Auction Mechanism to Allocate Air Traffic Control Slots

Marianne RAFFARIN *

April 28, 2003

Abstract

This article deals with an auction mechanism for airspace slots, as a means of solving the European airspace congestion problem. A disequilibrium, between Air Traffic Control (ATC) services supply and ATC services demand, are at the origin of almost one fourth of delays in the air transport industry in Europe. In order to tackle this congestion problem, we suggest modifying both pricing and allocation of ATC services, by setting up an auction mechanism. Objects of the auction will be the right for airlines to cross a part of the airspace, and then to benefit from ATC services over a period corresponding to the necessary time for the crossing. Allocation and payment rules have to be defined according to the objectives of this auction. The auctioneer is the public authority in charge of ATC services, whose aim is to obtain an efficient allocation. Therefore, the social value will be maximized. Another objective is to internalize congestion costs. To that end, we apply the principle of Clarke-Groves mechanism auction: each winner has to pay the externalities imposed on other bidders. The complex context of ATC leads to a specific design for this auction.

1 Introduction

The air transport industry in Europe is faced with the recurring problem of delays. Although delays slightly decreased in 2001, this was essentially due to the current international context. Delay and traffic levels are strongly connected. High rate of flight delays can again become very topical with

*EUREQua, CNRS, University of Paris-I and LEEA, ENAC-CENA, Toulouse, e-mail: raffarin@recherche.enac.fr

future growth of the air transport industry. Delays are very costly in terms of welfare.

The reason essentially advanced by airlines to explain delays is the insufficient capacity of the Air Traffic Control (ATC). A study estimates the cost of those specific delays, borne by passengers and airlines, between 6.6 and 10.7 billions of euros for 1999. It is very important to tackle this problem.

However, if ATC services are actually responsible for an important part of European delays, airlines are also at the origin of this situation: under-capacity is due to insufficient supply and too much demand.

The aim of this article is to propose a solution to the inadequacy of the supply to cope with the demand, by considering a fixed capacity. Pricing is the mechanism usually used to avoid such disequilibrium. We observe that ATC fees do not satisfied this principle. Indeed, ATC charges are a function of the weight of the aircraft in order to introduce cross subsidies between airspace users, and of the distance flown over in order to reflect the cost of service. The ATC fees provide incentives to operate flights on small aircraft and to supply frequent flights, but frequencies contribute to air congestion. Thus, ATC charges do not work to reduce delays.

Another problem is the organization of ATC services, such that the upper airspace is divided into "sectors" with a team of two or three controllers in charge of one sector. When the demand is greater than the supply for one sector, ATC authorities have to "regulate" the traffic and to allocate slots. The time an aircraft is allowed to enter in a regulated sector is specified in a slot. For each regulated sector, a list of slots is built. Those slots satisfied the capacity per hour announced by the regulated sector. For instance, a four hour long regulation associated with a rate of 30 flights per hour would result in a slot allocation list made up of 120 slots separated from one another by 2 minutes. The principle "first planned, first served", which presumes that flights should arrive over the restricted sector in the same order in which they would have arrived without regulation, is applied throughout the process.

ATC fees and allocation rules do not produce an efficient treatment of congestion. This article proposes a mechanism which combines allocation and pricing. Allocation of slots will be done in an efficient way, via optimal ATC fees.

First, we need to present the specific context of auction for airspace. Then, we will be able to design allocation and payment rules of the mechanism. The public authority is faced with some constraints, as capacity constraints, to determine the optimal allocation. Airlines' payoffs have to be defined according to the objectives. Finally, we will provide an example to understand how the mechanism works and to analyze the results.

2

2 Context of auction for airspace

In order to define an auction mechanism, we need to present objects and objectives for the auction. The wide complexity of the ATC organization, explained by a high level of security, is at the origin of a specific definition of objects. They will be presented in detail. Objectives are more simple: a public authority in charge of ATC services sets up an auction with the aim of reducing delays and promoting a better use of the existing capacity.

2.1 Objects of the auction

An auction for the airspace will be the selling of rights for airlines to cross a part of the airspace, and then to benefit from ATC services over a period corresponding to the necessary time to cross. Those rights will be called "slots", as for the airport slots, but with a different sense of current "ATC slots".

Obviously, there are many objects in this auction. Two elements are at the origin of this multiplicity: space and time.

2.1.1 Components of a slot

The airspace is divided into "sectors". The work of a team of controllers is to ensure security to flights crossing the sector they are in charge of. Most of air links cross many sectors. It means many different goods.

Moreover, a sector is defined by its capacity. This the highest number of flights that can be present in the same sector at the same moment. A given sector can be crossed by several flights during a given period. It means many identical goods. Those goods are perfect complements because airlines cannot run a risk of missing a sector to operate a link. It is absolutely necessary to allow package bids from individual sectors.

Another important dimension to constitute a slot is the time. Each package must also include periods at which sectors will be crossed.

For security reasons, air traffic flow must be spread over the day and cannot be concentrated over a short period. Thus, a sector capacity cannot be defined by day. It has to be fixed for a short period. We divided a day into 34 periods of half an hour: 6:00-6:30, 6:30-7:00, etc. The auction is organized for flights in airspace between 6:00 in the morning and 11:00 in the evening. The set of sub-periods of one day is:

$$T = \{t_1, \ldots, t_{34}\}$$

3

The division into airspace sectors already exists. The set of those sector is: $X = \{1, \ldots, x\}$. Each sector is characterized by a time capacity. Due to the activation of some military areas, where civil flights are not allowed to cross, this capacity varies over the periods. We consider $k_{s;t}$ as the capacity of the sector s during the period t.

2.1.2 Complementary objects

A slot is not necessary only for one flight or for only one air link. For strategic reasons, airlines can bid for a slot used for several air links. Sectors for a same flight are not the only perfect complements. Flight periods of several air links need to be consistent with each other, due to aircraft turnover. The existence of a hub explains also the strong complementarity between sectors. Then, slots can be for an air link, or for several air links of an aircraft, or for several air links of a group of passengers.

