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TECHNICAL MEMORANDUM

No. 1142

FUNDAMENTALS OF THE CONTROL OF GAS-TURBINE
POWER PLANTS FOR AIRCRAFT

PART I
STANDARDIZATION OF THE COMPUTATIONS RELATING TO THE
CONTROL OF GAS-TURBINE POWER PLANTS FOR AIRCRAFT
BY THE EMPLOYMENT OF THE LAWS OF SIMILARITY

By H. Kühl

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FUNDAMENTALS OF THE CONTROL OF GAS-TURBINE

POWER PLANTS FOR AIRCRAFT*

PART I

Standardization of the Computations Relating to the
Control of Gas-Turbine Power Plants for Aircraft
by the Employment of the Laws of Similarity

By H. Kühl

SUMMARY

It will be shown that by the use of the concept of similarity a
simple representation of the characteristic curves of a compressor
operating in combination with a turbine may be obtained with correct
allowance for the effect of temperature. Furthermore, it becomes
possible to simplify considerably the rather tedious investigations
of the behavior of gas-turbine power plants under different operating
conditions. Characteristic values will be derived for the most
important elements of operating behavior of the power plant, which
will be independent of the absolute values of pressure and temperature.
At the same time, the investigations provide the basis for scale-model
tests on compressors and turbines.

INTRODUCTION

In various analytical investigations on combustion turbines,
especially investigations on control, there arises the problem of
determining the behavior of the power plant under all possible oper-
ating conditions. In contrast to the internal-combustion engine,
where the influence of the various operating conditions is easily

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discernible and may be expressed by simple relations, the interactions
in gas turbines are very complicated. The number of variable factors
is large, namely, flight speed, atmospheric pressure, atmospheric tem-
perature, power-plant speed, and the settings of the adjustable devices
that may be available.

The computations will be made substantially easier if it is
possible to eliminate the variation of even one of these factors.
In the first place, it follows from the thermodynamic relations for
compressors and turbines (reference 1), because at any given time
only pressure relations arise, that a complete recomputation for each
different value of absolute pressure is not necessary so long as no
alteration of the characteristic curves occurs. On the basis of the
concept of similarity, it is also possible to dispense with recompu-
tation for various absolute values of the temperature, as will be
shown.

This concept of similarity serves at the same time as the basis
for the evaluation and carrying out of scale-model tests. As power-
plant outputs tend to higher and higher levels, it will be increasingly
necessary, especially in view of the great power required to drive
the test rigs for the investigations of the component units, to resort
to taking the required measurements on compressors and turbines not on
the full-scale equipment but on reduced scale models suitable to the
available test rigs and to extend the test results to the full-scale
equipment.

The following research is limited to the gas-turbine power plants
that are of the most immediate practical importance today (TL, PTL,
and ZTL power plants) with combustion occurring at constant pressure
and without a heat exchanger. [NACA comment:  jet, TL; turbine-
propeller jet, PTL; ducted-fan jet, ZTL.]


## I. FUNDAMENTALS AND PREMISSES OF CONCEPT OF SIMILARITY

Flow processes in gases are customarily said to be similar when
there is complete geometric similarity and all velocity and pressure
ratios are the same. According to the laws of similarity, gas-kinetic
processes in flow machines (references 2 to 6) are similar when,
assuming geometric similarity and equality of pressure and temperature
ratios at corresponding points of the inlet cross section, the fol-
lowing characteristic values are respectively equal:

1. Velocity ratio, ratio of inflow velocity to peripheral velocity

2. Mach number

3. Reynolds number at corresponding points in the machine

4. Prandtl number

5. Exponents of the adiabatic curve at corresponding points in the machine

6. Ratios of gas weights, especially that of combustion gas to air, at corresponding points in the machine

Because this report deals exclusively with power plants without heat exchangers, the heat transfer and consequently the Prandtl number, as well as the influence of Reynolds number on the heat transfer, need not be taken into account[1].

In order to obtain simple equations, it is furthermore necessary to disregard the influence of Reynolds number. (See section IV.)

The exponents of the adiabatic curves and the ratio of gas weight to air weight remain unchanged at various absolute pressures but they do vary at different absolute temperatures. The differences, however, are not too large. Because the temperatures at the inlet are variable only within certain limits, in this report differences in these factors will be disregarded when the operating states are otherwise similar. A more exact check of their effect will be made later (section IV).

Furthermore, the mechanical friction in the bearings and so forth will be taken to be proportional to the work done by the gas so that the similarity will extend not only to the internal processes (internal work output) but also to the over-all thermodynamic behavior (effective output) of the power plant.

## II. COMPRESSOR-CHARACTERISTIC CURVES WHEN OPERATING

### IN CONJUNCTION WITH THE TURBINE

In order to exhibit the characteristics of the compressor, a family of curves is generally used in which the adiabatic pressure

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[1]In the case of cooled turbine blades, so long as no additional regulation of the cooling air dependent upon the gas temperature is provided, the changes in heat transfer in the blade are without important effect on the thermodynamic behavior of the power plant.

head for various peripheral speeds of the compressor is plotted against the volume of inlet air. This method of representation is, however, not well adapted for the mathematical description of gas-turbine power plants with axial compressors of large pressure head because the adiabatic pressure head and the efficiency of the compressor vary considerably with the temperature of the inlet air. The variations in the inlet temperature are composed of the changes in the Ina [NACA comment: International standard atmosphere] temperature with altitude (which is 71.5° C between altitudes of 0 and 11 km), the deviations from the Ina temperature, and the varying heating of the air due to ram so the temperature range in question is approximately from -55° to 70° C.

