value of the voltage fluctuations thus computed will later be denoted by ΔP_v and will be termed the virtual voltage fluctuations because that would be their value if there were no thermal lag.

The second factor E on the right side of equation (12) is properly speaking the correction factor of the wire dynamic characteristic:

$$E = \frac{1 + \dfrac{\bar{a} + 1}{\bar{J}} \left(\dfrac{\partial J}{\partial a} \right)_r}{1 - \left(\dfrac{\partial q}{\partial a} \right)_r \left(\dfrac{\partial a}{\partial q} \right)_y}$$

It expresses the reaction of the fluctuations on the wire current.
Thus if we use a very large voltage source and a very large resistance

connected with the wire, the current would not vary. If this effect is not taken into account in the wire characteristic or the computation of the voltage we would have $E = 1$.

The third factor in the equation is

$$z = \frac{\overline{y}}{y'} = \frac{\overline{y}}{\overline{y} + y_0} = \frac{\overline{q} - q_0}{\overline{q}} < 1$$

where q_0 is the value of q corresponding to zero velocity.

We shall now compute the value of E. We shall first consider the simple case when the wire with a large resistance connected with it has a large source of voltage across it (fig. 5).

We shall denote the value of the resistance in the circuit not including that of the wire by R_0s. The total resistance is thus

$$R = R_0(1 + a + s) \quad J = \frac{U}{R_0} \frac{1}{1 + a + s}$$

$$\left(\frac{\partial J}{\partial a}\right)_r = -J \frac{1}{1 + \overline{a} + s} \tag{13}$$

$$\left(\frac{\partial q}{\partial a}\right)_r = 2J\left(\frac{\partial J}{\partial a}\right)_r = -2q \frac{1}{1 + \overline{a} + s} \tag{14}$$

or using equation (9)

$$E = \frac{1 + \dfrac{\overline{a} + 1}{\overline{J}}\left(\dfrac{\partial J}{\partial a}\right)_r}{1 - \left(\dfrac{\partial a}{\partial q}\right)_y \left(\dfrac{\partial q}{\partial a}\right)_r} = \frac{1 - \dfrac{\overline{a} + 1}{\overline{a} + 1 + s}}{1 + 2\overline{a}\dfrac{\overline{a} + 1}{\overline{a} + 1 + s}}$$

$$\frac{\overline{a} + 1}{\overline{a} + 1 + s} = \frac{R_0\overline{J}(\overline{a} + 1)}{R_0\overline{J}(\overline{a} + 1 + s)} = \frac{\overline{P}}{U} = e$$

$$E = \frac{1 - e}{1 + 2\overline{a}e}$$

where e denotes the ratio of the wire voltage to the total battery
voltage. Thus the coefficient E is very simply expressed since it
depends only on the percentage of the battery voltage used and on a.
Since the battery voltage is at least 10 to 20 times the wire voltage
and a is never greater than 1, the value of E is not much below
0.8.

The reaction of the fluctuations on the current circuit and the
fact that we must take into account the dynamic characteristic
explains this 10 to 20 percent deviation of E from unity.

NONSTATIONARY STATE OF THE WIRE

The results thus far obtained all apply to the case where the
changes are very slow or the heat generated in the wire is in
equilibrium with the heat conducted away. For rapidly occuring
changes the quantities of heat generated and conducted at any instant are not equal
and the difference is taken up by the wire heat capacity.

Let Q be the heat contained in the wire referred to unit length.
The rate of change of the heat is obtained from the difference between
heat produced and heat lost according to equation (1)

$$\frac{dQ}{dt} = J^2 r_0 \left[1 + \alpha(\theta_1 - \theta_0) \right] - (\theta_1 - \theta_0)(\beta \sqrt{v} + \gamma)$$

We can write the equation using the variables and constants introduced
in the first section. In addition let

$$Q = h \theta = 4.2 \, m \, \sigma \, \theta; \quad 1 \, cal = 4.2 \, W \, sec$$

where h is the heat capacity of the wire, m the mass of unit
length of the wire, σ the specific heat of the wire and
$n = \dfrac{h}{\alpha r_0}$ the thermal lag constant of the wire. With the variables
introduced above we have

$$n \frac{da}{dt} = q(1 + a) - acy'$$

As we see the above equation reduces to the steady state equation (2)
if $\frac{da}{dt} = 0$.

The equation of the unsteady state of the heated wire was first obtained by Dryden (reference 2) on the assumption that the heating current of the wire was constant. As was already shown even in quasi-stationary state this supposition does not correspond to actual condistions and therefore in the expression for the nonsteady state equation we must take into account the effect of the fluctuation of the current.

Since we are considering the measurements of the velocity fluctuations and since the magnitude of any fluctuation is small as compared with the average value, we shall again split up the variables into the mean value and the fluctuation component. The mean value satisfies the steady state equation:

$$0 = \bar{q}(1 + \bar{a}) - \bar{a}c\bar{y}'$$

In addition we write down the nonsteady state equation:

$$n\,\frac{da}{dt} = q(1 + a) - acy'$$

Subtracting the top equation from the bottom, since $\dfrac{da}{dt} = \dfrac{d\Delta a}{dt}$, we have

$$n\,\frac{d\Delta a}{dt} = \Delta q + \bar{q}\Delta a + \bar{a}\Delta q - \bar{a}c\Delta y' - c\bar{y}'\Delta a +$$

$$+ \Delta a\Delta q - c\Delta a\Delta y$$

Neglecting the second order small quantities and making use of equation (2) we are led to the following simple form

$$\frac{\bar{a}}{\bar{q}}\,n\,\frac{d\Delta a}{dt} = \frac{\bar{a}(\bar{a} + 1)}{\bar{q}}\,\Delta q - \Delta a - \frac{\bar{a}^2 c}{\bar{q}}\,\Delta y' \qquad (15)$$

Introducing the relations

$$\Delta q = \left(\frac{\partial q}{\partial a}\right)_r \Delta a; \quad -\frac{\bar{a}^2 c}{q} = \left(\frac{\partial a}{\partial y}\right)_q$$

and making use of equation (9)

$$\frac{1}{1 - \left(\dfrac{\partial a}{\partial q}\right)_y \left(\dfrac{\partial q}{\partial a}\right)_r}\,\frac{\bar{a}}{\bar{q}}\,n\,\frac{d\Delta a}{dt} + \Delta a = \frac{1}{1 - \left(\dfrac{\partial a}{\partial q}\right)_y \left(\dfrac{\partial q}{\partial a}\right)_r}\,\left(\frac{\partial a}{\partial y}\right)_q \Delta y$$

which with the aid of equations (9) and (14) we can reduce to the
following form

$$\frac{1}{1 + 2\bar{a}e} \frac{\bar{a}}{\bar{q}} \, n \, \frac{d\Delta a}{dt} + \Delta a = \left(\frac{da}{dy}\right)_{dyn} \Delta y$$

In what follows we shall require the voltage fluctuations. We
can express ΔP in terms of Δa on the basis of equations (11)
and (13)

$$\Delta P = R_o \bar{J}(1 - e) \, \Delta a$$

Multiplying the previous equation by $R_o J(1 - e)$ we obtain the
voltage fluctuation except that on the left side are the true voltage
fluctuations while on the right side appear the so-called virtual
voltage fluctuations (equation (12)) which can be derived from the
steady state equation and which would hold for very slow changes.
Thus the equation assumes the form

$$\frac{1}{1 + 2\bar{a}e} \frac{\bar{a}}{\bar{q}} \, n \, \frac{d\Delta P}{dt} + \Delta P = \Delta P_V$$

where

$$\frac{1}{1 + 2\bar{a}e} \frac{\bar{a}}{\bar{q}} \, n = \frac{M_o}{1 + 2\bar{a}e} = M \qquad (16)$$

the coefficient M_o being denoted as the ideal and M the actual
time constants, respectively. The former value was assumed by
Dryden without taking account of the reaction effect of the heating
current circuit. Thus, if the wire is fed from an infinitely large
source of voltage $1 + 2\bar{a}e = 1$ and we would have $M = M_o$. In the
actual apparatus used, however, such assumption leads to considerable
error. The value given by Dryden in his first report (reference 3)
$\frac{1}{1 + 2\bar{a}e} = 0.86 - 0.95$ depends on the operating state of the system.
This, however, gives an error of 7 to 14 percent which moreover is
not constant. With the aid of the time constant the general
fundamental equation of the wire can be written

$$M \, \frac{d\Delta P}{dt} + \Delta P = \Delta P_V \qquad (17)$$

The equation is entirely similar in form to that derived by Dryden
except that the time constant is different. Here ΔP_V denotes that

value of the voltage fluctuation which we considered in the previous sections. This value would hold for quasi-stationary or very slowly varying velocity fluctuations while ΔP is the actual voltage fluctuation of the wire under the distorting effect of the termal lag.

Equation (17) is a very simple linear first order equation. We shall solve it for the case that ΔP_v is a sine wave process. As can be easily seen ΔP is then also a sine wave except that it is of decreased amplitude and displaced in phase. Let

$$\Delta P_v = P_{vo}e^{i2\pi ft}; \quad \Delta P = P_o e^{i2\pi ft}$$

where P_{vo} and P_o are constant but may be complex. Substituting into the equation we can express the ratio of the two complex voltage amplitudes.

$$\frac{P_o}{P_{vo}} = \frac{1}{1 + iM2\pi f} = \frac{1}{\sqrt{1 + (2\pi fM)^2}} e^{-i \arctg M\pi 2f} \qquad (18)$$

Introducing the phase displacement angle φ

$$\varphi = \arctg 2\pi fM$$

the above ratio assumes the following simple form:

$$\frac{P_o}{P_{vo}} = e^{-i\varphi}\cos\varphi; \quad \left|\frac{P_o}{P_{vo}}\right| = \cos\varphi$$

or the amplitude ratio decreases with the frequency while the phase lag increases. This would be exactly the situation if the voltage ΔP_v is connected across a voltage divider consisting of a resistance R and an inductance L whose time constant L/R is equal to the time constant of the wire (fig. 6).