An airline bid specifies which sectors, at which periods, are necessary to form a package. Generally, flights will be operated over several periods. A slot z will be pairs of "sectors-period": $z = \{[(y_a, t_a)]_{a=1,\ldots,A}\}$, with y_a a package of sectors, t_a the time period needed for the package y_a and A the number of necessary periods to operate the flight.

Airlines can ask for many slots. So, this auction is for multiple packages.

2.2 Objectives of the auction

Many objectives justify an auction mechanism for airspace.

2.2.1 Internalize congestion costs

ATC services are not a public good. On one hand, fees paid by users involve possibility of exclusion. On the other hand, periodic situation of congestion involve rivalry. With the limited capacity of ATC services, airlines impose externalities on others.

One aim of this auction is to lead airlines to take into account the consequences of theirs flight choices. We need to know the value of each slot for airlines. If the demand of airlines for a slot is not satisfied, they bear an opportunity cost. This cost is equal to the profit that airlines would obtain if the demand has been satisfied, minus fees for the slot. The airlines ability to pay for a slot is the amount of this opportunity cost.

If this cost is revealed by the auction, it will be possible to charge winners the externality imposed on others.

2.2.2 Reach an efficient allocation of slots

Although ATC services are not a public good, a public authority is in charge of them. In the collective interest, this authority would prefer to reach the highest social surplus than the highest revenue. We look for an efficient mechanism.

The social surplus is equal to the sum of the airlines' net surplus, passengers' net surplus and ATC revenues. With "yield management" strategies, airlines capture passengers' surplus. ATC costs for airlines and ATC revenues cancel each other out. Thus, after simplification, social surplus is equal to the sum of bids of winners, because we saw, that airlines' net profit plus ATC cost are equal to their bids.

One objective is the maximization of this welfare.

2.2.3 Spread the traffic over time and space

We decided to study an auction mechanism for airspace slots in order to solve the present problem of congestion. The goal of this system is also to spread the traffic over time and space and not to cancel flights initially forecasted at a peak period.

The interest of an auction mechanism in this context is to incite airlines to modify either flight route, or flight hour, or both, when the capacity is insufficient, by means of prices. The optimal period to flight and the optimal sectors to cross will be determined according to ability to pay.

2.2.4 Balance the ATC services budget

France, as most in countries, decided to charge direct users of airspace. An auction mechanism is at the origin of transfers from bidders to the auctioneer. However, it is not sure that the budget of the civil aviation administration will be balanced.

For this reason, we suggest to separate the fees in two parts. One will be connected to the ATC service costs and the other will be linked to the congestion costs. This second part will be determined by the auction results.

3 Auction design

Due to multiple packages, the auction design specifies not only allocation and payment rules but also what bids look like. Airlines will have to announce which slots they want and how much they are able to pay for them.

An optimization of the total value of the bids, under constraints, will give the allocation and the objectives will induce a special payment rule. Indeed, externalities and services managed by a public authority are in favor of a mechanism such that winners pay the cost imposed on others. So, the auction mechanism for airspace slots will be an adaptation of a Clarke-Groves mechanism.

3.1 Bids of the auction

There is a lot of possible "sector-period" combination. So, we propose to leave airlines to define themselves their slots.

Bids of the auction will have two components. First, airlines will describe precisely the slots which are relevant to them. Second, they will announce their values for those slots.

3.1.1 Relevant slots

The auctioneer cannot propose an exhaustive list of all possible slots. Airline i will describe the M^i slots it wants. The first part of her announcement will be the list $Z^i = \{z_m^i\}_{m=1,...,M^i}$.

Considering the third objective, it is not possible that at the end of the auction, some forecasted flights at peak period are canceled and all capacity is not used at off-peak period. The auctioneer can allow airlines to modify a slot if their demand is not satisfied. We suggest to implement an auction with only one turn and to leave airlines asking for several slots for the same flight. At their "favorite" slot, they will add others slots in case of insufficient capacity to obtain the first one. Those alternatives will be different from the "first choice", either by the time period, earlier or later, or by sectors, a longer but less congestioned route, or by both.

Thus, for an air link, airlines ask for several slots. $z_m^i = \{z_m^{i;r}\}_{r=1,...,R_m^i}$ is a vector including all the slots described by airline i to operate the air link m.

For a given air link m, the smaller is r, the more the airline prefers this slot. It means that slots with $r = 1$ are "favorite" slots.

With the previous notation, the slot r asked by the airline i for its air link m is: $z_m^{i;r} = \{[(y_{m;a}^{i;r}, t_{m;a}^{i;r})]_{a=1,...,A_m^{i;r}}\}$.

3.1.2 Slots values

In addition to describing slots for their air links, airlines must also announce how much they are able to pay for them. For each airline i, bids are contained

in the list of price: $B^i = \{b^i_m\}_{m=1,\ldots,M^i}$.

As for the slots, b^i_m is a vector including bids for all the slots asked for a given air link. Bids are classified in order of preference: $b^i_m = \{b^{i;r}_m\}_{r=1,\ldots,R^i_m}$, such that $b^{i;r}_m$ is greater than $b^{i;r+1}_m$ for all r in the set $[1, R^i_m - 1]$.

According to information they have, airlines announce their bids. Consider that θ^i in Θ^i, is the exogenous private information of airline i. The set of all bidders' private information is $\Theta = \Theta^1 \times \ldots \times \Theta^n$ and the vector of all bidders' signals is $\theta = (\theta^1, \ldots, \theta^n)$, with θ in Θ.

For each airline i, abilities to pay is defined as following:

$$v^{i;r}_m : \Theta^i \longrightarrow \mathbb{R}^+, \forall r = 1, \ldots, R^i_m, \forall m = 1, \ldots, M^i,$$

where $v^{i;r}_m(\theta^i)$ is the willingness to pay of the airline i with the signal θ^i for her slot r, for its air link m. For notation: $V^{i;r}_m = \{v^{i;r}_m(\theta^i)/\theta^i \in \Theta^i\}$.

