

*Dr. Sundquist*

NATIONAL ADVISORY COMMITTEE  
FOR AERONAUTICS

TECHNICAL MEMORANDUM

No. 1086

DISTRIBUTION OF STRUCTURAL WEIGHT OF WING ALONG THE SPAN

By V. V. Savelyev

Central Aero-Hydrodynamical Institute



Washington  
August 1946



3 1176 01440 7770

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 1086

DISTRIBUTION OF STRUCTURAL WEIGHT OF WING ALONG THE SPAN<sup>1</sup>

By V. V. Savelyev

In the present report the true weight distribution law of the wing structure along the span is investigated. It is shown that the triangular distribution and that based on the proportionality to the chords do not correspond to the actual weight distribution. On the basis of extensive data on wings of the CAHI type airplane formulas are obtained from which it is possible to determine the true diagram of the structural weight distribution along the span from a knowledge of only the geometrical dimensions of the wing. At the end of the paper data are presented showing how the structural weight is distributed between the straight center portion and the tapered portion as a function of their areas.

INTRODUCTION

At the present time the greatest difficulty in structural design is not in determining the critical flutter velocity itself but in obtaining the computed data required for the latter. Included among these required data is the distribution of the structural weight of the wing along the span.

In making the strength computations for a wing it is necessary to take into account the decrease in the bending moments due to the air forces by the given weight of the wing. For this purpose it is likewise necessary to know the structural weight distribution of the wing along the span.

Both of these problems are generally solved at the very beginning of the design when usually only the geometric dimensions of the wing and its total weight are known.

---

<sup>1</sup>Report No. 381 of the Central Aero-Hydrodynamical Institute, Moscow, 1939.

For a solution of these problems the designer must assume the weight distribution of the wing according to some law without knowing how well the distribution law selected corresponds with actual fact. The object of the present paper is to provide an answer to this question. The work is based on the extensive data available on wings of the CAHI type of airplane.

The investigations of CAHI in this direction are being continued, and in the near future methods will be given for the rapid determination of other computed data required for determining the critical flutter velocity, such as the determination of the line of centers of gravity of the segments of the wing along the span and the moments and radii of inertia of the wing about its longitudinal axis. (See NACA TM No. 1052.)

#### NOTATION

Lengths along the span (fig. 1):

- $l$  length of the tapered part of the wing (outer wing)
- $l_1$  reduced length of the outer wing, equal to  $l - \Delta l_1$
- $\Delta l_1$  length of the rejected wing tip in reducing the area of the outer wing to an equivalent area of a trapezoid
- $l_3$  length of the straight center portion (center wing)
- $b_f$  maximum width of the fuselage,  $l_2 = \frac{l_3 - b_f}{2}$

Chords and profile thickness (fig. 1):

- $t_0$  fictitious chord of outer wing at the distance  $\Delta l_1$  from its tip
- $t_1$  chord along axis of the juncture of the center and outer wing
- $t_2$  chord at the end of the center wing, equal to  $cr$  (fig. 1)

- $a_0$  maximum fictitious profile thickness at end of outer wing
- $a_1$  maximum profile thickness at juncture of center and outer wing
- $a_2$  maximum profile thickness of center wing at fuselage

#### Determination of Weight of Structure and Areas of Wing

The term  $P_{ow}$  denotes the structural weight of the outer wing including the ailerons, flaps, and fillets of joint to center wing. The weight of the aileron and flap controls, the weight of all brackets joining the assemblages in the outer wing, the weight of the electric conductors, etc. are not included in the structural weight. In the structural weight of the center wing of a low-wing airplane  $P_{cw}$  there enters the total weight of the longerons, the weight of the stringers, the ribs and skin from the joint with the taper to the line  $cr$  (fig. 1), the weight of the fairings, and that of the flaps. The weight of the center part of the fuselage, the weight of the aileron and flap controls, the weight of brackets joining any assemblage in the center wing, and the weight of the electric conductors do not enter. In the weight of the center wing of a high-wing airplane  $P'_{cw}$  there enters in addition to the above-mentioned structures for the low wing, the weight of the ribs, stringers, and skin under the central part of the fuselage.

#### Wing Areas ( Fig. 1 )

area of center wing  $S_{cw} = cder \times 2$

area under the fuselage  $S_f = ocro \times 2$

geometric area of wing  $S_{gw} = odeo \times 2 + 2S_0$

$S_0$  area of one outer wing

$S_{fcw}$  area of fairings of center wing

#### DISTRIBUTION OF WEIGHT OF WING STRUCTURE ALONG SPAN

To obtain the true law of the weight distribution of the wing structure along the span a large number of wings of longeron construction with smooth metal skin of the CAHI type were taken and by making use of the actual weights of these

wings the spanwise weight distribution of the wing was drawn, the problem being divided into two parts, namely, the distribution of the weight over the outer wing and the distribution over the center wing.

### 1. Outer Wing

#### (a) Form of the weight distribution along the span.

To construct the diagram of the true weight distribution along the outer wing everything was excluded that did not enter the structural weight as defined above. A preliminary analysis showed that the weight of the joints between the two parts of the wing must be considered as a concentrated load and it was assumed that this concentrated load was located along the axis of the joint.

The rounded tip of each outer wing was reduced to a trapezoid by decreasing the length of each outer wing by the amount  $\Delta l_1$  (fig. 1). The weight of the cut-off tip of width  $\Delta l_1$  was included in the weight of the last segment.

To construct the diagram of the weight distribution each outer wing was divided into a number of segments. These were chosen in such a manner that in each of them entered only one rib which was placed at the center of the segment (fig. 2). The weight of each segment was divided by its width and the magnitude  $q_n$  thus obtained was laid off under this segment from a certain horizontal axis. Taking in succession all segments and constructing under each of them a rectangle of height  $q_n$  the diagram of the weight distribution of the outer wing along the span was obtained.

Figures 3 and 4 show the true distribution over the two outer wings of various land airplanes and figure 5 shows the true distribution for one of the marine aircraft. From these diagrams it is seen that the stepped upper line can be entirely replaced by a straight inclined line. The inclination and position above the horizontal axis of this line is determined from the following two conditions:

1. The area of the obtained trapezoid must be equal to the area under the stepped line.

2. The center of gravity of the obtained trapezoid must coincide with the center of gravity of the true distribution, that is, with the center of gravity of the area under the stepped line.

It is found that the substitution of the stepped line by the straight line may without impairment of accuracy be made also for heavy airplanes. Figure 6 gives the structural weight distribution for the "Maxim Gorky."

Figure 7 gives the weight distribution for a long-range airplane. In such airplanes fuel tanks must usually be located not only in the center wing but also toward the tips, and the fuel tanks may constitute part of the structure. In figure 7 in the left part of the diagram the part above the dotted line represents the weight of the fuel tank which in the given wing constitutes part of the structure. It may be thought that the substitution in this case of the stepped line by a straight line might lead to appreciable inaccuracy, but actually this is not so and in this case as in all the above cases the substitution may be made, provided the above two conditions are satisfied. This may easily be shown by constructing the diagram of the bending moments for the stepped and straight lines, respectively. The data are as follows:

Distance from wide end of taper (m)	Stepped line	Straight line
10	58	36
8	156	140
6	360	350
4	720	720
2	1260	1260
0	2030	2025

Note.— Bending moment in kgm.