As an example of the influence of inlet temperature, figure 1 shows the adiabatic pressure head and efficiency at various inlet-air temperatures of an eight-stage axial compressor, the characteristic curves of which were computed on the basis of measurements made by the AVA at Göttingen on single stages and with the use of simplifying assumptions. The particular contour of the curves will vary widely for different compressors; but a marked dependence upon the inlet temperature is, in general, to be expected because of the great influence of Mach number on the individual stages, the very steep contour of the characteristic curves, and the high total adiabatic pressure heads of the type of compressor under consideration.

As previously shown (reference 2), with the help of the concept of similarity, a characteristic diagram, which is valid at all temperatures for geometrically similar compressors[2], may be obtained by taking as the abscissa the value $V_1/D^2 \cdot \sqrt{g\kappa_L R_L T_1}$, as the ordinate either the value $\dfrac{H_{ad}}{\kappa_L R_L T_1}$ or $p_2/p_1$, and as the parameter the Mach number of the peripheral speed $u/\sqrt{g\kappa_L R_L T_1}$. (See fig 2(a).) The symbols are defined as follows:

$p_1$, $T_1$    pressure and temperature ahead of compressor

$p_2$, $T_2$    pressure and temperature behind compressor

$V_1$    volume of inlet air

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[2]The similarity must extend also to the supply and disposal of the air, that is, the distribution of velocity, pressure, and temperature must be similar especially at the inlet (in general, velocity, pressure, and temperature being constant).

u         peripheral speed

n         speed

$H_{ad}$       $= \dfrac{\kappa_L}{\kappa_L - 1} R_L T_1 \left[ \left( \dfrac{p_2}{p_1} \right)^{(\kappa_L - 1)/\kappa_L} - 1 \right]$, adiabatic heat rise

$\kappa_L$        ratio of specific heats at constant pressure and constant volume for air

$R_L$       gas constant for air

D         rotor diameter (for example, of the first stage)

g         acceleration of gravity

$T_{1n}$      selected reference temperature at which measurements (or computations) were made

Because for the compression of air in a given compressor some of these factors are constant, by giving up the pure nondimensional representation, the individual quantities may also be simplified and the abscissa taken as $V_1/\sqrt{T_1}$ or $V_1\sqrt{T_{1n}/T_1}$, the ordinate as $H_{ad}/T_1$ or $H_{ad}T_{1n}/T_1$, and the parameter as either $n/\sqrt{T_1}$ or $n\sqrt{T_{1n}/T_1}$ or else $u/\sqrt{T_1}$ or $u\sqrt{T_{1n}/T_1}$. (See figs. 2(b) and (c).)

The pressures, the temperatures, and the volumes are based on the gas at rest. Hence the values at the stagnation point are to be inserted in each case. The effect of Reynolds number and of the heat exchange with the surroundings are ignored in this representation. Of the two quantities mentioned as possible ordinates for the diagram the pressure ratio should generally be preferred because the representation of the joint operation of compressor and turbine is desired. In the characteristic diagrams given in figure 2, lines of equal efficiencies have also been plotted.

In the case of all the gas-turbine power plants considered in this report, the air leaving the compressor enters the combustion chamber, is there heated to a certain temperature by the combustion of fuel, and is then led to the turbine. The quantity of air compressed in the compressor may be computed from the air-flow capacity of the turbine nozzles. The following equation applies:

$$G_G = G_L (1 + m) = F_t \mu X \frac{p_3}{\sqrt{R_G T_3}}$$

The symbols are defined as follows:

$p_3, T_3$     pressure and temperature ahead of turbine

$G_G$          weight of gas per unit time

$G_L$          weight of air per unit time

$m$            mixture ratio (weight ratio) of fuel to air

$F_t$          smallest cross section of turbine nozzle

$R_G$          gas constant of combustion gases

$\kappa_G$     ratio of specific heats at constant pressure and at constant
               volume for combustion gases (dependent upon temperature
               and mixture ratio)

$\mu$          outlet coefficient

$X$            $\sqrt{g\ \kappa_G \left(\dfrac{2}{\kappa_G + 1}\right)^{(\kappa_G+1)/\kappa_G-1)}}$

when

$$\frac{p_{3a}}{p_3} < \frac{p_k{}'}{p_3}$$

or

$$\frac{F_{ta}}{F_t} \sqrt{\frac{2g\ \kappa_G}{\kappa_G - 1} \left[\left(\frac{p_{3a}}{p_3}\right)^{2/\kappa_G} - \left(\frac{p_{3a}}{p_3}\right)^{(\kappa_G+1)/\kappa_G}\right]}$$

when

$$\frac{p_{3a}}{p_3} > \frac{p_k{}'}{p_3}$$

$p_{3a}$       pressure turbine nozzle

$F_{ta}$       outlet cross section of turbine nozzle (in cylindrical non-
               flaring nozzle  $F_{ta} = F_t$)

$p_k{}'$      pressure at which maximum attainable velocity in $F_t$, or critical velocity, is reached. In cylindrical nonflaring

nozzle $\quad p_k{}'/p_3 = \left(\dfrac{2}{\kappa_G + 1}\right)^{\kappa_G/(\kappa_G-1)} \quad$ (The effect of friction

is unimportant in this connection.)

