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NEW METHOD OF DETERMINING THE POLAR CURVE OF AN AIRPLANE IN FLIGHT

By B. N. Yegorov

Central Aero-Hydrodynamical Institute

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SUMMARY

A fundamental defect of existing methods for the determination of the polar of an airplane in flight is the impossibility of obtaining the thrust or the resistance of the propeller for any type airplane with any type engine. The new method is based on the premise that for zero propeller thrust the mean angle of attack of the blade is approximately the same for all propellers if this angle is reckoned from the aerodynamic chord of the profile section. This angle was determined from flight tests. Knowing the mean angle of the blade setting the angle of attack of the propeller blade at zero thrust can be found and the propeller speed in gliding obtained. The experimental check of the new method carried out on several airplanes gave positive results. The basic assumptions for the construction of the polars and the method of analyzing the flight data are given.

INTRODUCTION

The determination of the polar of an airplane in flight under the gliding condition may be determined with a blocked or rotating propeller (reference 1). In order to obtain the polar of the airplane for zero propeller thrust it is necessary to exclude the thrust or the propeller drag. For this several methods are available all of which, however, possess some defect or another. A glide with propeller blocked is possible only for a very small number of airplanes due to the difficulty or impossibility of stopping and cutting out the engine in flight. A glide with rotating propeller may be carried out with the zero thrust indicator but then a necessary condition is the presence of a sufficient clear space at the crankshaft of the engine or the reducing gear shaft. This is not the case for all engines. A glide may

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be carried out, assuming for the propeller values of λ $(=V/2n)$ taken from wind tunnel tests on a propeller series. By this method large errors may be obtained since the data of model tests on propellers in the wind tunnel disagree in most cases with the flight data. At times, in order to determine the polars, the method of integrating the pressures is applied. This method has many defects, particularly the need for a large number of preliminary wind-tunnel tests. A glide with the hub dynamometer can finally be effected but up to the present time no suitable design for measuring the propeller thrust has been worked out.

From what has already been said it follows that the available methods are not entirely satisfactory for determining the polar of an airplane in flight.

1. FUNDAMENTAL ASSUMPTIONS OF THE NEW METHOD

Consider the work of an element of the propeller blade for zero thrust. It is then possible to neglect the additional axial and rotational velocities arising from the propeller slipstream. The velocity triangle in the plane of the propeller (fig. 1) will then consist of the axial velocity V, equal to the flight velocity, the rotational velocity of the element U, and the resultant of these W. The latter forms with an element of the blade the angle of attack α and is inclined β° with the plane of rotation. The blade element is acted on by the lift force $c_y s\ \rho W^2/2$ and the drag force $c_y s\ \rho W^2/2$ directed both perpendicular and parallel to the velocity W, respectively. The difference in the projection of these forces on the direction of the velocity V gives the thrust of the blade element. Evidently for zero thrust of the blade element the equation is true:

$$c_y s \frac{\rho W^2}{2} \cos \beta = c_x s \frac{\rho W^2}{2} \sin \beta$$

or

$$\frac{c_y}{c_x} = \tan \beta \qquad (1)$$

On the other hand, from the velocity triangle there is

$$\tan \beta = \frac{V}{U} = \frac{V}{2\pi r n_s} = \frac{\lambda}{\pi r} \qquad (2)$$

where λ is the advance ratio of the propeller and $\overline{r} = r/R$ is the nondimensional radius of the blade section.

From formulas (1) and (2) there is obtained

$$\frac{c_y}{c_x} = \frac{\lambda}{\pi \overline{r}} \qquad (3)$$

It is seen, from the above formula, that for a given radius $\overline{r}$ for zero thrust for the same values of λ the efficiency of the blade section is the same, independent of the shape of the profile or of the thickness and pitch. Moreover it follows from formula (3) that the efficiency of the various blade elements of the same propeller varies inversely proportional to $\overline{r}$ and is equal to $2.5\ \lambda/\pi$ at $\overline{r} = 0.4$. Remembering that the value of λ for present day propellers, with account taken of the increase in velocity of the airplane, may vary within the limits of 1 to π it may be assumed that the efficiency at the blade tip varies from $1/\pi$ to 1 and at $\overline{r} = 0.4$ from $2.5/\pi$ to 2.5. Thus, for a thrust equal to zero, the efficiency of the blade sections in the most divergent cases may vary within the small limits from $1/\pi$ to 2.5. Considering the wind tunnel tests of various profiles it may be seen that for such small values of the efficiency for profiles with equal values of c_y/c_x the angles of attack are approximately equal if they are reckoned from the aerodynamic chord of the section. By the latter the chord parallel to the direction of the velocity is meant for which the lift of the section is equal to zero. The effective blade sections are those lying at the radii from $\overline{r} = 0.3 - 0.4$ to $\overline{r} = 1.0$. At smaller radii the blade either is entirely ineffective or only slightly effective.

As has already been said, for a given value of λ the efficiency of the various elements for zero thrust varies inversely proportional to $\overline{r}$. The mean value of the efficiency will be possessed by the blade section at radius $\overline{r} = 0.65 - 0.7$. From a preliminary check on the basis of wind-tunnel tests on various series of propellers the conclusion was reached that the propeller blade may be replaced by a blade of the same section having a single blade setting and angle of attack. This section may be taken at the radius $\overline{r} = 0.67$.

Thus, on the basis of elementary considerations the following proposition was derived which is the underlying basis of the new method. <u>The mean angle of attack of the propeller blade at the section at radius $\overline{r} = 0.67$ for zero thrust is constant for all propellers for equal values of λ.</u> In other words λ = constant for different propellers.

$$\alpha = \text{constant} \tag{4}$$

where α is the mean angle of attack of the blade at radius $\overline{r} = 0.67$, this angle being computed from the aerodynamic chord of the section.

Thus for λ = constant, write

$$\alpha = \varphi - \beta + \alpha_a = \text{constant} \tag{5}$$

where φ is the mean setting of the blade at radius $\overline{r} = 0.67$, β is the mean angle of approach of the flow at the same radius (fig. 1) and α_a is the angle between the aerodynamic chord of the blade section at radius $\overline{r} = 0.67$ and the geometric chord from which the blade setting φ is reckoned.

Formulas (4) and (5) are true only for λ = constant. As has already been shown above, on the basis of formula (3) the efficiency of the section at radius $\overline{r} = 0.67$ varies from zero to approximately 1.5 for a change in λ from 0 to 3.0. Thus, for the same series of propellers which differ only in the various pitches the mean angles of attack should be different for different values of λ at zero thrust. Since the efficiency of the blade section at $\overline{r} = 0.67$ varies slightly for the rather considerable change in λ from 0 to 3.0 the change in the mean angles of attack should likewise be slight. Analysis of wind tunnel tests on various profiles showed that with variation in the section efficiency from zero to 1.5 the angles of attack change approximately by 0.6°. For this reason it may be approximately assumed that the change in the mean angle of attack for varying λ is given by the following relation

$$\Delta\alpha \cong 0.2\lambda \qquad (6)$$

Thus formulas (4) and (5) may be generalized by taking account of the change in angle of attack with λ. Then

$$\alpha = \varphi - \beta + \alpha_a - \Delta\alpha = \text{constant} \qquad (7)$$

The above formula may be applied to all propellers.

The above conclusion may now be formulated as follows. The mean angle of attack of the propeller blade at radius $\bar{r} = 0.67$ at zero thrust is the same for all propellers.

All that has been said above applies to isolated propellers. The above results are extended to propellers mounted on the airplane. The propeller mounted on the airplane gives an additional drag to the parts of the airplane located in the propeller slipstream. As is known, this additional drag arises from the increased pressure and from the increased flow velocity behind the propeller as compared with the flight velocity. Under gliding conditions of the airplane, when the propeller thrust is equal to zero, the velocity of the flow behind the propeller should not differ from the flight velocity. For this reason there should be no increase in the drag of the airplane. The additional drag due to the increase in pressure in the propeller slipstream may be expressed as follows (see references 3 and 4):

$$R = hP$$

where

R compressive force on the nose portion of the fuselage or on other parts of the airplane located directly behind the propeller

h coefficient depending on the reduction in the flight velocity by the body

P propeller thrust

Evidently for $P = 0$, R is also zero. Hence for zero thrust the propeller has no effect on the airplane.

The effect of the airplane on the propeller is now considered. This effect shows up in the change in the axial velocities of the air flow in the propeller disk. If V denotes the flight velocity and V_0 the velocity in the plane of the propeller then

$$V = V_0 (1 + \epsilon) \tag{8}$$

where ϵ is a coefficient of increase equal in turn to (reference 4)

$$\epsilon = h \sqrt{1 + 2B}$$

where

h coefficient depending on the reduction by the body of the flight velocity in the plane of the propeller

B coefficient for the area swept out by the propeller

At zero thrust $B = 0$ and $\epsilon = h$. Therefore

$$V = V_0 (1 + h) \tag{8'}$$

It is evident that

$$\lambda = \lambda_0 (1 + h) \tag{9}$$

To determine the angle β between W and the plane of rotation for the isolated propeller formula (2) may be used. Evidently for the propeller mounted on the airplane this formula becomes

$$\tan \beta_0 = \frac{\lambda_0}{\pi r} \tag{10}$$

where λ_0 is determined by formula (9). Formula (7) becomes

$$\alpha_0 = \varphi - \beta_0 + \alpha_a - \Delta\alpha = \text{constant} \tag{11}$$

The previously mentioned formula should be applicable to all propellers mounted on various airplanes. The result finally obtained may be formulated as follows: The mean angle of attack of the propeller blade at radius $\overline{r} = 0.67$ for zero thrust is constant for all propellers on all airplanes. Knowing the mean angle of attack of the blade α, it is not difficult to determine the values of λ for gliding flights of the airplane at zero thrust. From formula (11) the angle β_o is determined:

$$\beta_o = \varphi - \alpha_o + \alpha_a - \Delta\alpha \qquad (12)$$

On the right side of the above formula all the terms are known from the dimensioning of the propeller and experimental data. Then there is determined λ_o:

$$\lambda_o = \pi\overline{r} \tan \beta_o \qquad (13)$$

and λ:

$$\lambda = \lambda_o (1 + h) \qquad (14)$$

With the value of λ known the flight velocity and the rotational speed of the propeller for gliding at zero thrust are known. Practical application of formulas (12) and (14) will be shown later.