Then:

$$v^i : \Theta^i \longrightarrow \mathbb{R}^{\left(\sum_{m=1}^{M^i} R^i_m\right)+}$$
$$\text{such that} \quad v^i(\theta^i) = \{v^{i;r}_m(\theta^i)/r = 1, \ldots, R^i_m, m = 1, \ldots, M^i\}$$

Airlines' bids are based on their own willingness to pay for goods:

$$b^{i;r}_m : \Theta^i \longrightarrow \mathbb{R}^+$$
$$\hat{b}^{i;r}_m[v^{i;r}_m(\theta^i)] = b^{i;r}_m(\theta^i)$$

The complete airline i's bid is $D^i = (Z^i; B^i)$, with:

$$\begin{cases} Z^i &= \{\{[(y^{i;r}_{m;a}, t^{i;r}_{m;a})]_{a=1,\ldots,A^{i;r}_m}\}_{r=1,\ldots,R^i_m}\}_{m=1,\ldots,M^i} \\ B^i &= \{\{b^{i;r}_m\}_{r=1,\ldots,R^i_m}\}_{m=1,\ldots,M^i}. \end{cases}$$

The component $z^{i;r}_m = \{[(y^{i;r}_{m;a}, t^{i;r}_{m;a})]_{a=1,\ldots,A^{i;r}_m}\}$ of Z^i is associated with the component $b^{i;r}_m$ of B^i.

3.2 Results of the auction

With the objectives we fixed to design an auction mechanism, a Clarke-Groves mechanism will be suitable for airspace slots auction. Such a system can be used when either a single public good is sold or many private goods are sold to many people. In a Clarke-Groves mechanism, payoff of an agent is connected to its bid only through the consequences its bid has on the final allocation or decision. The price he pays is independent of its bid. Let's see how this price will be computed for airspace slots, after the announcement of the final allocation.

3.2.1 Allocation rule

Once airlines passed on their bids, made of slots and willingness to pay, the auctioneer computes the maximum social surplus. Then, the auctioneer announces to airlines which slots they obtained. The allocation list of airline i is given by $H^i = \{h_1^i, \ldots, h_{M^i}^i\}$, such that $h_m^i = \{h_m^{i;r}\}_{r=1,\ldots,R_m^i}$ is equal to one if it won the slot and to zero otherwise:

$$h_m^{i;r} \in \{0,1\}, \quad \forall i, \forall m, \forall r \tag{1}$$

Moreover, h_m^i cannot contain more than one element equal to one, because all bids are for the same air link:

$$\sum_{r=1}^{R_m^i} h_m^{i;r} \in \{0,1\}, \quad \forall i, \forall m \tag{2}$$

Finally, capacity constraint must be satisfied by the final allocation:

$$\sum_{i=1}^{n} \sum_{m=1}^{M^i} \sum_{r=1}^{R_m^i} \sum_{a=1}^{A_m^{i;r}} \left(h_m^{i;r} \times \mathbf{1}_{s \in y_{m;a}^{i;r}} \times \mathbf{1}_{t=t_{m;a}^{i;r}} \right) \leq k_{s;t}, \quad \forall s \in X, \forall t \in T \tag{3}$$

We obtain the allocation of slots among airspace users by maximizing the sum of the ability to pay for slots, under constraints (1), (2) and (3). The allocation is given by $H = (H^1, \ldots, H^i, \ldots, H^n)$, with $H^i = \{\{h_m^{i;r}\}_{r=1,\ldots,R_m^i}\}_{m=1,\ldots,M^i}$, solution of the auctioneer program:

$$\max_{\left\{\{\{h_m^{i;r}\}_{r\in[1,R_m^i]}\}_{m\in[1,M^i]}\right\}_{i\in N}} \sum_{i=1}^{n} \sum_{m=1}^{M^i} \sum_{r=1}^{R_m^i} b_m^{i;r} \times h_m^{i;r} \tag{4}$$

under the constraints:

$$\begin{cases} \sum_{i=1}^{n} \sum_{m=1}^{M^i} \sum_{r=1}^{R_m^i} \sum_{a=1}^{A_m^{i;r}} \left(h_m^{i;r} \times \mathbf{1}_{s \in y_{m;a}^{i;r}} \times \mathbf{1}_{t=t_{m;a}^{i;r}} \right) \leq k_{s;t}, \forall s \in X, \forall t \in T \\ h_m^{i;r} \in \{0,1\}, \forall i, \forall m, \forall r \\ \sum_{r=1}^{R_m^i} h_m^{i;r} \in \{0,1\}, \forall i, \forall m \end{cases}$$

3.2.2 Payment rule

The auctioneer informs winners of the slots cost. To compute the price paid by agent i for slot m, he needs to know what would be the allocation $L(i; m)$, if agent i did not bid for slot m. It is a vector such that:

$$
\begin{aligned}
L(i; m) &= (L^1(i; m), \ldots, L^i(i, m), \ldots, L^n(i; m)) \\
\text{with} \quad L^j(i; m) &= \{l_1^j(i; m), \ldots, l_{M^j}^j(i; m)\} \\
\text{and} \quad l_m^i(i; m) &= \{l_m^{i;r}(i; m)\}_{r=1,\ldots,R_m^i} = \{0\}_{r=1,\ldots,R_m^i}
\end{aligned}
$$