.For gas-turbine power plants in general, only the region in which X is constant is of practical importance with regard to the required flight performance especially as the variation of X, when the pressure is slightly above $p_k{}'$, is practically insignificant. This applies also to multistage turbines and to turbines operating with great reaction, because in this case with diminishing adiabatic heat drop there is a sharp increase in the ratio of the drop converted in the nozzle to the total drop.

If the value of $G_L$ obtained from the flow equation for the turbine nozzle is inserted into the equation

$$\frac{V_1}{\sqrt{T_1}} = \frac{G_L R_L T_1}{p_1 \sqrt{T_1}}$$

the following equation is obtained:

$$\frac{V_1}{\sqrt{T_1}} = F_t \, \frac{p_2}{p_1} \sqrt{\frac{T_1}{T_3}} \, \frac{X}{1 + m} \, \mu \, \frac{R_L}{\sqrt{R_G}} \, \frac{p_3}{p_2}$$

or, if

$$\frac{X}{1 + m} \, \mu \, \frac{R_L}{\sqrt{R_G}} \, \frac{p_3}{p_2} = \alpha$$

$$\frac{V_1}{\sqrt{T_1}} = F_t \, \frac{p_2}{p_1} \sqrt{\frac{T_1}{T_3}} \, \alpha$$

If the critical velocity is reached in the turbine nozzle, then, because $p_3/p_2$ is also nearly constant, the factor $\alpha$ varies only slightly with various operating conditions. The difference amounts in extreme cases to only a few percent. If the temperature ahead of the compressor varies within the range encountered in practice and if, on the other hand, the ratio $T_3/T_1$ remains constant, then the variation of $\alpha$ will be less then 1 percent. Hence for a given value of $T_3/T_1$, the value of $\alpha$ may be taken as constant (equal to an average value).

If the critical velocity is attained in the nozzle of the turbine at a constant value of $T_3/T_1$ and with unvarying nozzle cross section, all operating states will lie on a straight line through zero in the compressor-characteristic diagrams constructed as in figure 2. This is shown in figure 3.

The slope of any one of the straight lines for $T_3/T_1$ = constant, that is, the tangent of the angle between it and the abscissa, is proportional to $\sqrt{T_3/T_1}/\alpha$ (or roughly, proportional to $\sqrt{T_3/T_1}$), provided that an average value for $\alpha$ is inserted in each case with the respective temperature ratio.

If the critical velocity is not attained, then the operating states of the compressor at a given temperature ratio can no longer be represented by one line; they are then also influenced by the back pressure in the turbine, that is, by atmospheric pressure.

These considerations apply to all gas-turbine power plants in which the whole quantity of air drawn in by the compressor flows through the turbine. The influence of a small quantity of air diverted in the intermediate stages for the cooling of the turbine may generally be disregarded if the compressor is appropriately designed. However, this quantity of air is to be deducted in the calculation of $V_1$ and $G_L$.

In drawing a characteristic diagram for the turbine, the general laws of similarity must also be taken into consideration. For example, an unobjectionable representation will be obtained if, as is generally done, the efficiency, or other desired characteristics, is plotted against the ratio of the peripheral speed $u$ to the speed $c_{ad}$ corresponding to the adiabatic heat rise for various pressure ratios in the turbine (reference 7). For the computation of the jet power plant, the appropriate representation is found to be one in which the efficiency is plotted against the ratio of the peripheral speed to the speed $\sqrt{2g\ L_t}$ corresponding to the turbine output $L_t$ for various values of $L_t/R_G T_3$, inasmuch as these values are known in this case.

## III. SIMILAR OPERATING STATES OF THE POWER PLANT

The principles of similarity will now be extended to the whole power plant. For this purpose it must first be determined when similar operating states exist. The first requirement, geometric similarity of the flow processes, means first of all that all the adjustable regulating devices with which the power plants are equipped and which

directly influence the flow in the power plants shall be fixed at the same settings. The fuel-supply devices are not counted among these adjustable regulating devices, as the quantity of fuel supplied only indirectly affects the gas flow; it does so through the temperature rise (expressed by $T_3/T_1$), which takes place during combustion.

As the compressor-characteristic diagram of the type shown in figure 2 is constructed on the principles of similarity, the requirements of similarity for the compressor are automatically fulfilled when it is operating at the same point of its characteristic diagram. If similarity is to apply to the processes in the whole power plant, then (see section II) the velocity relations, Mach numbers, pressure relations and so forth in the other parts of the power plant must also coincide. If

$w_0$        flight velocity

$Ma_0$      Mach number of flight velocity

$$= w_0/\sqrt{g\kappa_L R_L T_0}$$

and

$p_0$, $T_0$    pressure and temperature of atmosphere

the following requirements for the scoop must be fulfilled:

$$\frac{w_0}{u} = \text{constant}$$

$$Ma_0 = \text{constant}$$

or

$$\frac{p_1}{p_0} = \text{constant}$$

These requirements can be expressed in terms of each other so the determination of one of these values is sufficient.

By means of one of these values, for example $p_1/p_0$, plus the operating point in the compressor-characteristic diagram, for example $n/\sqrt{T_1}$ and $T_3/T_1$, the pressure and the velocity ratios in the turbine and in the jet nozzle are uniquely determined; a fact which is readily apparent if the turbine and the jet nozzle are regarded as a unit. As the individual pressure and temperature ratios are constant and the speed is proportional to $\sqrt{T_1}$, the velocity $c_{ad}$ corresponding

to the adiabatic heat rise is therefore proportional to $\sqrt{T_1}$, the peripheral speed is proportional to $\sqrt{T_1}$, and the velocity of sound is also proportional to $\sqrt{T_1}$, so that $u/c_{ad}$ and the Mach number of the peripheral speed are constant. But under these conditions, similarity exists for the flow processes in the turbine and the jet nozzle. Thus the similarity extends to the whole power plant.

