Consumer expectations of capacity constraints and their effect on the demand for multi-class air-travel

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Abstract

This paper argues that a consumer's decision on ticket class takes into account the expected likelihood of obtaining a seat in a particular class which, in turn, partially depends on an optimum "transaction cost". Taking into account the preferences of the consumer and the information that the consumer is endowed with, the consumer will select a ticket that includes its own optimal transaction cost. This motivates the inclusion of the capacity constraint as a proxy independent variable for these consumer expectations. This then forms the basis of a model of air-travel demand with specific reference to Australia. A censored likelihood function allowing for correlation in the disturbance term across *k* classes is introduced. The correlation in the disturbances arises as a result of the interdependence of the capacity constraints in *k* different ticket classes on each flight.

KEYWORDS: Travel demand, airline markets, limited dependent variables.

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Introduction

This paper advances the research that was presented at the Air Transport Research Society Conference in Seattle in 2002. The purpose, here, is to present empirical findings for demand on one particular route in the State of New South Wales, Australia, that utilizes capacity both in terms of an indicator of the expected probability of availability for a particular ticket and as an upper bound in the econometric estimation of demand for the route.

This paper's core objective is to estimate demand for seven different ticket classes on one regional route in Australia. The data is captured over one year where all seat sales on each flight for one of the airlines on the route have been made available. Further, the capacity constraint for each class is derivable through the setting that the airline uses for the class. Complexity enters the problem through the interaction of the demands for each of the ticket classes. That is, it should be expected that there will be a correlation between the demands for each of the ticket classes on individual flights. The interaction of the capacity constraints for each of the different ticket classes adds a further dimension of intricacy.

There has been a significant body of research on air-travel demand, particularly as a component of yield management research. The analyses traditionally have focused on aggregate data which does not allow a sufficiently robust analysis of different consumer types – an integral factor in the price discrimination practiced by the airlines. This paper seeks to take another step towards a deeper understanding of consumers and their responses in the air-travel market. While individual consumer data is not available, the availability of data on individual flights has encouraged the exploration of alternative estimation procedures and allowed the extraction of a number of interesting results.

The econometric models that are analysed are broken down into two categories. In the first category, the entire data set is estimated together to gather outcomes that otherwise would not be attainable because of unmoving data values – for instance, price in each of the classes does not change over the course of the year and the only way to gather a useful price effect is to estimate an overall model of demand at the flight level. The second category of models examines demand at the class level. At this level, variation is only high in the variables of sales and capacity. Nonetheless, value can be found, particularly through a model that estimates demand for each class as part of a demand system. In this paper, a quasi-maximum bivariate Tobit system is also used so that correlations between the classes can be evaluated. This evaluation of the correlations should also allow for more efficient parameter estimates of the variables in the model.

This paper is divided into a number of sections. The next section reviews some of the recent work done in this area – particularly focussing on the role of the capacity constraint and transaction cost in the consumer's utility function. The third section reviews the econometric approaches that are used in the creation and analysis of the demand models with the fourth section briefly outlining the data and manipulation required prior to estimation. The fifth section presents the results from this initial work and the sixth section focusses on some directions for further work in this area.

The air-travel consumer, the capacity constraint and the transaction cost.

In 2002, a model of consumer choice was presented at both the Air Transport Research Conference in Seattle and at the Australian Conference of Economists in Adelaide (see Battersby, 2002a and 2002b). This model noted that consumers are now becoming more capable in their understanding of the pricing methods used by the airlines and this provided for the interaction of an expectation on capacity. This section briefly summarises those ideas as a basis for the empirical outcomes that are discussed in this paper.

As consumers become more aware of the methods used by airlines to discriminate between them, their ability to form expectations on the availability of different types of tickets improves. If we let π represent the expected probability of seat availability in ticket class i for consumer n, and ζ represent the expected capacity or number of seats in that ticket class, then we can form a simple equation that relates the two:

$$\pi_{in} = f(\zeta_{in}, \tau_{in}, d_{in}) \qquad \text{Equation 1}$$

In this model, two other variables are included that also help to explain the expected probability of seat availability for the consumer. They are the expected demand for those seats by other consumers in the market, d, and the expected transaction cost involved in procuring a ticket in that ticket class, τ.

