

Topography modeling in atmospheric flows using the immersed boundary method

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1. Motivation and objectives

Numerical simulation of flow over complex geometry needs accurate and efficient computational methods. Different techniques are available to handle complex geometry. The unstructured grid and the multi-block body-fitted grid techniques have been widely adopted for complex geometry in engineering applications. In atmospheric applications, terrain fitted single grid techniques have found common use. Although these are very effective techniques, their implementation, coupling with the flow algorithm, and efficient parallelization of the complete method are more involved than a Cartesian grid method. Oftentimes, the grid generation can be tedious and one needs to pay special attention in numerics to handle skewed cells for conservation properties. Researchers have long sought for alternative methods to ease the effort involved in simulating flow over complex geometry. A good example is the work by Peskin (1977). He has developed the immersed boundary method (IBM) to simulate blood flow in a heart/mitral valve system, where the boundary is represented by a body force and the equations are solved on a Cartesian grid. However, formulating a suitable body force term has proved to be a challenging issue.

In recent years, IBM has been significantly improved. Mohd-Yusof (1997) has proposed the direct forcing method in which the body force is implicitly taken into account by reconstructing the velocity field around the immersed boundary. Essentially, this new approach has eliminated the issue of explicit formulation of a suitable body force representing the boundary. IBM with the direct forcing approach has been further developed by adopting higher order reconstruction schemes, and it has been successfully applied to complex flow problems in engineering applications (e.g. flow over a truck, flow in piston cylinder assembly and flow in a stirred tank) by Fadlun *et al.* (2000); Verzicco *et al.* (2000); Iaccarino & Verzicco (2003). For high Reynolds number flow simulations, IBM coupled with an adaptive mesh refinement technique has been effective to provide the required near-wall resolution (Kalitzin & Iaccarino 2003). Adaptive mesh refinement helps reduce the overall number of computational nodes to achieve the desired near-wall resolution, but the problem can still be very demanding for higher Reynolds number flow simulations.

Wall/surface modeling in LES or the wall-function formulation in Reynolds-averaged Navier-Stokes (RANS) computations have been proposed to decrease the computational load due to near-wall resolution. In these approaches, coarse resolution grids can be employed, and no-slip conditions are not applied directly at the surface, because the implied stress would be overestimated on a coarse resolution grid. Instead, stresses at the surface are imposed as boundary conditions, alleviating the need to resolve the thin turbulent boundary layer. If one wants to extent the IB method for topography modeling

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in LES of atmospheric flows, for which numerical resolutions have been poor, then it is necessary to develop a reconstruction scheme that can take into account surface modeling. This issue has not been addressed within the context of IBM and it is one of our main objectives in this paper.

In what follows, we briefly describe the immersed boundary method, and apply it to low Reynolds number laminar flow cases to test our implementation. To take into the LES surface modeling issue within the IB method, we develop a reconstruction scheme based on the mean logarithmic wind profile assumption near the surface. To test this new scheme, we perform LES of a neutrally stratified atmosphere. Specifically, we compare the results of the IB method with the new reconstruction scheme to the results of the commonly adopted surface modeling approach, in which surface stresses are imposed as boundary conditions.

2. Governing equations

The governing equations for LES of a neutrally stratified atmospheric boundary layer are the filtered Navier-Stokes equations written as follows

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0, \quad \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial (\bar{u}_i \bar{u}_j)}{\partial x_j} = \frac{\partial \bar{p}}{\partial x_i} + \epsilon_{ijk} f_j \bar{u}_k - \frac{\partial \tau_{ij}}{\partial x_j}, \quad (2.1)$$

where turbulent stresses are defined as $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$, f_j is the Coriolis parameter, ϵ_{ijk} is the permutation tensor, \bar{u} and \bar{p} are the filtered velocity and filtered dynamic pressure, respectively.

To solve the above governing equations, we use the LES atmospheric research code DHARMA. The numerical method adopted in DHARMA is described in detail in Stevens & Bretherton (1996) and Stevens *et al.* (2000). The governing equations are integrated using a forward-in-time projection method based on an explicit second-order Runge-Kutta scheme (Bell & Marcus 1992). The spatial discretisation is performed on a staggered grid. A third-order accurate upwind-biased monotonic scheme is used for the advection terms, whereas diffusion and pressure gradient terms are discretized using second-order accurate central differencing schemes. A direct solver (FFT) is utilized for solving the pressure Poisson equation, and the code has been parallelized to run on various platforms using MPI.