We obtain the voltage ΔP, distorted by the thermal lag, from the resistance. This can be seen from the substituted circuit. The considerations are naturally true for any sine component and therefore the fluctuations in the above substitution circuit can be expressed by a Fourier series or a Fourier integral. Table 1 and figure 7 show the effect of the thermal lag on the amplitude decrease and phase displacement for three values of the time constant that occur in practice. The time constants refer to platinum wires (Wollaston) of 6 to 10 μ diameter in normal use (v = 2 to 10 m/sec, and a = 1). Figure 7 shows the amplitude distortion in logarithmic

scale, the absolute value of the above complex ratio naturally being
used. It is seen that at large frequencies the absolute amplitude
varies linearly with the frequency.

The turbulence, as we mentioned in the introduction, is entirely
random and consists of velocity fluctuations which can be expressed
by a Fourier integral. Thus the separate components are each
differently distorted. We cannot use a thin enough wire with small
thermal lag in such a way that in reproducing the velocity fluctua-
tions the distortion due to the effect of the thermal lag is reduced
to the limit of error. Thus the only recourse is to design an
electric compensating circuit whose distortion effect just balances
that of the wire both as regards amplitude and phase.

To determine the compensation however an accurate knowledge of
the time constant for all conditions under which the wire is used is
indispensable. The computation of the thermal lag constant n from
its diameter is on the other hand very uncertain because the mass of
the wire is proportional to the square root of its diameter and thus
for example if we determine the diameter of a 0.006 millimeter wire
with an error of 0.0003 millimeter or a 5 percent error this alone
would introduce a 10 percent error in the determination of the time
constant. This error of 0.0003 millimeter can be easily committed
however since this length is quite short within the wave lengths of
the visible spectrum. The unevenness of the wire contributes to the
difficulties of the measurement as well as the fact that the density
required for the computation of the thermal lag constant and
particularly the value of the specific heat, are not with certainty
at our disposition because the chemical purity of the wire prepared
by the Wollaston method is not perfect.

In view of the above difficulties a solution had to be worked
out which on the one hand permitted direct measurement of the wire
time constant and on the other hand the control of the proper
functioning of the compensation in operation.

THE EXPERIMENTAL CONTROL OF THE COMPENSATION AND THE

DIRECT MEASUREMENT OF THE TIME CONSTANT

We have seen in the previous section that the thermal lag of
the wire under any operating condition is characterized by the time
constant which depends on the thermal lag constant of the wire and
on the temperature coefficient and wire current (a and q); further-
more the magnitude of the heat source (e) also gives a correction.

Hence the time constant values obtained from computation alone and the determination of the desired compensation is very uncertain. We could convince ourselves most directly of the correctness of the compensation if we could permit velocity fluctuations of known form and magnitude to act on the wire. From the distortion of these we could determine the correctness of the compensation. We cannot however solve this satisfactorily, especially at the higher frequency numbers (above 200/sec). /Dryden (reference 2) employed only a small frequency $f = 60$/second for the fluctuations of the air speed in calibrating the wire but neither was the desired sine wave process assured nor was the amplitude sufficiently well defined)

A somewhat less direct method is that of fluctuating the heating current of the wire. Ziegler (reference 12) used a method in which the wire time constant was measured experimentally by a variable current bridge.

In what follows an apparatus is described which reproduces the fluctuations faithfully and with the aid of which the desired compensation measurements can easily and rapidly be controlled. The underlying idea is that the character of the amplitude response provides the most sensitive indication of the faithfulness of the response.

The wire heating current is produced as a square wave form. This is distorted by the thermal lag of the wire into a wave form consisting of segments of exponential curves. From the record obtained with an oscillograph any value of the time constant can be determined as in the case of other heating characteristics (the heating of an electric motor). The distorted voltage, being led to an amplifier provided with a compensating circuit is in the case of a properly designed compensator, regained at the output terminals in its original square form after amplification. The compensation of the square voltage form can be very accurately controlled.

We can read off from the scale of the separately calibrated amplifier the time constant for which it has been compensated and therefore we can determine the wire time constant for the condition under consideration.

We shall investigate the wire placed in a stream of uniform velocity and, as we shall later see, connect it to a simple Wheatstone bridge. It turns out that we can repeat our considerations with respect to the nonstationary condition of the wire for the case where, with the stream velocity constant the heating

current of the wire is made to fluctuate about a mean value by means
of a series-connected resistance which varies periodically and with
the wire connected to a bridge (fig. 8).

The bridge is so chosen that the ratio of the currents in the
two branches is sufficiently large (1:20 - 1:100). We shall call
this ratio T. We shall express all resistances in terms of the
cold resistance of the wire. Between the points X and Y of the
bridge there is no direct voltage if the bridge is balanced with
direct current. The current in the wire is made to fluctuate about
the mean value by short-circuiting part of the resistances outside
the bridge. From the condition of bridge balance

$$\frac{b}{d} = \frac{1 + a}{c} = \frac{J'}{J} = T; \quad J_O = J + J' = J(1 + T)$$

the bridge resistance itself is

$$R_{br} = R_O \frac{1 + a + b}{1 + T}$$

and the total resistance of the circuit is

$$\Sigma R = R_O \left(\frac{1 + a + b}{1 + T} + s' \right)$$

substituting $s = s'(1 + T)$ or the resistance which gives that
voltage drop which would correspond to the current flowing in the
wire. Thus the wire current is

$$J = \frac{J_O}{1 + T} = \frac{U}{R_O} \frac{1}{1 + a + b + s}$$

From this we see that we obtain the same formula for the wire
current as for the simple current circuit except that it is
necessary to take into account the resistance outside the bridge in
obtaining the wire current. The ratio T must be chosen as small
as possible in order that changes in the wire resistance due to
changes in the ratio should not play a large part. We may then
neglect the variation of T in the equation.

The changes in resistance in the circuit are determined by the
derivatives $\left(\frac{\partial q}{\partial a} \right)_r$ and $\left(\frac{\partial q}{\partial s} \right)$. We note that

$$\left(\frac{\partial q}{\partial a}\right)_r = \left(\frac{\partial q}{\partial s}\right)$$

because in the formula a and s are interchangeable.

Let us write down the equation (15) for the unsteady state noting that $\Delta y = 0$

$$n \frac{d\Delta a}{dt} = \Delta q(1 + \bar{a}) + (\bar{q} - c\overline{y'}) \Delta a$$

Substituting the equalities

$$\Delta q = \left(\frac{\partial q}{\partial a}\right)_r \Delta a + \left(\frac{\partial q}{\partial s}\right) \Delta s; \quad \overline{cy'} - \bar{q} = \frac{\bar{q}}{\bar{a}}$$

$$n \frac{d\Delta a}{dt} = (1 + \bar{a}) \left(\frac{\partial q}{\partial s}\right) \Delta s - \left[\frac{\bar{q}}{\bar{a}} - \left(\frac{\partial q}{\partial a}\right)_r (1 + \bar{a})\right] \Delta a$$

$$n \frac{\bar{a}}{\bar{q}} \frac{d\Delta a}{dt} = \frac{\bar{a}(1 + \bar{a})}{\bar{q}} \left(\frac{\partial q}{\partial s}\right) \Delta s - \left[1 - \frac{\bar{a}(1 + \bar{a})}{\bar{q}} \left(\frac{\partial q}{\partial a}\right)_r\right] \Delta a$$

From equations (9) and (14)

$$\frac{\bar{a}(1 + \bar{a})}{\bar{q}} \left(\frac{\partial q}{\partial a}\right)_r = \frac{\bar{a}(1 + \bar{a})}{\bar{q}} \left(\frac{\partial q}{\partial s}\right) = -2\bar{a}e$$

Substituting the true time constant from equation (16) we have

$$M \frac{d\Delta a}{dt} + \Delta a = -\frac{2\bar{a}e}{1 + 2\bar{a}e} \Delta s \tag{19}$$

The above equation gives the relation between Δs and Δa. From the equation we see that the time constant characterizes the current fluctuations the same way as the velocity fluctuations.

We must now determine the voltage fluctuations. In the present case however the voltage fluctuations of the wire are not proportional to the resistance and therefore to Δa. The current follows the square form of the wave corresponding to the short circuiting of the resistance while the wire resistance follows the distorted wave form consisting of the exponential segments.