2. EXPERIMENTAL CHECK OF THE NEW METHOD

The new method was worked out on the basis of elementary considerations and for this reason it was necessary to subject it to a complete experimental check. Moreover, it was possible to determine the mean angle of attack of the propeller blade α_o only experimentally. The experimental check was to establish the correctness of formula (11) regarding the constancy of the mean angle of attack of the blade and the agreement between the polars obtained by the new and old methods. Since the results obtained refer to propellers of different shapes with different profiles and mounted on different airplanes the test was undertaken for several types of airplanes with different propellers.

To check the constancy of the mean angle of attack of the propeller blade flights were undertaken with the object of determining the coefficients of effective thrust. The tests were conducted so as to obtain the values of the effective thrust coefficients for a large range of values of λ including the conditions approaching the zero thrust condition. As is known, the effective thrust is determined from the formula

$$P = Q + G \sin \theta$$

where

P effective thrust of the propeller

Q drag of the airplane in gliding

G weight of the airplane

θ flight path angle.

Thus to determine the thrust coefficient it is necessary to know the drag of the airplane in gliding, or in other words, the polar of the airplane. For this reason, on each airplane, its polar in flight was determined with the aid of the zero thrust indicator or in a glide with blocked propeller. For illustration the values of the thrust coefficients experimentally obtained are shown in figure 2 for the Northrup airplane. Similar curves were obtained for all propellers. From the curves $\alpha = f(\lambda)$ the values of λ were determined for $\alpha = 0$. With the values of λ and the angles φ, α_a, and $\Delta\alpha$ known the values of the mean angles of attack of the blade α_0 can be computed. The obtained results for all propellers on all airplanes tested are given in table 1.

As may be seen in table 1 seven airplanes and eleven propellers in all were tested. The airplanes were of the most diversified types and for this reason the mutual interference between propellers and airplanes was different. This is indicated by the different values of the interference coefficient h; the values of h varied from 0.015 to 0.1. The propellers likewise differed from each other; some propellers were of large blade thickness (wood) and some of small thickness (metal). The various blade sections were of three types; namely, Clark-Y, RAF-6, and a type obtained from RAF-6 by increasing its thickness. The propellers had two- and three-blades. The tests thus embraced the most diversified airplanes and propellers. As is seen in table 1 the values of the blade angles of attack were obtained near each other. The mean value of the angle α_0 was found to be 0.66. The maximum deviations from the mean value of α_0 did not exceed 0.15. The obtained scatter of the points is explained, on one hand, by a certain inaccuracy in the fundamental assumptions and on the other by inaccuracy in the test itself. The probable error in determining α_0 was obtained as $\Delta\alpha_0 = 0.075^\circ$. Consider what systematic error in the coefficient c_x may be obtained as a result of the error $\Delta\alpha_0 = 0.075^\circ$. It is not difficult to show (fig. 3) that to an error in the angle α_0 of 0.075° for propellers of the usual types there correspond the following errors in the thrust coefficient:

For the three-blade propeller $\Delta\alpha = 0.0006$

For the two-blade propeller $\Delta\alpha = 0.0004$

The error in the coefficient c_x of the airplane may be computed by the formula

$$\Delta c_x = \frac{2\Delta\alpha D^2}{\lambda^2 s}$$

Evidently Δc_x will be greater the greater the value of D and the smaller the values of s and λ.

For the present day high speed airplane with powerful engines it is difficult to assume less than 10 square meters for the wing area. The corresponding propeller diameter does not exceed 3.0 meters (for the single engine airplane). For multi-engine airplane propellers of larger diameter may be expected but there also is a corresponding increase in the wing area. For sport and trainer airplanes the wing area may be less than 10 square meters but the diameters of the propellers are also considerably smaller than 3.0. In other words, it may be assumed that for present day airplanes the ratio D^2/s will not be larger than 0.9. The values of λ for present day airplanes usually exceed 1.0 for zero thrust. Only for low speed airplanes do the values of λ lie between 0.7 and 0.8 but the ratio D^2/s is then considerably less than 0.9. Thus if the assumed values are $D = 3.0$ meters, $s = 10$ square meters, and $\lambda = 1.0$ it may be said that such combination of values gives the extreme limiting value of the error. Determining Δc_x for values of D, s, and λ for the three-blade propeller there is obtained:

$$\Delta c_x = \frac{2 \times 0.0006 \times 9}{10} = 0.00108$$

and for the two-blade propeller:

$$\Delta c_x = \frac{2 \times 0.0004 \times 9}{10} = 0.00072$$

For a coefficient c_x of the entire airplane equal to 0.02 these errors constitute for the three-blade propeller, 5.5 percent and for the two-blade propeller, 3.6 percent. In practice it is difficult to expect such combination of values of D, s, and λ which were assumed in these computations and the values of Δc_x should be less than those obtained. For the airplanes and propellers tested in the present investigation the probable systematic errors obtained were the following:

Northrup 2E airplane	$\Delta c_x = 0.00026$
Biplane	$\Delta c_x = 0.00016$
Four-engine airplane	$\Delta c_x = 0.0002$
Two-engine airplane	$\Delta c_x = 0.00016$

Thus it may be concluded that the probable systematic error in the coefficient c_x of the airplane arising from the error in determining the angle α_0 will not exceed 2 to 3 percent, which of course, may be considered entirely satisfactory since this error does not exceed the experimental accuracy in determining the polar in flight.

A slight digression will precede a comparison of the polars determined by the new and old methods. In practice, in determining the polars in flight by the new method, it is difficult to assume the values of λ. For this purpose it is necessary that the values of velocity and rotational speed of the engine and the value of the air density should be given very accurately. This is practically impossible and deviations from the assumed conditions, although not large, are always possible. For this reason it is necessary to have a method for making corrections on the deviations from the assumed conditions or a method of taking account of the thrust or the drag of the rotating propeller. With this object the result obtained of the constancy of the mean angle of attack of the blade of the propeller α_0 for all propellers and airplanes at zero thrust was extended to flight conditions near the zero thrust condition. It was assumed here that some other mean angle of attack of the blade corresponds to the deviation of λ from the assumed value for a nonzero thrust, depending on the value of the propeller thrust. Evidently the propeller thrust depends not only on the blade angle of attack but also on other factors, mainly, the area of the blade. In other words, for equal angles of attack of the blade the thrust will be greater for a propeller with greater area. In order to make the correction coefficient for the thrust or drag of the propeller, independent of the blade area, it was decided to introduce the value of the relative thrust coefficient α_s in the form

$$\alpha_s = \frac{\alpha}{i s_l} \tag{15}$$

where α is the propeller thrust coefficient, i the number of blades, and s_l a coefficient depending on the shape of the blade:

$$s_l = \frac{s_f}{D^2} \tag{16}$$

where s_f is the developed area of the blade and D the propeller diameter. Since the area of the blade is determined by the formula

$$s_f = \frac{D^2}{2} \int_{\bar{r}=0.15}^{\bar{r}=1.0} \bar{b}\,d\bar{r} \tag{17}$$

where $\bar{b} = \frac{b}{D}$ the nondimensional width of the blade, therefore,

$$s_l = \frac{1}{2} \int_{\bar{r}=0.15}^{\bar{r}=1.0} \bar{b}\,d\bar{r} \tag{18}$$

The previously mentioned data may be formulated as follows: The corrections on the small values of the thrust or drag of the propeller are made on the basis of the assumption that for all propellers on all airplanes for equal mean angles of attack of the blade α_o the thrust coefficients α_s are equal.

The above assumption is approximate because the result, with regard to the constancy of the mean angle of attack of the blade for zero thrust, is, strictly speaking, applicable to propellers at zero thrust. Moreover, the propeller thrust for equal angles of attack, as has already been said, depends not only on the blade area but on other factors as well. For this reason discrepancies in the test results were to be expected, the discrepancies being larger the more the thrust differed from zero. However, for small values of the thrust coefficient these discrepancies have practically no effect on the polars and, as will be seen from what follows, the corrections based on the above assumptions are entirely permissible. To determine the corrections from the thrust coefficient curve $\alpha = f(\lambda)$ the values of the thrust coefficient α were taken for various values of λ. By formula (15) the values of α_s were computed and by formula (11) the values of the mean angle of attack of the blade α_o for given values of λ. The obtained values $\alpha_s = f(\alpha_o)$ are given on figure 3. The correction curve is drawn through the test points. From this curve the corrections for the condition of nonzero thrust may be made. In place of the correction curve its equation may be used:

$$\alpha_s = 0.1245\alpha_o - 0.0822 \qquad (19)$$

The test curve showed that the curve of figure 3 or formula (19) may be used with sufficient reliability. However, it is recommended that the values of α_o be limited between -4° and 5° because for values of α_o outside of these limits appreciable errors may be obtained. If the values of λ in gliding are correctly assumed, the deviations of the values of α from 0.66° do not exceed 1 to 2 percent.

The experimental check of the agreement of the polars determined by the new method with those obtained with the aid of the zero thrust indicator or with blocked propeller was carried out on the airplanes listed in table 1.