We obtain this allocation by solving:

$$
\max_{\left\{\left\{\{l_{m'}^{j;r}(i;m)\}_{r\in[1,R_{m'}^j]}\right\}_{m'\in[1,M^j]}\right\}_{j\in N}} \sum_{j=1}^{n}\sum_{m'=1}^{M^j}\sum_{r=1}^{R_{m'}^j} b_{m'}^{j;r} \times l_{m'}^{j;r}(i; m) \tag{5}
$$

under constraints:

$$
\begin{cases}
\displaystyle\sum_{j=1}^{n}\sum_{m'=1}^{M^j}\sum_{r=1}^{R_{m'}^j}\sum_{a=1}^{A_{m'}^{i;r}}\left(l_{m'}^{i;r}(i; m) \times \mathbf{1}_{s\in y_{m';a}^{j;r}} \times \mathbf{1}_{t=t_{m';a}^{j;r}}\right) \leq k_{s;t}, \forall s \in X, \forall t \in T \\
l_{m'}^{j;r}(i; m) \in \{0, 1\}, \forall j \neq i, \forall m', \forall r \\
l_{m'}^{i;r}(i; m) \in \{0, 1\}, \forall m' \neq m, \forall r \\
l_m^{i;r}(i; m) = 0, \forall r \\
\displaystyle\sum_{r=1}^{R_{m'}^j} l_{m'}^{j;r}(i; m) \in \{0, 1\}, \forall i, \forall m'
\end{cases}
$$

Then, the agent i must pay for the slot m:

$$
\begin{aligned}
p_m^i &= \sum_{j=1}^{n}\sum_{m'=1}^{M^j}\sum_{r=1}^{R_{m'}^j} b_{m'}^{j;r} \times l_{m'}^{j;r}(i; m) \\
&\quad - \left(\sum_{j=1}^{n}\sum_{m'=1}^{M^j}\sum_{r=1}^{R_{m'}^j} b_{m'}^{j;r} \times h_{m'}^{j;r} - \sum_{r=1}^{R_m^i} b_m^{i;r} \times h_m^{i;r}\right) \\
&= \sum_{j=1}^{n}\sum_{m'=1}^{M^j}\sum_{r=1}^{R_{m'}^j} b_{m'}^{j;r} \times l_{m'}^{j;r}(i; m) \\
&\quad - \left(\sum_{\substack{j=1 \\ j\neq i}}^{n}\sum_{m'=1}^{M^j}\sum_{r'=1}^{R_{m'}^j} b_{m'}^{j;r'} \times h_{m'}^{j;r'} + \sum_{\substack{m'=1 \\ m'\neq m}}^{M^i}\sum_{r'=1}^{R_{m'}^i} b_{m'}^{i;r'} \times h_{m'}^{i;r'}\right)
\end{aligned} \tag{6}
$$

9

This price is the difference between the total bids of all airlines, except i, when airline i does not bid for the air link m and when it bids for this air link. Then p_m^i reflects the amount of which airline i deprives other bidders with its demand for m.

The price can also be written:

$$p_m^i = \sum_{r=1}^{R_m^i} b_m^{i;r} \times h_m^{i;r} - \sum_{j=1}^{n} \sum_{m'=1}^{M^j} \sum_{r=1}^{R_{m'}^j} \left(b_{m'}^{j;r} \times h_{m'}^{j;r} - b_{m'}^{j;r} \times l_{m'}^{j;r}(i;m) \right)$$

In this way, another interpretation is possible. Airline i pays its bid for the slot it won, for its air link m, and it benefits from a discount equal to the amount it increases the final allocation value with its demand.

Prices correspond to externalities that airlines imposed on other bidders.

Proposition 1 *This mechanism is a direct revealing and efficient mechanism.*

Proof is given in annex A.

4 Example

In order to understand what the process is of the auction, we give a simple example with less parameters than in an actual situation.

4.1 Data of the simulation

We consider:

- 4 airlines: $i = 1, 2, 3, 4$;

- 6 ATC sectors: $s = s_1, s_2, s_3, s_4, s_5, s_6$, laid out as on the figure (1):

- 3 time periods: $t = t_1, t_2, t_3$;

- a capacity for each sector at each time period equal to 2 aircraft: $k_{s;t} = 2, \forall s, t$.

Each airline describes the slots it wants and the substitute slots in case it would not obtain its favorite slot. The slot r of the airline i for its air link m is $z_m^{i;r}$ and is associated with the ability to pay $v_m^{i;r}$. Given that the mechanism is efficient and direct revealing, we know that the airlines' bids
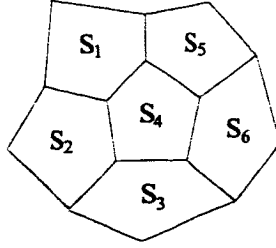
Figure 1: Layout of the 6 sectors.

are equal to their ability to pay. We suppose that the airlines' bids are the following:

Airline 1: ★ 1$^{\text{st}}$ link - 1$^{\text{st}}$ choice:
$$\begin{cases} z_1^{1;1} &= \{[(s_1, s_2, s_4), t_1], [(s_3, s_6), t_2]\} \\ v_1^{1;1} &= 70 \end{cases}$$

- 2$^{\text{nd}}$ choice:
$$\begin{cases} z_1^{1;2} &= \{[(s_1, s_2), t_1], [(s_3, s_4, s_6), t_2]\} \\ v_1^{1;2} &= 62 \end{cases}$$

- 3$^{\text{rd}}$ choice:
$$\begin{cases} z_1^{1;3} &= \{[(s_1, s_2, s_4), t_2], [(s_3, s_6), t_3]\} \\ v_1^{1;3} &= 53 \end{cases}$$

★ 2$^{\text{nd}}$ link: -1$^{\text{st}}$ choice:
$$\begin{cases} z_2^{1;1} &= \{[s_2, t_1], [s_1, t_2]\} \\ v_2^{1;1} &= 16 \end{cases}$$

- 2$^{\text{nd}}$ choice:
$$\begin{cases} z_2^{1;2} &= \{(s_1, s_2), t_2\} \\ v_2^{1;2} &= 13 \end{cases}$$

- 3$^{\text{rd}}$ choice:
$$\begin{cases} z_2^{1;3} &= \{[s_2, t_2], [s_1, t_3]\} \\ v_2^{1;3} &= 10 \end{cases}$$

