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Reliability Quantification of the Flexure: A Critical Stirling Convertor Component

Ashwin R. Shah and Igor Korovaichuk
Sest, Inc., Middleburg Heights, Ohio

Edward J. Zampino
Glenn Research Center, Cleveland, Ohio

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Sest, Inc.
Middleburg Heights, Ohio 44130

Edward J. Zampino
National Aeronautics and Space Administration
Glenn Research Center
Cleveland, Ohio 44135

Summary

Uncertainties in the manufacturing, fabrication process, material behavior, loads, and boundary conditions results in the variation of the stresses and strains induced in the flexures and its fatigue life. Past experience and the test data at material coupon levels revealed a significant amount of scatter of the fatigue life. Owing to these facts, the design of the flexure, using conventional approaches based on safety factor or traditional reliability based on similar equipment considerations does not provide a direct measure of reliability. Additionally, it may not be feasible to run actual long term fatigue tests due to cost and time constraints. Therefore it is difficult to ascertain material fatigue strength limit. The objective of the paper is to present a methodology and quantified results of numerical simulation for the reliability of flexures used in the Stirling convertor for their structural performance. The proposed approach is based on application of finite element analysis method in combination with the random fatigue limit model, which includes uncertainties in material fatigue life. Additionally, sensitivity of fatigue life reliability to the design variables is quantified and its use to develop guidelines to improve design, manufacturing, quality control and inspection design process is described.

1. Introduction

Ongoing joint efforts between the NASA Glenn Research Center (GRC) and the Department of Energy (DOE) to develop a free-piston Stirling radioisotope power system has made it possible to be considered as an energy conversion system option for future NASA deep space exploration missions. The 110 watt Stirling Radioisotope Generator (SRG110) now under development has a multifold increase in efficiency and would reduce the inventory of the radioisotope fuel by a factor of four compared to the Radioisotope Thermoelectric Generators (RTG's) for an equivalent level of power. GRC has been conducting research for decades on Stirling power conversion technology and is currently providing technical support to DOE and their system integration contractor for the SRG110, Lockheed Martin, at Valley Forge, PA. SRG110, a candidate for NASA's deep space exploration missions, require power systems that are reliable and operate for a long time (at least 14 years) without failure, interruption of power, or preventative maintenance. Overall reliability of the SRG110 hinges upon the reliability of its sub-systems and its components.

The Stirling Convertor Assembly (SCA) transforms the thermal energy from a radioisotope into the electrical energy needed to power spacecraft functions. The Stirling convertor is a free-piston device with a minimal number of moving parts, and the design has eliminated and/or minimized all principal wear and life limiting mechanisms. The salient features of the free-piston Stirling power convertor and the measures taken in the design and development to enhance reliability have been described in earlier publications (ref. 1). In order to ascertain reliability of the Stirling convertor during pre-launch handling, assent, space flight, and possible descent to a planetary surface, it is critical to quantify and assure the reliability of its components. The flexure and flexure stacks have been identified as one of the most critical mechanical components of the SCA and will have to undergo billions of cycles during the planned 14 years long mission. Successful performance of the flexures depends heavily on its long reliable fatigue life.

Use of flexures eliminates rubbing seals and lubrication, gas-bearing failures and provides good predictability, repeatability, reliability and stability as proven by its use in multiple space cryocoolers. Flexures used in many other applications have also demonstrated its efficiency, reliability, robustness, and safe operation over fairly good range of environmental conditions. However, like any material it is prone to uncertainties in the manufacturing and fabrication process, which affects its strength and long term behavior under cyclic loads. Majority of structural materials have shown considerable scatter in the fatigue strength due to probable existence of flaws in the material, material processing variations, granular structure, and fabrication and to some extent variations in the quality control and inspection (human aspect). Fatigue resistant material used for flexure is no exception from variation in the fatigue strength as evidenced by the vendor supplied data. Additionally, since the fatigue strength of a material is a function of the stress level it is subject to, any scatter in the stress affects the fatigue life and as a result its reliability for desired life. The scatter in stresses is mainly derived from uncertainties in the material properties, geometry, loads and boundary conditions. Therefore, uncertainties in the flexure manufacturing and fabrication process, loading conditions, assembly, material strength, etc. need to be considered in the reliability assessment of the flexure fatigue life. The present paper briefly describes the methodology used to quantify the reliability of flexure, typical geometry of flexure used in the linear alternator section of the SCA, figure 1, and results of the analysis. Also, significance of the results and sensitivity of the reliable fatigue life to the design variables have been discussed and described.