In reality, however, an arbitrary fixing of the values of three independently variable quantities such as $p_1/p_0$, $n/\sqrt{T_1}$, and $T_3/T_1$ is not possible without violating the work balance. When two independent variables, for example $p_1/p_0$ and $n/\sqrt{T_1}$, are given, the third can no longer be arbitrarily chosen but on the contrary is determined by the work balance.

Thus, if there exists geometric similarity of the power plant, that is, equal settings of all adjustable regulating parts and if two characteristic values (either nondimensional ones or proportional to nondimensional ones) are given, all other characteristic values are thereby fixed. In addition to the pressure, the temperature, and the velocity ratios, the Mach numbers, and the characteristic values derived from them, such as $n/\sqrt{T_1}$, as nondimensional characteristic values must also be included the efficiency and above all the characteristic values that serve to determine the thrust force and the specific and absolute fuel consumptions, which values must therefore be formulated next.

Because of the similarity of the velocity ratios, the general Newtonian law of similarity applies to the thrust force:

$$\frac{S}{\rho_0 F w_0^2} = \text{constant}$$

[NACA comment: S, thrust force.]

If for the reference area $F$ the flow area of the turbine nozzle $F_t$ is used, $Ma_0$ is substituted for $w_0$, and the gas equation is used in place of the density $\rho_0$

$$\frac{S}{\dfrac{p_0}{g R_L T_0} F_t \, Ma_0^2 \, g \kappa_L R_L T_0} = \text{constant}$$

or

$$\frac{S}{F_t p_0} = \sigma = \text{constant}$$

In this equation there is a very useful characteristic value for the thrust, which will be called the thrust coefficient $\sigma$. The thrust coefficient $\sigma$ expresses the thrust per unit of nozzle area and at a pressure of 1 atmosphere.

An important consequence of the above is that given similar operating states (that is, constant pressure and temperature ratios) the thrust is independent of the absolute temperature.

As all pressure and temperature ratios are constant in the case of similar operating states, the characteristic values may be based on pressures and temperatures other than $p_0$ and $T_0$. In dealing with the whole power plant, the state of the atmosphere is generally given so that $p_0$ and $T_0$ are naturally preferred. In other cases $p_1$ and $T_1$, for example, may be more appropriate (compressor-characteristic diagram); thus $\sigma_1 = S/F_t\, p_1$.

For the specific fuel consumption $b_s$, that is, the weight of fuel required per unit time to produce one unit of thrust (generally given in g/kg sec), a simple equation is obtained from

$$sw_0 = \frac{H_u}{A}\, m\eta_{ges}$$

in which $s$ denotes the momentum per unit weight of air, $H_u/A$ the work corresponding to the lower heating value of the fuel, and $\eta_{ges}$ the over-all efficiency of the power plant. By passing to the limit, this equation may even be extended to the value $w_0 = 0$. The equation is

$$\frac{m}{s} \cdot \frac{\eta_{ges}\, H_u/A}{Ma_0\, \sqrt{g\kappa_L R_L T_0}} = constant$$

and hence for similar operating states

$$b_s \cdot \frac{H_u/A}{\sqrt{g\kappa_L R_L T_0}} = constant$$

or likewise $b_s/\sqrt{T_0} = constant$ or $b_s\sqrt{T_{on}/T_0} = constant$.

The specific fuel consumption for similar operating states is thus proportional to the square root of the absolute temperature.

Finally, from $G_K = b_s S$ is obtained an equation for the fuel consumption $G_K$ per unit time. Given similar operating states

$$b_s \frac{H_u/A}{\sqrt{g\kappa_L R_L T_o}} \frac{S}{F_t p_o} = G_K \frac{H_u}{A} \frac{1}{F_t p_o} \frac{1}{\sqrt{g\kappa_L R_L T_o}} = \text{constant}$$

or likewise

$$\frac{G_K}{p_o \sqrt{T_o}} = \text{constant}$$

or it may be expressed otherwise as required, for example, $G_K/p_3 \sqrt{T_1}$ = constant. The fuel consumption per unit time is thus, for similar operating states, proportional to the absolute pressure and to the square root of the absolute temperature.


## IV. ACCURACY OF RESULTS

In the derivation of the various formulas the simplifying assumptions that have been made and the factors that have been disregarded have been specified. Therefore the order of magnitude of the errors thus introduced must next be determined.

In the first place, the influence of Reynolds number was disregarded. The variation in Reynolds number encountered in operating states of the same power plant, which here have been regarded as similar, is not inconsiderable. Between 0 and 10 kilometers it varies for the compressor over a ratio of about 2.5:1 and for the turbine over a ratio of almost 4:1. In scale-model tests of water turbines, the difference in Reynolds number as compared with the full-scale construction is generally very much higher. In general the efficiency of the compressor (reference 8) and the turbine will increase with increasing Reynolds number, as will also the pressure ratio in the compressor[3] and the volume of flow through the turbine; but a variation in the other direction is not excluded (references 6, 9, and 10). More precise investigations of this question, such as are in preparation at various places, particularly investigations of the quantitative influence of Reynolds number in compressors and turbines of various designs, are therefore an important task in this field.