Airline 2: ★ 1$^{\text{st}}$ link: - 1$^{\text{st}}$ choice:
$$\begin{cases} z_1^{2;1} &= \{[(s_1, s_4), t_1], [s_6, t_2]\} \\ v_1^{2;1} &= 32 \end{cases}$$

- 2$^{\text{nd}}$ choice:
$$\begin{cases} z_1^{2;2} &= \{[(s_1, s_5), t_1], [s_6, t_2]\} \\ v_1^{2;2} &= 27 \end{cases}$$

- 3$^{\text{rd}}$ choice:
$$\begin{cases} z_1^{2;3} &= \{[s_1, t_1], [(s_4, s_6), t_2]\} \\ v_1^{2;3} &= 25 \end{cases}$$

- 4$^{\text{th}}$ choice:
$$\begin{cases} z_1^{2;4} &= \{[(s_1, s_5), t_2], [s_6, t_3]\} \\ v_1^{2;4} &= 21 \end{cases}$$

$\star$ 2nd link: - 1st choice: $\begin{cases} z_2^{2;1} &= \{[(s_3, s_6), t_1], [s_2, t_2]\} \\ v_2^{2;1} &= 26 \end{cases}$

- 2nd choice: $\begin{cases} z_2^{2;2} &= \{[s_3, t_1], [(s_2, s_6), t_2]\} \\ v_2^{2;2} &= 24 \end{cases}$

- 3rd choice: $\begin{cases} z_2^{2;3} &= \{[(s_3, s_6), t_2], [s_2, t_3]\} \\ v_2^{2;3} &= 21 \end{cases}$

- 4th choice: $\begin{cases} z_2^{2;4} &= \{[s_3, t_2], [(s_2, s_6), t_3]\} \\ v_2^{2;4} &= 17 \end{cases}$

$\star$ 3rd link: - 1st choice: $\begin{cases} z_3^{2;1} &= \{(s_3, s_4), t_1\} \\ v_3^{2;1} &= 22 \end{cases}$

- 2nd choice: $\begin{cases} z_3^{2;2} &= \{[s_3, t_1], [s_4, t_2]\} \\ v_3^{2;2} &= 19 \end{cases}$

- 3rd choice: $\begin{cases} z_3^{2;3} &= \{(s_3, s_4), t_2\} \\ v_3^{2;3} &= 13 \end{cases}$

Airline 3: $\star$ 1st link: - 1st choice: $\begin{cases} z_1^{3;1} &= \{[(s_4, s_5), t_1], [(s_4, s_5), t_2]\} \\ v_1^{3;1} &= 52 \end{cases}$

- 2nd choice: $\begin{cases} z_1^{3;2} &= \{[(s_4, s_5), t_2], [(s_4, s_5), t_3]\} \\ v_1^{3;2} &= 34 \end{cases}$

- 3rd choice: $\begin{cases} z_1^{3;3} &= \{(s_4, s_5), t_3\} \\ v_1^{3;3} &= 20 \end{cases}$

$\star$ 2nd link: - 1st choice: $\begin{cases} z_2^{3;1} &= \{[s_2, t_1], [s_1, t_2]\} \\ v_2^{3;1} &= 21 \end{cases}$

- 2nd choice: $\begin{cases} z_2^{3;2} &= \{(s_1, s_2), t_2\} \\ v_2^{3;2} &= 17 \end{cases}$

- 3rd choice: $\begin{cases} z_2^{3;3} &= \{[s_2, t_2], [s_1, t_3]\} \\ v_2^{3;3} &= 13 \end{cases}$

Airline 4: $\star$ 1st link: - 1st choice: $\begin{cases} z_1^{4;1} &= \{[(s_3, s_6), t_1], [s_5, t_2]\} \\ v_1^{4;1} &= 20 \end{cases}$

- 2nd choice: $\begin{cases} z_1^{4;2} &= \{[s_3, t_2], [(s_5, s_6), t_2]\} \\ v_1^{4;2} &= 17 \end{cases}$

- 3rd choice: $\begin{cases} z_1^{4;3} &= \{[(s_3, s_6), t_2], [s_5, t_3]\} \\ v_1^{4;3} &= 14 \end{cases}$

$\star$ 2nd link: - 1st choice: $\begin{cases} z_2^{4;1} &= \{(s_1, s_2), t_2\} \\ v_2^{4;1} &= 18 \end{cases}$

- 2nd choice: $\begin{cases} z_2^{4;2} &= \{[s_2, t_2], [s_1, t_3]\} \\ v_2^{4;2} &= 15 \end{cases}$

- 3rd choice: $\begin{cases} z_2^{4;3} &= \{(s_1, s_2), t_3\} \\ v_2^{4;3} &= 12 \end{cases}$

Interpretation of those demands is appended to (see annex B).

4.2 Results

From those bids and capacity constraints, we can solve the program (4). The optimal allocation is:

$$H = \{[(1,0,0),(0,1,0)], [(0,1,0,0),(0,0,0,1),(0,1,0)],$$
$$[(1,0,0),(1,0,0)], [(1,0,0),(0,1,0)]\}$$

Results are resumed in the following tabular:

	1st link	2nd link	3rd link
Airline 1	1st choice	2nd choice	-
Airline 2	2nd choice	4th choice	2nd choice
Airline 3	1st choice	1st choice	-
Airline 4	1st choice	2nd choice	-

The social value, computed by adding the abilities to pay for allocated slots, is equal to 254 monetary units.

Then, we have to compute the price of each slot. We solve the program (5) several times, by removing alternatively an air link of an airline. The tabular 1 gives all the new allocations. If the airline i does not receive any slot for its air link m, the allocation is $L(i, m)$. It means that components of the slot z_m^i are available to make other slots. Then, new allocations are possible.