This last component deserves some further explanation. The transaction cost is effectively a method for the consumer to manipulate their expected probability of availability for a particular ticket class. In a comparative statics analysis, it would be expected that the higher the transaction cost, the higher is the probability of availability. Indeed, the transaction cost concept is similar to the concept of manipulability by consumers in rationing schemes (see Benassy, 1993). By entering the queue for a ticket earlier, paying in advance for the ticket, or by having someone else organise the ticketing for them, the consumer spends resources in the attempt to maximise the probability of acquiring a particular seat.

The consumer will not, however, endlessly spend on these transaction costs. It turns out that the consumer will have an optimal transaction cost for each of the available choices If we define the consumer choice problem as a random utility model, the basic model would suggest that the consumer, n, will most likely be observed to choose the alternative i from the choice set C that has the highest of the random utilities, U:

$$P(i \mid C_n) = \Pr\left[U(x_{in}, m - p_{in}, \varepsilon_{in}) = \max_{j=1..k} U(x_{jn}, m - p_{jn}, \varepsilon_{jn}) \right], \quad \forall j \in C_n \qquad \text{Equation 2}$$

Where x is the vector of the various attributes of the ticket and characteristics of the consumer, m is the budget constraint, p is the price of the ticket and ε is the randomness associated with the observation of the choice probabilities.

If we incorporate the concept of the expected probability of ticket availability, the consumer will make a choice under a modified framework:

$$P(i \mid C_n) = \Pr\left[\pi_{in} U(x_{in}, m - p_{in} - \tau_{in}, \varepsilon_{in}) = \max_{j=1...k} \pi_{jn} U(x_{jn}, m - p_{jn} - \tau_{jn}, \varepsilon_{jn})\right], \quad \forall j \in C_n \qquad \text{Equation 3}$$

Using this approach, it can be seen that the choice that the consumer makes depends on the product of the expected probability of ticket availability and the expected utility of the ticket. Moreover, it can also be seen that the expected utility of the ticket incorporates the expected transaction cost for that ticket.

Because the transaction cost has a negative effect on the utility function while positively affecting the expected probability of ticket availability, the consumer will not try to maximise the expected probability of ticket availability. Rather, the consumer will endeavour to optimise the transaction cost such that the expected probability of availability and the expected utility combined produce the highest result. Their optimal transaction cost for a particular ticket, then, will be that transaction cost where the negative change in utility from an increase in the transaction cost is equal to the positive change in expected probability of ticket availability.

The question then arises as to how this affects the way demand is observed in the market. Clearly, if consumers are factoring in the transaction cost in their choices, then an empirical model of demand should somehow take account if this. Equation 1 highlights the interactivity of the transaction cost with the expected probability of availability and consequently the expected capacity constraint and the expected level of demand. Before proceeding it is worthwhile to note the implications of the information endowment of the consumer in their decision making process.

It is fair to assume that, up until recently, consumed were relatively naïve in their understanding of the pricing and capacity policies of the airlines. This naïvety has two clear impacts on the model presented. First, the consumer may not have full information on the available choice set and secondly, the consumer may have little information on the effect of their transaction cost on the expected probability of fulfilment. While this paper does not detail the interaction of information availability[1], it can be assumed that as the information endowment of the consumer increases, so too does the available choice set and their understanding of the interaction of the transaction cost with the expected probability of availability.