Only the influence of Reynolds number is disregarded in the characteristic diagram as represented in figure 2. Slight errors at

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[3]It is possible that Reynolds number also has an influence on the boundary of the region of stable operation in the case of the axial compressor.

various temperatures may in addition be introduced due to the fact
that the tip clearances vary with temperature due to the expansion
of different materials, so that the requirement of geometrical simi-
larity is no longer fulfilled.

The further simplifications used in the investigation of the
power plant as a whole, namely, the disregard of the variations that
occur at a constant temperature ratio $T_3/T_1$ in the fuel-air mixture
ratio and above all in the specific heat of the combustion gases, were
introduced solely for the purpose of simplifying the computation. In
order to determine the order of magnitude of the error thus caused,
in figure 4 the most important characteristic values of a jet power
plant for example, namely $\sigma$, $b_s/\sqrt{T_o}$, and $G_K/p_o\sqrt{T_o}$, are calcu-
lated for a constant temperature ratio $T_3/T_1$ and for various
absolute temperatures ($T_o = 202^o$, $249^o$, and $296^o$ K, corresponding
to $T_3 = 600^o$, $800^o$, and $1000^o$ C). Because temperature differences
of such magnitude occur only rarely, the errors are, of course,
generally smaller in practice.

In the case of the most important quantity, the thrust coefficient,
the errors are less than 1 percent for smaller pressure ratios $p_2/p_1$
and reach approximately 1.5 percent only at the highest pressure ratio.
These differences may certainly be disregarded.

The errors introduced are greatest for the characteristic value
for the quantity of fuel $G_K$, because in this case the marked
increase in the specific heat with increasing temperature produces a
direct effect. The variations compared with the mean value amount,
at the temperatures investigated, to about 3 or 3.5 percent. It would
be possible, if for example a control of the quantity of fuel should
be necessary, to make allowance for this error by inserting in place
of $\sqrt{T_o}$ the factor $T_o^{\alpha_b}$, in which for this case $\alpha_b$ should be
taken as 0.66. However, the expression $G_K/T_o^{\alpha_b}p_o$ is to be evaluated
empirically.

In the case of the characteristic value for the specific fuel
consumption, the effect produced by a change in the specific heat
on the quantity of fuel to be supplied is partly canceled by the
concurrant change in the work of expansion. The errors in $b_s/\sqrt{T_o}$
as compared with the mean value amount to only about 2 percent at the
temperatures investigated; they are for the most part insignificant

in practice. A good approximation would be obtained here by means of the expression $b_s/T_o^{\alpha_b}$ with $\alpha_b = 0.61$.[4]

## V. SCALE-MODEL TESTS

The presentation of data as in figure 2(a) shows how scale-model tests on geometrically similar compressors are to be carried out, insofar as the influence of Reynolds number can be ignored or the required corrections are known. The required driving power for the compressor is proportional to $D^2 p_2 \sqrt{T_1}$; at the same time Reynolds number varies in proportion to $Dp_2/T_1\bar{\eta}_1$, if $\bar{\eta}_1$ is the viscosity at the inlet temperature. Should the Reynolds number for the model be as large as for the full-scale construction, then at equal inlet temperatures the pressures would be inversely proportional to the diameters. The required driving power is then proportional to the diameter. Fundamentally, the geometric similarity should extend even to the surface quality, that is, the model should have smoother surfaces than the full-scale construction. However, where the Reynolds number is the same, a difference in roughness will have no influence on the flow relations (reference 11).

In scale-model tests on turbines it is desirable to operate the model at the same gas temperature as the actual turbine. If the influence of Reynolds number is disregarded, then agreement, for example, in the pressure ratio and in the peripheral speed (or, if the temperatures are different, in the ratio of the peripheral speed $u$ to the speed $c_{ad}$ that would correspond to the adiabatic heat rise), will suffice. Agreement of Reynolds number, as well, may be obtained by increasing the pressures in inverse proportion to the diameters. A certain increase in pressures can also be advantageous by way of simplifying the test rig if thus the exhaust-section equipment for the combustion gases, which is required in the case of back pressures that are lower than atmospheric pressure may be dispensed with.

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[4]The fact that marked changes in individual characteristic values may be observed even with the limited changes in absolute temperature, which may occur while a constant temperature ratio $T_3/T_1$ is maintained, shows that the influence of the variability of the specific heat with the temperature and that of the variation of the gas composition in the combustion are not really negligible in the gas-turbine power plant either, and must be taken into account if a more exact agreement of the calculated with the observed relations is desired.

In the case of air cooling of the turbine blades, if the turbines are geometrically similar, pressure and velocity relations are equal, and there is agreement in Reynolds numbers, similarity also exists in respect to heat transfer to the blades and to the rotor as well as heat conduction in the metal (assuming that the same materials were used); at equal gas temperatures the temperatures in the case of the model are thus the same as in the full-scale construction.

Scale-model tests of this sort also yield valuable information on strength of materials (reference 3). For the same materials, the corresponding natural periods of vibration of the individual parts occur at the same peripheral speeds; likewise the stresses in the material due to centrifugal force are the same in model and full-scale construction at equal peripheral speeds whereas the stresses in the material due to the gas forces are proportional to the pressures. Scale-model testing is therefore evidently a very valuable aid in the development of flow machines.