Thus it is found finally that the structural weight distribution along the span of the tapered part follows a linear law independent of the type and dimensions of the airplane.

(b) Comparison of the true distribution with the triangular distribution and the distribution proportional to the chords.— To compare the true distribution of the structural weight along the outer wing with the triangular distribution and the distribution proportional to the chords, respectively, bending moment diagrams were constructed for a series of airplanes.

The most characteristic of these diagrams are given in figures 8, 9, 10, and 11 where the diagrams of the true weight distribution and bending moment due to this distribution are indicated by the solid lines, the diagrams corresponding to the triangular distribution by the dot-

dash lines and those based on proportionality to the chords by the dotted lines.

In table 1 are given the values of the bending moment at the joint of the outer wing due to the weight of the wing structure for a number of airplanes. It is seen from table 1 that for the first group of airplanes use of the triangular distribution of the weight yields a smaller moment than is actually the case, while use of the distribution based on proportionality to the chords yields too large a moment, the true moment being nearer the triangular distribution than that based on proportionality to the chords.

The second group refers to long-range airplanes having wings with large aspect ratio, and heavy, large airplanes with very great airfoil thickness at the joint. For this group the triangular weight distribution and the distribution proportional to the chords give too great a bending moment, the actual moment being nearer to the triangular distribution.

Thus is obtained the result that the true weight distribution along the tapered part for all the airplanes lies closer to the triangular distribution and the difference between the first and second groups consists only in the fact that for the first group the triangular distribution gives too small a bending moment while for the second group it either gives accurate agreement with the moment (fig. 10) or too large a value (fig. 11).

## 2. Center Wing

Weight distribution diagram of the center wing.- In constructing the diagrams of the true structural weight distribution along the span of the center wing for each center wing considered everything was excluded that did not enter into the structural weight proper, as defined above. Moreover, it was assumed that the weight of the joints of the center wing to the outer wing constituted a concentrated load along the axis of the joint. All the other elements of the center wing structure were uniformly distributed over the corresponding segments.

The general shape of the diagram of true weight distribution of the center wing of a low-wing airplane is shown in figure 12, where the small rectangle at the left represents the weight of the longerons passing through

the fuselage and the other two rectangles represent the weight of the center wing protruding from the fuselage. By analogy with the tapered part the obtained stepped line from the protruded part is replaced by a straight line. The inclination and position above the axis of this straight line are determined from the two above-mentioned conditions.

### 3. Complete Wing

#### Diagram of true structural weight distribution.

Adding the obtained results for the center wing and outer wing there is obtained the shape of the diagram of the true weight distribution along the wing span as shown in fig. 13. The shape of this diagram in no way corresponds with the diagram for the triangular distribution or the distribution based on proportionality to the chords. Figure 14 shows the distribution curves over the wing according to the various distributions and below are given the bending moment diagrams due to the weight of the wing according to the three types of distribution.

## DERIVATION OF FORMULAS FOR DETERMINING THE TRUE SPANWISE STRUCTURAL WEIGHT DISTRIBUTION

### 1. Outer Wing

It was concluded above that the true structural weight distribution over the outer wing approached most nearly the triangular distribution. Nevertheless this law can not be used for constructing with its aid the diagram of the bending moments due to the weight of the wing and for determining the position of the center of gravity of the outer wing on the assumption that this agrees with the true conditions.

The true spanwise weight distribution is the trapezoidal, the center of gravity of the trapezoid always lying between the centers of gravity of the triangular distribution and the distribution proportional to the chords, the center of gravity corresponding to the latter distribution lying nearer to the tip of the wing and the center of gravity corresponding to the triangular distribution lying nearer the wide end of the taper (figs. 8, 9, 10, 11).

In obtaining formulas for the true weight distribution over the taper it was first of all proposed to connect these formulas with the fundamental dimensions of the taper, and on the other hand, to take account of the above-mentioned position of the true position of the center of gravity between the centers of gravity corresponding to the other two distributions.

In order to satisfy the above-given conditions it was assumed that the center of gravity of the true weight distribution lies on the same vertical with the center of gravity of the trapezoid constructed under this curve, the large base of which is the cross-section of the taper at the joint with the center wing, equal to  $F_1$  the other base being the cross-section equal to  $f_0$  of the taper at the distance  $l_1$  from the wide end (fig. 15).

The determination of the areas  $F_1$  and  $f_0$  in practice is rather difficult. For this reason an additional investigation was made with the object of replacing these areas by more accessible magnitudes, namely, by the product of the chord by the airfoil thickness at these sections. As a result of this investigation it was found that

$$F_1 = kt_1 a_1$$

$$f_0 = kt_0 a_0$$

where the coefficient  $k$ , denoted by us as the fullness factor of the profile, was found for a larger number of airfoils to be approximately 0.675 independent of the thickness of the profile. For this reason in what follows the magnitudes  $F_1$  and  $f_0$  were respectively replaced by the products  $t_1 a_1$  and  $t_0 a_0$ .

To obtain formulas determining the magnitudes  $q_1$  and  $q_0$  (fig. 15) we denote by  $P_{ow}$  the structural weight of the outer wing and by  $p_1$  the weight of the bolted joints.

We then have

$$\frac{P_{ow} - p_1}{l_1} = \frac{q_1 + q_0}{2} \quad (1)$$

But since the centers of gravity of these two trapezoids lie on one vertical, therefore

$$\frac{q_1}{q_0} = \frac{t_1 a_1}{t_0 a_0} \quad (2)$$

Denoting the ratio  $t_1 a_1 / t_0 a_0$  by  $n_0$  and calling it the taper coefficient

$$q_1 = n_0 q_0 \quad (3)$$

Substituting the obtained value of  $q_1$  in formula (1)

$$\frac{P_{ow} - P_1}{l_1} = \frac{n_0 q_0 + q_0}{2} = \frac{q_0}{2} [n_0 + 1]$$

whence

$$q_0 = \frac{2(P_{ow} - P_1)}{(n_0 + 1)l_1} \quad (4)$$

The obtained formulas (3) and (4) for determining the magnitudes  $q_1$  and  $q_0$  are valid only for the case that  $n_0$ , determined from the dimensions of the outer wing is equal to  $n_{tr}$ ,  $n_{tr} = q_1 / q_0$ . A comparison of  $n_0$  and  $n_{tr}$  for a large number of tapers showed that these magnitudes as a rule are not equal but that there is the relation between them shown in figure 16. Examining this curve for land airplanes we see that  $n_0$  as compared with  $n_{tr}$  has a wide variation of values. At two points of this curve near the origin and at  $n_0 \approx 34$ ,  $n_0 = n_{tr}$ , between these two values  $n_0 > n_{tr}$  and over the remaining part of the curve  $n_0 < n_{tr}$ . This is particularly clearly seen on figure 17 where along the vertical axis are laid off the absolute values of the difference

$$\Delta n = n_0 - n_{tr}$$

as a function of  $n_0$ .