Illustrations of the data obtained on polars of two airplanes are given but similar results in agreement were obtained on the remaining airplanes. The polar of a single-engine biplane with water-cooled engine is given on figure 4. This polar was obtained by the new method from a single flight. The scatter of the test points does not exceed the scatter in obtaining the polar by other methods. The polar of the same airplane is given on figure 5 for three propellers: a two-blade metal propeller with diameter $D = 3.1$ meters and angular setting $\varphi = 25.5^\circ$, the same propeller with angle $\varphi = 21.5^\circ$, and a three-blade metal propeller with diameter $D = 2.8$ meters with angle $\varphi = 30.2^\circ$. The curve on figure 5 is the polar of the same airplane that was obtained by the zero thrust indicator. The scatter of the test points in the tests with the three-blade propeller was somewhat greater than for the other propellers. This is explained by the fact that for a three-blade propeller the given values of λ were not suitable and the values of the blade angles of attack α_0 lay outside the above indicated limits (from -4° to 5°). The same results are given on figure 6 for the Northrup 2E airplane with propeller $D = 2.95$ meters and blade settings 25.8° and 17.8°. It is seen from figures 5 and 6 that the agreement of the results obtained by the new and old methods is good.

The polars obtained by the new method from individual flights with various propellers mounted on the same single-engine biplane are given on figure 7. As is shown on figure 7 the agreement of the polars obtained from the various flights is good. The difference in the values of the coefficients c_x for equal values of the coefficients c_y does not exceed the value 0.002 to 0.003 which is explained, evidently, by the accuracy of the test itself. From figure 8 on which the polar of the same airplane with two propellers is obtained from separate flights with the aid of the zero thrust indicator, it is seen that the deviations between the curves is of the same order as for figure 7. The polars obtained from separate flights on the Northrup airplane with two propellers are given on figure 9. These polars were obtained by the new method. On figure 9 the deviations between the curves is of the same order as was obtained for figures 7 and 8.

Thus, the experimental check has shown that the accuracy of determining the polars by the new method is not less than the accuracy in determining by other methods. An additional experimental confirmation of the new method was obtained on still another airplane not indicated in table 1. On this airplane, due to the peculiar properties of the shaft bearings of the reduction gear it was not possible to determine the polar in flight with the aid of the zero thrust indicator. On this airplane the polar in flight was determined by the new method. The tests were conducted with two positions of the blade angles differing by 9°. Both polars were in complete agreement, a fact which is an indirect confirmation of the new method.

3. APPARATUS AND TEST PROCEDURE

To determine the polars in flight the following apparatus must be used:

(1) A velocity indicator with pitot tube

(2) An altimeter with double pointer or accurate barograph, the accuracy of the measurements of the pressure drop by the barograph being not less than 0.5 millimeters of mercury

(3) A thermometer for measuring the temperature of the outside air

(4) A thermometer for measuring the temperature of the air in the cabin near the apparatus

(5) A stop watch

(6) A longitudinal inclinometer with wide base

(7) A tachometer or rotational speed indicator. The tachometer should be sufficiently accurate and the position of the pointer stable. The reading accuracy should not be less than 10 rpm.

If self-recorders are available part of the apparatus may be replaced or duplicated.

All apparatus should first be carefully calibrated, the calibration of the speed indicator, altimeter, and thermometers being carried out over the temperature range within which the flights are made. Moreover, the speed indicator or the speedograph with the pitot tube should be calibrated over a measured kilometer, or better still, with the aid of an "aerolag" for measuring the velocity in gliding under the conditions at which the test will be conducted in determining the polars. In preparing for the tests it is necessary to determine the values of λ, for the gliding conditions at zero thrust. For this purpose it is first necessary to determine the blade setting at the radius $\bar{r} = 0.67$. With this object the propeller should be dimensioned. If there are geometric data available in the form of curves of the blade setting distribution and blade thickness and width distribution the necessary dimensions may be taken from these curves. The propeller blade setting at the radius $\bar{r} = 0.67$ is checked for all blades. From the obtained results the mean arithmetic value is taken. Simultaneously with measuring the angles it is necessary to take the dimensions of the width of the blade at $\bar{r} = 0.15$ to $\bar{r} = 1.0$. Moreover, at the radius $\bar{r} = 0.67$ the thickness of the blade section should be measured for determining afterward the angle of the aerodynamic chord with the geometric chord. Thus on the basis of the geometric characteristics of the propeller there should be obtained the following data: the blade setting at radius $\bar{r} = 0.67$, the width and thickness of the blade section at the same radius, and the width distribution of the blade from $\bar{r} = 0.15$ to $\bar{r} = 1.0$. Moreover, it is necessary to know the shape of the blade section.

Having the shape of the section and relative section thickness $c = \delta/b$ at the radius $\bar{r} = 0.67$ (c is the relative thickness, δ the thickness, and b the width of the section) the angle between the aerodynamic and geometric chords may be determined. This angle may be determined by wind-tunnel tests on a series of propeller profiles. For this purpose the value of the angle of attack of the section is taken from wind-tunnel tests at $c_y = 0$ (reference 5). Moreover, ahe angle between the aerodynamic chord and the geometric chord may be determined, approximately, by a simple geometrical construction (reference 5, pp. 111-113). On figures 10 and 11 are given the values of the angles α_a as a function of the thickness of the section for the most widely used sections RAF-6 and Clark-Y. On these curves the angles α_a are reckoned from the chord passing through the flat lower side of the section. By a geometric construction the angle α_a is determined in the following manner: The section at the radius $\bar{r} = 0.67$ is sketched and chord AB is drawn (fig. 12) connecting the leading and trailing edges. The axial arc ACB is then drawn. For this purpose draw the radii of curvature, bisect them and draw the arc ACB through the centers of the segments of the radii of curvature. The maximum distance of the arc ACB from the chord AB is denoted by f. The relative curvature is then equal to

$$\bar{f} = \frac{f}{AB} 100 \; [\%]$$

and the angle α_a in degrees is approximately equal to

$$\alpha_a{}^o = \bar{f} \; \%$$

It may be noted that in determining α_a geometrically the line connecting the front and rear points of the profile is taken for the chord of the section. For this reason the angle α_a refers to this chord and in this case the blade setting φ must also be measured from the same chord.

The angle $\Delta\alpha$ is determined by formula (6) but since the value of λ before the flight is not known the angle $\Delta\alpha$ may be approximately determined by the following formula

$$\Delta\alpha \cong 0.2 \; h \tag{20}$$

where $h = \pi\bar{r}\tan\varphi$ is the relative pitch at radius $\bar{r} = 0.67$. In determining $\Delta\alpha$ by formula (20) a large error is not made which might affect the results. Thus all data for determining the angle β_o is obtained:

$$\beta_o = \varphi - \alpha_o + \alpha_a - \Delta\alpha \tag{21}$$

where $\alpha_o = 0.66$ is an experimental constant.

Further, λ_o and λ are determined:

$$\lambda_o = \pi \; \bar{r} \; \tan \beta_o \tag{22}$$

$$\lambda = \lambda_o \; (1 + h) \tag{23}$$

where $\bar{r} = 0.67$ and h is the interference coefficient in the propeller plane. The value of h is determined from test data or by the approximate theory of the mutual interference between propeller and airplane (references 3, 4, and 5). On figures 13 to 19 the curves of the coefficient h for several cases of propeller mounting are given. On figures 13 to 15 the curves of h are given as a function of the ratio f/F where f is the area of the middle section of the fuselage and F the area swept by the propeller. Such curves are also given in figures 16 to 19 for the cases where the engine is mounted ahead of the wings. On these curves the values of h are given as a function of the sum $f/F + e/D$ where f is the area of the middle section of the engine nacelle, F is the area swept by the propeller, e the maximum thickness of the wing, and D the propeller diameter. On all curves ξ denotes the relative radius of the noneffective part of the blade (hub and tip). If the airplane type on which the tests are conducted differs from the types shown in figures 13 to 19 it is necessary, in determining h, to refer to the abovementioned original sources.

With the value of λ for gliding known it is necessary to assume a mean altitude of flight. It is recommended to conduct flights at altitudes 2000 to 4000 meters since at a lower altitude the atmosphere is usually not quiet. From the mean flight altitude the value of the relative density Δ is found from the standard atmosphere tables. Next assume a series of values of the velocities V_{ins} read on the instrument, correct them by the calibration of the speed indicator over the measured kilometer, and obtain the value of the indicated velocity V_i. With V_i known compute the true velocity $V_{tr} = V_i/\sqrt{\Delta}$ and determine the rotational speed of the propeller (or engine) by the formula

$$n = \frac{60\ V_{tr}}{3.6\lambda D}$$

where V_{tr} is taken in kilometers per hour, D in meters, and n in revolutions per minute. For geared engines the reduction coefficient is used in order to obtain the speed of the engine and not of the propeller.

By computing the engine speed by the above method large corrections may be obtained for the propeller thrust. This is because the mean flight altitude and the relative air density from the standard atmosphere conditions are assumed. In practice the temperature conditions at the given altitude often differ from the standard and, therefore, the relative density values will differ from the given values. For this reason it is recommended that the mean altitude from the given density be chosen. For this purpose the crew takes with them in flight, a previously prepared curve of dependence of flight altitude on the temperature for a given value of the relative density Δ. This curve may be easily drawn by computing, according to the formula:

$$p = \frac{\Delta T}{0.379}$$

where p is the air pressure in millimeters of mercury and T the temperature of the outside air. A series of values of the outside air temperature near the standard temperature for the given value of the relative air density Δ is assumed. The value of p is computed and by the standard atmosphere tables the value of the altitude is found. The value of the latter is plotted as a function of the temperature. With the aid of this curve the pilot at the start of the flight makes three or four level flights at altitudes near the given altitude. At these altitudes the outside air temperature is measured and the obtained values are laid off for the different altitudes on a curve. The point of intersection of the given altitude-temperature curve with the corresponding curve obtained in flight gives the flight altitude at which the air density coincides with the assumed density. The curve $H = f(T)$ should be prepared with account taken of the calibration corrections of the altimeter. On the curve, in other words, it is necessary to give the dependence of the indicated altitude on the temperature. It is recommended to prepare similar curves and also values of the rotational speed for two or three altitudes so as to enable the crew to choose a flight altitude for quiet atmospheric conditions.