The application of the principles of similarity to gas-turbine power plants does not extend to the processes of fuel supply, atomization, and chemical conversion in the combustion chamber because it is generally impossible to secure similarity of these processes as between model and full-scale construction. For similarity in the over-all behavior of the power plant, it is sufficient if the quality of the combustion and the distribution of temperatures at the inlet into the turbine are the same.

## SUMMARY

As earlier investigations had already shown, the behavior of a compressor at various temperatures, which is not correctly reflected with the usual construction of the compressor-characteristic diagram, can be accurately represented by a single diagram through the introduction of nondimensional characteristic values. For a particular air compressor, it is sufficient to divide the inlet volume as the abscissa and the speed as the parameter by the square root of the absolute inlet temperature and correspondingly to use the pressure ratio of the compression as the ordinate. In this type of chart with constant turbine-nozzle cross section and constant temperature ahead of the turbine—provided the critical velocity is attained in the turbine nozzle—the operating points lie very nearly on a straight line through the zero point of the compressor-characteristic diagram. The angle of this line with the abscissa axis is approximately proportional to the square root of the ratio of temperature ahead of the turbine to temperature ahead of the compressor. Similar operating

states of the turbine are likewise determined by two characteristic values, for example, the ratio of the peripheral speed to the speed corresponding to the adiabatic heat rise and the pressure ratio.

The investigation of gas-turbine power plants in various operating states is considerably simplified by use of the concept of similar operating states because it then becomes unnecessary to compute various absolute temperatures. In similar operating states all nondimensional characteristic values, such as the velocity and the pressure relations, the Mach numbers, and the characteristic values derived therefrom remain constant. If two independent characteristic values are given and if there exists geometric similarity, that is, equal settings of all devices directly influencing the flow, all the other characteristic values are also determined. As an appropriate nondimensional characteristic value for the thrust, the thrust coefficient is used, that is, the thrust per unit of turbine-nozzle cross-sectional area for a pressure of 1 atmosphere. As a simplified characteristic value for the specific fuel consumption (calculated on the thrust), the ratio of specific fuel consumption to the square root of the temperature of the atmosphere can be used.

In construction of the characteristic diagrams for a compressor and turbine, the influence of Reynolds number, which presumably is not important has been disregarded; and in definition of similarity as applied to the whole power plant the variations of the specific heat and of the gas composition, which may occur if the ratio of temperatures remains constant have been disregarded. The errors in the thrust and in the specific fuel consumption that are caused by these simplifications are negligible.

The concept of similarity serves at the same time as the basis for scale-model tests of compressors and turbines. More precise investigations on the influence of Reynolds number must be made in the future. By increasing the absolute pressure, if the necessary power is available, the same Reynolds number may be attained in the model as in the full-scale construction. At equal Reynolds number, the cooling relations in the turbine in the case of air cooling (for example, interior cooling of the blades) are also the same in the model as in the full-scale construction. Furthermore, scale-model tests give important information on the question of material strengths. Therefore they are to be regarded as a valuable aid in the development of gas-turbine power plants.


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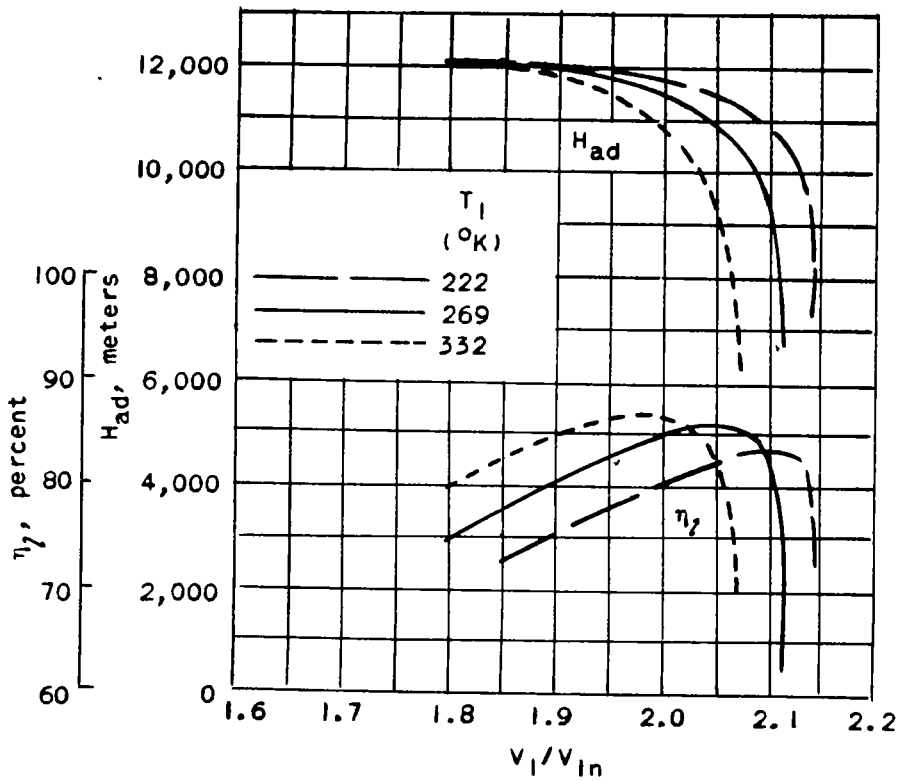    Forsch. Ing.-Wes., Bd. 8, 1937, p. 25.