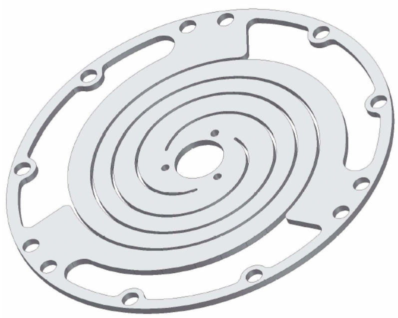

Figure 1.—Spiral flexure geometry.

2. Flexure Analysis

Flexures have been used in both the linear alternator piston and displacer assembly of SCA. Flexures are critical in the energy conversion of thermal energy in to the electrical energy. Spiral geometrical configuration with one or more legs, with one end clamped at the stationary stator and other at the mover rod at the center provides a high radial stiffness and soft spring in the axial direction. The high radial stiffness allows maintaining non-contacting close clearance seal and provides linear motion without rocking. SRG110 flexures are made from the fatigue resistant material, a material proven for long life. SRG110 is designed to run at 82Hz frequency and therefore the flexures will undergo alternating stress cycles of over 35 billion loading cycles during the 14 years of a planned mission without failure. Although the SRG110 flexures are fabricated with surface treatments such as precise machining to remove nicks and gouges, shot peening, etc. in order to improve fatigue life, the uncertainties described in the introduction section need to be quantified in the reliability evaluation for a successful long term performance. A further description of uncertainties considered in the analysis has been described in the following section.

(a) Design variable uncertainties.—Flexure design variables relate to geometry, fabrication process, material properties, loads, boundary conditions, etc. Current analysis results presented herein include uncertainties related to these variables in the form of different physical parameters such as thickness, elastic modulus, pressure, piston stroke, degree of fixity, etc. Uncertainties in these variables used are based on the available test data, allowable tolerances in the drawings, operational loads bounds, engineering experience, judgment and SCA technology development team opinions. As will be explained in the methodology description, some of the variable uncertainties having insignificant impact (based on preliminary evaluation) on reliability have been omitted for discussion in this paper. The significant variables and their uncertainties have been listed in table 1 below.

The numerical values of many variables related to SCA design have been omitted herein in compliance with ITAR regulations. Uncertainties in the variables such as degree of fixity, Poisson's ratio, geometric shape, etc. have been found to be small enough not to cause any effect on the life of the flexure and therefore have not been listed here. Also, magnitude of the temperature on flexures is small enough to cause any thermal fatigue on flexure material and hence it is neglected in the analysis as well.

Table 1.—Uncertainties in the design variables

Random design variable	Mean Value	Coefficient of Variation, %	Probability Distribution
Thickness	–	0.08	Lognormal
Elastic modulus	–	2.50	Weibull
Stroke	–	3.30	Normal