In order that we obtain the fluctuations of the voltage (temperature) proportional to the resistance fluctuations we derive the voltage fluctuation between the points X, Y of the Wheatstone bridge in figure 8. This fluctuation consists of two parts. Between the points X and Z the voltage fluctuation due to the voltage fluctuation of the wire, equation 10 is

$$\Delta P_{ZX} = R_0(1 + \bar{a}) \, \Delta J + \bar{J}R_0 \Delta a$$

Between the points Y and Z the resistance (cR_0) is constant, only the current fluctuating

$$\Delta P_{ZY} = cR_0 \Delta J'$$

$$\Delta J' = JT \, \frac{\Delta a}{1 + \bar{a} + b} + T\Delta J; \quad \frac{\Delta T}{T} = \frac{\Delta a}{\bar{a} + 1 + b}$$

$$\Delta P_{ZY} = R_0 \bar{J} \, \frac{1 + \bar{a}}{1 + \bar{a} + b} \, \Delta a + (1 + \bar{a}) \, R_0 \Delta J$$

$$\Delta P_{XY} = \Delta P_{ZX} - \Delta P_{ZY}$$

$$\Delta P_{XY} = R_0(1 + \bar{a}) \, \Delta J + \bar{J}R_0 \Delta a - R_0(1 + \bar{a}) \, \Delta J -$$

$$- \bar{J}R_0 \, \frac{1 + \bar{a}}{1 + \bar{a} + b} \, \Delta a$$

$$\Delta P_{XY} = JR_0 \left(1 - \frac{1 + \bar{a}}{1 + \bar{a} + b}\right) \Delta a = JR_0(1 - f) \, \Delta a$$

$$\text{where} \quad f = \frac{1 + \bar{a}}{1 + \bar{a} + b} = \frac{\bar{J}R_0(1 + \bar{a})}{\bar{J}R_0(1 + \bar{a} + b)} = \frac{\bar{P}}{P_{br}}$$

In the last equation we substitute the quantity f which is quite analogous to the quantity e which is the ratio of the wire voltage to the bridge voltage. If we multiply equation (19) by $\bar{J}R_0(1 - f)$ we obtain

$$M \frac{d\Delta P_{XY}}{dt} + \Delta P_{XY} = - R_0 \bar{J}(1 - f) \, \frac{2\bar{a}e}{1 + 2\bar{a}e} \, \Delta s \qquad (20)$$

The above equation is entirely similar in form to equation (17)
except that here the fluctuating resistance of the heated circuit   i
enters in place of the virtual voltage fluctuations. If the changes
in the resistance follows the purely square form (fig. 18) then the
fluctuations of the resistance consist of exponential segments
(fig. 18). If we lead the voltage fluctuation ΔP_{XY} which is
accurately proportional to the resistance fluctuation, to an
amplifier which is provided with a compensating circuit and determine
the compensation correctly then the amplified voltages are propor-
tional to the resistance fluctuations, that is, we obtain a purely
square voltage form at the amplifier output terminals.

COMPENSATION OF THE THERMAL LAG BY A TRANSFORMER CONNECTION

The simplest compensation is obtained given by a voltage
divider consisting of a resistance and inductance, the voltage being
taken off from the inductance (fig. 9). This adaptation was also
the one used by Dryden.

This solution however possesses two disadvantages. The first
is that the time constant is altered by the variation of the resist-
ance R and thus to the value of a single time constant corresponds
a different absolute amplifications and such a long series of measure-
ments is very inconvenient. The greater disadvantage however lies in
the fact that the errors given by the connection can only be corrected
if we change all the other elements of the system to correspond to the
fixed time constant. The compensation of the circuit is accurate only
if on the one hand the load of the compensating circuit can be regarded
as a pure resistance load or if the frequency is small, and on the
other hand if we are far from the resonance point of the inductance
coil L.

The increase in the resistance R_t is at the expense of the
amplification, but we can displace the resonance point toward greater
frequencies only with very great difficulty if the time constant is
given. There is thus no other way except either to be content with
the greater accuracy obtainable with a lower frequency range or to
seek a solution where the above errors are eliminated without inserting
further circuits which would have to be varied when determining the
value of the time constant.

The fundamental circuit of the compensated amplifier of Dryden
is shown in figure 10. With this circuit it was possible for Dryden
and his coworkers, after further improvements on it, to produce an
amplifier (reference 5) in which through the compensation the frequency

characteristic in the range 25-1000/second shows a deviation of
10 percent and in the range of 1000-3000/second a deviation of
40 to 45 percent in comparison with the measured value at 100/second.
The magnitude of the above value is due to resonance. This linear
distortion does not answer our requirements because in the above
distortion the faithfulness of the response is not satisfactory.

If in equation (17) which expresses the effect of thermal lag
we give the value of $\bar{P}$ and substitute

$$\frac{d\Delta P}{dt} = \frac{dP}{dt}; \quad \bar{P} + \Delta P_V = P_V$$

we obtain the equation

$$M \frac{dP}{dt} + P = P_V \tag{21}$$

This shows that equation (17) holds not only for the voltage fluctua-
tions but also for the voltages themselves. In the equation the
term $M \frac{dP}{dt}$ can also be called the compensating voltage because we
must add this to the distorted wire voltage P in order to obtain
the initial undistorted or so-called virtual voltage.

We shall amplify the voltage P by two amplifiers connected in
parallel (fig. 11). Let one of the voltage amplifying factors be g_1
so that the amplified voltage is $g_1 P$. Let the other amplifying factor
be a differential operator $g_2 \frac{d}{dt}$ so that the other voltage obtained
is $g_2 \frac{dP}{dt}$. If we add the two voltages which act in parallel we obtain

$$g_1 P + g_2 \frac{dP}{dt} = g_1 \left(\frac{g_2}{g_1} \frac{dP}{dt} + P \right) = g_1 P_V, \quad \text{if} \quad \frac{g_2}{g_1} = M$$

From this we see that the amplification is determined by g_1 alone
while the time constant which results from the compensation is
determined by the ratio of the two parallel amplifiers. We can vary
the quantity $\frac{dP}{dt}$ to any degree by varying the compensation.

Let us see what amplitude distortion and phase displacement the
differentiation gives for a sine wave voltage. The differentiation
from our point of view can be conceived as a linear distortion and we
can determine the amplitude ratio and phase displacement as a function
of the frequency. Let

$$P = P_o e^{i2\pi ft}; \quad \frac{dP}{dt} = i2\pi f P_o e^{i2\pi ft} = i2\pi fP$$

or the amplitude ratio must increase with the frequency while the phase must be displaced $\pi/2$ ahead. Such a frequency characteristic must be possessed by the amplifiers within the intended frequency range. The condition itself also shows that it is not possible for the compensator to have frequencies of any magnitude whatever for such high frequencies require a proportionately greater amplification.

For the differential apparatus we choose a transformer, since in the no-load condition of the transformer the primary current is proportional to the derivative of the secondary current. Therefore by putting the primary side of the transformer in a preponderantly ohmic resistance circuit the primary current is proportional to the voltage of the voltage source and thus in the voltage of the secondary we obtain a time derivative. The practical solution was to connect the primary side to the anode circuit of a pentode with large internal resistance (R_b = 2 megohms) while the secondary side is connected nearly without load by means of a direct tube amplifier to the grid of a following tube (fig. 12).

The actual circuit connections are shown in figure 14, the elements being so connected that in the direct amplifier a uniform amplification is obtained and in the differential stage the corresponding amplitude and phase ratios accurately correspond to the differentiation.

The amplification by the direct stage amplifier can be increased. The increase is limited however by the condition that the value of the desired time constant can only reach the value corresponding to the value of $\dfrac{g_1}{g_2}$ or we must also be able to increase g_2. The latter however is limited in value and thus we amplify further the already compensated voltage.

The value of the time constant can be varied by regulating either g_1 or g_2. We can obtain a large range of regulation if we vary both. It is best to vary g_1 in several fixed stages but g_2 continuously but between smaller limits in such a manner that the regulated values of g_2 just comes in between the fixed jumps of g_1. By the sufficiently fine variation of g_2 we can control the time constant to which the compensation gives rise while g_1, which determines the absolute amplification of the compensated voltage, has only several fixed values.

It can be verified directly with an oscillogram that the above
described amplifier with transformer assures a faithful response.
These oscillograms are shown in figures 18(a) to (g). Figure 18(a)
shows the square-form fluctuation impressed on the heating current
circuit (the voltage being taken off from the 5 ohm normal resist-
ance). The current fluctuation depends on the wire resistance and
therefore the variation of the voltage fluctuation that is taken
from the points XY of the bridge follows the exponential form
(fig. 18(b).

With the connected compensating circuit we can set the compensa-
tion for different values. In the case of perfect compensation we
regain the original square-wave form (fig. 18(e)). Figures 18(c)
and (d) show the condition of undercompensation $(M_1 < M)$ and
figures 18(f) and (g) the condition of overcompensation $(M_1 > M)$.
The wire data were $d = 0.008$ millimeter, $M = 1.6.10^3$ second; the
frequency was 200/second.

DESCRIPTION OF THE MEASURING CIRCUIT

The arrangement of the measuring circuit is sketched in fig-
ure 13. The wire is connected to a Wheatstone bridge. For control-
ling the heating of the wire, in rough measurements, as well as for
various connections, a special junction box was employed (K. K.)
which could be used with various wires (not shown in the figure for
greater clarity). A 5 ohm normal resistance which was connected in
series with the wire forming one branch of the bridge was connected
in series with the wire forming one branch of the bridge was connected
to the junction box. For the other two branches of the bridge there
was used a part of the resistances of a measuring bridge (W. bridge).
The ratio of the two branches was chosen as 5:100 and thus satisfied
the condition $(T = 0.05)$ under which we obtained our results. The
galvanometer required for the balancing (G) was connected to the
junction box. For current source there was used a 20 or 40 V
storage battery. The current was measured by measuring the voltage
across a 5 ohm normal resistance in series with the wire. The
voltage was measured with a direct current compensating apparatus (F. K.).