Figure 1. – Adiabatic pressure head $H_{ad}$ and adiabatic efficiency $\eta_\ell$ of 8-stage axial compressor (calculated values) plotted for various inlet temperatures $T_1$ against inlet volume $V_1/V_{1n}$. (At the design point $V_1/V_{1n} = 1$.) Constant speed.
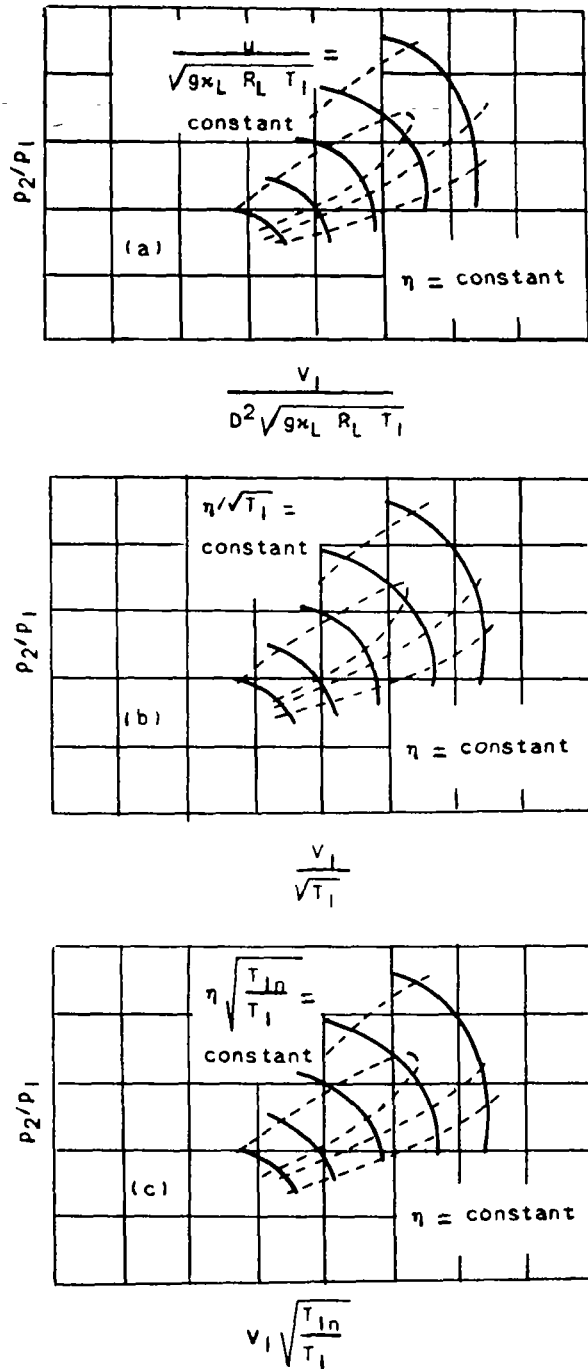
Figure 2. - Compressor performance charts for
   air.
   (a) Universally valid representation for
       similar compressors with nondimensional
       characteristic values.
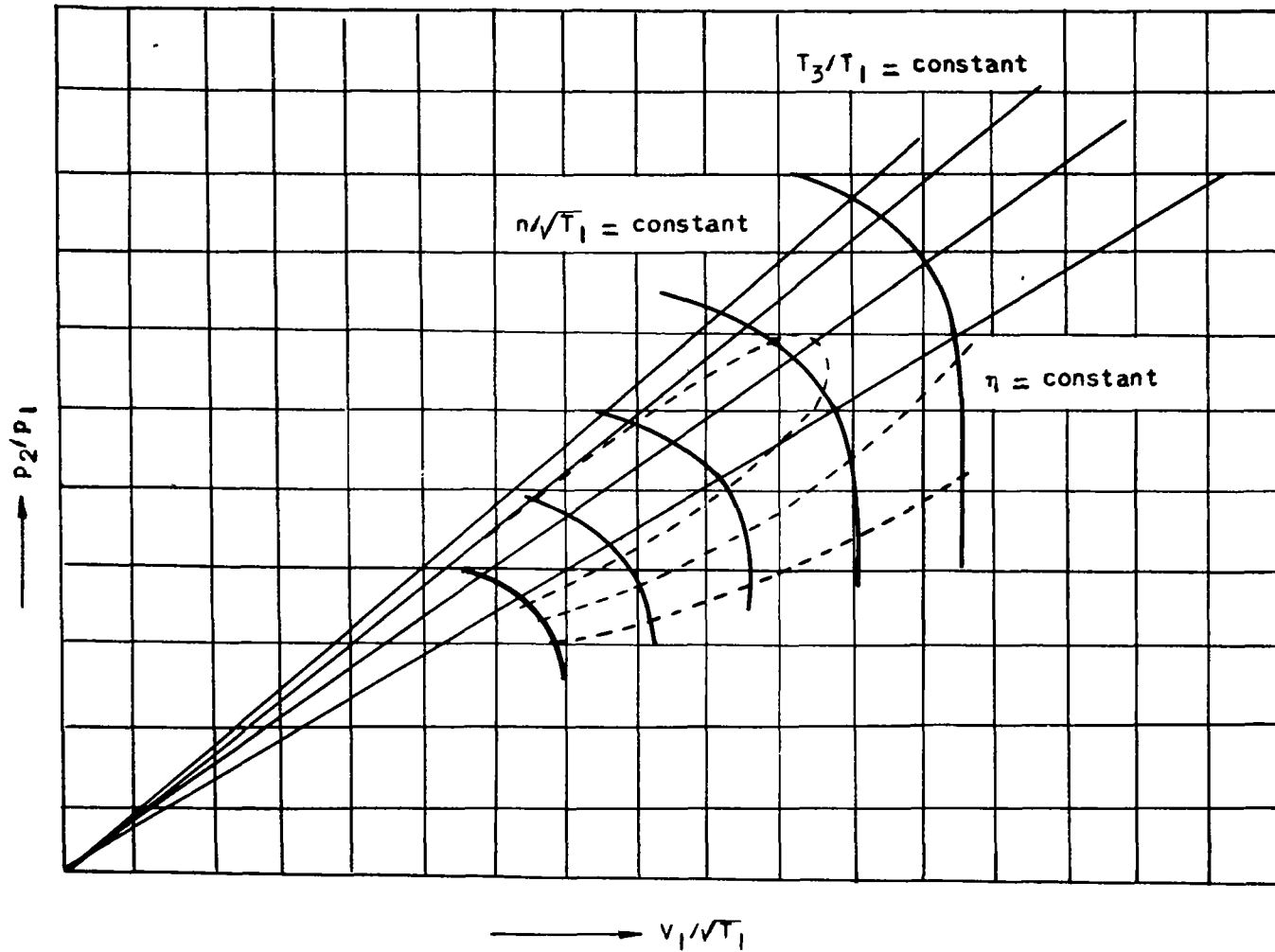   (b) and (c) Simplified representation for a
       particular compressor.