It may be noted that for a multiengine airplane with different propellers it is necessary to assume the speed for each propeller. If a variable-pitch propeller is mounted on the airplane the polars must be determined for a single definite pitch for which all computations of λ and engine speed must be made. The computation of all the data required for determining the engine speed in gliding is preferably made in the following order:

(1) φ the blade setting at the radius $\bar{r} = 0.67$, determined for measurement

(2) δ blade thickness at radius $\bar{r} = 0.67$

(3) b blade width at radius $\bar{r} = 0.67$

(4) $c = \frac{\delta}{b}$ relative blade thickness

(5) α angle between the aerodynamic chord and the geometric chord of the section, determined by wind tunnel tests or geometric constructions

(6) $\Delta\alpha$ determined by the formula

$$\Delta\alpha = 0.2\ h$$

where

$$h = \pi\ \bar{r}\ \tan\ \varphi$$

(7) β_o angle between W and U:

$$\beta_o = \varphi - \alpha_o + \alpha_a - \Delta\alpha$$

where

$$\alpha_o = 0.66^\circ$$

(8) $\lambda_o = \pi\ \bar{r}\ \tan\ \beta_o$

(9) h coefficient of interference, determined from figures 13 to 19

(10) $\lambda = \lambda_o\ (1 + h)$

(11) H_{ins} given mean gliding altitude from the pilot's altimeter reading

(12) H mean flight altitude corrected for the calibration of the apparatus

(13) p pressure of the air in millimeter of mercury from the standard atmosphere tables corresponding to the altitude H

(14) T absolute temperature of the air at altitude H

(15) $\Delta = 0.379\ \frac{p}{T}$

All computations are tabulated.

(1) V_{ins} given values of the gliding velocity by the pilot's speed indicator

(2) V_i indicated velocity

(3) V_{tr} true velocity $V_{tr} = V_i/\sqrt{\Delta}$

(4) n engine speed in gliding for the corresponding values of V_{tr}

$$n = \frac{60\, V_{tr}}{3.6 \lambda D i}$$

where i gear reduction factor of the engine

(5) n_{ins} speed of the engine by the pilot's tachometer corrected by calibration

$$n_{ins} = n - \Delta n_{ins}$$

The polars are determined for established gliding conditions. Taking an altitude 300 to 400 meters higher than the chosen mean altitude, the pilot cuts down the gas and establishes the given velocity and rotational speed by the instruments. After the condition has been established the readings can be started including either the self-recorders or the visual apparatus. For a whole division of the scale of the altimeter the observer sets the stop watch, records the initial altitude of the glide, and reads the mean values for the flight off the other instruments. Finally, for a whole division of the altimeter scale, the stop watch is set and a record of the final gliding altitude and the time of the glide is made. In practice the time during which the measurements are made should be about 40 to 70 seconds. The number of flights should be about 12 to 15 and the flights should be repeated on another day for checking since the possible vertical currents may introduce a systematic error in the results. A good means for excluding the effect of vertical currents is the recorder of the flight path angles (reference 6). Thus to obtain sufficient accuracy it is necessary to make about 25 to 30 runs.

4. FUNDAMENTAL CONSIDERATIONS ON DETERMINING THE POLARS

For an established gliding condition the airplane is acted upon by the following forces (fig. 20): the weight of the airplane G; the lift force $c_y\ s\ \rho V^2/2$; the drag force $c_x\ s\ \rho V^2/2$; the propeller thrust P.

With the flight path and the horizontal forming the angle θ and the forces projected in the direction of motion and perpendicular to it

$$c_x s \frac{\rho V^2}{2} = G \sin \theta + P \cos (\alpha + \varphi) \tag{24}$$

$$c_y s \frac{\rho V^2}{2} = G \cos \theta - P \sin (\alpha + \varphi) \tag{25}$$

where α is the angle of attack and φ the angle between the longitudinal axis of the airplane and the propeller axis. Since the angles α and φ are small, it may be assumed in the case of gliding, that $\cos(\alpha + \varphi) = 1$. And since the thrust in gliding is small, it may be assumed that $P\sin(\alpha+\varphi)=0$. Equations (24) and (25) become:

$$c_x s \frac{\rho V^2}{2} = G \sin \theta + P \tag{26}$$

$$c_y s \frac{\rho V^2}{2} = G \cos \theta \tag{27}$$

Dividing both sides of these equations by $s\ \rho V^2/2$ there is

$$c_x = \frac{2 G \sin \theta}{\rho s V^2} + \frac{2P}{\rho s V^2} = c'_x + \Delta c_x \tag{28}$$

$$c_y = \frac{2\ G \cos \theta}{\rho s V^2} \tag{29}$$

where

$$c'_x = \frac{2\ G\ \sin\ \theta}{\rho s V^2} \quad (30)$$

$$\Delta c_x = \frac{2P}{\rho s V^2} \quad (31)$$

an added coefficient of the airplane drag due to the propeller thrust. On the basis of what has previously been mentioned

$$\Delta c_x = \frac{2 i s_1 D^2}{s} \frac{\alpha_s}{\lambda^2} \quad (32)$$

where i is the number of propeller blades, s_1 the blade area coefficient depending on the shape of the blade, α_s is the relative thrust coefficient. Thus to determine c_y and c_x of the airplane in gliding it is necessary to know the weight of the airplane, the angle which the flight path makes with the horizontal and Δc_x. Moreover, for plotting the angles of attack on the polar it is necessary to know the angles of attack of the airplane α. From the velocity triangle of figure 20 there is

$$\sin\ \theta = \frac{u}{V} \quad (33)$$

where u is the vertical velocity of the airplane, determined by the pressure drop dp in time dt

$$u = \frac{11.1}{\Delta} \frac{dp}{dt} \quad (34)$$

In practice in computing the pressure drop by the altimeter it is more convenient to compute the vertical velocity by the formula:

$$u = \frac{H_1 - H_2 + \Delta H_1 - \Delta H_2}{dt} \frac{T_m}{T_{st}} \quad (35)$$

where H_1 and H_2 are the altimeter readings and ΔH_1, ΔH_2 the corresponding corrections from the calibrations of the altimeter, T_m the mean true temperature in gliding and T_{st} the standard temperature at the same altitude. Knowing the angle of inclination of the airplane to the horizontal there is

$$\alpha^{\circ} = \theta^{\circ} + \gamma^{\circ} \tag{36}$$

The weight of the airplane during the glide may be determined from the weight of the airplane before and after the flight and from the distribution of the fuel consumption between the flights. The distribution of the fuel consumption is computed from the time between flights and from the consumption data of the engine. It is necessary to take into account the fuel consumption when the engine is running on the ground, during the climb to a given altitude, and after carrying out the last glide up to the stopping of the engine. An inaccuracy in determining the weight of the order of 10 to 20 killograms gives a negligible error. The determination of Δc_x was discussed in section 3.

5. ANALYSIS OF THE TEST RESULTS

The form in which the data on the determination of the polar from flight tests is presented is shown in table 2. The data presented refer to the Y-2 airplane. There are first given the altimeter readings H_1 at the start of the glide and H_2 at the end of the glide and the corresponding corrections for the altimeter calibration ΔH_1 and ΔH_2. In column 6 the values of $dH = H_1 - H_2 + \Delta H_1 - \Delta H_2$ are computed. In the next column the gliding times dt in seconds are given. This is followed by the true temperature of the outside air T_m and the standard temperature T_{st} at the mean gliding altitude $H_m = H_1 - dH/2 + \Delta H_1$. In column 10 for the same altitude H_m are written the values of p, the air pressure in millimeters of mercury. In column 12 the values of the vertical velocity calculated by the formula are given.

$$u = \frac{dH}{dt} \frac{T_m}{T_{st}}$$

Further, the relative air density is determined by the formula

$$\Delta = 0.379 \frac{p}{T_m}$$

In column 14 the values of the velocity by the speed indicator V_{ins} (km/hr) are given. From the calibration of the speed indicator over the measured kilometer the values of the indicated velocities V_i are obtained. These values are corrected for the temperature of the air near the instrument panel. For this purpose it is necessary to have the calibration of the speed indicator for various temperatures. This calibration is carried out in the laboratory. From the calibration curve for the value of V_{ins} the corrections for the temperature in the pilot's cabin during the flight for measuring out the kilometer and for taking the polar are obtained. The difference of these corrections ΔV_i is added to the readings V_i. Thus $V_{i1} = V_i + \Delta V_i$ where V_{i1} is the value of the indicated velocity V_i corrected for the temperature calibration. From the corrected values V_{i1} there are computed the true values $V_{tr} = V_{i1}/\sqrt{\Delta}$. In column 19 the values of $\sin\theta$ are computed.

$$\sin\theta = 3.6 \frac{u}{V_{tr}}$$

In columns 20 and 21 the values of θ and $\cos\theta$ are given. The values of the weight G and the computed values $G\sin\theta$ and $G\cos\theta$ are further given. In columns 25 and 26 c_y and c'_x are computed.