The nonagreement of  $n_0$  with  $n_{tr}$  indicates that the center of gravity of the outer wing, determined from its dimensions, does not lie on the same vertical as the center of gravity of the diagram of true weight distribution. For small values of  $n_0$  the center of gravity of the diagram of true weight distribution lies to the left of the vertical  $o - o$ , drawn through the center of gravity of the trapezoid constructed from the dimensions of the outer wing (fig. 15); then with

increasing  $n_0$  the center of gravity rapidly approaches this vertical, passes it to the right at 13 percent of  $l_{c.g. ow}$  (fig. 18) and rather slowly returns to this vertical, again exceeding it at  $n_0 \approx 34$  but now by less than 1 percent of  $l_{c.g. ow}$ . To determine correctly the magnitudes  $q_1$  and  $q_0$  (fig. 15) in formulas (3) and (4), it is necessary instead of  $n_0$  to substitute the corresponding value  $n_{tr}$ , determined by figure 16, so that finally

$$q_1 = n_{tr} q_0 \quad (5)$$

$$q_0 = \frac{2(P_{ow} - P_1)}{(n_{tr} + 1)l_1} \quad (6)$$

To construct the curve of dependence of  $n_{tr}$  on  $n_0$  a large number of tapers of various types of airplanes were observed, from sesquiplanes to 42-ton airplanes, including land and sea aircraft. From this study it was found that for seaplanes, for the same  $n_0$  as for the land airplanes,  $n_{tr}$  has a smaller value; that is, the center of gravity of the outer wing of seaplanes having the same dimensions as the land airplanes lies nearer the tip of the wing than in the case of the land airplane. This is understandable since each outer wing of a seaplane generally has at the wing tip a water-tight segment which shifts the center of gravity toward the tip of the wing. Hence, the dependence of  $n_{tr}$  on  $n_0$  was constructed in the form of two curves, one for seaplanes and the other for land planes. (See fig. 16.)

The smallest value of  $n_0$  for outer wings of a trapezoidal plan form was equal to 10. In order to be able to construct the initial part of the curve shown in figure 16, the data were taken on the outer wing having a rectangular plan form (P-6) where  $n_0 = 1$  and there were also plotted the data for the center wings (fig. 16 denoted by crosses).

Inspection of the curves in figure 16 shows the almost complete absence of scatter and this indicates the existence of a definite law. Therefore, knowing the weight and dimensions of any outer wing of longeron construction of the CAHI type, it is possible with the aid of these curves to determine with great accuracy and very rapidly the law of spanwise weight distribution and the position of the center of gravity.

## 2. Center Wing

All formulas given for determining the distribution over the span of the center wing were empirically derived and are presented below. The sharp drop of the diagram at the middle of the center wing (fig. 13) is due to the fact that only longerons pass through the fuselage for low-wing airplanes, ribs, stringers, and skin, being absent over the entire width  $b_f$ .

### FORMULAS FOR DETERMINING THE SPANWISE STRUCTURAL WEIGHT DISTRIBUTION OF THE WING

#### 1. General Observations

All formulas given below for determining the structural weight distribution of the wing along the span refer to wings of the longeron construction of the CAHI type and are independent of the tonnage of the airplane and location of the engines. The latter may be located in the fuselage or in the center or outer wing. The weight content of the center wing and taper entering the formulas given below should be reduced to conform with the structural weight as defined above. The presence of fuel tanks in the center wing and in the outer wing does not affect the formula for the structural weight distribution.

A check of the formulas for the distribution and center of gravity location gave excellent results, the formulas for the distribution on the center wing giving somewhat less favorable agreement since in these formulas it is necessary to substitute the magnitude  $n$  determined from the dimensions of the center wing, and a curve which establishes the relation between  $n_{cw}$  and  $n_{cw\ tr}$  does not yet exist as for the case of the outer wing. To illustrate to what extent  $n_{cw}$  fails to agree with  $n_{cw\ tr}$  the following data on four different airplanes is given:

Aeroplane	1	2	3	4
$n_{cw}$	1.00	1.05	1.13	1.21
$n_{cw\ tr}$	1.20	1.10	1.05	1.60

In figure 19,  $p_1$  is the weight of the joints of the outer wing parts including the weights of the bolts and nuts, and  $p_2$  the weight of the joints on one side of the center wing.

## 2. Outer Wing

Determination of  $q_0$  and  $q_1$

$$q_0 = \frac{2(P_{ow} - p_1)}{(n_{tr} + 1)l_1} \quad (7)$$

The weight of the joints of the outer wing is determined by the curve in figure 20 as a function of the taper ratio given by the formula

$$n_0 = \frac{t_1 a_1}{t_0 a_0} \quad (8)$$

The value of  $n_0$  is determined by the curve shown in figure 16 as a function of the magnitude  $n_0$ .

$$q_1 = n_{tr} q_0 \quad (9)$$

The curves in figure 21 show the change of the ordinate  $q_0$  expressed in percent of  $q_1$  as a function of the change in the taper ratio  $n_0$ .

Determination of the distance from the wide end of the outer wing to the center of gravity of the outer wing.-

$$l_{c.g.ow} = \frac{l_1}{3} \frac{n_{tr} + 2}{n_{tr} + 1} \quad (10)$$

The distance to the center of gravity of the outer wing may be also determined graphically by the curve shown in figure 22 as a function of the taper coefficient  $n_0$ .

## 3. Center Wing of Low-Wing Airplane

Determination of  $q_2$ ,  $q_3$ , and  $q_4$  (fig. 19).

$$q_2 = \frac{P_{cw} - 2p_2}{(n_{cw} + 1)(l_2 + 0.2b_f)} \quad (11)$$

The weight of the joints  $p_2$  is determined by the curve shown in figure 23 as a function of the taper coefficient of the center wing  $n_{cw}$  determined by the formula

$$n_{cw} = \frac{l_2 a_2}{t_1 a_1} \quad (12)$$

$$q_3 = n_{cw} q_2 \quad (13)$$

$$q_4 = \frac{(P_{cw} - 2p_2) - [(q_2 + q_3)l_2]}{b_f} \quad (14)$$

Determination of the distance to the center of gravity of half the center wing (fig. 19).-

$$l_{c.g. cw} = \frac{l_2 n_{cw} + 2}{3 n_{cw} + 1} \quad (15)$$

#### 4. Center Wing of a High-Wing Airplane

(Wing above Fuselage)

Determination of  $q'_2$  and  $q'_3$  (fig. 24) and the distance to center of gravity of half the center wing.

$$q'_2 = \frac{P'_{cw} - 2p_2}{(n_{cw} + 1)0.5l_3} \quad (16)$$

The magnitude  $2p_2$  is determined by the curve shown in figure 23. The magnitude  $n_{cw}$  is determined by formula (12)

$$q'_3 = n_{cw} q'_2 \quad (17)$$

The distance to the center of gravity of half the center wing is determined by the formula

$$l_{c.g. cw} = \frac{0.5l_3}{3} \frac{ncw + 2}{ncw + 1} \quad (18)$$

APPORTIONMENT OF WEIGHT OF WING BETWEEN THE CENTER  
AND OUTER WINGS

In a preliminary design of an airplane when the total weight of the airplane is obtained from available data the problem often arises how this weight is to be apportioned between the center and outer wing, knowing the areas of the latter. The solution of the problem is necessary in the first place in order to construct the diagram of the span-wise weight distribution of the wing, and in the second place for finding the theoretical center of gravity of the airplane.