For example, with the components of the slot $z_1^{3;1}$ available, it becomes optimal to allocate to the second airline its first choice instead of its second choice for its first air link, with all other slots still allocated in the same way. The airline 2 can benefit from the sector s_4 at the period t_1 to operate its flight avoiding to make a detour through the sector s_5. With the allocation $L(3, 1)$, the sum of the willingness to pay is equal to 277.

We can compute the price of each slot. It is the willingness to pay for it minus the difference between the sum of the willingness to pay of allocated slots, 254 monetary units, and the one of slot that would be allocated if the airline did not bid for this air link. Prices of the nine slots are given by the set of equations (7).

Table 1: Allocations $L(i,m)$ and H.

Air Link Choice	Airline 1		Airline 2			Airline 3		Airline 4		Value total
	1	2	1	2	3	1	2	1	2	
L(1,1)	[0 0 0]	[1 0 0]	[1 0 0 0]	[0 0 1 0]	[0 1 0]	[1 0 0]	[1 0 0]	[1 0 0]	[0 1 0]	196
L(1,2)	[1 0 0]	[0 0 0]	[0 1 0 0]	[1 0 0 0]	[0 0 0]	[1 0 0]	[1 0 0]	[1 0 0]	[1 0 0]	247
L(2,1)	[1 0 0]	[0 1 0]	[0 0 0 0]	[0 0 0 0]	[0 1 0]	[1 0 0]	[1 0 0]	[1 0 0]	[0 1 0]	231
L(2,2)	[1 0 0]	[0 1 0]	[0 1 0 0]	[0 0 0 0]	[0 1 0]	[1 0 0]	[1 0 0]	[1 0 0]	[0 1 0]	237
L(2,3)	[1 0 0]	[0 1 0]	[0 1 0 0]	[1 0 0 0]	[0 0 0]	[1 0 0]	[1 0 0]	[1 0 0]	[0 0 1]	241
L(3,1)	[1 0 0]	[0 1 0]	[1 0 0 0]	[0 0 0 1]	[0 1 0]	[0 0 0]	[0 0 0]	[1 0 0]	[0 1 0]	207
L(3,2)	[1 0 0]	[0 0 0]	[0 1 0 0]	[1 0 0 0]	[0 0 1]	[1 0 0]	[1 0 0]	[1 0 0]	[1 0 0]	242
L(4,1)	[1 0 0]	[0 1 0]	[0 1 0 0]	[1 0 0 0]	[0 1 0]	[0 0 0]	[0 0 0]	[0 0 0]	[0 0 1]	240
L(4,2)	[1 0 0]	[0 1 0]	[0 1 0 0]	[1 0 0 1]	[0 1 0]	[1 0 0]	[1 0 0]	[1 0 0]	[0 0 0]	242
H	[1 0 0]	[0 1 0]	[0 1 0 0]	[0 0 0 0]	[0 1 0]	[1 0 0]	[1 0 0]	[1 0 0]	[0 1 0]	254

$$\begin{cases} p_1^1 &=& 70 - (254 - 196) &=& 12 \\ p_2^1 &=& 13 - (254 - 247) &=& 6 \\ p_1^2 &=& 27 - (254 - 231) &=& 4 \\ p_2^2 &=& 17 - (254 - 237) &=& 0 \\ p_3^2 &=& 19 - (254 - 241) &=& 6 \\ p_1^3 &=& 52 - (254 - 207) &=& 5 \\ p_2^3 &=& 21 - (254 - 242) &=& 9 \\ p_1^4 &=& 20 - (254 - 240) &=& 6 \\ p_2^4 &=& 15 - (254 - 242) &=& 3 \end{cases} \qquad (7)$$

Let us understand how those prices are defined. If airline 3 would not win a slot for its first air link, the only change in the optimal allocation will be in favor of airline 2 for its first air link. Instead of winning its second choice, with a value equal to 27 monetary units, it would obtain its first choice, with a value equal to 32 monetary units. Since there would be no bid from airline 3 for its first air link, the increase in the total value would be of 5 monetary units. Then, the price of this slot is: $p_1^3 = 5$.

4.3 Analysis of the results

Some comments can be done on payoffs and on the competition consequences on allocations.

4.3.1 Payoffs

We note that the price of the slot z_2^2 is zero. It means that airline 2 does not deprive any bidders with its slot. Since it would not bid for this slot, any of its components would be allocated to another airline: $L(2, 2) = H$. Airline 2 imposes no externality with its slot for its second air link.

With other slots, airlines deprive at least one bidder of getting a better choice. The price they have to pay is different from zero. Nevertheless, sometimes airlines impose externality on themselves. The price paid by an airline is computed for each slot. The authority can define the payoff differently. For example, the price can be computed for the whole set of slots won by airlines. Then, the new allocation we look for is the one when an airline is completely out of the auction. In this way, results would be different from previous ones. To observe the difference, we need to compare the two methods.

The price for the pool of slots obtained by airline 1 is computed by looking for the allocation $L(1)$ of the auction with no demand from airline 1. This is the following allocation with a total value equal to 183:

15

$$L(1) = \{[(0,0,0),(0,0,0)],[(1,0,0,0),(0,0,1,0),(0,1,0)],$$
$$[(1,0,0),(1,0,0)],[(1,0,0),(1,0,0)]\}$$

The total value of slots won by airline 1 is 83 $(v_1^{1;1} + v_2^{1;2} = 70 + 13)$. The global price is then:
$$p^1 = 83 - (254 - 183) = 12$$

$$p^1 = 12 \neq p_1^1 + p_2^1 = 18$$

This price is less than the sum of the prices for the two slots computed separately. It means that airline 1 imposes externalities not only on others airlines but also on herself, and this latter externality is equal to 6. Indeed, we observe from the allocation $L(1,1)$, with the available slot $z_1^{1;1}$, airline 1 gets a better slot for its second air link.