In terms of empirically examining this and noting that the expected transaction cost can be considered a partial function of the expected capacity for a particular ticket, a useful solution becomes evident. Hence, it should be expected that the consumer has become more responsive to the actual capacity constraint over time as their information endowment has improved. But the methodology of examining this is not straightforward, capacity imposes an upper limit in the econometric model as well as being explanatory of demand and the demand for any particular ticket class is understandably correlated with the demand for the other ticket classes. As such, the rest of this paper is dedicated to providing a workable approach to empirically examining outcomes based on this approach to understanding consumer choices in the air-travel market.

Establishing empirical outcomes in the face of consumer expectations on the capacity constraints

This research focuses on a number of approaches that are useful when sales and capacity data are available at the ticket level for one firm and all other data is sourced from standard areas. If more disaggregated data were available, the techniques associated with the random utility model such as the multinomial logit would serve quite well. Here, however, the approach recognises the lack of individual consumer data and instead

[1] The presentation of the research that examines the interaction of the information endowment of the consumer in relation to the expected probability of fulfilment and the transaction cost is forthcoming.

provides a number of alternative options. In this analysis, we presuppose that there are seven ticket classes with unique capacity constraints, which are tied to various ticketing restrictions.

There are two broad approaches that can be taken, and, from within those, there are at least two techniques that can be applied to examine the data. The first approach is to estimate a model across pooled data and the second approach entails establishing demand equations for each of the classes. Both approaches have their strengths and weaknesses. The first enables an inspection of the interaction of price in the demand for air-travel on the route under inspection and also allows a comparison of the restrictions and their effect on demand. This approach also allows a general analysis of the interaction of the capacity constraint on overall air-travel demand.

The second approach involves the creation of individual demand functions for each of the classes. While there is no price variability in these classes (therefore making the price effect impossible to deduce), there is regular variability in the capacity of the classes. It is possible, using this approach, to examine the sensitivity of demand for tickets in a particular class to the capacity constraint and therefore deduce some outcomes relating to the transaction cost involved in purchasing tickets for that particular class. Moreover, recognising that in any one flight, it is highly likely that the demand equations for each of the seven ticket classes will be correlated, it is possible to create a system of equations that accounts for this correlation.

In this paper, findings of the censored approach are developed using a tobit specification. In this case, a straightforward tobit model application with upper censoring is used on the pooled data where d_i is the demand for the observation on class i, x is a vector of the attributes of the ticket and the characteristics of the consumer, ζ is the capacity constraint, k is the number of classes and u is a residual:

$$d_i = \beta_1' x_i + \beta_2 \zeta_i + \beta_3 \frac{\sum_{\forall j \neq i} \zeta_j}{k-1} + u_i \; \forall j \neq i \text{ if } d_i < \zeta_i$$ Equation 4

$d_i = \zeta_i$ otherwise.

Such that the likelihood function is:

$$L = \prod_{d_i < \zeta_i} f(d_i) \prod_{d_i = \zeta_i} \int_{\zeta}^{\infty} f(d_i) dd_i$$ Equation 5

Where $f(d_i)$ is the density function for d_i. If we assume the standard Tobit with normal distributions (Φ representing the normal distribution function), the log-likelihood is:

$$\ln L = \sum_{d_i < \zeta_i} -\frac{1}{2} \left[\ln(2\pi) + \ln \sigma^2 + \frac{\left(y_i - \beta_1' x_i - \beta_2 \zeta_i - \beta_3 \frac{\sum_{\forall j \neq i} \zeta_j}{k-1} \right)^2}{\sigma^2} \right]$$
$$+ \sum_{d_i = \zeta_i} \left[1 - \Phi \left(\frac{\zeta_i - \beta_1 x_i - \beta_2 \zeta_i - \beta_3 \frac{\sum_{\forall j \neq i} \zeta_j}{k-1}}{\sigma} \right) \right]$$ Equation 6

An alternative way to examine the problem is, as mentioned, by individual classes. In this case, estimation is required for k likelihood functions similar to those in equation 8 but for individual classes only. The key difference is that the data is partitioned into each individual class which creates the undesirable characteristic of invariability across many of the independent variables. Nevertheless, there is significant variability in the capacities of the classes and this does allow for the development of some useful results.