Another important variable in the design is material fatigue strength/resistance. It is a common practice to define the fatigue life (strength/resistance/ limit) as the stress/strain range, below which tested coupons will not fail before a certain number of cycles, e.g., 10^6 or 10^7 is attained. Generally, it has been observed that materials having fatigue life over 10^6 or 10^7 under a given set of load conditions tend to have extremely low failure rate and said to have infinite life[1]. This limit is often called as endurance limit. However, in real world it is impossible that there would never be a failure for components designed lower than the endurance limit. In general, designing for endurance limit is sufficient for a classical safe-life design concept. In many cases, fatigue data is reported as a median S-N curve, which represents 50 percent probability of failure (*PoF*) as a function of applied stress/strain. Conventional approach assigns knock-down factors/factors of safety (FOS) to these limits to design components. However, it is evident from the tests and experience that the fatigue life exhibits a large uncertainty and scatter to the order of 200 percent and more from the mean/median values. Designing using FOS may result in highly conservative design or sometime un-conservative. However, design-for-reliability requires an approach, so-called S-N-P curves or *p*-quantiles (probability quantiles) of fatigue life, that account for uncertainties in the fatigue strength in a rational manner. For highly reliable components such as flexure for SCA, the 0.001- or 0.0001- quantile of fatigue life must be taken into account.

In practice, a limited amount of p-quantiles test data, for instance, such as p-0.05, 0.10, 0.50 may be available to the designer. Therefore, a methodology to predict lower p-quantiles of fatigue life must be addressed and developed. In this paper, an approach based on application of random fatigue limit model is presented. The distinguishable feature of this model is that the fatigue limit is considered as random variable with certain cumulative distribution function parameters which are estimated using available test data and statistical and probability methods. SCA flexures use material for which coupon level fatigue test data is available from the vendor. The uncertainties in the fatigue life of this material have been quantified for lower p-quantiles using the available test data. The outline of the approach used to quantify uncertainties is given in the subsequent sections.

(b) Reliability analysis methodology.—The reliability methodology developed and used has been divided in three parts: (i) quantification of uncertainties in the structural response (stresses), (ii) quantification of uncertainties in the material fatigue strength, and (iii) reliability analysis.

(*i*) *Structural response uncertainties quantification*: A probabilistic analysis method that simulates uncertainties in the design variables through mechanics-based model combined with finite element analysis forms a rational basis for structural response uncertainties simulation. The flexure shown in figure 1 was modeled using finite elements and structural analysis was performed using ANSYS FEA (finite element analysis) software package. The geometry of the flexure leaf profile was obtained from the drawings and quad shell elements were used in the analysis. An appropriate element size was determined by performing a non-linear elastic finite element analysis using different mesh sizes until the computed results compared with the measured non-linear spring rate of the flexure. Boundary conditions consisted of clamping the flexure peripheral area while allowing reversed axial motion of the center of the flexure with frequency of 82 Hz. Note that owing to the flexure structure and imposed constrains, each leaf of the flexure will experience reversed bending as well as axial elongation. Several finite element runs were made for different variations in the random variables listed in table 1 to generate the equivalent stress response surface (since equivalent stress will be compared with the fatigue strength) required to perform probabilistic analysis using fast probability integration. A dynamic analysis was also performed in frequency domain to evaluate the dynamic stress amplification factor. It was observed that the dynamic amplification factor is less than 5 percent and has been neglected in the analysis. Fast probability integration of input random variables and their corresponding equivalent stress response was performed to compute cumulative probability distribution function of the equivalent stress, figure 2 and its sensitivity to the random variables quantified. The scatter computed in the equivalent stress is 4.2 percent.

(*ii*) *Fatigue strength uncertainty quantification*: Generally available fatigue tests are pure reversed bending for coupon geometry specified by the American Society of Testing Materials (ASTM). Cyclic reversed bending tests are easier to design and perform than cyclic axial fatigue tests (ref. 2). Vendor supplied reversed bending fatigue test data for flexure material for 0.05, 0.10 and 0.50 p-quantiles were available. Meaning each curve represents fatigue life for 5, 10, and 50 percent of probability of failure for different stress magnitudes. The fatigue life of a large class of steel material has been found to be log-normally distributed. Using the available limited tests data, the fatigue life of the material used for flexure is also verified to have log-normal distribution. Since a complete probability distribution is required for reliability assessment, it is necessary to characterize the entire distribution from the 5, 10, and 50 percent p-quantile life test data. A random fatigue limit model (ref. 3), eq. (1), described below was used to characterize the parameters that define the distribution.