3. Subgrid scale turbulence modeling

Since the Reynolds number of a typical atmospheric boundary layer (ABL) flow is very high, LES of ABL with near-surface resolution is not a practical option with current computer resources. To alleviate this obstacle one resorts to LES with surface modeling. For a planar surface, a common approach is to set the vertical component of the velocity to zero, and to define the horizontal components of the turbulent stresses based on the mean logarithmic wind profile assumption (Moeng 1984). The surface stresses can be written as follows

$$\tau_{i3} = - \left(\frac{\kappa}{\ln \frac{z_1}{z_0}} \right)^2 |u| \bar{u}_i, \quad (3.1)$$

where z_1 , u_i are the distance in the vertical direction and the velocity of the first grid point away from the surface, respectively. z_0 represents the roughness height. It should be noted that, in the immersed boundary method, we do not utilize the above approach

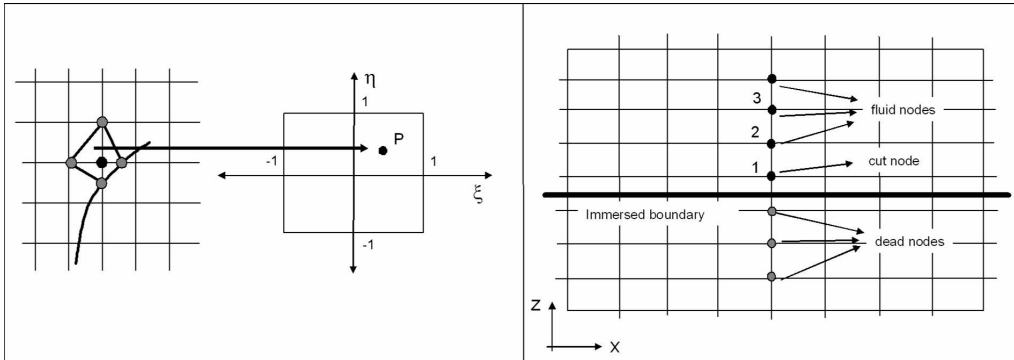


FIGURE 1. Schematics of the reconstruction schemes. Bilinear inverse interpolation (left), log-law (right)

for surface modeling. Instead, we employ the log-law reconstruction scheme, which is described in the next section.

Turbulent stresses that appear in equation 2.1 are modeled based on the Boussinesq eddy viscosity assumption. As discussed in Senocak *et al.* (2004), in LES of ABL near-surface models are needed to improve the predictions in the vicinity of the surface. Based on the results of that study we employ the hybrid RANS/LES model for the present computations. The hybrid RANS/LES model adopts the Prandtl's mixing length model (Prandtl 1925) near the surface and blends in with the dynamic Smagorinsky (Germano *et al.* 1991) away from the surface. The resulting form of the turbulent eddy viscosity can be written as

$$\nu_t = [(1 - \exp(-z/h))^2 \cdot (C\Delta^2) + \exp(-z/h)^2(\kappa z)^2]|S|, \quad (3.2)$$

where z is the distance from the surface, Δ is the filter width, κ is the von Karman constant with a value of 0.41, and $|S|$ is the magnitude of the strain rate tensor. The dimensionless parameter C is computed dynamically during the solution, making it a function of space and time (Germano *et al.* 1991). In the above equation h is the altitude, corresponding roughly to the upper edge of the surface layer. A value of 150(m) is used for the present computations. It should be mentioned that a logarithmic velocity profile is expected within the surface layer, which is approximately the bottom 10 percent of the atmospheric boundary layer height for neutrally stratified conditions (100 m -200 m) (Stull 1988).

4. Immersed boundary method

In IBM the solid boundary is represented by a body force F_i . The discretized form of the momentum equation given in 2.1 can be written as follows.