An analysis of the weights of wings of airplanes of the CAHI type having a longeron construction showed that between the weight of the center wing, expressed in percent of weight of the entire wing, and the area of the center wing, expressed in percent of the area of the entire wing, there exists a sufficiently sharply expressed relation. This relation is represented in figure 25 and may be expressed by the following formulas:

1. For a Low Wing

$$\frac{P_{cw}}{P_{wing}} \times 100 = \frac{S_{cw} + 0.5S_f}{S_{wing} - 0.5S_f} 100 + 12 \quad (19)$$

From this the formula for the determination of the structural weight of the center wing is obtained

$$P_{cw} = P_{wing} \left[ \frac{S_{cw} + 0.5S_f}{S_{wing} - 0.5S_f} + 0.12 \right] \quad (20)$$

2. For a High Wing

The formula for determining the structural weight of the center wing is

$$P'_{cw} = P'_{wing} \left[ \frac{S_{cw} + S_f}{S_{wing}} + 0.12 \right] \quad (21)$$

Translation by S. Reiss,  
National Advisory Committee  
for Aeronautics.

TABLE I.

Group	Bending moment at joint (percent)			Remarks
	True distrib- ution	Triangu- lar distrib- ution	Distribution proportional to chords	
I	100	93.0	115.0	Single-engine land plane
	100	97.0	121.0	Two-engine land plane
	100	93.0	121.0	Two-engine land plane
	100	93.0	107.6	Marine airplane
Mean percent	100	94.0	116.2	
II	100	100.2	116.1	Long range-airplane
	100	112.0	121.0	"Maxim Gorky"
Mean percent	100	106.1	118.6	

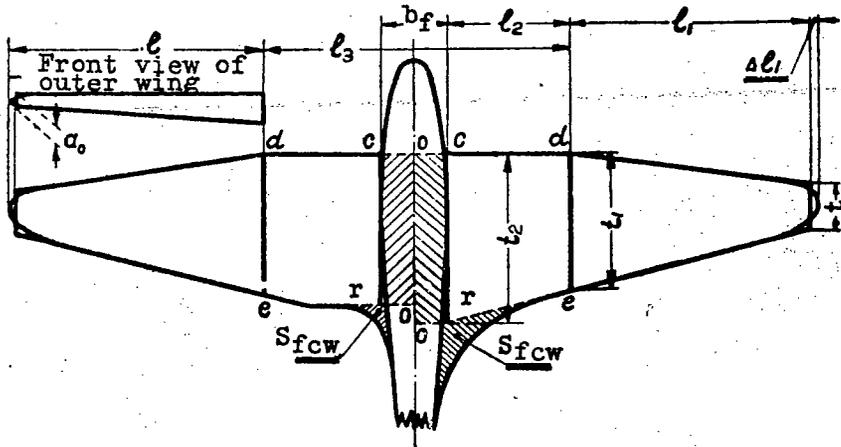


Figure 1

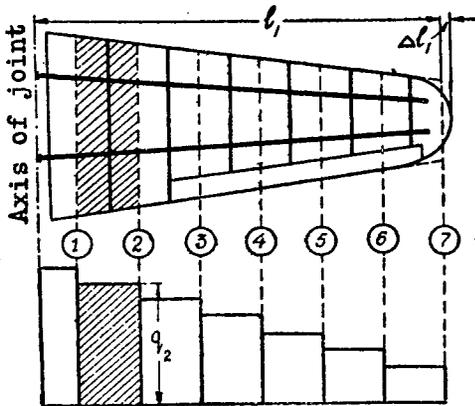


Figure 2



Figure 3

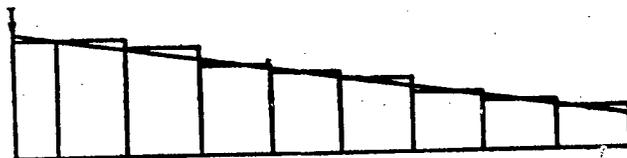
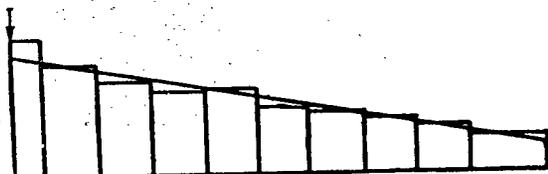