$$c_y = \frac{2G\cos\theta}{\rho_o s V_{i1}^2}$$

$$c'_x = \frac{2G\sin\theta}{\rho_o s V_{i1}^2}$$

The readings of the tachometer and the corrected values of the engine speed by the tachometer calibration are given in columns 27 and 28. There the values λ are computed:

$$\lambda = \frac{60 V_{tr}}{3.6 \, nDi}$$

where i is the reduction factor of the engine. The values of λ_o are given in column 30:

$$\lambda_o = \frac{\lambda}{1 + h}$$

where h is the interference coefficient computed from figures 13 to 19. The values of $\tan \beta_o$ and β_o are then computed:

$$\tan \beta_o = \frac{\lambda_o}{\pi \bar{r}}$$

where $\bar{r} = 0.67$. Then $\Delta\alpha$ is computed by:

$$\Delta\alpha = 0.2 \lambda$$

The values of the blade angle of attack α_o are computed in column 34 by the formula:

$$\alpha_o = \varphi - \beta_o + \alpha_o - \Delta\alpha$$

From figure 3 or formula $\alpha_s = 0.1245 \alpha_o - 0.0822$ there are computed the values of α_s from α. In columns 36 and 37 are computed λ^2 and Δc_x:

$$\Delta c_x = \frac{2 \, i s_l D^2}{s} \frac{\alpha_s}{\lambda^2}$$

where i is the number of blades, s_l a coefficient depending on the shape of the blade. It may be noted that for multi-engine airplanes it is necessary to compute Δc_x separately for each propeller and combine their sum with c'_x in order to obtain c_x for the entire airplane. In column 38 are computed the values of c_x of the entire airplane:

$$c_x = c'_x + \Delta c_x$$

To measure the angles of attack it is necessary, from the readings of the inclinometer γ in millimeters (column 39) and the calibration of the inclinometer, to write down the values of the slope angle γ of the airplane. Knowing the values of θ (column 20) and γ (column 40) the values of the angle of attack of the airplane α are computed:

$$\alpha^o = \theta^o + \gamma^o$$

In computing the angles of attack it is necessary to take into account the angle between the wing chord and the airplane axis since the angles of inclination of the airplane are generally measured from the airplane axis. The obtained values of c_y and c_x are plotted in the form of curves $c_y = f(\alpha^o)$, $c_x = f(\alpha^o)$, and $c_y = f(c_x)$.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

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6. Kachanov, A. S.: Self Recorder of Trajectory Angle. Technika Vozdushnogo Flota, no. 4, 1938.

TABLE 1.– SHOWS THAT THE ANGLE α_0, THE AVERAGE ANGLE OF ATTACK OF THE BLADE AT ZERO THRUST, IS APPROXIMATELY THE SAME FOR VARIOUS PROPELLERS AND AIRPLANES

	Airplane	Propeller data	Blade section	h	α_0
1.	Northrup 2E, single engine monoplane, air-cooled engine	Three-blade, metal D = 2.95 φ = 25.8°	Clark-Y	0.07	0.5
2.	Same	Same φ = 17.8°	Clark-Y	.07	.7
3.	Same	Two-blade, metal D = 2.8 φ = 27.9°	Modified RAF-6	.07	.7
4.	П-5, single engine biplane, water-cooled engine	Two-blade, wood D = 3.3 φ = 15°	RAF-6	.02	.75
5.	Single-engine biplane	Two-blade, metal D = 3.1 φ = 25.5°	RAF-6	.015	.7
6.	Same	Same; φ = 21.5°	RAF-6	.015	.55
7.	Same	Three-blade metal D = 2.8 φ = 30.2°	RAF-6	.02	.8
8.	Four-engine monoplane, water-cooled engine	Two-blade, wood D = 3.5 φ = 15°	RAF-6	.1	.6
9.	Two-engine monoplane, water-cooled engine	Two-blade, wood D = 3.1 φ = 21.3°	RAF-6	.7	.5
10.	У-2, trainer biplane with air-cooled engine	Two-blade, wood D = 2.4 φ = 16.7°	RAF-6	.06	.78
11.	Single-place monoplane with air-cooled engine	Two-blade, metal D = 2.9	RAF-6	.08	.7

Table II

№	H_1	H_2	ΔH_1	ΔH_2	dH	dt	T_m	H_m	P	T_{st}	u	Δ	V_{ins}	V_i
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	1 450	1 250	30	30	200	69,6	296	1 380	643,6	279,0	3,05	0,824	72,5	84,6
2	1 450	1 250	30	30	200	63,1	296	1 380	643,6	279,0	3,37	0,824	81,5	92,8
3	1 450	1 250	30	30	200	52,7	296	1 380	643,6	279,0	0,04	0,824	92	102,3
4	1 350	1 150	30	30	200	43,6	296	1 280	651,5	279.7	4,85	0,834	104	112,9
5	1 400	1 200	30	30	200	36,0	296	1 330	647,5	279,4	5,89	0,829	114	121,9
6	1 350	1 150	30	30	200	30,7	296	1 280	651,5	279,7	6,90	0,834	123	129,9
7	1 300	1 100	30	30	200	25,8	296	1 230	655,5	280,0	8,20	0,839	133	138,6

№	ΔV_i	V_{i1}	V_{tr}	$\sin\theta$	$\theta°$	$\cos\theta$	G	$G\sin\theta$	$G\cos\theta$	c_y	c'_x	n	n_{cor}	λ
	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	2	86,6	95,6	0,115	6,60	0.993	845	97,2	839	0,699	0,0810	600	620	1,07
2	2	94,8	104,4	0.116	6,66	0,993	840	97,5	834	0,580	0,0678	710	700	1,035
3	2	104,3	114,9	0,1263	7,25	0,992	835	105,5	828	0,475	0,0606	780	770	1,035
4	2	114,9	125,8	0,1388	8,00	0,990	830	115,3	822	0,389	0,0546	850	860	1,016
5	2	123,9	136,1	0,1558	8,97	0,988	825	128,6	815	0,332	0,0523	920	940	1,005
6	2	131,8	144,3	0,1720	9,90	0,985	820	141,1	808	0,291	0,0509	1000	1 000	1,002
7	2	140 6	153,4	0,1925	11,1	0,981	815	157,0	800	0,253	0,0496	1075	1 090	0,977

№	λ_0	$\mathrm{tg}\,\beta_0$	β_0	$\Delta\alpha$	$\alpha_\bullet$	α_s	λ^2	Δc_x	c_x	γ	$\gamma°$	$\alpha°$
	30	31	32	33	34	35	36	37	38	39	40	41
1	1,009	0,481	25,7	0,21	—4,0	—0,58	1,145	0,01243	0,0686	129	— 0,6	6
2	0,976	0,465	24,95	0,2	—3,25	—0,485	1,071	0,01114	0,0567	113	— 2,7	4
3	0,976	0,465	24,95	0,2	—3,25	—0,485	1,071	0,01114	0,0495	100	— 4,4	2,85
4	0,958	0,456	24,5	0,2	—2,8	—0,43	1,032	0,01022	0,0444	85	— 6,6	1,4
5	0,948	0,452	24,35	0,2	—2,65	—0,41	1,010	0,01000	0,0423	72	— 8,1	0,87
6	0,946	0,451	24,26	0,2	—2,56	—0,396	1,004	0,0097	0,0412	56	—10,15	—0,25
7	0,921	0,438	23,7	0,2	—2,0	—0,33	0,955	0,0085	0,0411	45	—11,0	—0,5

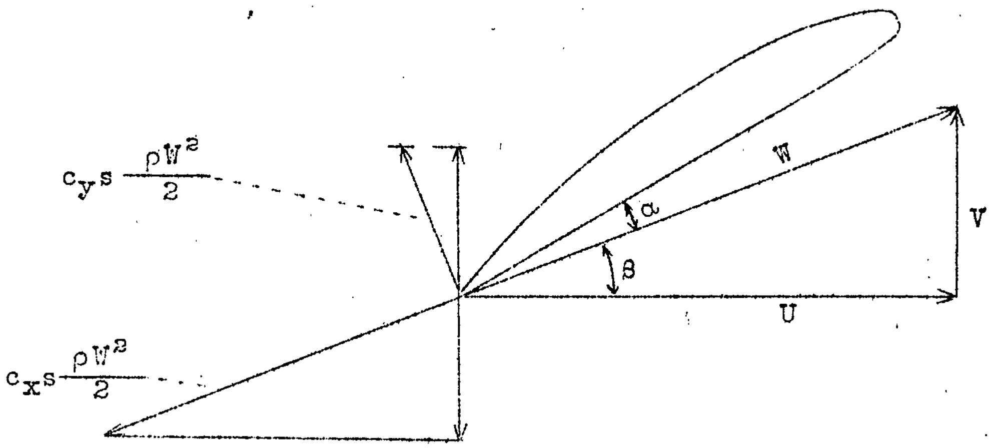


Figure 1.- Vector diagram for propeller blade at zero thrust.

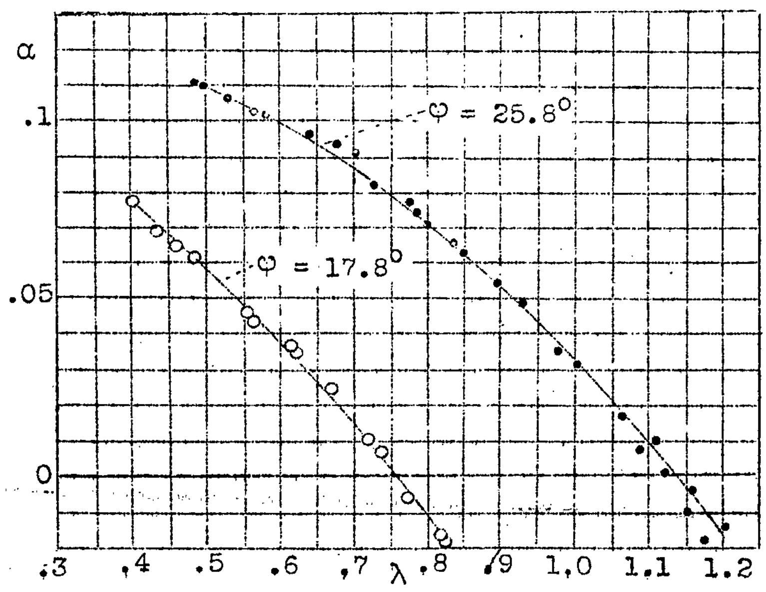


Figure 2.- Thrust coefficients of propellers of Northrup airplanes.