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = RHS_i + F_i, \quad (4.1)$$

where RHS_i includes the pressure gradient, convective, diffusive, and Coriolis terms. F_i is the body source term that gives the desired velocity at the boundary. In the direct forcing technique (Mohd-Yusof 1997; Fadlun *et al.* 2000) if the desired velocity at the

boundary is $u_i^{n+1} = V_i^{n+1}$ then one can write the explicit form of the body force as

$$F_i = -RHS_i + \frac{V_i^{n+1} - u_i^n}{\Delta t}. \quad (4.2)$$

Hence, instead of imposing the body force F_i explicitly to obtain the desired velocity at the boundary, one can impose the desired velocity and take into account the body force implicitly. For complex geometry, the boundary is not coincident with the Cartesian grid nodes, so one needs to reconstruct the velocity field using the values from neighboring nodes and the desired value at the boundary.

The steps involved in applying the immersed boundary method can be summarized as follows.

- (a) Preprocessing: Determine the Cartesian cells that are cut by the boundary. Tag the nodes as *dead*, *fluid* and *cut*.
- (b) Predictor stage: Solve the discretized momentum equations.
- (c) Set zero velocity field on the *dead* nodes, and apply the reconstruction scheme on the *cut* nodes.
- (d) Solve the pressure Poisson equation
- (e) Update the velocity and the pressure field, and impose the reconstruction on the *cut* nodes.

Several reconstruction schemes have been suggested in literature (Fadlun *et al.* 2000; Iaccarino & Verzicco 2003). The simplest reconstruction scheme is the nearest neighbor. For instance, if we consider a 2-D geometry, a linear interpolation can be employed in either the x (horizontal) or the z (vertical) direction depending on the distance from the boundary. To preserve the local maxima or the minima a bilinear inverse interpolation can also be adopted. As depicted in the left part of figure 1, a quadrilateral element can be constructed from the neighboring nodes and the boundary surrounding the cut node. This quadrilateral element and the coordinate of the cut node is then mapped onto a square element, where an area weighted average is used to interpolate the value of the velocity at the cut node. A quadratic equation needs to be solved for this mapping, which can be derived as follows

$$x_P = \sum_{i=1}^4 x_i N_i, \quad y_P = \sum_{i=1}^4 y_i N_i, \quad (4.3)$$

$$N_i = \frac{1}{4}(1 + \xi\xi_i)(1 + \eta\eta_i), \quad \xi_i = \pm 1, \eta_i = \pm 1.$$

Eliminating ξ gives a quadratic equation for η .

$$a\eta^2 + b\eta + c = 0, \quad (4.4)$$

Upon solution of this equation the root that lies within the mapped zone is taken as the solution. A stable algorithm for bilinear inverse interpolation is given by Felippa (2004). However, the extension of this method to 3-D (trilinear inverse interpolation) is not straightforward. An iterative algorithm is needed to solve the set of equations defining the mapping of a hexahedral element to a cubic element

For high Reynolds number flows, both the nearest neighbor and the bilinear inverse interpolation schemes would require a very fine resolution near the boundary. The computational grid should be fine enough to resolve the viscous sublayer, where the velocity variation is physically linear. However, this is an overwhelming requirement, which is not

practical for atmospheric flows. Instead one should consider LES of atmospheric flows with surface modeling.

We propose the *log-law reconstruction scheme* for incorporating surface modeling into the immersed boundary method. It should be emphasized that this scheme addresses the surface modeling issue through reconstruction of the velocity field, whereas in the commonly adopted surface modeling approach turbulent stresses are imposed at the surface as boundary conditions. In the following, we explain the formulation of the log-law reconstruction scheme.