The system of independent classes would then be:

$$d_{it} = \beta_1 x_{it} + \beta_2 \zeta_{it} + \beta_3 \zeta_{jt} + u_{it} \quad \forall j \neq i \text{ if } d_{it} < \zeta_{it}$$
$$d_{it} = \zeta_{it} \text{ otherwise} \qquad \text{Equation 7}$$

for class $i \in C_n$ at observation t. Here, β_3 represents a vector of coefficients conformable to the vector of class capacities, ζ_j. The system of likelihood functions would therefore be:

$$L_1 = \prod_{d_{1t} < \zeta_{1t}} f(d_{it}) \prod_{d_{1t} = \zeta_{1t}} \int_{\zeta}^{\infty} f(d_{1t}) dd_{1t}, \forall j \neq i$$
$$\vdots \qquad \vdots \qquad \vdots \qquad \text{Equation 8}$$
$$L_k = \prod_{d_{kt} < \zeta_{kt}} f(d_{it}) \prod_{d_{kt} = \zeta_{kt}} \int_{\zeta}^{\infty} f(d_{kt}) dd_{kt}, \forall j \neq i$$

The log-likelihood for each of the equations in the system is as equation 9.

The probability becomes apparent, however, that each of these likelihood functions is correlated because the total demand for seats on a particular flight is the sum of the demands for each of the individual classes and those individual demands are partially explained by a class capacity constraint which is, itself, constrained by a total capacity constraint. That is, each of the capacity constraints sum to the total aircraft capacity[2]. To account for these expected correlations, the system of equations may be estimated somewhat differently.

The approach that is used here parallels that used by Yen and Lin (2002) which examines approaches to overcome the numerical difficulties inherent with multiple probability integrals that arise in this type of censored problem. The approach that Yen and Lin

[2] In fact, the sum of the capacities may actually exceed the capacity constraint of the aircraft if the settings used by the airline creates that effect. Nonetheless, the interrelated nature of the capacity constraints continues to produce correlative pressure across the individual demand functions.

suggest follows the quasi-maximum likelihood method that has been used in multivariate probit estimations. In this case, however, the quasi-likelihood function is used in a tobit estimation so that the cross-equation error correlations are taken into account, therefore improving the efficiency of the model.

Following Yen and Lin (2002), the quasi-likelihood function is therefore specified as:

$$L = \prod_{i=2}^{k} \prod_{j=1}^{i-1} L_{ij} \qquad \text{Equation 9}$$

Where L_{ij} is the bivariate tobit likelihood for d_i and d_j specified as:

$$\begin{aligned} L_{ij} = & \prod_{d_i=\zeta_i, d_j=\zeta_j} \int_{\zeta_i}^{\infty}\int_{\zeta_j}^{\infty} g(d_i, d_j)\, dd_j\, dd_i \\ & \times \prod_{d_i<\zeta_i, d_j=\zeta_j} f(d_i) \cdot \int_{\zeta_j}^{\infty} f(d_j)\, dd_j \\ & \times \prod_{d_i=\zeta_i, d_j<\zeta_j} f(d_j) \cdot \int_{\zeta_i}^{\infty} f(d_i)\, dd_i \\ & \times \prod_{d_i<\zeta_i, d_j<\zeta_j} f(d_j) \cdot f(d_i) \end{aligned} \qquad \text{Equation 10}$$

Yen and Lin (2002) find that this approach is useful as an alternative to full information maximum likelihood (FIML) in the case where there are three alternatives. In the case under investigation in this paper there are seven alternatives and the derivation of a full information maximum likelihood result is not possible because of the intractability of the equation. Yen and Lin (2002) also suggest the use of simulator methods for the calculation of higher dimension FIMLs.

In this paper, the quasi-maximum likelihood estimation is carried using the Broyden-Fletcher-Goldfarb-Shanno descent algorithm. The standard errors for the parameter estimates are calculated using White's (1982) covariance matrix.