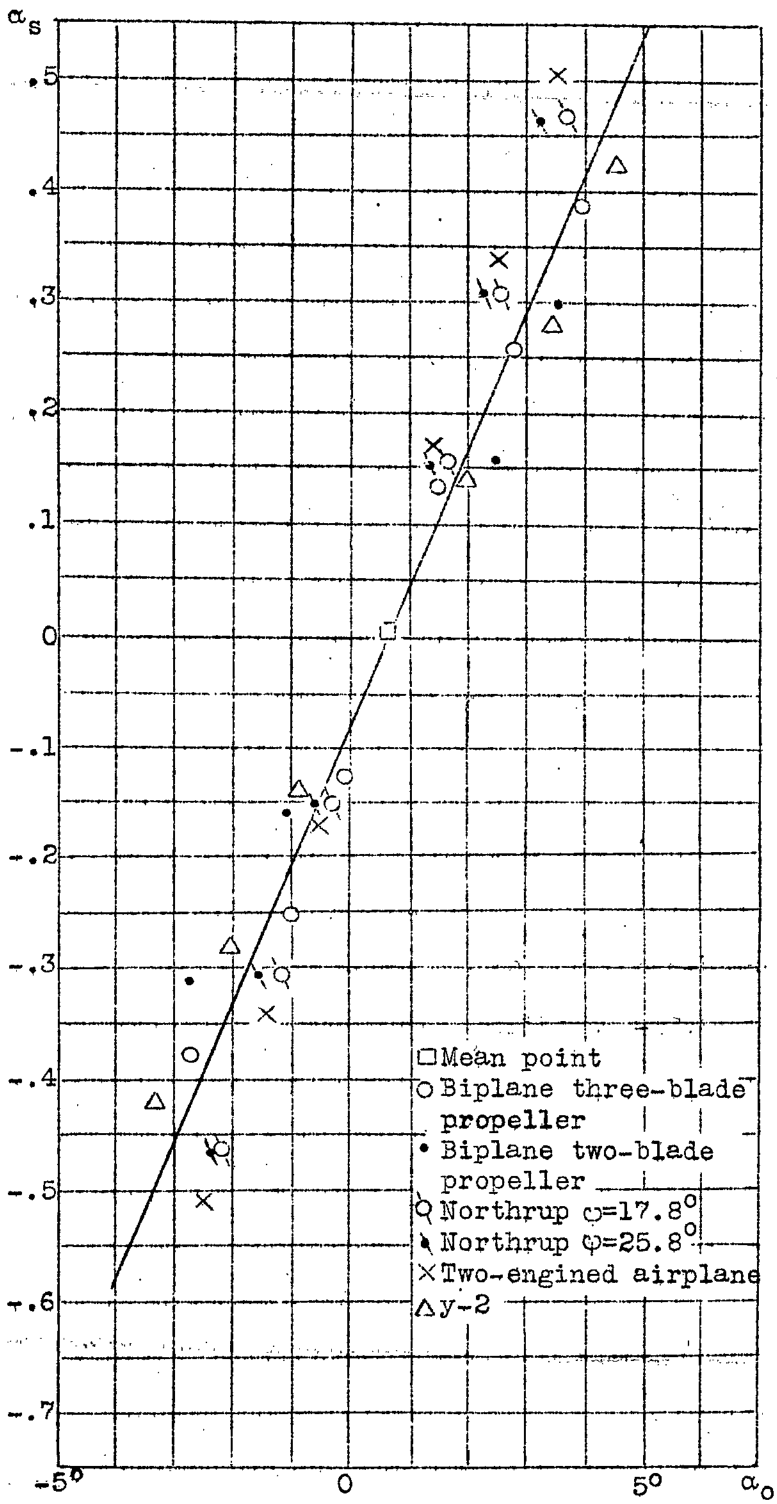


Figure 3.- Correction curve.

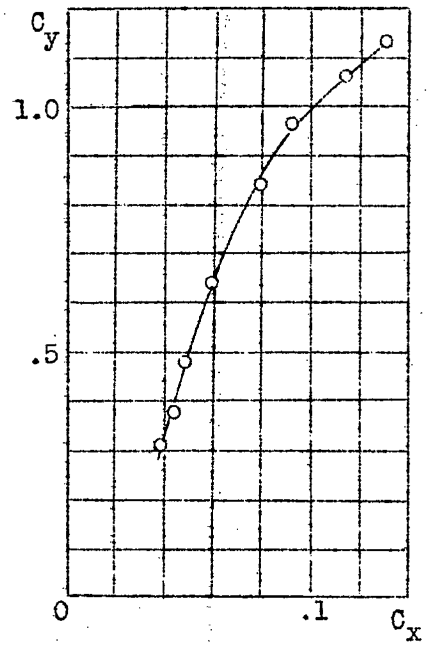


Figure 4.- Polar of single-engined biplane by the new method.

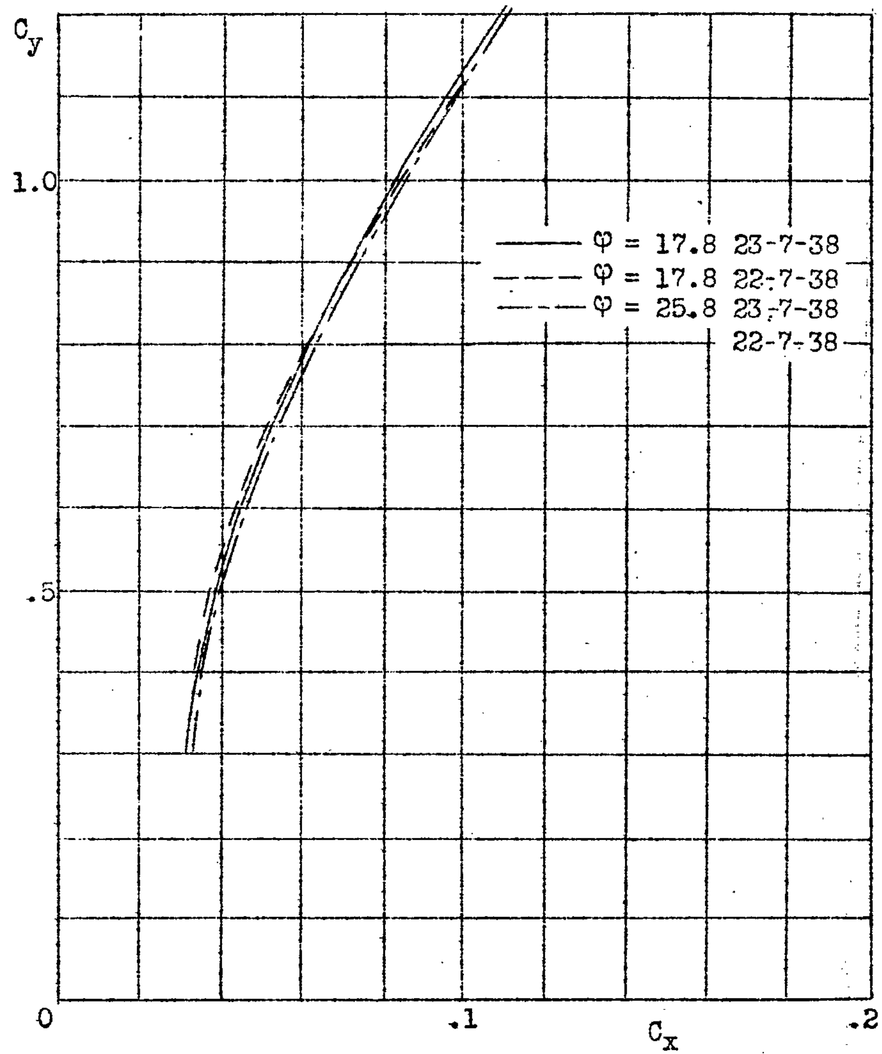


Figure 9.- Polars of Northrup airplane by new method.

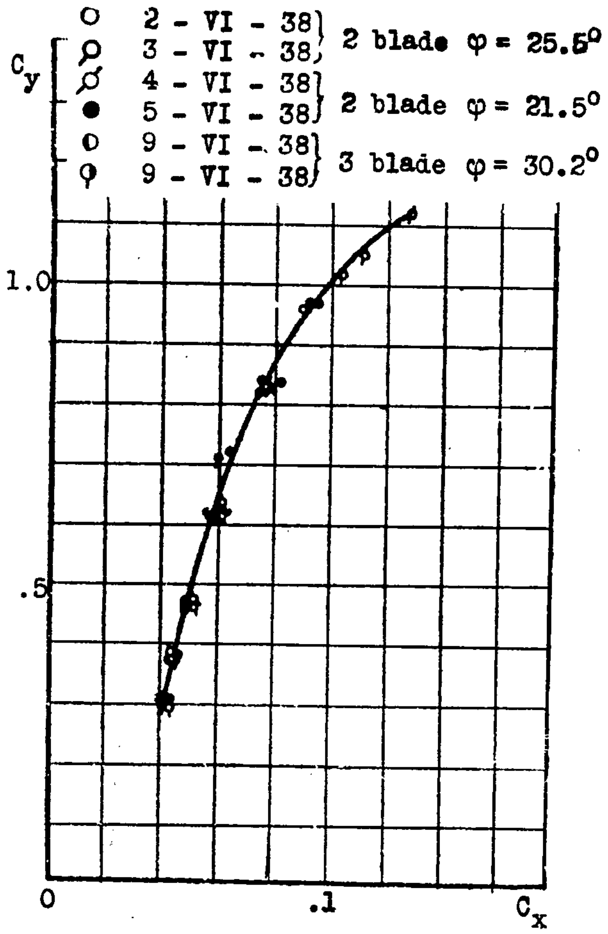


Figure 5.- Polar of single-engined biplane by old method.