$$\log N_F = \beta_0 + \beta_1 \log(S - S_F),\ S > S_F \tag{1}$$

where N_f , S and S_F are number of cycles to failure, applied stress value and material fatigue limit, respectively. It is assumed that fatigue limit S_F is random with lognormal distribution. This distribution was chosen based on analysis of fatigue data and experience. Coefficients β_0, β_1 and parameters of the fatigue limit distribution function were computed using maximum likelihood estimation method.

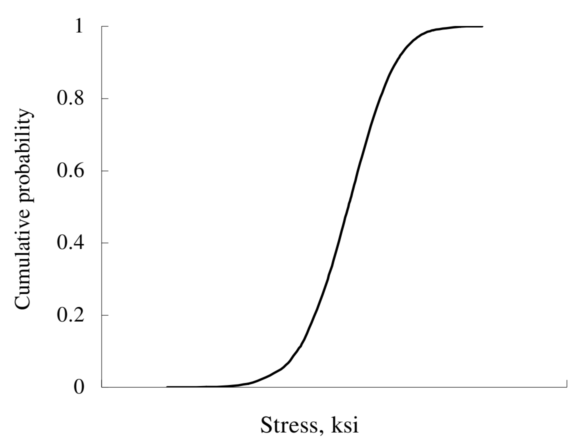

Figure 2.—Cumulative probability function of maximum equivalent stress.

Results of uncertainties in the fatigue life are shown in figure 3. As expected, the model clearly captures larger scatter in the long term life and small scatter in the short fatigue life.

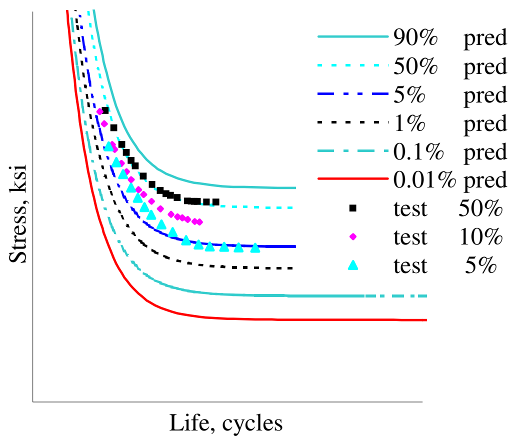

Figure 3.—Flexure material fatigue life distribution.

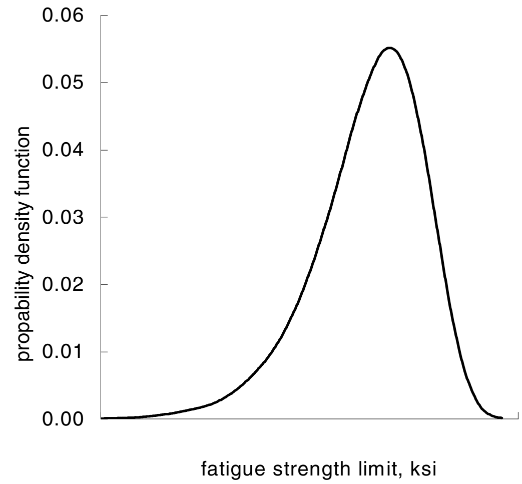

Figure 5.—Probability density function of fatigue stress limit.