Data and Market Background

The route under analysis in this research is the Wagga Wagga to Sydney one-way regional route in New South Wales, Australia. Wagga Wagga is located on the Sturt Highway, which connects Sydney and Adelaide, and on the Murrumbidgee River. It is also located on the train line that connects Sydney and Melbourne. The city is also regarded as the gateway to the Riverina, one of the richest agricultural areas in New South Wales.

Sydney, the other city in the pair under examination in this study, is the largest city in Australia and is the capital of New South Wales. Sydney is internationally known as one of the key cities in Australia and is also recognised as the international gateway to Australia. While Sydney has a number of airports, Sydney Kingsford Smith International airport is the only one that offers services by the primary airlines.

Usefully, at the beginning of this research project, Kendell Airlines provided a series of data for the 1997 financial year which contained a record of sales for each class on each flight of the year. The respective settings for the classes were also provided from which capacity constraint information could be developed. The method outlined in Battersby (2002a) for the calculation of the capacity constraint for individual classes based on settings data is used in this paper.

Kendell Airlines also provided information on the prices for each of the classes. No variability in the prices was recorded over the financial year. This has a key implication for the single class models as it becomes impossible to determine a useful price elasticity. Nevertheless, in the pooled data analysis, the variability across classes provides some feedback on broad response to price. Income data is the average weekly employee earnings in New South Wales.

A dummy variable is used to identify whether there is a significant event on in Wagga Wagga on the day of the flight. Events such as major race meetings, commercial events and government events were recorded from the regional newspaper, the Daily Advertiser. A level of arbitrariness was used in defining which events were appropriate. Other dummy variables are also included for Monday and Friday flights, weekend flights, public holidays, and school holidays.

In the pooled data, further dummy variables were used to control for the various restrictions on the each of the ticket classes. These dummy variables included the requirements to book 21 days, 14 days and 7 days in advance, non-refundable ticket type, availability only to senior citizens, and the transferability of ticket restrictions (seasonal or completely flexible).

A summary of the pooled data is presented in Table 1 while Table 2 presents a summary for data at the individual class level. It should be also pointed out that sales equals the capacity constraint in 3280 (32.17%) of the observations. In the individual class data, the following ticketing class regime exists:

Class 1	Fully flexible airfare. Highest priced.
Class 2	
Class 3	
Class 4	↕
Class 5	
Class 6	
Class 7	Restrictive Airfare. Lowest priced. 21 day advance requirement. Not refundable.

Table One
Summary of Pooled Data

	Mean	Std. Dev	Minimum	Maximum
Sales (SALES)	3.394	4.630	0	34
Capacity (CAP)	10.983	7.919	0	34
Ticket Price (PRICE)	139.578	29.693	93	186
Weekend (WEEKEND)	0.286	0.452	0	1
Monday or Friday (MONFRI)	0.286	0.452	0	1
21 days advance purchase (21DAY)	0.143	0.350	0	1
14 days advance purchase (14DAY)	0.143	0.350	0	1
7 days advance purchase (7DAY)	0.143	0.350	0	1
Not refundable (NOREFUND)	0.714	0.452	0	1
Available to senior citizens (SNRCIT)	0.143	0.350	0	1
Completely flexible (FLEX)	0.143	0.350	0	1
Flexibility restricted to season (SEASFLEX)	0.143	0.350	0	1
Average Weekly Earnings (AVEARN)	573.133	4.102	568.5	580
Major Event in Wagga (WGAEVENT)	0.041	0.199	0	1
Public Holiday (PUBHOL)	0.036	0.185	0	1
School Holidays (SCHHOL)	0.231	0.421	0	1
NUMBER OF OBSERVATIONS	10193			