For amplifying the voltage fluctuations taken off the wire and
compensating the thermal lag there was used the first stage
amplifier (E. E.). The amplified and compensated voltage taken from
the amplifier was led to the principal amplifier (F. K.) to whose
output connecting posts was connected the thermocouple voltmeter.
(Th. V.) which measured the effective value of the fluctuations and
to a cathode ray oscillograph (Osc.) which permitted viewing the

fluctuation process. For calibrating the voltage an audible frequency
generator (Gen.) was used. The calibration voltages were adjusted
with a one way reading millimeter. The square wave fluctuation for
measuring the time constant was obtained by a mechanical interrupter
driven by a synchronous motor. The velocity measuring wire itself
was connected to the junction box by a shielded cable. For accurately
adjusting the position of the wire at the place at which the velocity
survey was made an arrangement capable of fine adjustment and movable
in two directions was at our disposal. The velocities were measured
in a wind tunnel.

By means of the cross connections to the junction box it was
possible to connect the primary amplifier to the wire (a), or in
measuring the time constant to the bridge junctions (b), or to the
generator producing the calibrating voltage (c).

The wind tunnel in the investigation was that used for turbulence
investigation by the Aerodynamic Institute. It is 500 millimeters by
500 millimeters cross section and 2200 millimeters in length and
capable of wind velocities of 0.2 to 30 meters per second.

With the above described arrangement the following measurements
could be made.

1. Measurement of the wire resistance, obtained by balancing the
galvanometer in the bridge.

2. Measurement of the wire heating current. The voltage across
the 5 ohm normal resistance is measured with the voltage compensator
when the bridge is balanced and no current flows in the galvanometer.

3. Measurement of the wind velocity, with a Prandtl tube and
micromanometer.

4. Measurement of the time constant. The voltage proportional
to the resistance fluctuations of the wire is led from the post b
to the first stage amplifier. In the case of correctly adjusted
compensation we can read off the value of the time constant from the
calibration scale of the primary amplifier circuit. The correct
compensation is controlled in an objective manner with the aid of the
picture appearing on the oscillograph screen.

5. Measurement of the degree of turbulence, or the effective
value of the velocity fluctuations. With the first amplifier set for
the correct time constant we determine the compensated voltage
fluctuation proportional to the velocity fluctuations. After further

amplification the voltage is measured with the thermocouple volt-
meter. Corresponding to the adjustment of the compensation and the
magnitude of the fluctuations we also measure the magnitudes
characterizing the state of the wire under test (heating current,
resistance).

6. Calibration of the amplification. We set the alternating
current delivered by the frequency generator to a fixed value and
lead the produced voltage fluctuation across a 1 ohm normal resist-
ance to the amplifiers (terminal c of the K. K. switch). With the
thermal voltmeter we measure the voltage obtained from the amplifiers.

CALIBRATION OF THE AMPLIFIER CIRCUITS

Since the apparatus used in the measurements except for the
amplifiers can be obtained commercially, only the amplifiers must
be carefully calibrated especially the compensating stages of the
thermal lag of the initial amplifier. The amplifier was constructed
by Karoly Pulvari of the Mechanical Measurements Laboratory on the
basis of the principles of the present investigation. The detailed
drawing of the connections is shown in figure 14.

The object of the main amplifier is to further amplify the
voltage compensated by the primary amplifier and already proportional
to the velocity fluctuations and render the voltage measurable with
a thermocouple voltmeter. Thus the only requirement imposed on the
main amplifier is that it should amplify uniformly within the frequency
range used.

It was possible to perfect the primary amplifier, especially to
adjust it by varying the resistances R_8 and R_{17}, so that the
desired trend of the characteristic curve of the compensating stage
(corresponding to the differentiation) is obtained. Special difficulty
however was encountered in the response at small frequency numbers
($f < 50/sec$). It was found possible to increase the faithfulness of
the response with the aid of a branch consisting of a resistance R_6
and capacitance C_6 and with the connections used in the primary and
main amplifiers (R_4 and $C_3 = 0.01 \ \mu F$). The resonance of the
transformer for the differential stage could be lowered by the increased
resistances R_8 and R_{17}. The switch K_2 was used for the case where
the compensation was not required (in different measurements) and it
was desired that the amplification in the frequency range of 5000-
15000/second should not be impaired by the damping effect of the
transformer.

The calibration was effected with a sine voltage supplied by the frequency generator. There were separately calibrated the direct and differential (indirect) stages of the primary amplifier.

Let the complex amplification of the direct stage be

$$g_1(f) = g_{10}\eta_1(f) \, e^{i\psi_1(f)}$$

and that of the indirect (differential) stage

$$g_2(f) = g_{20}\eta_2(f) \, e^{i\psi_2(f)} \, \frac{d}{dt}$$

where g_{10} and g_{20} are the nominal amplification factors (referred to f_o)

ψ_1 and ψ_2 the phase displacement angles

η_1 and η_2 the relative amplifications

From equation (17), for the sine wave

$$\Delta P = \frac{\Delta P_V}{1 + i2\pi fM}$$

Let P_1 be the amplified voltage by the direct stage

P_2 the amplified voltage by the indirect stage

P_k the sum of the two

$$P_1 = g_1 \Delta P; \quad P_2 = g_2 \Delta P; \quad P_k = P_1 + P_2$$

Thus the amplified and compensated voltage is

$$P_k = \Delta P_V \frac{g_{10}\eta_1 e^{i\psi_1} + ig_{20}2\pi f\eta_2 e^{i\psi_2}}{1 + i2\pi fM} \tag{22}$$

Let

$$\frac{g_{20}}{g_{10}} = \mu \quad \text{and} \quad \frac{\mu}{M} = k \quad \text{so that} \quad \mu = kM$$

$$P_k = \Delta P_v g_{10} \frac{\eta_1 e^{i\psi_1} + i2\pi fkM\eta_2 e^{i\psi_2}}{1 + iM2\pi f}$$

If $\eta_1 = \eta_2 = 1$ and $\psi_1 = \psi_2 = 0$ then with $k = 1$ perfect compensation would be obtained. The deviation from this is shown by the relative amplification and phase displacement of the compensated voltage or η_k and ψ_k. These we can express as follows:

$$P_k = \Delta P_v g_{10} \eta_k(f) \, e^{i\psi k(f)} \tag{23}$$

The values of η_k and ψ_k depend naturally on η_1, η_2, ψ_1, ψ_2 as well as on the choice of M and k. With the above data, η_k and ψ_k can be determined most simply graphically (fig. 15).

First we obtain the value of $\dfrac{1}{1 + i2\pi fM}$. The circle described with the unit vector $\overline{OA}$ as diameter intersects the radius vector $\overline{OB}$ at angle $\varphi = \operatorname{arctg} 2\pi fM$. The vector $\overline{OC}$ at right angles to it has the value $\dfrac{i2\pi fM}{1 + i2\pi fM}$. We multiply the vector $\overline{OB}$ by η_1 and rotate by the angle ψ_1 and thus obtain the vector $\overline{OB'}$ while we multiply the vector $\overline{OC}$ by $k\eta_2$ and rotate by the angle ψ_2 thus obtaining $\overline{OC'}$.

$$\overline{OB'} = \frac{\eta_1 e^{i\psi 1}}{1 + iM2\pi f}; \quad \overline{OC'} = ik \frac{2\pi fM\eta_2 e^{i\psi 2}}{1 + iM2\pi f}$$

The sum of the two vectors $\overline{OX}$ gives the relative amplification and phase displacement:

$$\overline{OX} = \overline{OB'} + \overline{OC'} = \frac{\eta_1 e^{i\psi 1} + i2\pi fkM\eta_2 e^{i\psi 2}}{1 + iM2\pi f} = \eta_k e^{i\psi k}$$

The segment YX shows by how much η_k deviates from unity. These values naturally hold for a single frequency. If we draw the vector $\overline{OX}$ for various frequencies we obtain a polar diagram for the relative amplification and phase displacement. From this it is clear that we do not choose the value of k as unconditionally unity but smaller or larger according to the trend of the polar curve. With the chosen value of k we obtain the value of the time constant

$$M = \frac{g_{20}}{g_{10}} \frac{1}{K}$$

which is essentially the ratio of the nominal amplifications. Thus we see that the calibration of the time constant scale can be obtained independently of the hot wire measurements.

We adjust the value of the time constant in the primary amplifier so that first the direct amplification (g_{10}) can be varied in 7 stages and secondly the indirect (differential) stage magnification (g_{20}) is regulated continuously.

The values of η_1 and η_2 were measured for constant-amplitude frequencies. The amplified voltage could be measured with the thermo-couple voltmeter independently of the frequency. In the case of η_2 however where it is necessary to investigate the relative deviation from the differentiation it is more suitable to integrate the voltage with constant amplitude but different frequencies so that after differentiation we obtain a constant amplitude and from the deviation from the constant amplitude we measure η_2.

The integrating stage can be obtained in a relatively easy manner with great accuracy. The stage was made up of a voltage divider consisting of a large resistance and a large condenser (fig. 16) with the following data:

$$R = R_e = 0.5 M\Omega, \quad C = 1\mu F$$

Thus the maximum angular error of the connection at the lowest frequency used, 50/second, is $\delta = 1°$. This is of the order of magnitude consistent with the other measurement errors.