The log-law for a rough surface can be written as follows (Panofsky & Dutton 1984)

$$\frac{U}{u_*} = \frac{1}{\kappa} \ln\left(\frac{z}{z_0}\right), \quad (4.5)$$

where z is the distance from the surface measured along the surface normal direction, z_0 is the roughness height, u_* is the friction velocity, and U is the magnitude of the velocity. Within the log-layer the friction velocity, u_* is constant in the surface normal direction. Using this property, one can write the following between two points lying on the same surface normal direction.

$$\frac{U_2}{U_1} = \frac{\ln(z_2/z_0)}{\ln(z_1/z_0)}, \quad (4.6)$$

The above formula is based on the magnitudes of the velocity. One needs to decompose it into velocity components in each direction (u, v, w), in order to impose the direct forcing in equation 4.1. For instance, if we consider a neutrally stratified atmospheric boundary layer over a planar surface, the velocity direction in the horizontal plane θ changes with altitude due to the rotation of the Earth. Since one has the magnitude of the velocity at hand due to equation 4.6, θ needs to be extrapolated from the fluid nodes to the cut node lying along the same vertical direction. The direction of the velocity on the horizontal plane is computed as follows

$$\theta = \arctan(v/u) \quad (4.7)$$

For simplicity, we suggest a linear extrapolation to compute θ at the cut node. Once θ_{cut} is determined, then horizontal components of the velocity is computed as shown below

$$u = U \cos \theta_{cut}, \quad v = U \sin \theta_{cut} \quad (4.8)$$

The log-law reconstruction scheme in the case of a planar surface is described schematically on the right part of figure 1. In the preprocessing stage, first, the nodes are tagged as dead, fluid and cut. Next, the magnitude of the velocity at the cut node, numbered as 1, is updated based on equation 4.6, using the information from the fluid node numbered as 2. Then θ at fluid nodes 2 and 3 are computed based on equation 4.7. Using θ_2 and θ_3 , a linear extrapolation gives θ_1 , which is then used to compute the horizontal components of the velocity at the cut node, based on equation 4.8. Note that, the vertical component of the velocity (w) has zero value at the boundary and a linear interpolation is used to impose it on the cut node.

For a three dimensional complex surface the above scheme needs to be reformulated based on normal and tangential directions to the surface. We are currently working on extension of our scheme to complex topography.

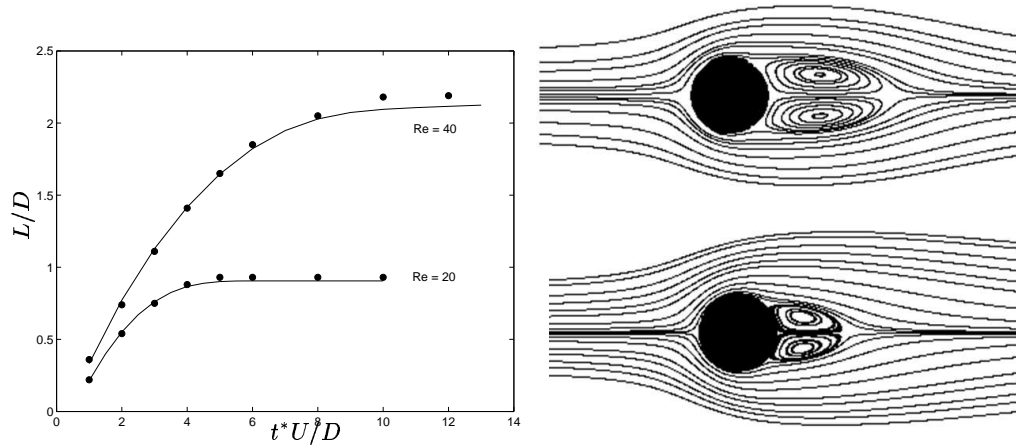


FIGURE 2. Time evolution of the recirculation zone length and the flow structure. — : present computations, • : experiment (Coutanceau & Bouard 1977a).