Table Two
Summary of Individual Class Data

	Mean	Std. Dev	Minimum	Maximum
Sales Class One (SALES1)	9.7445	6.6561	0	34
Sales Class Two (SALES2)	1.4629	1.8803	0	22
Sales Class Three (SALES3)	1.0137	1.4549	0	21
Sales Class Four (SALES4)	0.9464	1.2869	0	16
Sales Class Five (SALES5)	1.4478	2.123	0	20
Sales Class Six (SALES6)	4.4732	3.7867	0	21
Sales Class Seven (SALES7)	4.7685	4.1056	0	26
Capacity Class One (CAP1)	19.886	5.8956	0	34
Capacity Class Two (CAP2)	4.3764	4.5992	0	24
Capacity Class Three (CAP3)	3.5	4.346	0	23
Capacity Class Four (CAP4)	13.0934	5.8536	0	25
Capacity Class Five (CAP5)	13.6078	5.6103	0	25
Capacity Class Six (CAP6)	16.6236	4.8485	5	24
Capacity Class Seven (CAP7)	5.5357	4.5408	0	26
NO. OF OBSERVATIONS / CLASS	1456			

Results

The results presented in this section are preliminary. At this stage, diagnostic testing on each of the models has not taken place and there remain a number of key issues surrounding the computation of the quasi-maximum likelihood model. Central to these latter issues are a discovered "bug" in the underlying calculation of distribution functions at extreme values in the software that is used. Nonetheless, the results do provide some useful insights into the multi-class environment and are instrumental in defining the future direction of this research.

The results for the pooled data set are presented in Table 3. These results are particularly useful because they highlight the increasing cost of the restrictions – particularly the advance purchase restrictions. More importantly, however, is the effect of an increase in the capacity constraint.

Table Three
Tobit Application over Pooled Data

	SALES
ONE	14.2193*** (3.110)
PRICE	-0.1928*** (-34.786)
CAPACITY	0.4500*** (61.546)
OTHERCAPACITY	-0.9997*** (-89.463)
EARNINGS	0.0518*** (6.604)
WGAEVENT	0.0481 (0.305)
MONFRI	0.0955 (1.247)
WEEKEND	0.4297*** (5.644)
DAY21	-19.0323*** (-60.411)
DAY14	-15.9514*** (-76.754)
DAY7	-11.1109*** (-79.614)
FLEX	2.2297*** (7.340)
SEASFLEX	2.1875*** (12.049)
PUBHOLS	0.0612 (0.321)
SCHHOLS	0.0429 (0.539)

Note: Figures in brackets indicate Standard Error / Estimate. *** indicates significance at the 0.01 level. ** indicates significance at the 0.05 level. * indicates significance at the 0.10 level.

Both of the capacity variables have signs as expected suggesting that an increase in the capacity for a particular class, other things held constant, will increase the demand for that class. The cross capacity coefficient further supports the theoretical base suggesting that an increase in the capacity in an alternative class will have a negative impact on demand for the class under analysis.

Table 4 presents findings from the univariate Tobit models. In this case, models are estimated for each of the classes in the choice set. Price is left out of the model because there is no variability throughout the year in the price of the tickets within each ticket class.

One of the clear outcomes using this approach is the differing parameters for the cross capacity constraint estimates. Indeed, the own capacity coefficient is significant for all of the seven classes. One of the other interesting outcomes is that, in some cases, there are positively signed cross-capacity coefficients. This may be a result of the correlations that exists between the demands in each of the classes, in which case controlling for this correlation may provide more efficient and understandable estimates.

The remaining variables provide their own indications on their relationship with the sales variable. There is also some use in examining these results in conjunction with those results in table 3.

At the time of writing, a significant problem had been discovered in the estimation of the multiple bivariate Tobit model using QML. This problem was not a result of the specification, but rather the result of a discovery of a "bug" in the mathematical software. This bug was related to the calculation of the log of the distribution function. Dialogue with the software developer established that:

> "After some investigation it appears that some functions are not terminating their loops as promptly as they should for extreme arguments." (Horecny, 2003)

Nonetheless, the QML multiple bivariate Tobit model was estimable using three classes and a highly constrained independent variable set. These results are presented in table 5.