During the measurements the phase displacement angles of the amplifiers (ψ_1, ψ_2) were adjusted by leading the input voltage to the horizontal plate of the cathode ray tube and the output voltage to the vertical plate. Thus in the case of a sine voltage we obtain an ellipse on the screen. If we choose the two amplitudes to be equal (on the screen) the axes of the ellipse include an angle of $45°$ with the directions of the deviations and in this case the phase angle between the incoming and outgoing voltages is

$$\psi = \text{arctg} \frac{2}{\frac{a}{b} + \frac{b}{a}}.$$

where a and b are the major and minor axes of the ellipse. We can readily see that if the ellipse degenerates into a straight line, $\psi = 0$ and if into a circle, $\psi = 90°$.

In this manner we determined the values of η_1, η_2, ψ_1, and ψ_2 and prepared the polar diagram for several values of k. In the polar diagram η_k and ψ_k are functions of the frequency number. This is seen in figure 17 for the time constant value $M = 2.10^{-3}$ second, $k = 0.97$. It is more suitable however in place of the phase displacement ψ_k to use the quantity

$$\tau_k = \frac{\psi_k}{2\pi f}$$

as an indicator of the trueness of the response.

From this we can see that up to about 3000/second frequency number the response is very good (about 5 percent deviation) and satisfactory up to about 5000/second. The greatest difference $\Delta\tau_k$ is 14.10^{-6} seconds or less than 1 percent for the time constant value of 2.10^{-3} seconds.

PRACTICAL CARRYING OUT OF THE TURBULENCE MEASUREMENTS

In measuring the turbulence a large number of points must be taken so that we can determine the space distribution of the turbulence fluctuations, the dependence of the latter on the velocity or the turbulence boundary layer properties.

The cold resistance is measured from the heating characteristic of the wire. The square of the wire current (q) is plotted as a function of the measured reciprocal of the resistance. According to equation (8) we then obtain a downward sloping straight line. Extrapolating to zero current we obtain the value of the cold resistance.

The two heat dissipation constants used in the measurements at the desired temperature coefficient are determined $(a = 0.8 - 1.0)$. With the apparatus shown in figure 13 we measure the current at which the temperature coefficient of the wire and therefore its resistance remains constant for various velocities. The intercept on the horizontal axis gives the value of y_0 and the slope the value of the constant c (fig. 1).

For obtaining the thermal lag constant n we measure the time constant under different conditions. The state constants of the wire are measured in the usual manner while the time constant is determined with the square voltage form as described in detail above. There are thus at our disposal the corresponding values of a, q and M. With these expressed in equation (16) we have

$$n = M\,\frac{\bar{q}(1 + 2\bar{a}e)}{\bar{a}}$$

so that n can be determined from one measurement alone. More than one measurement is made for the purpose of equalizing the deviation of the measurements.

From the values of the constants we prepare the table which gives the corresponding compensation for the different states of the wire. Since in the measurements the mean resistance of the wire is constant (the bridge adjustment is kept constant) only the heating current of the wire is changed corresponding to the various velocities. Thus the state of the wire is uniquely characterized by the current.

The table is computed as follows. From the current and the resistance of the wire we know the value of $\overline{P}$. Assuming a constant voltage source (20 or 40 volts) we compute the value of e. The value of a is previously computed; the value of n is known from the thermal lag and $M = M(J)$ can be computed.

Having thus prepared the table we adjust the wind tunnel to the corresponding velocity, and adjust the wire to the point for which we wish to measure the velocity fluctuations. In the amplifiers we then adjust the compensation corresponding to the wire current and read the thermocouple voltmeter. The reading of the latter is proportional to the degree of turbulence. The effective voltage fluctuations (the square root of the mean square value) is expressed in percent of the fundamental velocity. The amplification of the apparatus is throughout controlled with the frequency generator and the possible correction factor determined.

Since during the measurements we keep the wire resistance and thus the value of a constant we note down only the value of the current, the reading of the thermocouple voltmeter, and the adjustment of the amplifiers.

In evaluating the results we compute two voltage values. The first is the voltage which the wire would have under the condition considered if the degree of turbulence were exactly 1 percent. Denoting this by ΔP_1 we have from equation (12)

$$\Delta P_1 = \frac{\overline{P}}{100} \frac{\overline{a}}{2} zE \tag{24}$$

so that the ΔP_1 can be computed. The second voltage is the input voltage which for a certain setting of the amplifiers gives one division reading of the thermocouple voltmeter. This is given by the above amplifier calibration table which we modify by a certain correction percentage in controlling the measurements. The ratio of the two voltages shows what percent of the degree of turbulence gives a one division reading of the thermal voltmeter. We multiply the above value by the voltmeter reading to obtain the value of the degree of turbulence itself. This system has the advantage that for

the same mean velocity ($\bar{v}$) the state of the wire ($\bar{a}$, $\bar{q}$) does not vary and therefore the reading of the thermal voltmeter need be multiplied only by the obtained constant.

If we wish to obtain a visual picture of the turbulence fluctuation we connect the compensated voltage fluctuations from the primary amplifier to the oscillograph. The above is important particularly for observing regular flow phenomena (Kármán vortex street, etc.).

With the above turbulence investigating apparatus the probable error is 3 to 4 percent of the measured value (degree of turbulence). The greatest part of this error is given by the amplifier and the thermal voltmeter while the direct current part (W. bridge F. K.) is essentially smaller. This value of the error corresponds to that in usual technical measurements and appears satisfactory for the purposes of turbulence investigation especially since this accuracy is not attained with the apparatus used in other countries. This estimated error is confirmed by the scatter of the measurements which is of similar magnitude.

Figure 19 shows the free turbulence of the wind tunnel. There is clearly seen the quite arbitrary character of the fluctuation. Figure 20 shows the turbulence research apparatus set up by Szerző for the Aerodynamic Institute.

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Table I

f	P_0/P_{vo} ($M = 0.5$ m sec)	$\varphi°$	P_0/P_{vo} ($M = 1.5$ m sec)	$\varphi°$	P_0/P_{vo} ($M = 5$ m sec)	$\varphi°$
10	0.999	1.8	0.996	5.4	0.954	17.5
20	0.998	3.6	0.983	10.7	0.847	32.3
50	0.988	8.8	0.903	25.4	0.537	57.5
100	0.954	17.5	0.729	43.2	0.302	72.4
200	0.847	32.2	0.469	62.0	0.156	81.0
500	0.537	57.5	0.208	78.0	0.063	86.4
1000	0.302	72.4	0.106	83.5	0.032	88.2
2000	0.156	81.0	0.052	87.0	0.016	89.0
5000	0.063	86.4	0.021	88.5	0.006	89.6
10000	0.032	88.2	0.010	89.2	0.003	89.7

Table II

f $1/sec$	η_1	ψ_1	η_2	ψ_2	η_k	ψ_k	τ_k $10^{-6}sec$
20	0.980	$-5°$	0.900	—	0.970	—	65
50	0.995	—	1.000	1°	0.990	1°	70
100	1.000	—	1.000	2°	1.000	2°	70
200	1.000	1°	1.000	4°	1.000	4°	68
500	1.005	3°	1.005	12°	1.010	11°	61
1000	1.020	6°	1.040	27°	1.035	25°	70
2000	1.020	12°	1.035	52°	1.030	51°	75
5000	0.850	30°	0.580	150°	0.580	150°	83

$$I^2 = q$$

$$q = \frac{a}{a+1} cy'$$

$$a = \frac{R - R_o}{R_o}$$

$$tg\, \varphi_\infty = c$$

$$q_\infty = cy'$$

CONSTANT T

Figure 1.

$$a = \frac{q}{cy' - q}$$

Asymptote to curve $q = 1$

$$y_a' = \tfrac{1}{2} y_1'$$

Horizontal asymptote

$$y_1' = 2y_a$$

CONSTANT I

Figure 2.