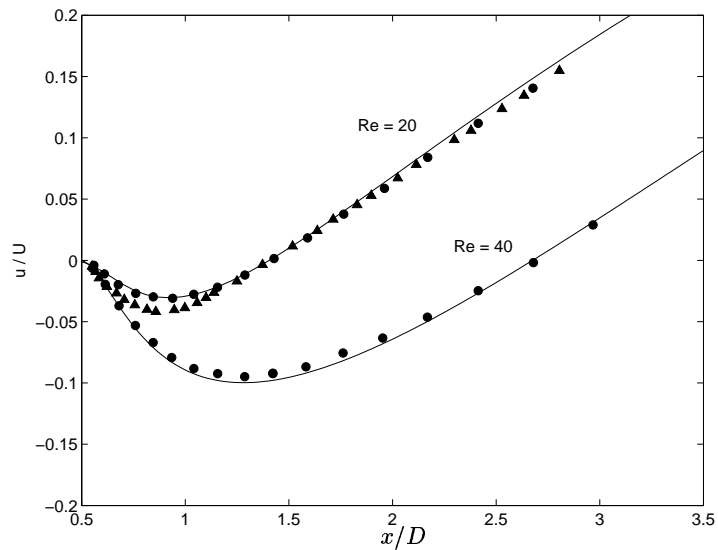


FIGURE 3. Streamwise component of the velocity field in the wake of the circular cylinder. — : present computations, • : computations of Nieuwstadt & Keller (1973), Δ : experiment (Coutanceau & Bouard 1977a).

5. Results

In this section we, first, present results of low Reynolds number laminar flow computations adopting the linear reconstruction scheme in the IB method. Following this, we present results of LES of a neutrally stratified atmospheric boundary layer adopting the log-law reconstruction scheme in the IB method.

5.1. Linear reconstruction scheme

To test our implementation of the immersed boundary method, we perform simulations of laminar flow past circular cylinders and compare our results with the available exper-

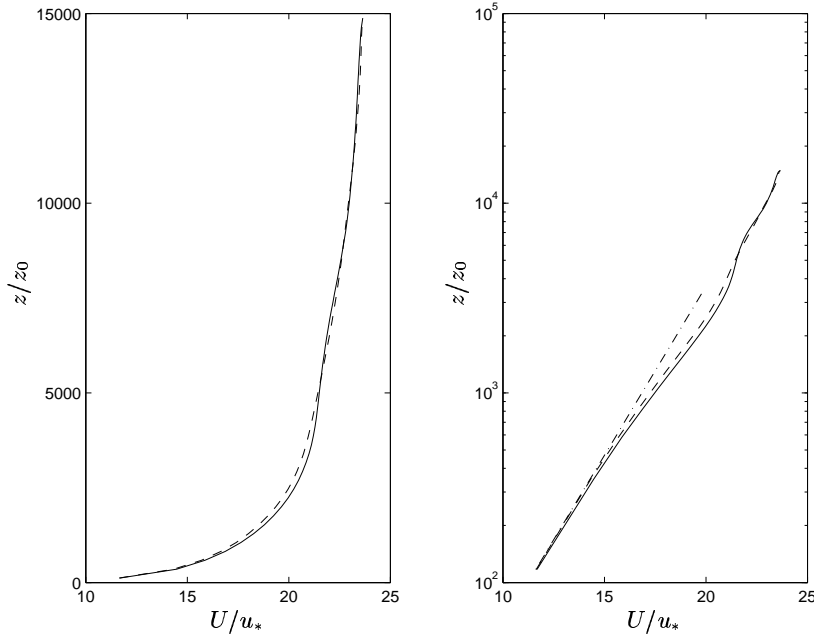


FIGURE 4. Comparison of the mean wind profiles. Linear plot (left), semi-log plot (right). — : surface-stress boundary condition, --- : immersed boundary with log-law reconstruction scheme, $\text{-}\cdot\text{—}$: log-law

imental and computational studies in literature. We consider flows at Reynolds number of 20 and 40. The Reynolds number is defined as

$$Re = \frac{UD}{\nu}, \tag{5.1}$$

where U is the upcoming freestream velocity, D is the diameter of the circular cylinder, and ν is the kinematic viscosity of the fluid. The wake behind the circular cylinder becomes unsteady above Reynolds number of approximately 40, (Coutanceau & Bouard 1977a,b).

Figure 2 shows the time evolution of the length of the recirculation zone that forms in the wake at Reynolds number of 20 and 40. In figure 3, the streamwise component of the velocity is compared with both the experimental data of Coutanceau & Bouard (1977a) and the computational data of Nieuwstadt & Keller (1973). The present results obtained with the IB method agrees well with both the experimental and the computational data. We have also compared the results of linear reconstruction scheme with the results of bilinear inverse reconstruction scheme. Since the results are nearly identical, we do not include the comparisons in this study.