Since the reliability assessment involves integrating effective stress with the fatigue strength distributions, it is necessary to convert fatigue life distribution in the form of stress distribution. Therefore, a probabilistic approach was used to compute the probability distribution of fatigue strength in terms of stress for a desired life say 36 billion cycles in the case of flexures. All of the computed spectra of estimated *p*-quantiles were used to calculate cumulative distribution function and probability density function of fatigue life as functions of fatigue stress limit, as shown in figures 4 and 5, respectively. It was found that coefficient of variation in the material fatigue life is 6.7 percent. Thus, the scatter in the transformed fatigue stress limit is lower than that one would notice when represented in terms of life. According to our prediction, safe-life design based on 3-sigma approach as the lowest fatigue limit corresponds to 1 percent of failure. However, design-for higher reliability of survival may require an even lower value of failure and therefore the design fatigue limit will be lower. Thus, for the studied material, fatigue limit corresponding to 99.99 percent of survival or 0.01 percent of failure is about 15 percent lower.

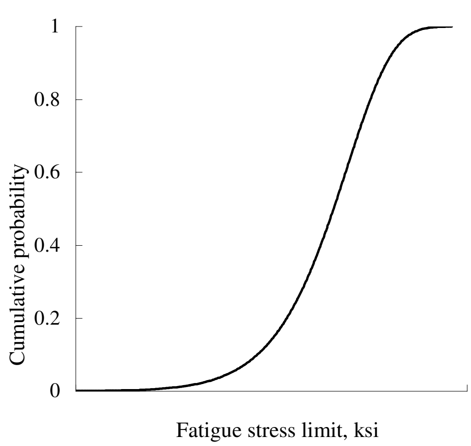

Figure 4.—Cumulative probability distribution of fatigue stress limit.

(iii) Reliability assessment: Reliability is defined as a cumulative probability that the structural strength/resistance will exceed the load effect (stress). As mentioned earlier, the flexures are subject to a multi-axial state of stress whereas the strength test data is for pure reversed bending stress only. Additionally, the probability of flaws in the material increase with the volume and as a result uncertainties in the fatigue life increases with the increase in size of components. Fatigue life under axial bending or pure reversed bending is generally higher than that under the multi-axial state of stress. Load factors have been applied to the results of pure reversed bending test data to compare with the multi-axial state of stress, represented in the form of von-Mises or effective stress, in order to assess the components life for design purpose. Generally the load factor has been found to be in 0.7~0.9 range (ref. 4). Determination of the appropriate load factor for a given state of multi-axial stress is critical and requires careful attention. Fatigue test data of the real components together with the computational results is helpful to determine the magnitude of the load factor. Since, enough fatigue test data of SRG110 flexures is not available at this time, a reliability evaluation has been performed considering the above uncertainties and the vendor supplied reversed bending fatigue test data using load factors 0.7 and 0.8.

Reliability or the probability of survival is represented mathematically using eq. (2).

$$P_S = P\left[\left(S_F - S\right) > 0\right] \qquad (2)$$

where P_s is reliability, S_F is fatigue strength limit and S is the maximum effective stress in flexures. Since all experimental data values were obtained from the reversed bending tests and considering the complexity of the flexure deformation and multi-axial state of stress, appropriate adjustment is made in their use in reliability analysis. The conventional and widely used approach to employ load conversion factor, K has been adopted. This factor depends on the nature of loading conditions and deformation state of the components. Generally in the case of axial loading, it is expressed mathematically as