$$a = \frac{q}{cy' - q} ,$$
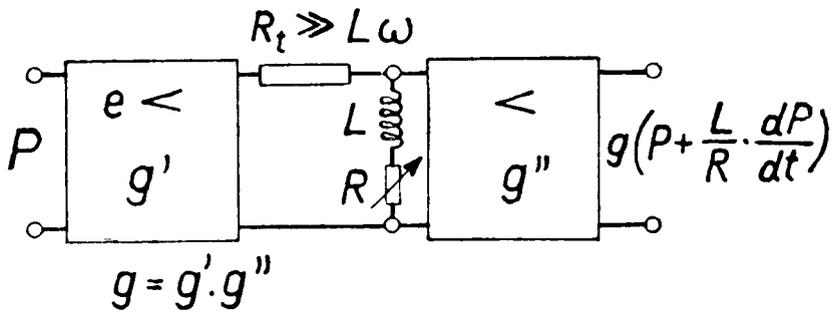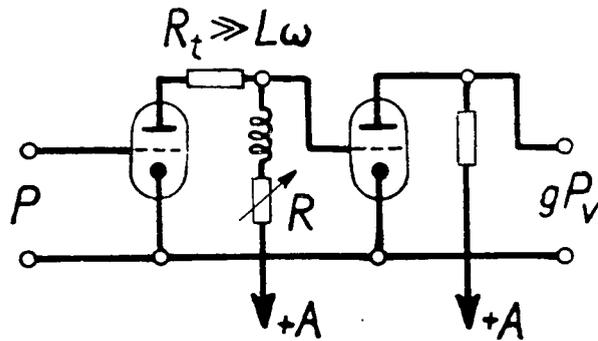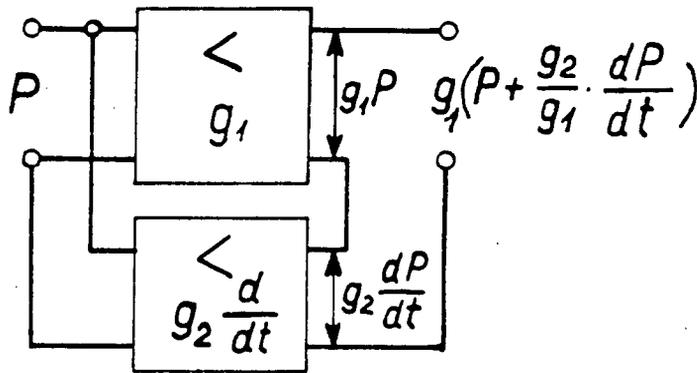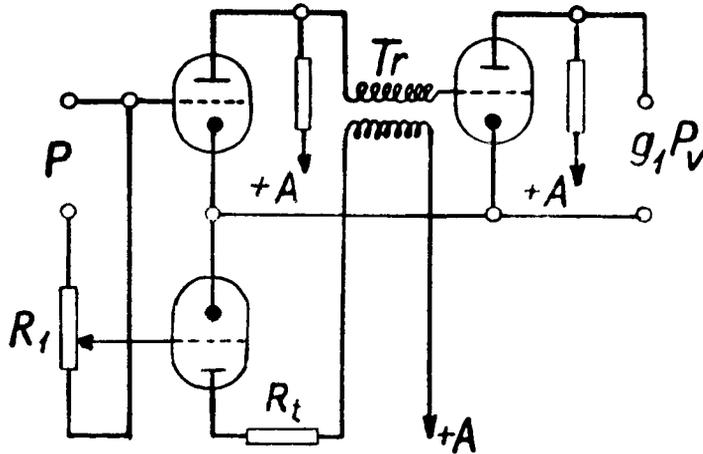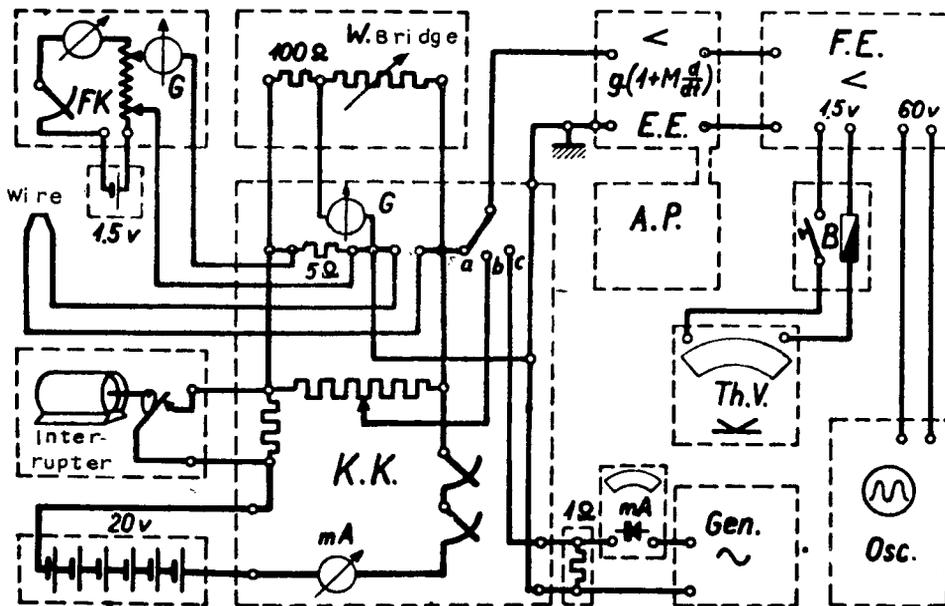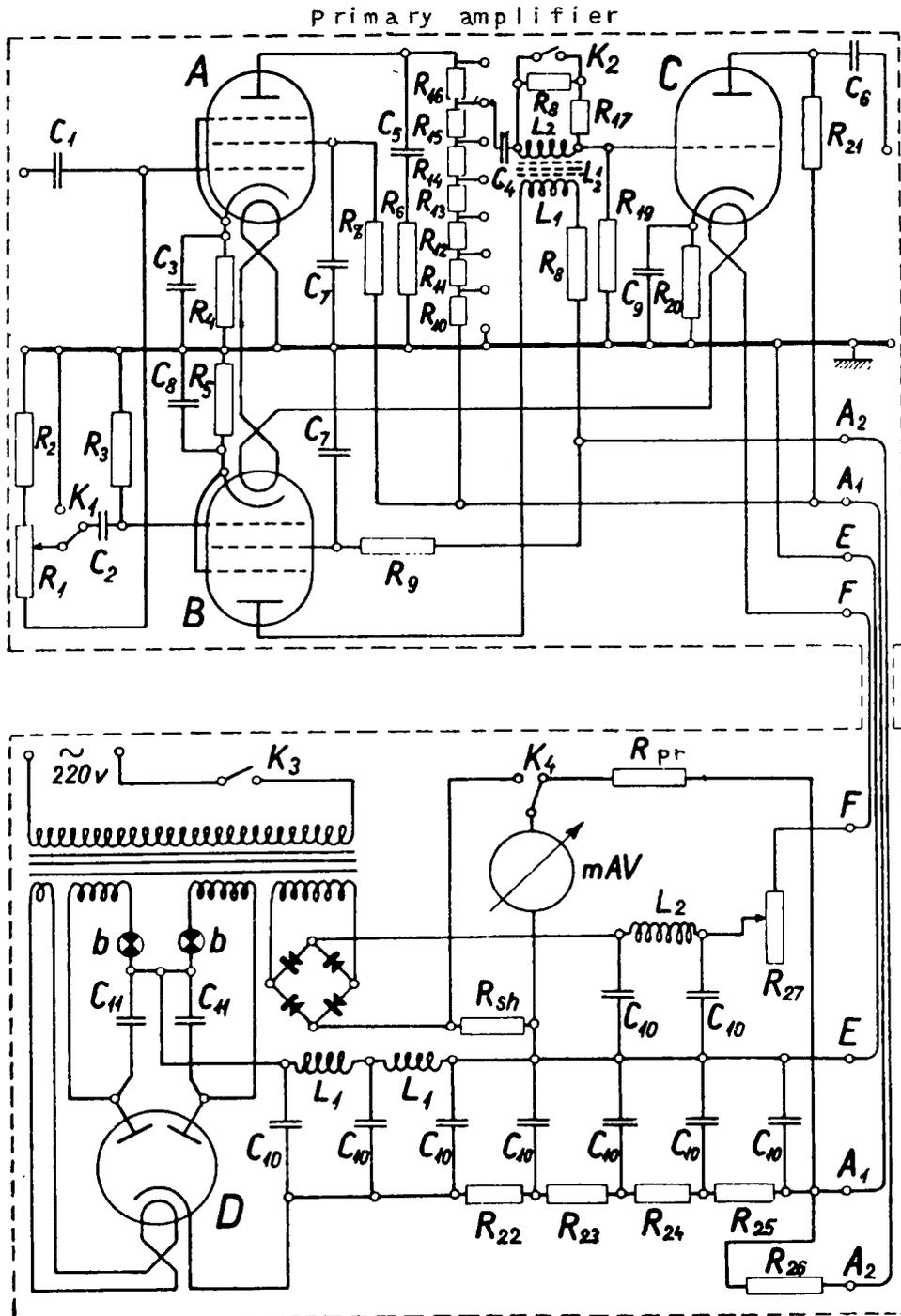
$$q_a = 2q_1$$