Figure 4

Figure 5



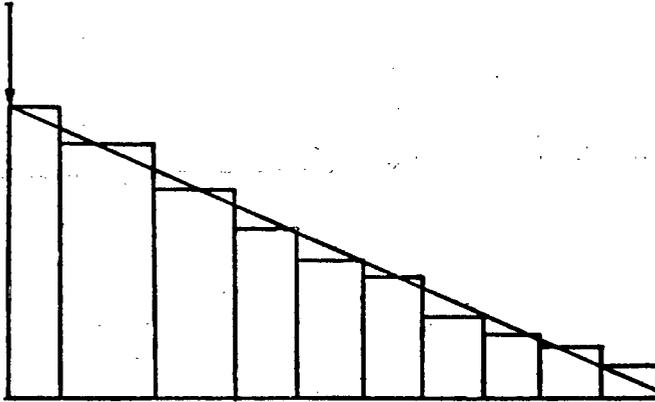


Figure 6

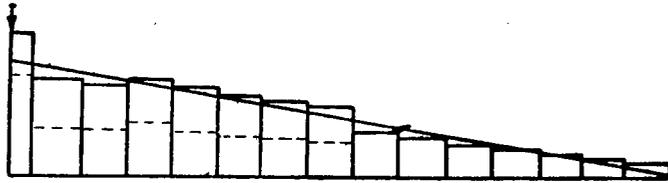


Figure 7

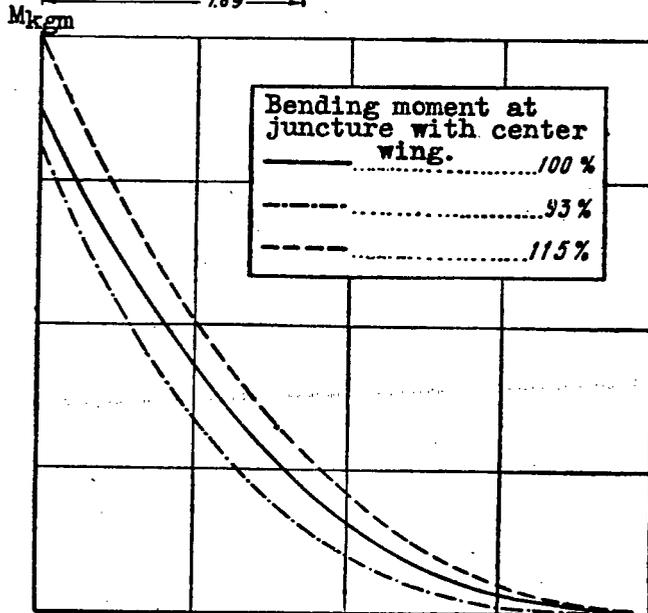
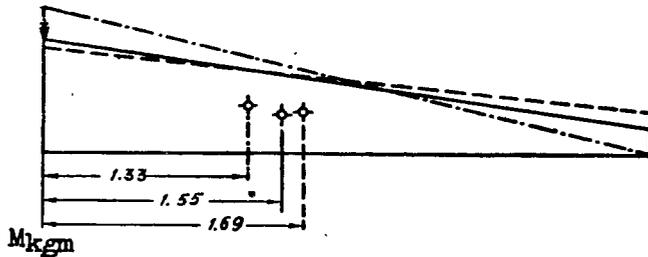