Fig. 3



Figure 3. – Compressor performance chart showing lines of constant temperature $T_3/T_1$ for case of uniform turbine–nozzle cross section.
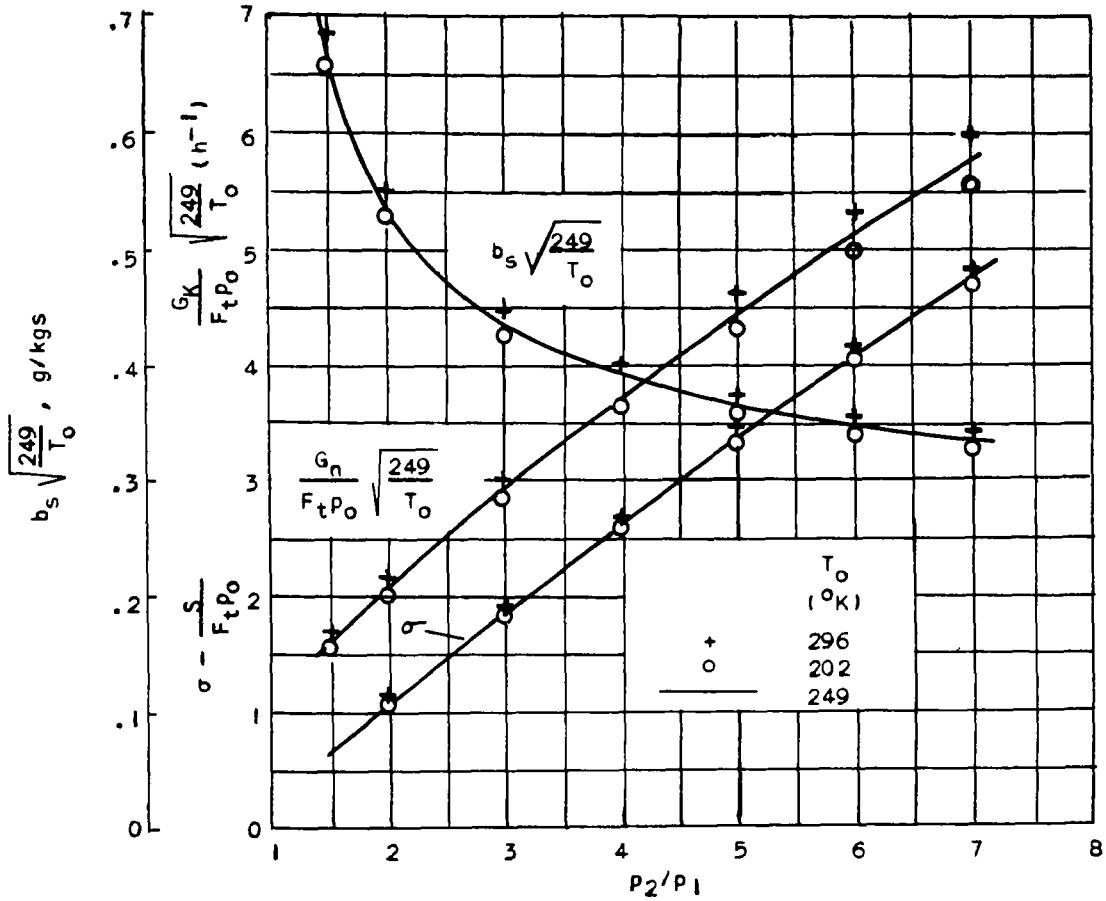
Figure 4. – Characteristic values for thrust and specific and hourly fuel consumption of jet power plant plotted against pressure ratio $p_2/p_1$ in the compressor at various absolute temperatures $T_0$.

$$Ma_0 = 0.632; \quad T_3/T_1 = 3.99$$

(Compressor efficiency, 0.85; turbine efficiency with reference to adiabatic heat rise minus outflow energy, 80 percent.)

H
Author

Turbine, Gas
, Theories - Similitude
Compre...

Engine - 'compressor' turbine Comb...
Flow - Comp...
" - Tu...
Reynolds number