$$q_1$$

Asymptote to curve $y = 1$

Horizontal asymptote
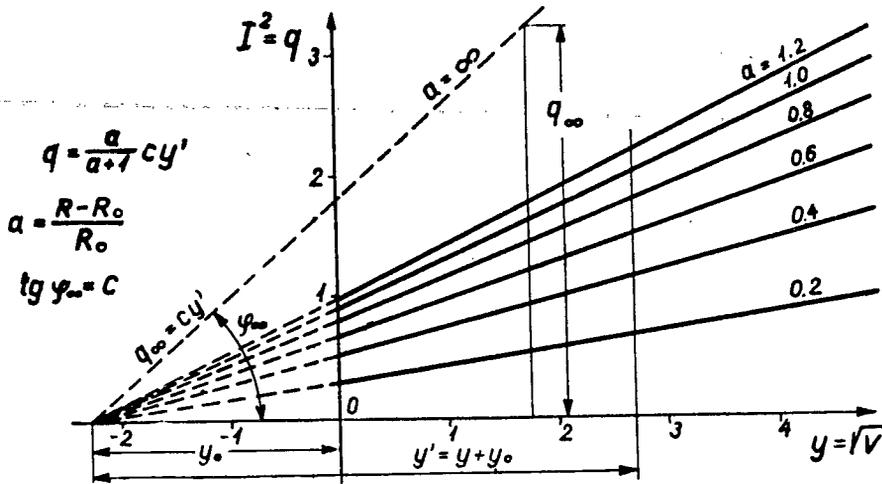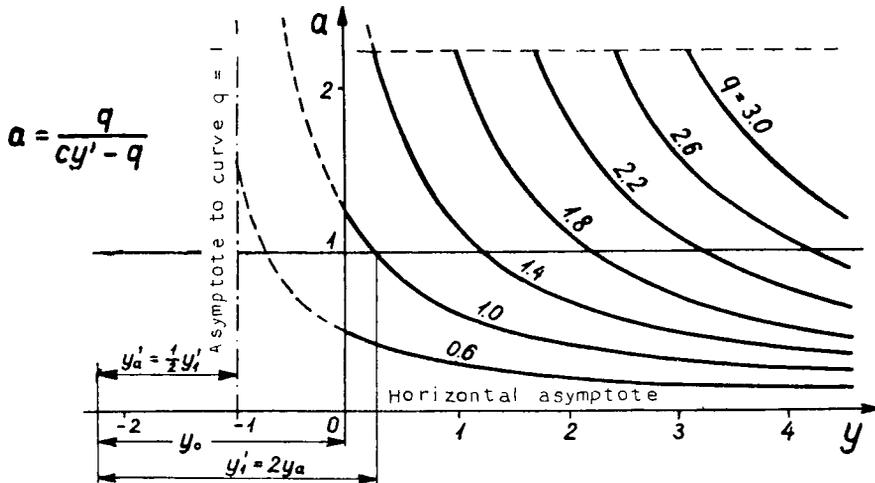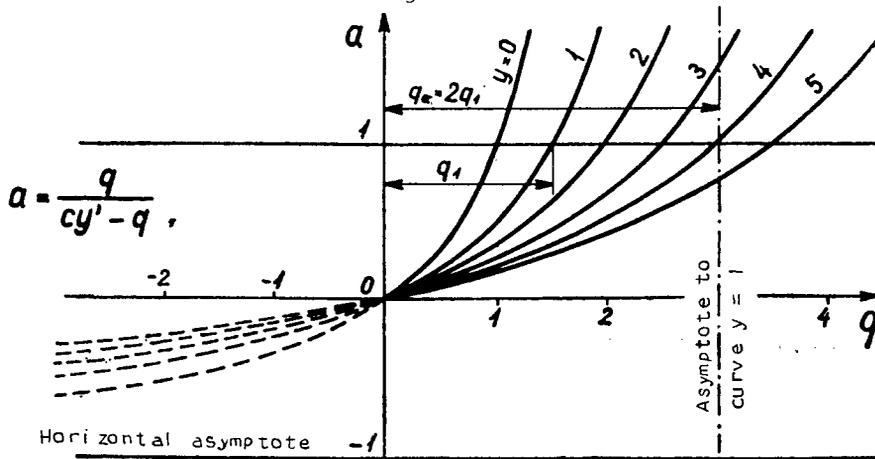
CONSTANT V
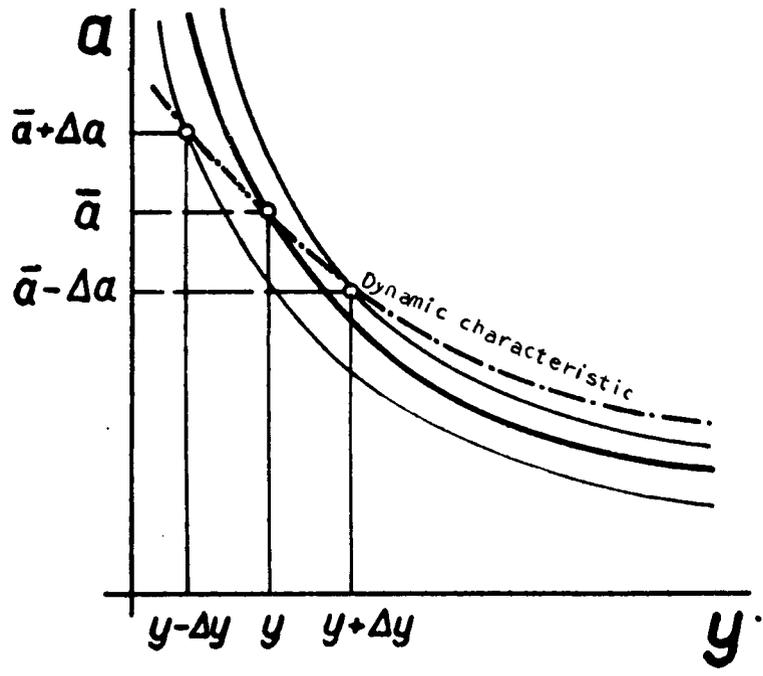
Figure 3.

Figure 4.
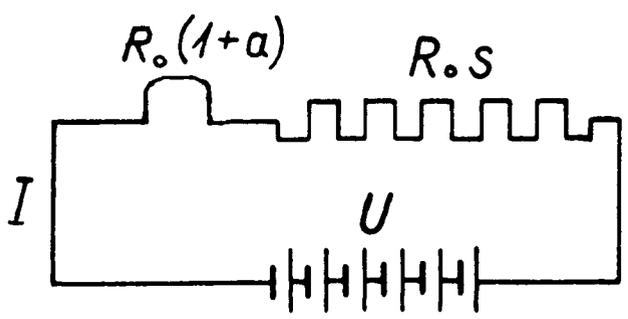
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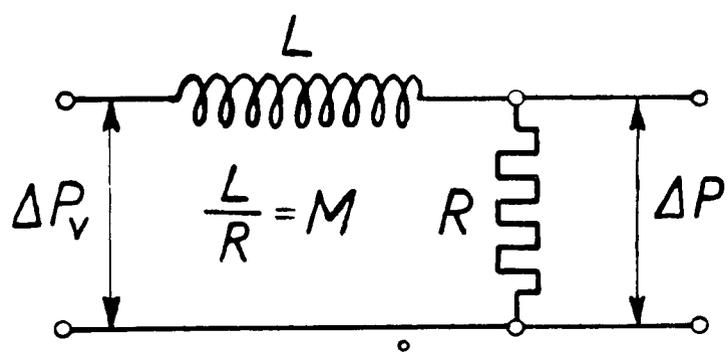
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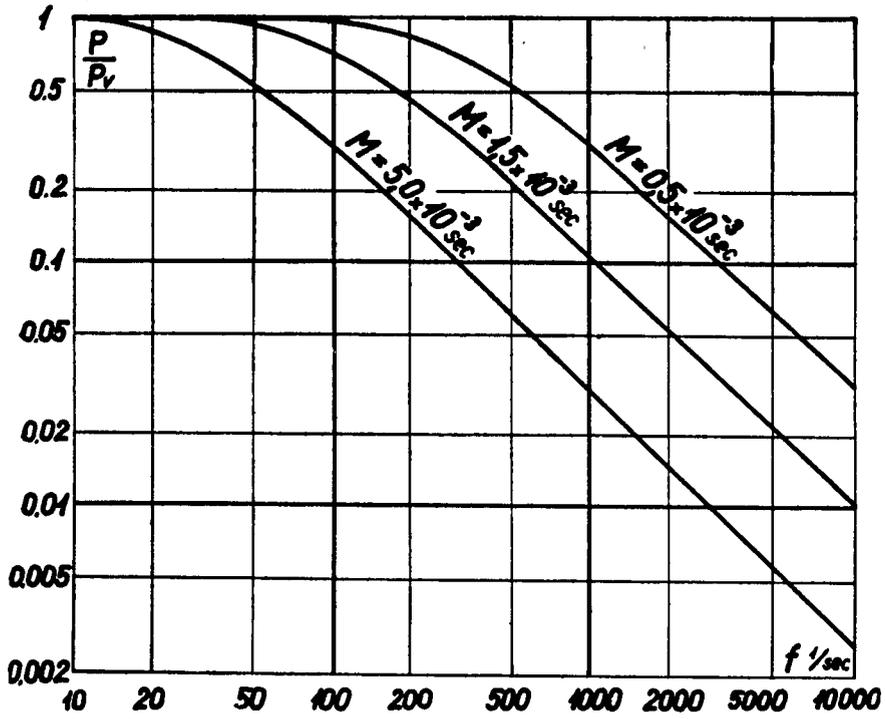
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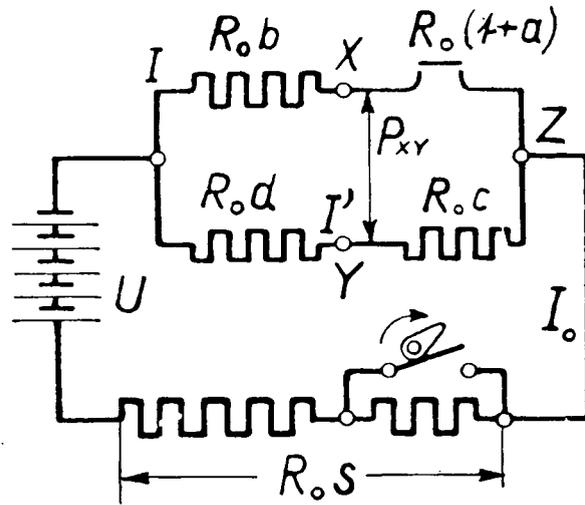
Figure 8.

Figure 9.

Figure 10.

$$g_1 P \qquad g_1\left(P + \frac{g_2}{g_1} \cdot \frac{dP}{dt}\right)$$

$$g_2 \frac{dP}{dt}$$

Figure 11.

Figure 12.

Figure 13.