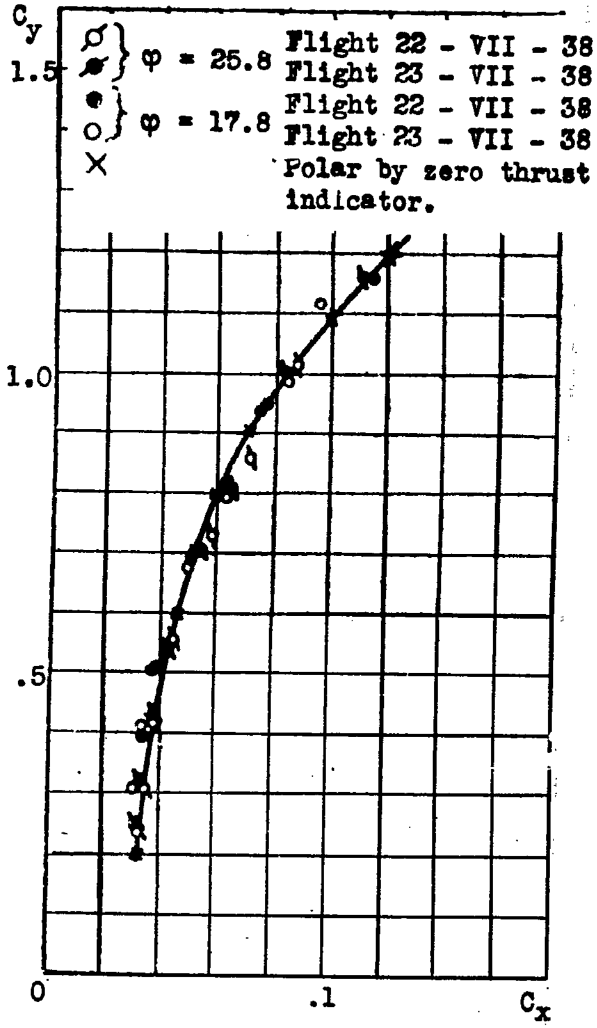


Figure 6.- Polar of Northrup airplane 2E by new method.

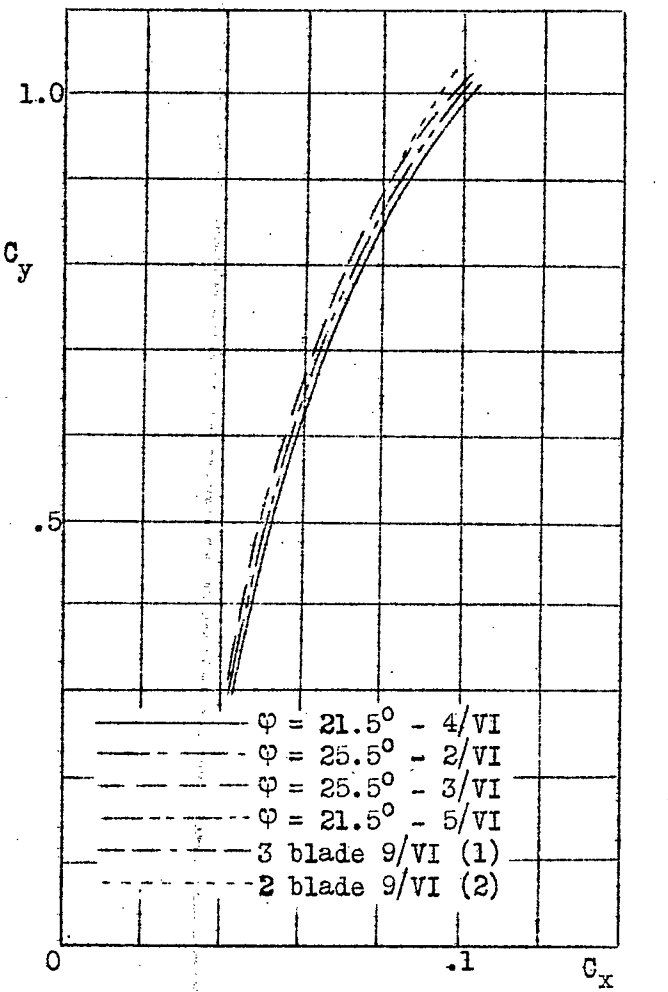


Figure 7.- Combined polar curves of single-engined biplane by new method.

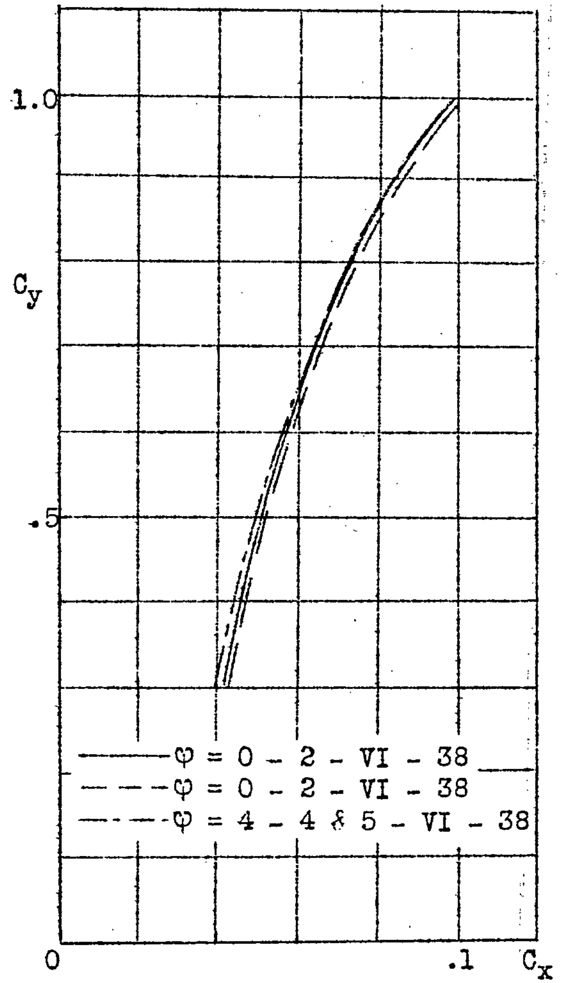


Figure 8.- Polars of single-engined biplane by zero thrust indicator.

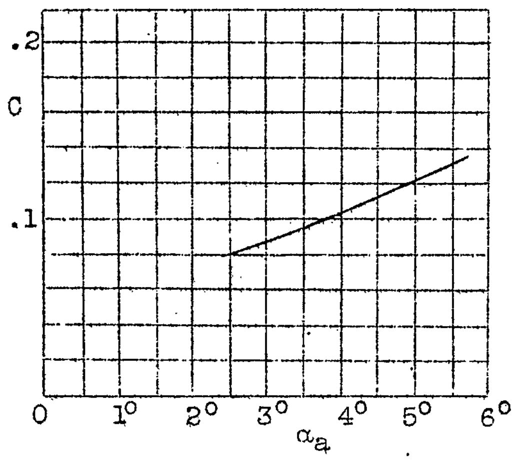


Figure 10.- Values of the angle α_a for the RAF-6 section.

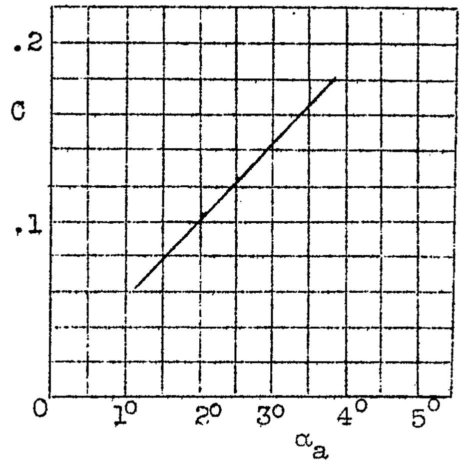


Figure 11.- Values of the angle α_a for the Clark-Y section (model).

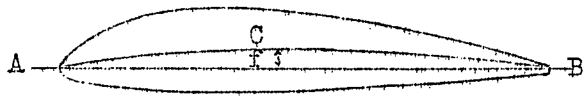
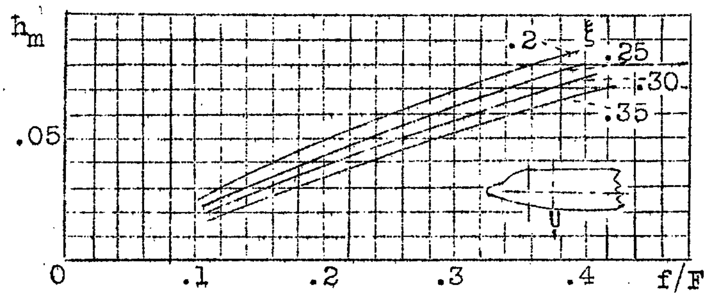


Figure 12.

Figure 13



Figures 13 to 19.- Curves of the coefficient h_m the relative pitch at $r = 0.67$ against f/F, the ratio of the middle section of the fuselage to the propeller disk area. e = maximum thickness of wing, D propeller diameter.