Figure 8

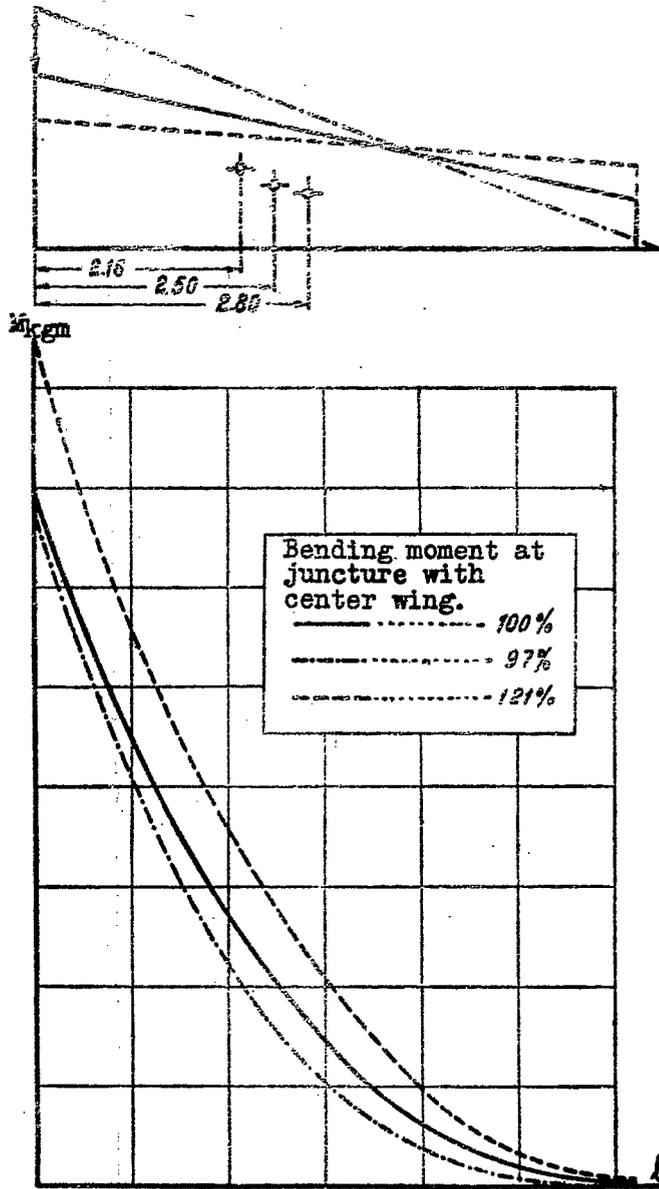


Figure 9

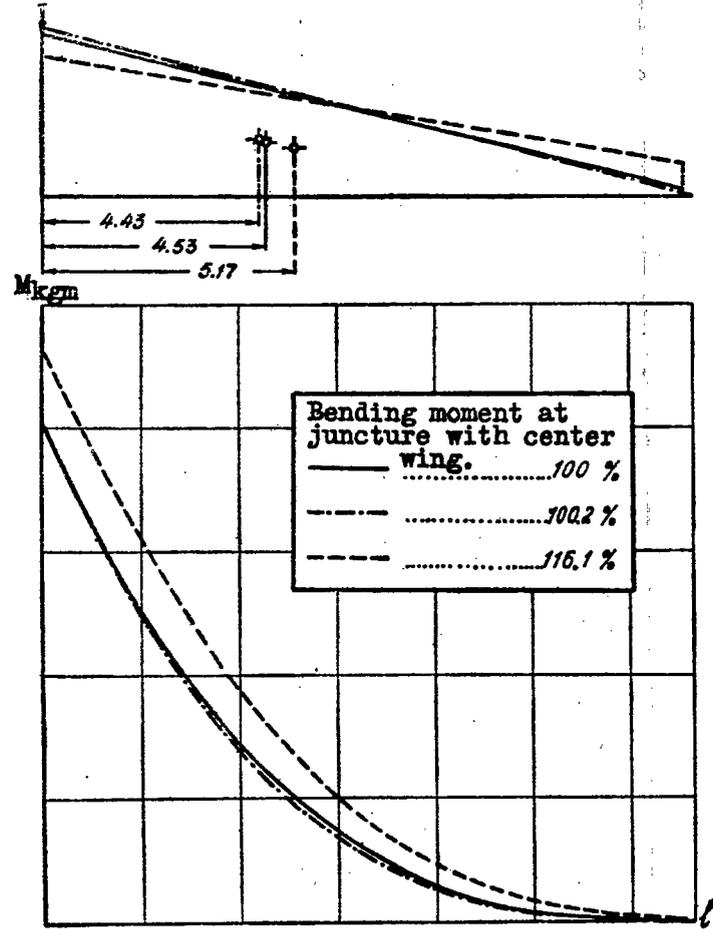


Figure 10

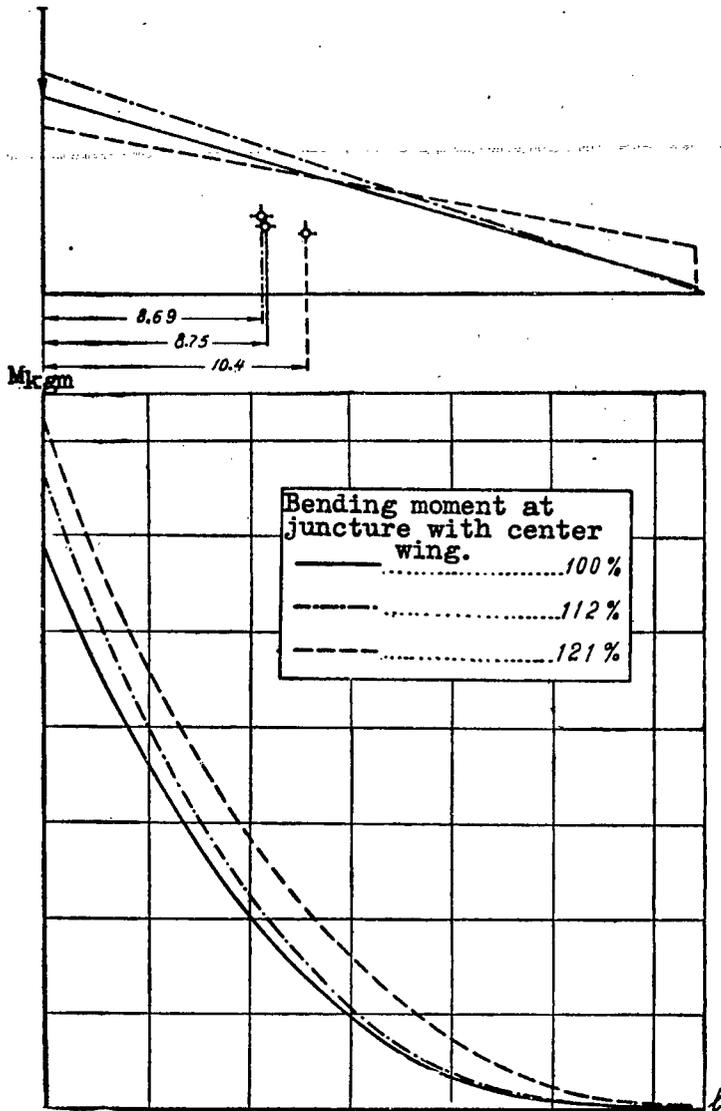


Figure 11

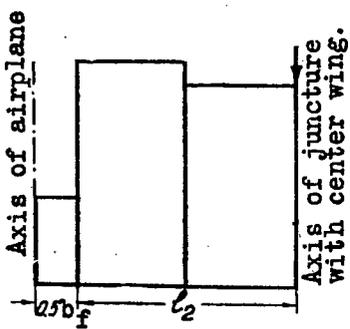


Figure 12

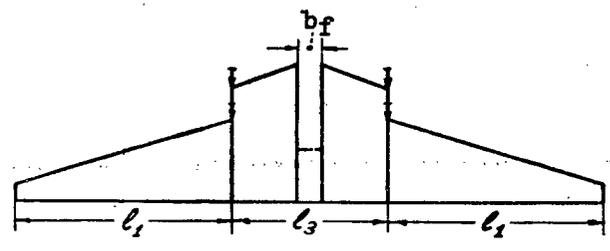


Figure 13

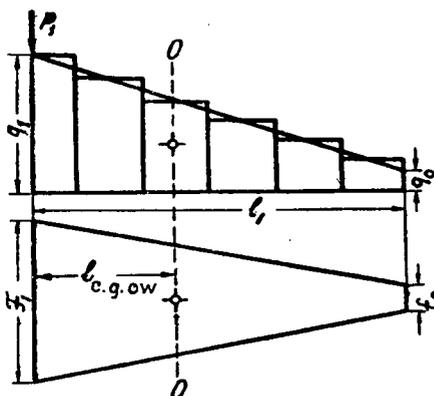
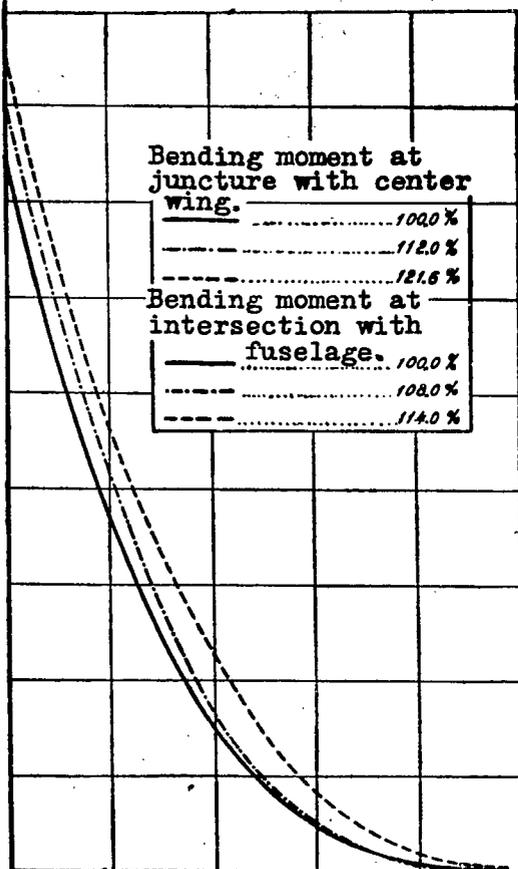
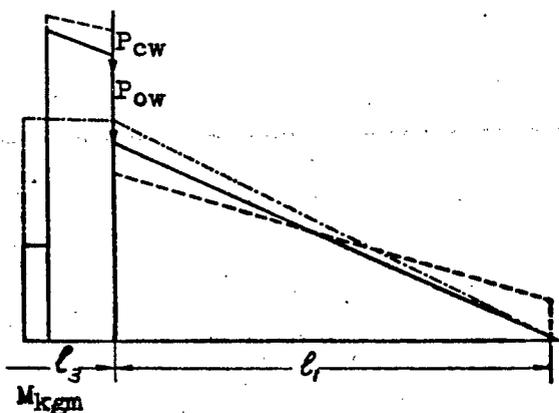


Figure 15

Figure 14

1. ——— Triangular distribution.
2. ..... Distribution proportioned to chords.
3. ——— True distribution.