Another example is for airline 2. If it did not bid in this auction, the allocation would be:

$$L(2) = \{[(1,0,0),(0,1,0)],[(0,0,0,0),(0,0,0,0),(0,0,0)],$$
$$[(1,0,0),(1,0,0)],[(1,0,0),(0,1,0)]\}$$

For airlines 1,3 and 4, the allocation is the same as H. Airline 2 imposes no externality on others bidders. We can guess that the price p^2 is equal to zero. Indeed, the total value of $L(2)$ is 191 and the sum of slots' values of airline 2 is $v_1^{2;2} + v_2^{2;4} + v_3^{2;2} = 63$. Then the price is:

$$p^2 = 63 - (254 - 191) = 0$$

For the other airlines, the global price for slots is equal to the sum of individual price.

4.3.2   Competition

In our example, we observe that airlines are in competition for some air links. The same slot is relevant for airlines 1 and 3. Obviously, due to under-capacity, the airline with the highest value gets the better possible choice.

It is different when the competition is only for a part of the slot. For example, for a slot made of three sectors, airlines 2 and 4 have respectively for their air link 2 and 1, two common sectors. Airline 4 with a value equal to 20 gets the slot, although the value of airline 2 was 26. The slot is in this case, not necessary for the airline with the highest value. This result is due to the other components of the slot. More than sector s_3 and s_6 at

16

period t_1, airline 4 needs sector s_5 at period t_2. But there is few demand for this sector. Allocating the slot to airline 4 and the sector s_2 at period t_2 to another airline than airline 2, leads to a greater total value of the objective than otherwise.

5 Concluding remarks

An auction mechanism seems well-suited to the ATC situation. A scarce resource, the airspace, can be efficiently allocated because willingnesses to pay of airlines are revealed. An example shows us consequences of such a mechanism on payoffs and on competition. A price computed by slot and not by airline leads airlines to internalize congestion costs imposed on themselves. From a competition point of view, it is not necessary that the airline with the highest value gets the slots. It can be a solution for airlines to reroute their air link from congestioned areas, keeping a large part of sectors in common with their competitors and to bid a lesser value. Such an airline may win the slot and its competitors not.

To do a computer simulation, we need to have a lot of data about airlines. For the moment, we only know their favorite slots. But we need their alternative slots and their willingness to pay.

Nevertheless, this chosen mechanism leads to reach objectives of this auction. It is a direct revealing and efficient mechanism. The payment rule, as for a Clarke-Groves auction, is at the origin of the incentive effects. A dominant strategy for airlines is to bid their actual ability to pay. But such a mechanism is complex due to multiple packages and alternatives slots.

This auction will be repeated and in each European country a major airline is at the origin of a majority of flights. Collusion problems may also appear.

Moreover, with a Clarke-Groves mechanism we supposed that airlines' values was independent. But those values may be in fact interdependent. The airline's would be a function, not only of her own signal, but also of other airlines' signals and to "collective judgments". A personal characteristic, useful only for itself would be the cost to operate an air link. An airline's ability to pay is connected to this cost. The competition in prices, flights times, frequencies and on board services would be at the origin of interdependency between airlines. For example, if two airlines bid for the same air link, the announcement of one of them would be linked to the effect of its own airline on the network of the other.

In such a case, airlines are no more able to bid their willingness to pay, because it depends on the others. Moreover, the payment rule is no more

17

suitable because we can not know what would be the allocation if an airline did not bid.

Dasgupta and Maskin (2000) show that an efficient, but constraint equilibrium exists. From a practical point of view, the mechanism would be more and more complex. Now, research on ATC auction have to take into account this interdependency of values and to remove problems of collusion.

A Proof of Proposition 1

Let us show that it is a dominant strategy for airline i to bid truthfully for each relevant slot, whatever are bids for other airlines $b_m^{j;r}$. We need to compare net surplus of airline i between the case it bids truthfully for slots of its air link m and the case it lies. Note that the net surplus is equal to the difference between the actual value of the agent and the amount it has to pay.

- $\hat{H}$ is the final allocation when airline i bids truthfully for the R_m^i slots for whom she announced $v_m^{i;r}$, whatever are bids for other airlines:

$$
\hat{H} = \underset{\{h_m^{i;r}\}_{r;m;i}}{\arg\max} \sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{m'=1}^{M^j} \sum_{r'=1}^{R_{m'}^j} b_{m'}^{j;r'} \times h_{m'}^{j;r'} + \sum_{\substack{m'=1 \\ m' \neq m}}^{M^i} \sum_{r'=1}^{R_{m'}^i} b_{m'}^{i;r'} \times h_{m'}^{i;r'} + \sum_{r=1}^{R_m^i} v_m^{i;r} \times h_m^{i;r} \quad (8)
$$

under constraints:

$$
\begin{cases}
\displaystyle \sum_{i=1}^{n} \sum_{m=1}^{M^i} \sum_{r=1}^{R_m^i} \sum_{a=1}^{A_m^{i;r}} \left(h_m^{i;r} \times \mathbf{1}_{s \in y_{m;a}^{i;r}} \times \mathbf{1}_{t=t_{m;a}^{i;r}} \right) \leq k_{s;t}, \quad \forall s, \forall t, \\[4mm]
h_m^{i;r} \in \{0,1\}, \quad \forall i, \forall m, \forall r, \\[2mm]
\displaystyle \sum_{r=1}^{R_m^i} h_m^{i;r} \in \{0,1\}, \quad \forall i, \forall m.
\end{cases}
$$

$\hat{L}(i;m)$ is the allocation when i gets not slot for its air link m:

$$
\hat{L}(i;m) = \underset{\left\{ \left\{ \{l_{m'}^{j;r}(i;m)\}_{r \in [1,R_{m'}^i]} \right\}_{m' \in [1,M^j]} \right\}_{j \in N}}{\arg\max} \sum_{j=1}^{n} \sum_{m'=1}^{M^j} \sum_{r=1}^{R_{m'}^j} b_{m'}^{j;r} \times l_{m'}^{j;r}(i;m) \quad (9)
$$