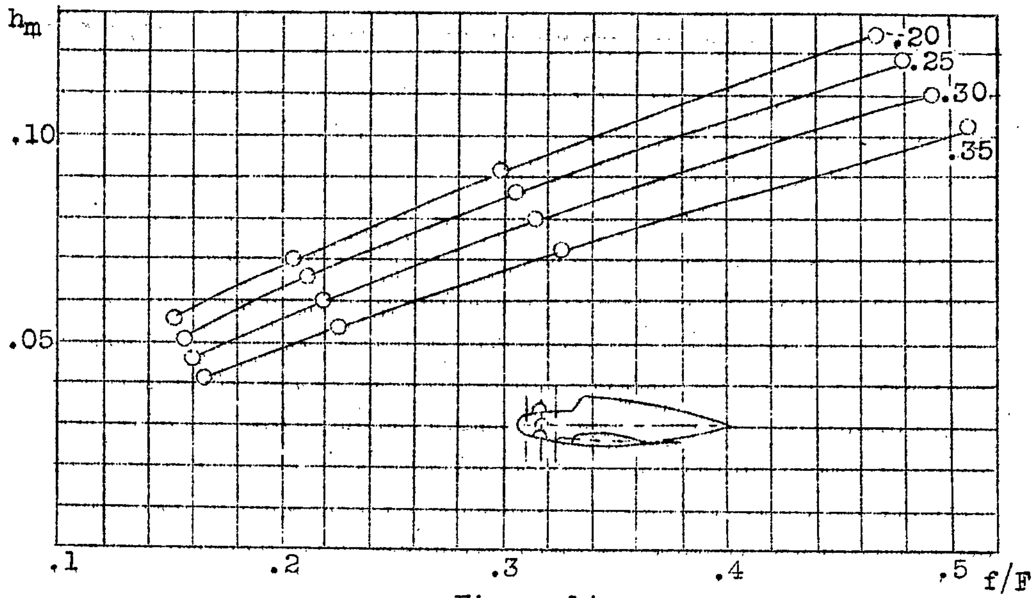


Figure 14

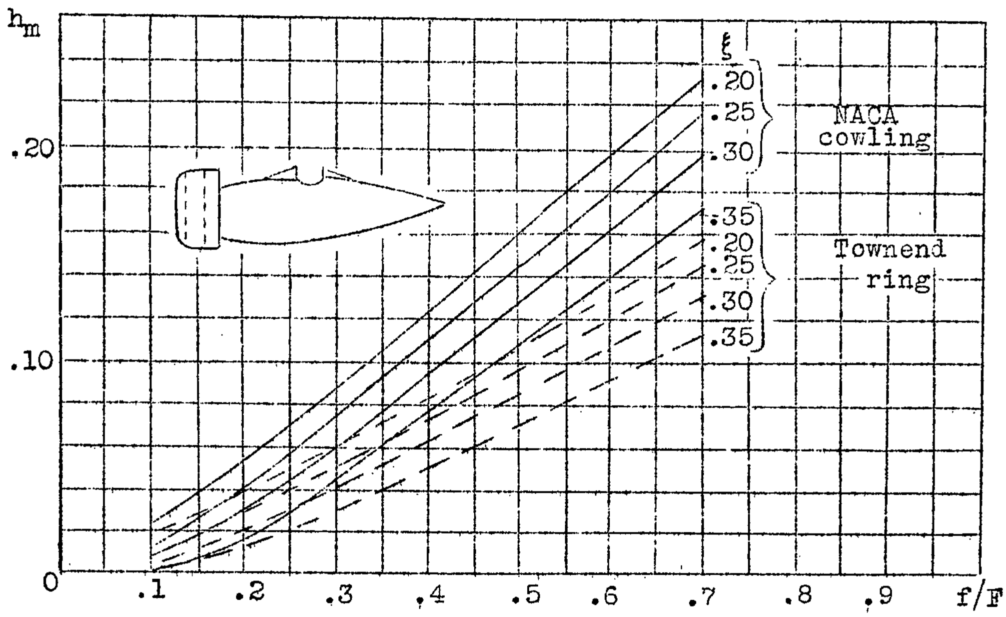


Figure 15

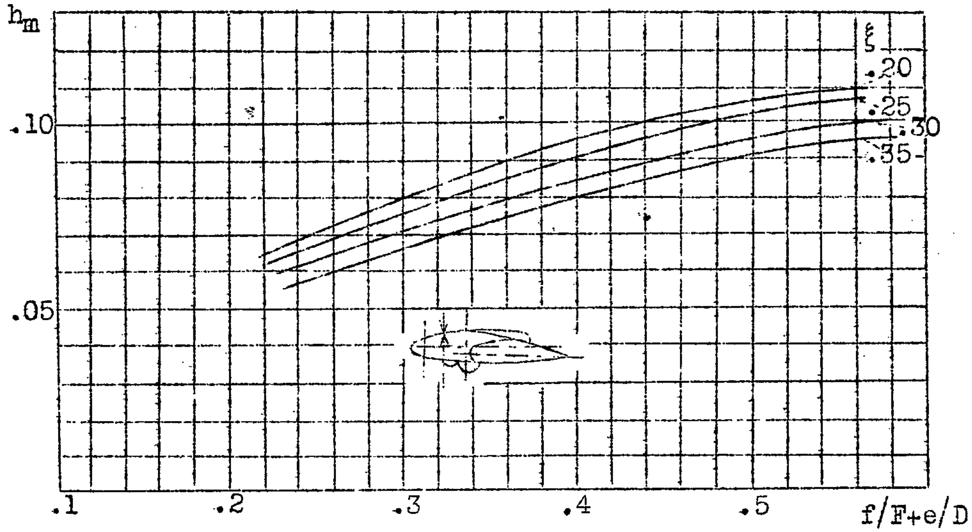


Figure 16

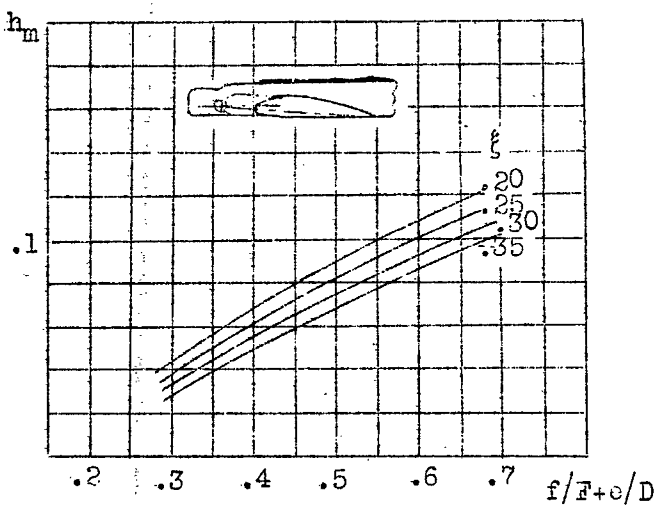


Figure 17

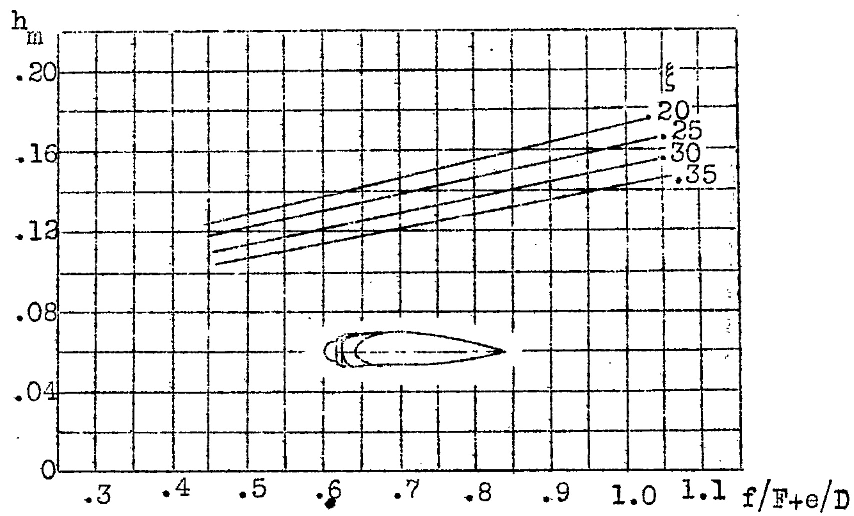


Figure 18

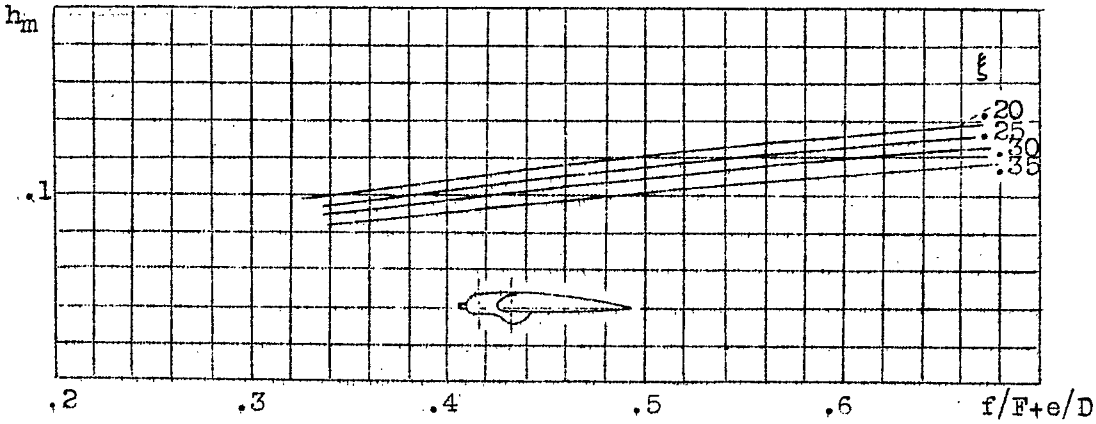


Figure 19

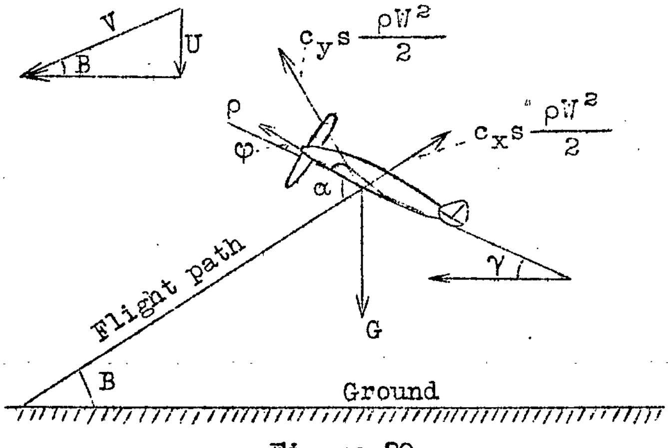


Figure 20