$$S_{axial} = KS_{bend} \approx (0.7, 0.9)\, S_{bend} \qquad (3)$$

A detailed description of the above factor is discussed in reference 4. The load conversion factor for torsion and rotating bending tests ranges from 0.5 to 0.6. Since the state of stress in flexures is not predominantly torsion, one can use the load conversion factor in the range (0.7 to 0.9). Analysis using a load factor of 0.7 and 0.8 was performed using fast probability integration methods. Figure 6 shows the cumulative probability distribution function for the limit state (S_F-S). It can be seen from this figure that the limit state less than zero defines the failure since it corresponds to the probability that the load effect will exceed the fatigue limit strength. Thus the y-axis divides the safe and unsafe regions. As expected it clearly shows the probability of failure is 0.42 for load conversion factor 0.7 and that for load factor 0.8 is 0.015. Thus the reliability of flexures for load factors 0.7 and 0.8 are 58 and 98.5 percent respectively. It is also apparent that the reliability increases exponentially with increase in the load conversion factor. The sensitivity of the reliable life to the design random variables is shown in figure 7. It shows that the uncertainties (listed in the order of importance) in the material fatigue strength, the piston stroke, elastic modulus and thickness of the flexure are most significant for 98.50 percent reliability (load conversion factor = 0.8 case). It means that the uncertainties in theses variables should be reduced in order to improve the reliability. Better quality control, stricter inspection procedures and testing could be implemented to have control over scatter in these variables. Also, evaluation of the load conversion factor should be performed using actual flexure fatigue test data to represent multi-axial state of stress more realistically. Past experience at Stirling Technology Company suggests flexure being highly reliable for life over 10 years (ref. 5) and therefore, the analysis reported in this paper need to be supported using actual flexure test data.

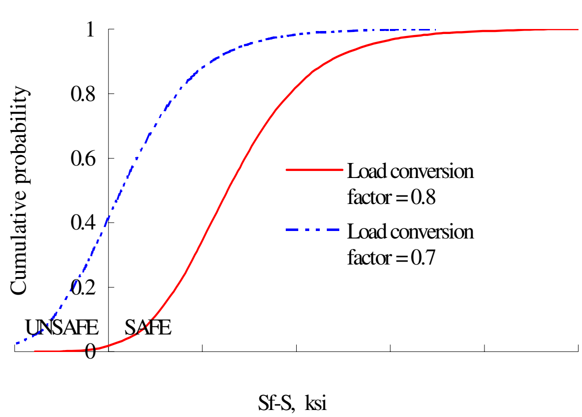

Figure 6.—Cumulative probability function of flexure limit state.

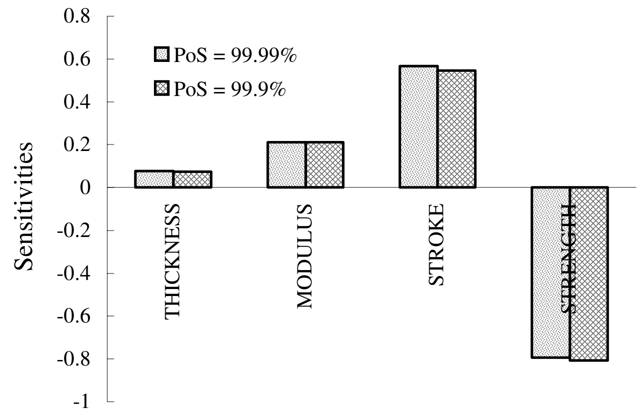

Figure 7—Sensitivity of flexure reliability to design variables (Load conversion factor = 0.8).

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13. ABSTRACT ***(Maximum 200 words)***

Uncertainties in the manufacturing, fabrication process, material behavior, loads, and boundary conditions results in the variation of the stresses and strains induced in the flexures and its fatigue life. Past experience and the test data at material coupon levels revealed a significant amount of scatter of the fatigue life. Owing to these facts, the design of the flexure, using conventional approaches based on safety factor or traditional reliability based on similar equipment considerations does not provide a direct measure of reliability. Additionally, it may not be feasible to run actual long term fatigue tests due to cost and time constraints. Therefore it is difficult to ascertain material fatigue strength limit. The objective of the paper is to present a methodology and quantified results of numerical simulation for the reliability of flexures used in the Stirling convertor for their structural performance. The proposed approach is based on application of finite element analysis method in combination with the random fatigue limit model, which includes uncertainties in material fatigue life. Additionally, sensitivity of fatigue life reliability to the design variables is quantified and its use to develop guidelines to improve design, manufacturing, quality control and inspection design process is described.

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