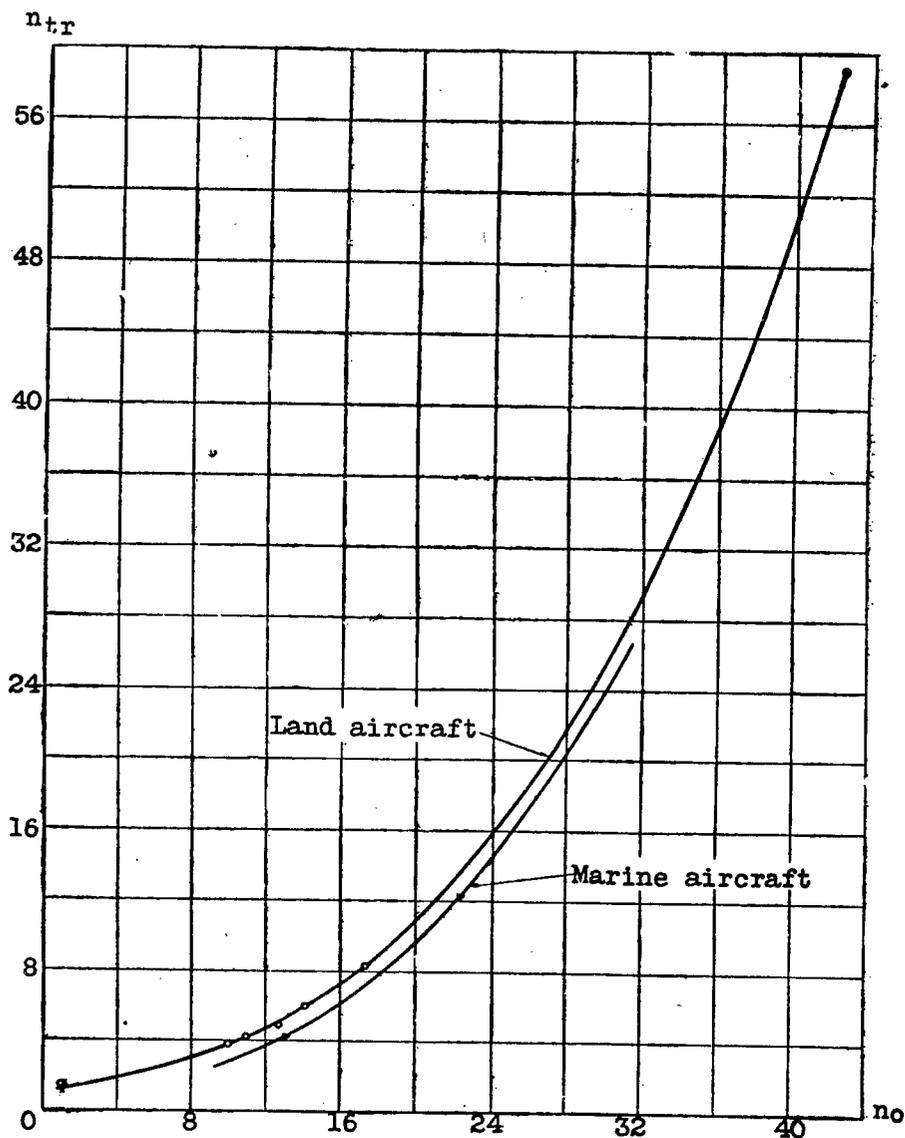


Figure 16

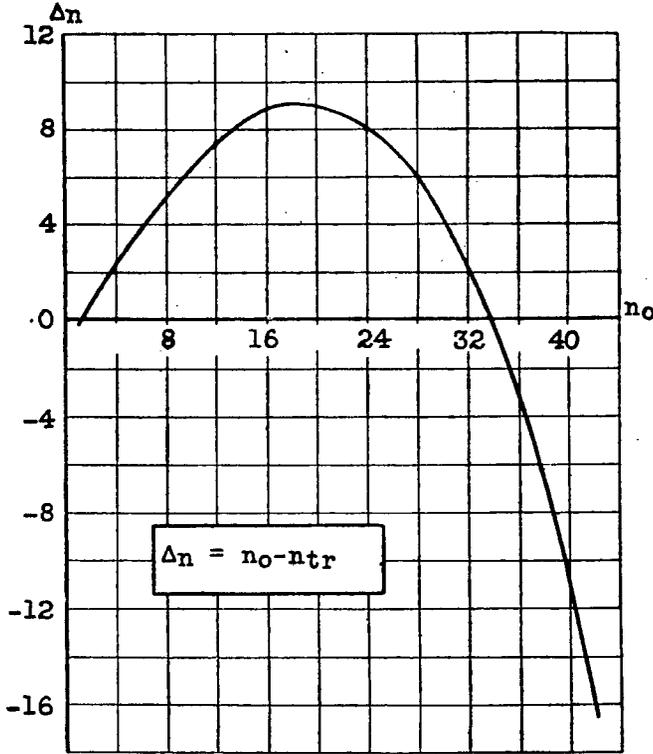


Figure 17

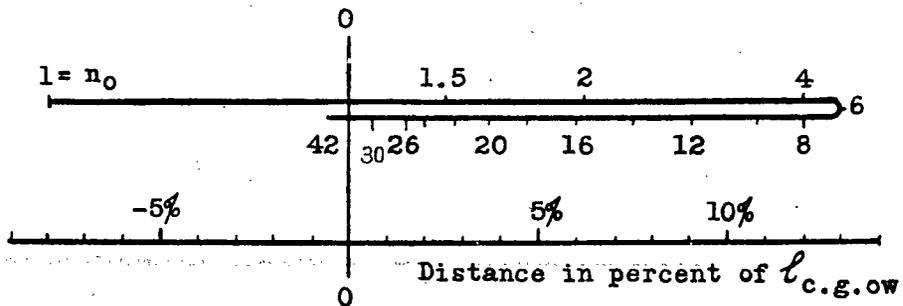


Figure 18

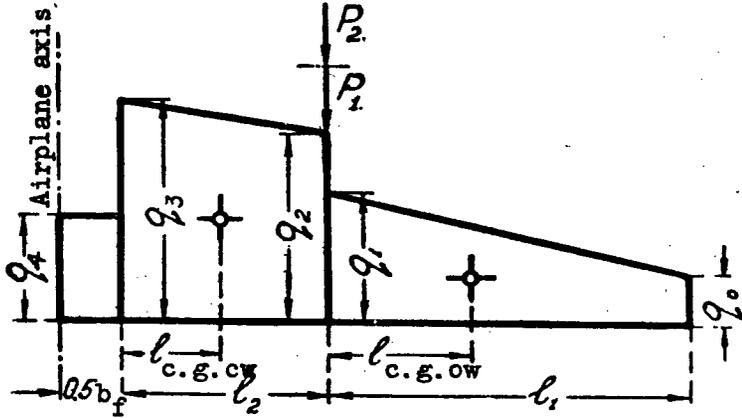


Figure 19

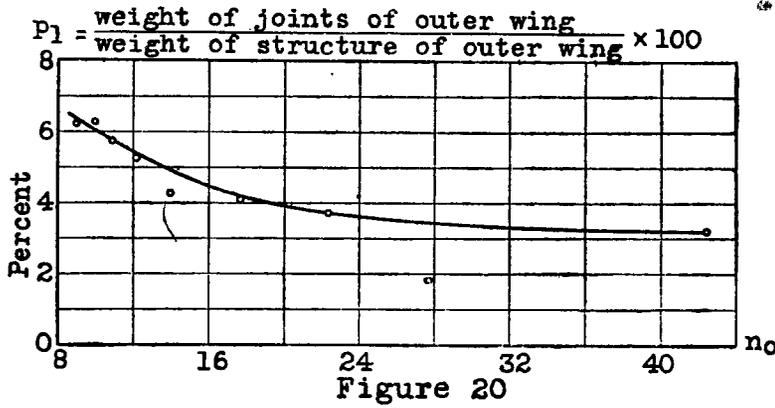