5.2. Log-law reconstruction scheme

To test the performance of IB method in LES of atmospheric flows, we consider a neutrally stratified atmospheric boundary layer. We compare the results of the IB method with log-law reconstruction scheme to the results obtained by adopting the surface-stress boundary condition approach, which is described in equation 3.1. For surface-stress boundary condition approach, the computational domain size is $3000 \text{ m} \times 1500 \text{ m} \times 1500 \text{ m}$ with $64 \times 32 \times 64$ grid points uniformly distributed in x, y, and z directions, respectively. For

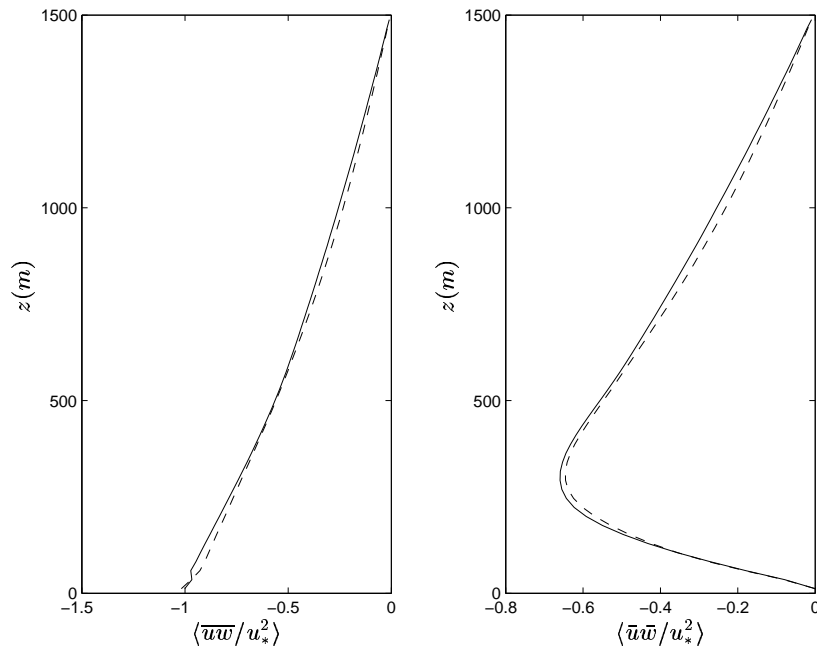


FIGURE 5. Comparison of streamwise component of the total (left) and resolved (right) stress. — : surface-stress boundary condition, ---- : immersed boundary with the log-law reconstruction

immersed boundary simulations, we consider a domain height of $1687.5m$ with 72 grid nodes in the vertical direction. We place the immersed boundary at a height of $187.5m$ so that we have a computational domain, which is identical to the computational domain adopting the surface-stress boundary condition approach. The flow is driven by a mean pressure gradient that would balance a 10 m s^{-1} geostrophic wind in the x direction. Coriolis parameter is set equal to 10^{-4} s^{-1} , and the roughness parameter z_0 has a value of 0.1 m . A dimensionless time unit can be defined based on the Coriolis parameter ($1/f$). The simulations were run over a period of $10f^{-1}$, and statistics are collected during the time period of last the $4f^{-1}$. Ensemble averaged vertical profiles have been obtained by collecting data at every 6×10^{-3} dimensionless units (60secs), and averaging them both in time and in horizontal space.

In figures 4 and 5 the mean wind profile and the streamwise component of the total and the resolved stress obtained with the commonly adopted surface-stress boundary condition are compared with the results of IB method. Clearly, the IB method with the log-law reconstruction is able to reproduce the results of the surface stress boundary condition.

Figure 6 compares the spanwise components of the total stress and the velocity. The noticeable differences seen in this plot is because of the extrapolation of the velocity direction in the IB method with log-law reconstruction, given in 4.7. As discussed in Senocak et al. (2004) the adoption of near-surface models results in non-vanishing spanwise component of the velocity. In that respect, the IB method with log-law reconstruction does a better job by predicting a lower value of the spanwise velocity.