Comparison results for the three individual classes using the same specification under the univariate approach are presented in table 6.

Table Four
Univariate Tobit Estimates

	SALES1	SALES2	SALES3	SALES4	SALES5	SALES6	SALES7
ONE	-104.41*** (-6.781)	-3.3541 (-0.522)	10.399** (2.031)	25.196*** (5.6)	4.7288*** (2.849)	-5.0906*** (-1.969)	-2.0372 (-0.263)
CAP1	0.5867*** (18.230)	-0.3235*** (-23.541)	-0.2831*** (-25.476)	-0.1497*** (-16.007)	-0.0277*** (-8.009)	-0.0616*** (-11.456)	-0.5419*** (-19.925)
CAP2	-0.5622*** (-12.006)	0.4646*** (24.710)	-0.4454*** (-29.917)	-0.0068 (-0.498)	0.0026 (0.520)	-0.0060 (-0.767)	-0.3216*** (-7.436)
CAP3	-0.3009*** (-5.743)	-0.3834*** (-18.829)	0.5459*** (34.762)	0.0247 (1.629)	-0.0149*** (-2.651)	0.0437*** (5.000)	-0.1711*** (-4.075)
CAP4	-0.0786 (-1.274)	0.1088*** (3.441)	0.1671*** (6.545)	0.1367*** (7.325)	-0.9253*** (-134.920)	-0.0841*** (-8.135)	0.4846*** (9.528)
CAP5	0.0548 (0.897)	0.0630* (1.977)	0.0261 (1.011)	0.0275 (1.486)	0.9528*** (140.830)	-0.9007*** (-86.606)	-0.0116 (-0.219)
CAP6	-0.3910*** (-8.280)	-0.4343*** (-19.789)	-0.3753*** (-20.691)	-0.1586*** (-11.474)	-0.0542*** (-10.613)	0.9585*** (119.931)	-0.6537*** (-16.028)
CAP7	-0.1357*** (-3.714)	-0.1241*** (-7.652)	-0.1323*** (-10.029)	-0.1327*** (-12.450)	-0.0231*** (-5.850)	-0.0614*** (-10.011)	0.4518*** (20.080)
EARNINGS	0.1980*** (7.372)	0.0307*** (2.751)	0.0038 (0.425)	-0.0350*** (-4.465)	-0.0044 (-1.511)	0.0142*** (3.149)	0.0452*** (3.423)
WGAEVENT	0.8102 (1.512)	0.3548 (1.627)	0.1484 (0.855)	0.0076 (0.048)	-0.0676 (-1.168)	-0.0963 (-1.059)	0.0856 (0.342)
MONFRI	0.2027 (0.794)	-0.2166** (-1.967)	-0.2891*** (-3.176)	-0.0047 (-0.064)	0.0080 (0.289)	0.1322*** (3.077)	0.7839*** (5.102)
WEEKEND	0.8820*** (3.035)	-0.2329** (-2.012)	-0.5331*** (-5.814)	0.1161 (1.370)	-0.0493 (-1.573)	0.9106*** (18.638)	0.9234*** (6.366)
PUBHOL	-0.6230 (-0.963)	-0.1936 (-0.582)	-0.2586 (-0.974)	-0.1313 (-0.695)	0.0492 (0.703)	0.0048 (0.045)	0.2258 (0.628)
SCHHOL	-0.5826*** (-2.038)	0.3950*** (-3.381)	-0.2167 (-2.298)	-0.1994** (-2.380)	0.1059*** (3.421)	0.0425 (0.881)	0.1538 (1.151)
Likelihood	-3893.439	-1198.404	-894.053	-2226.128	-828.497	-1455.053	-290.064

Note: Figures in brackets indicate Standard Error / Estimate. *** indicates significance at the 0.01 level. ** indicates significance at the 0.05 level. * indicates significance at the 0.10 level.