Fig. 14   NACA TM No. 1130

Primary amplifier
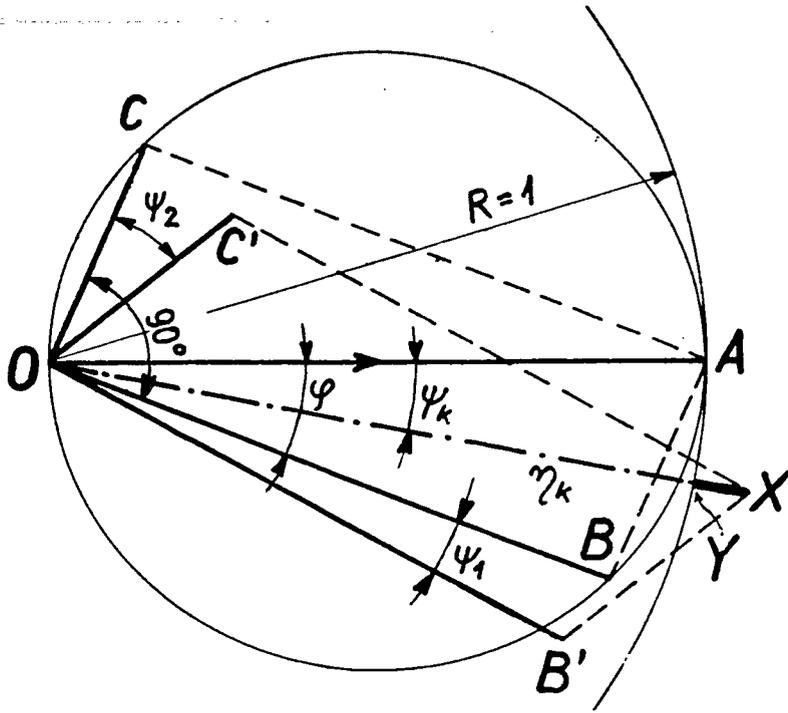
Figure 14.

Figure 15.

Figure 16.

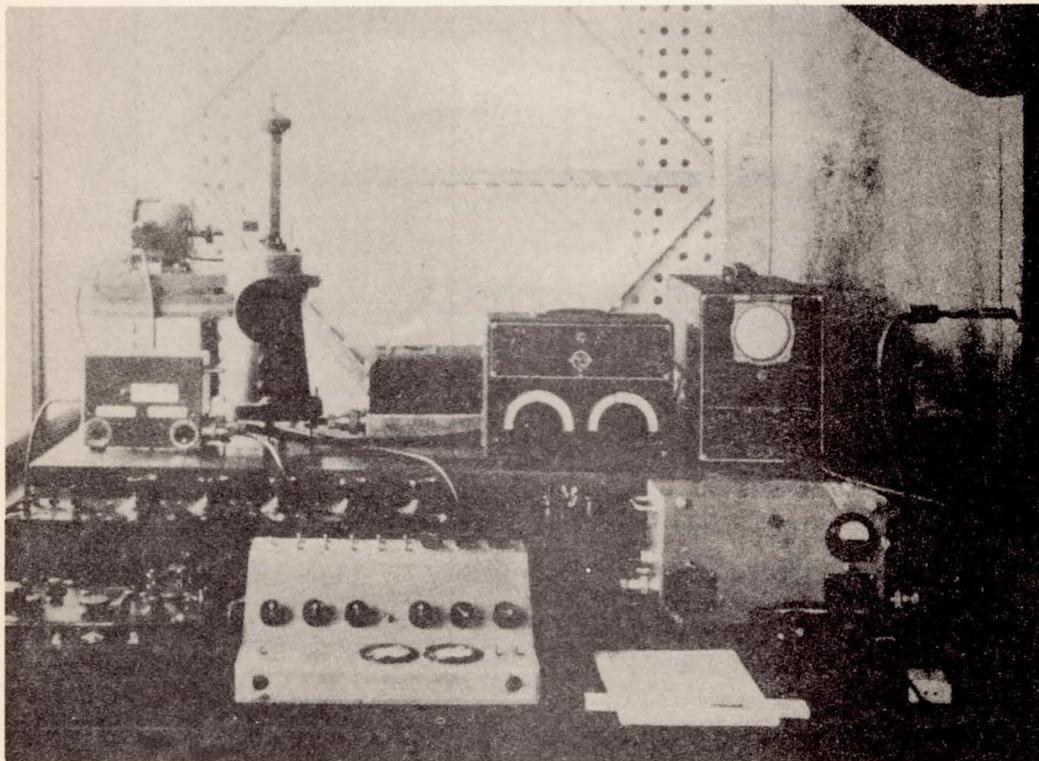Fig. 17   NACA TM No. 1130
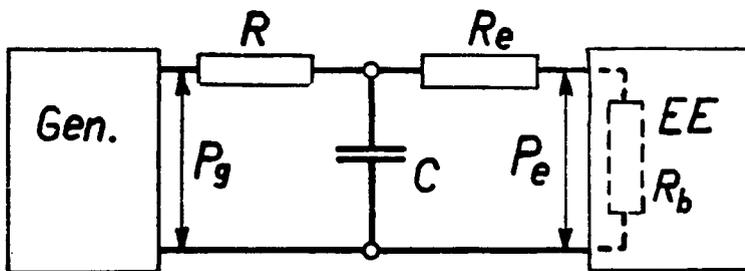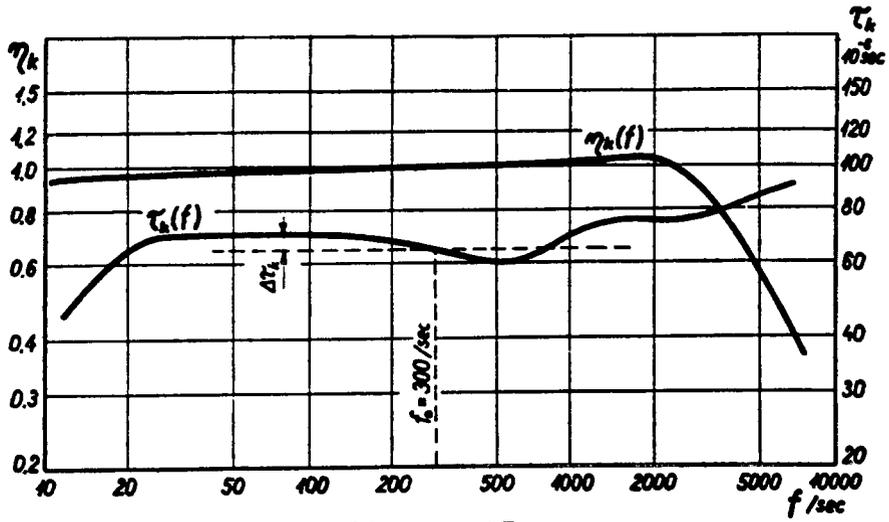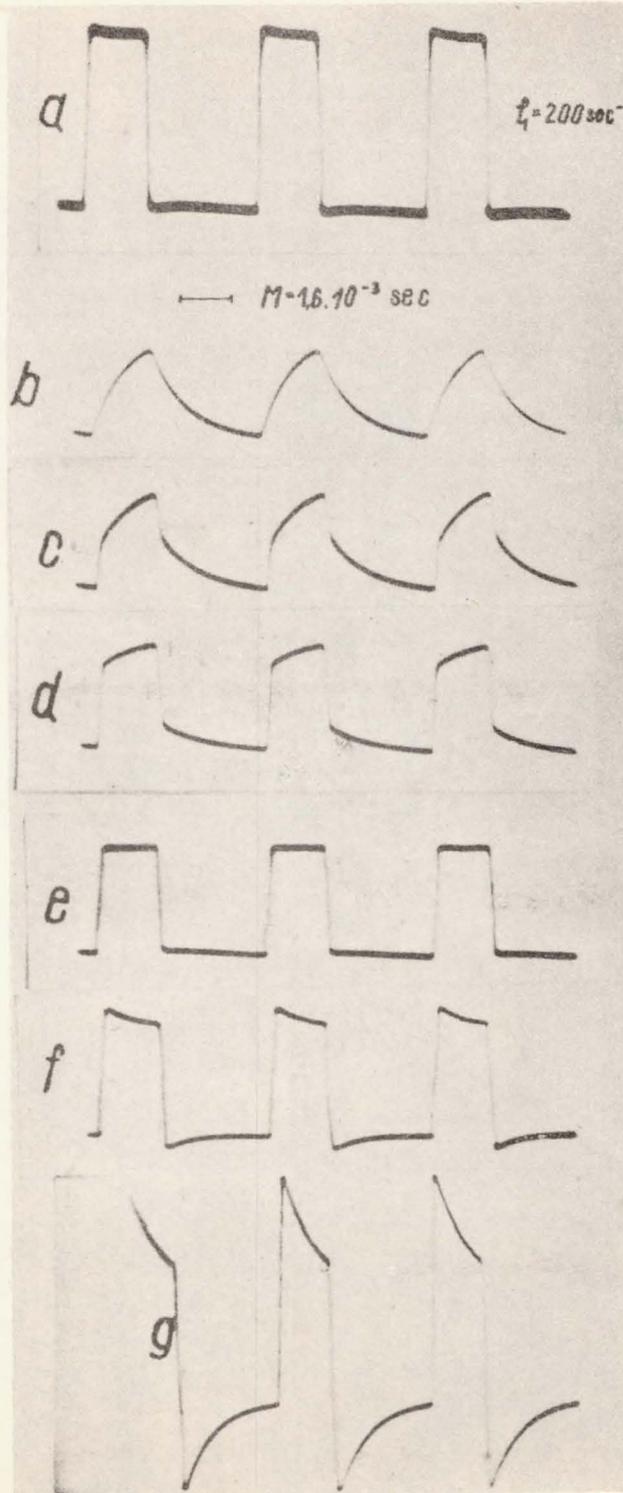
Figure 17.

Figure 18.

Figure 19.

Figure 20.