Finally, we compare turbulent eddy viscosity profiles because it is an important quantity in atmospheric modeling. In figure 7, we see that the IB method with log-law recon-

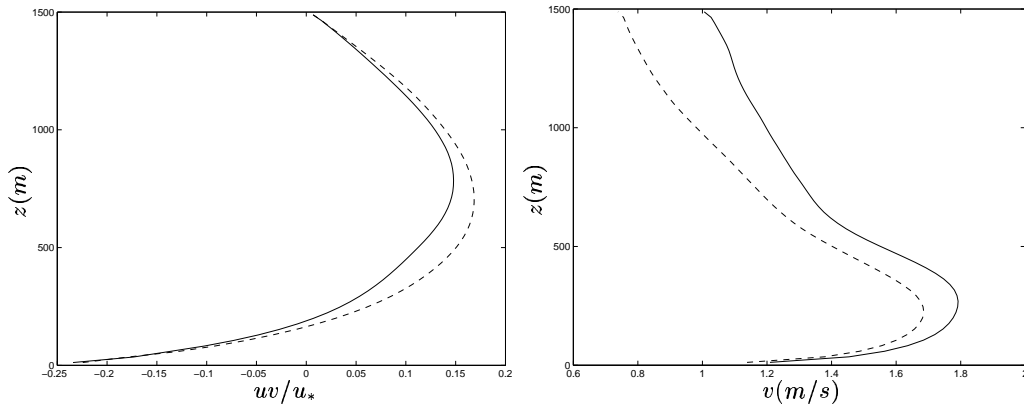


FIGURE 6. Comparisons of spanwise component of the total stress (left) and the velocity (right). — : surface stress boundary condition, ---- : immersed boundary with the log-law reconstruction

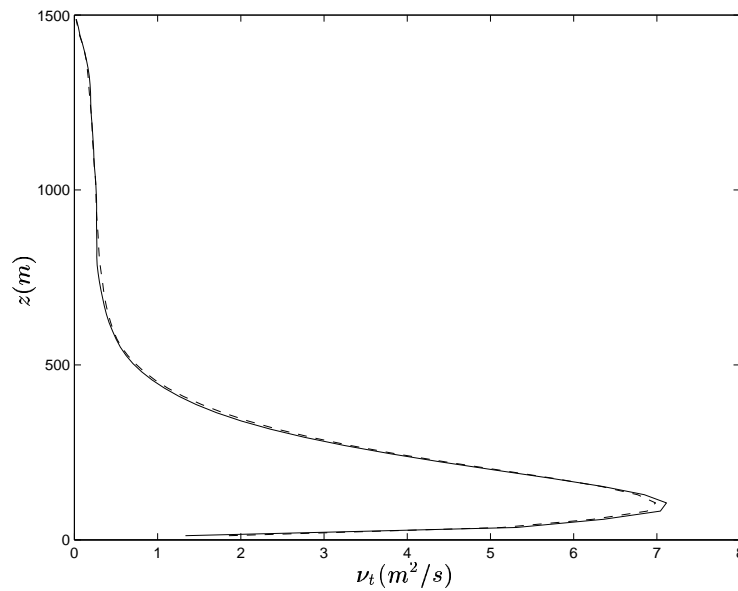


FIGURE 7. Comparison of turbulent eddy viscosity profiles. — : surface stress boundary condition, ---- : immersed boundary with the log-law reconstruction

struction gives results that are almost identical to results of the surface-stress boundary condition.

6. Summary and conclusions

In this study, we have presented a Cartesian grid immersed boundary method for LES of a neutrally stratified atmospheric boundary layer flow. The IB method is appealing for atmospheric flow modeling because its implementation is easy and does not alter the computational structure of the Cartesian grid code, which can involve extensive modeling for cloud microphysics and radiation modeling.

For validation purposes, first, we have considered laminar flows over a circular cylinder

at low Reynolds numbers and demonstrated good agreements with available experimental and computational studies in literature. To address the surface modeling issue in LES of ABL using the immersed boundary method, we have proposed the log-law reconstruction scheme. This scheme enables us to employ the IB method for high Reynolds number flows without the need for fine near-surface resolution. We have shown that the IB method with log-law reconstruction scheme can produce results that are nearly identical to the results obtained with the commonly adopted surface-stress boundary condition approach.

Our future work will focus on applying the IB method to atmospheric flow problems that involve complex topography, cloud coverage and radiative energy transfer.

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