Table Five
QML Estimates

	SALES1	SALES2	SALES3
ONE	-1.4800*** (-3.675)	0.5453*** (3.717)	-0.0267 (-0.189)
CAP1	0.7708*** (30.927)	-0.0804*** (-7.348)	-0.0245*** (-2.862)
CAP2	-0.6627*** (-10.670)	0.6160*** (13.021)	-0.2690*** (-5.787)
CAP3	-0.5040** (-8.191)	-0.4795*** (-10.555)	0.4191*** (8.040)
ρ21	-0.3775***		
ρ31	-0.5188***		
ρ32	0.8897***		

Note: Figures in brackets indicate Standard Error / Estimate. Log-Likelihood = -9606.38. *** indiciates significance at the 0.01 level. ** indicates significance at the 0.05 level. ρij identifies the estimated correlation coefficient between i and j.

Table Six
Comparative Univariate Tobit Specification to QML Estimates

	SALES1	SALES2	SALES3
ONE	0.0605 (0.143)	6.3854*** (22.435)	5.8145*** (23.538)
CAP1	0.7362*** (33.033)	-0.1526*** (-11.279)	-0.1143*** (-9.533)
CAP2	-0.6463*** (-12.703)	0.4210*** (16.310)	-0.4814*** (-22.517)
CAP3	-0.5666*** (-10.736)	-0.6222*** (-24.303)	0.3310*** (15.607)
Log-Likelihood	-4248.875	-1289.875	-1109.613

Note: Figures in brackets indicate Standard Error / Estimate. *** indiciates significance at the 0.01 level. ** indicates significance at the 0.05 level

It is important to note that the QML estimates are from a joint estimation. The estimates for ρ21, ρ31 and ρ32 indicate the correlations between the three equations. There is a clear positive correlation between class one and class three while the other two combinations have a negative correlation coefficient. Unfortunately, because of the constrained size of the number of classes and variables, more detailed information is unavailable at this stage.

While this paper give some theoretical motivation for the use of the QML method in this type of estimation, the preliminary nature of the results suggest that further work is required on this modelling approach to validate its use.

Conclusion

This paper has presented three different approaches to examining consumer demand in a multi-class capacity constrained discrete choice environment. Those three approaches were based on the premise that consumers carry some expectations on the probability of having their choice fulfilled. That probability depends on the expected capacity, the expected demand and the optimal transaction cost for the particular choice.

The three approaches to examining demand in this environment were a Tobit panel approach, a series of univariate Tobit models for individual choices, and a quasi-maximum likelihood approach that corrects the univariate Tobit models for the correlations that exist between each of the individual class demands. While the results provided some insights into the usefulness of the various approaches, there remain a number of issues surrounding the computational application of the quasi-maximum likelihood method.

The direction of this research is to refine the QML method for more than three classes and many independent variables. Further, the research is currently undertaking the construction of a model that utilises a maximum simulated likelihood approach to simulate the high dimensional integrals present in the multi-class system.

References

Battersby, B.D. (2002a) "A Model of Demand for Regional Air Travel in New South Wales." Paper presented at Air Transport Research Society Conference, July 2002, Seattle, WA.

Battersby, B.D. (2002b) "Utility maximisation in a multi-class ticket environment with capacity constraints: The case of the air-travel market." Paper presented at the 2002 Australian Conference of Economists, October 2002, Adelaide.

Benassy, J. P. (1993) "Nonclearing Markets: Microeconomic Concepts and Macroeconomic Applications" *Journal of Economic Literature*, **XXXI**, 732-761.

Horecny, G. (2003) *Personal Communication.*

Yen, S.T., and Lin, B.H. (2002) "Beverage Consumption Among U.S. Children and Adolescents: Full-Information and Quasi Maximum-Likelihood Estimation of a Censored System." *European Review of Agricultural Economics*. **29**, **1**, 85-103.

White, H. (1982) "Maximum likelihood estimation of misspecified models", *Econometrica*, **50**, **1**, 1-25.