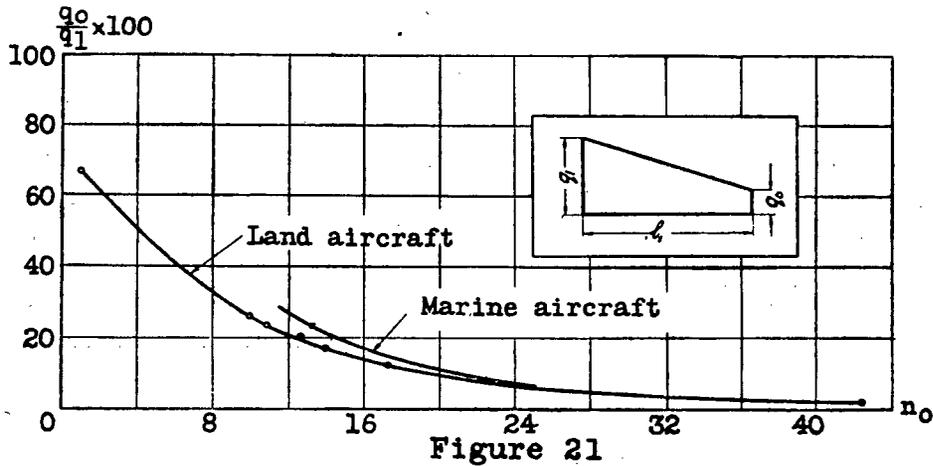
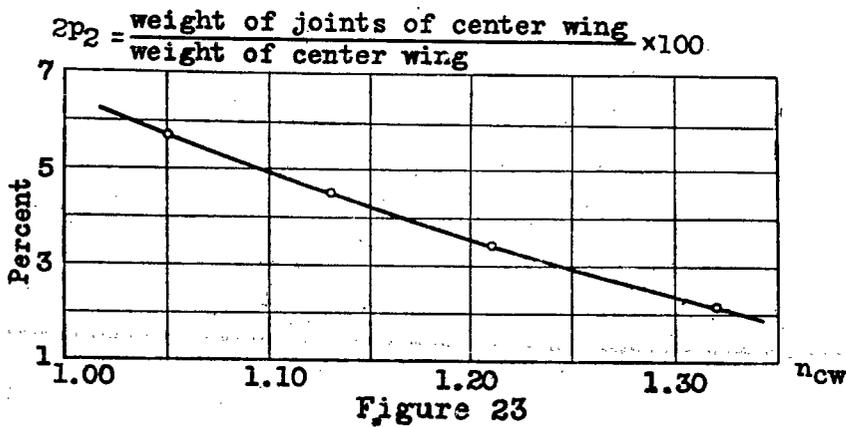
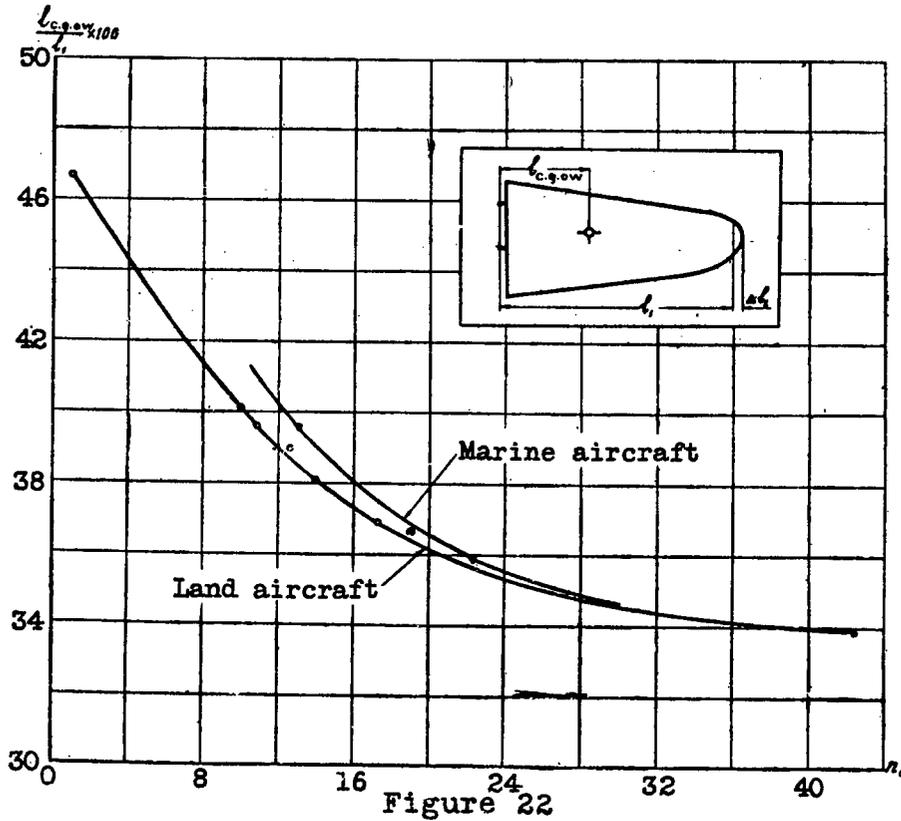


Figure 20





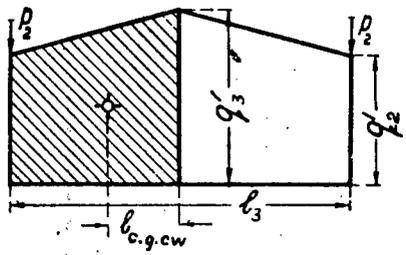


Figure 24

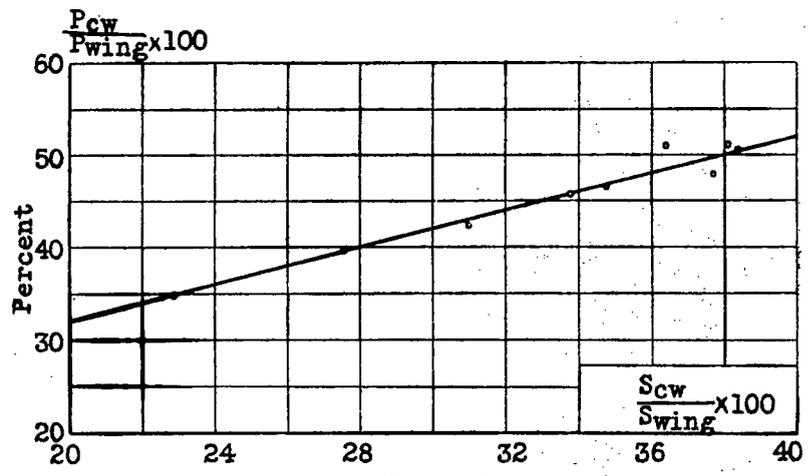


Figure 25

NASA Technical Library



3 1176 01440 7770