18

under constraints:

$$
\begin{cases}
\displaystyle\sum_{j=1}^{n}\sum_{m'=1}^{M^j}\sum_{r=1}^{R_{m'}^j}\sum_{a=1}^{A_{m'}^{i;r}}\left(l_{m'}^{i;r}(i;m)\times 1_{s\in y_{m';a}^{j;r}}\times 1_{t=t_{m';a}^{j;r}}\right)\le k_{s;t},\ \forall s,\forall t,\\[4mm]
l_{m'}^{j;r}(i;m)\in\{0,1\},\ \forall j\ne i,\forall m',\forall r,\\[2mm]
l_{m'}^{i;r}(i;m)\in\{0,1\},\ \forall m'\ne m,\forall r,\\[2mm]
l_{m}^{i;r}(i;m)=0,\ \forall r,\\[2mm]
\displaystyle\sum_{r=1}^{R_{m'}^j}l_{m'}^{j;r}(i;m)\in\{0,1\},\ \forall i,\forall m'
\end{cases}
$$

If bidder i bids truthfully, when its net surplus is:

$$
\begin{aligned}
S\left(\hat H\right) &= \sum_{r=1}^{R_m^i}v_m^{i;r}\times\hat h_m^{i;r}-\left(\sum_{j=1}^{n}\sum_{m'=1}^{M^j}\sum_{r=1}^{R_{m'}^j}b_{m'}^{j;r}\times\hat l_{m'}^{j;r}(i;m)\right.\\[2mm]
&\quad-\left.\left(\sum_{\substack{j=1\\j\ne i}}^{n}\sum_{m'=1}^{M^j}\sum_{r'=1}^{R_{m'}^j}b_{m'}^{j;r'}\times\hat h_{m'}^{j;r'}+\sum_{\substack{m'=1\\m'\ne m}}^{M^i}\sum_{r'=1}^{R_{m'}^i}b_{m'}^{i;r'}\times\hat h_{m'}^{i;r'}\right)\right)
\end{aligned}
\tag{10}
$$

• Now, let us see the case of a liar airline i for slots of its air link m. $\bar H$ is the final allocation when airline i announces $b_m^{i;r}\ne v_m^{i;r}$ for all slots r in $\{1,\ldots,R_m^i\}$:

$$
\bar H = \arg\max_{\left\{\left\{\{h_m^{i;r}\}_{r\in[1,R_m^i]}\right\}_{m\in[1,M^i]}\right\}_{i\in N}}\sum_{i=1}^{n}\sum_{m=1}^{M^i}\sum_{r=1}^{R_m^i}b_m^{i;r}\times h_m^{i;r}
$$

under constraints:

$$
\begin{cases}
\displaystyle\sum_{i=1}^{n}\sum_{m=1}^{M^i}\sum_{r=1}^{R_m^i}\sum_{a=1}^{A_m^{i;r}}\left(h_m^{i;r}\times 1_{s\in y_{m;a}^{i;r}}\times 1_{t=t_{m;a}^{i;r}}\right)\le k_{s;t},\ \forall s,\forall t,\\[4mm]
h_m^{i;r}\in\{0,1\},\ \forall i,\forall m,\forall r,\\[2mm]
\displaystyle\sum_{r=1}^{R_m^i}h_m^{i;r}\in\{0,1\},\ \forall i,\forall m.
\end{cases}
$$

Given that airlines' values are private and independent, airlines announce only one bid by slot and their bids are not functions of the other airlines.

Then, vector $\hat{L}(i;m)$, solution of the program (9), does not change and the airline i's net surplus is:

$$S\left(\tilde{H}\right) = \sum_{r=1}^{R_m^i} v_m^{i;r} \times \tilde{h}_m^{i;r} - \left(\sum_{j=1}^{n} \sum_{m'=1}^{M^j} \sum_{r=1}^{R_{m'}^j} b_{m'}^{j;r} \times \hat{l}_{m'}^{j;r}(i;m) \right.$$
$$\left. - \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{m'=1}^{M^j} \sum_{r'=1}^{R_{m'}^j} b_{m'}^{j;r'} \times \tilde{h}_{m'}^{j;r'} + \sum_{\substack{m'=1 \\ m' \neq m}}^{M^i} \sum_{r'=1}^{R_{m'}^i} b_{m'}^{i;r'} \times \tilde{h}_{m'}^{i;r'} \right) \right) \tag{11}$$

We compare the two net surplus (10) et (11):

$$\Delta S = \left(\sum_{r=1}^{R_m^i} v_m^{i;r} \times \hat{h}_m^{i;r} + \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{m'=1}^{M^j} \sum_{r'=1}^{R_{m'}^j} b_{m'}^{j;r'} \times \hat{h}_{m'}^{j;r'} + \sum_{\substack{m'=1 \\ m' \neq m}}^{M^i} \sum_{r'=1}^{R_{m'}^i} b_{m'}^{i;r'} \times \hat{h}_{m'}^{i;r'} \right) \right)$$
$$- \left(\sum_{r=1}^{R_m^i} v_m^{i;r} \times \tilde{h}_m^{i;r} + \left(\sum_{\substack{j=1 \\ j \neq i}}^{n} \sum_{m'=1}^{M^j} \sum_{r'=1}^{R_{m'}^j} b_{m'}^{j;r'} \times \tilde{h}_{m'}^{j;r'} + \sum_{\substack{m'=1 \\ m' \neq m}}^{M^i} \sum_{r'=1}^{R_{m'}^i} b_{m'}^{i;r'} \times \tilde{h}_{m'}^{i;r'} \right) \right) \tag{12}$$

As $\hat{H}$ is defined (program (8)), ΔS is positive.

Whatever are the other airlines' bids, it is a dominant strategy for an air link to bid truthfully for each slot of each air link relevant for her. Truthful bids constitute an equilibrium in dominant strategy. This mechanism is direct revealing.

An efficient mechanism is such that all goods are allocated to agents with highest values. An efficient allocation is the one defined by $\hat{H}$ (program (8)). We just show that airlines bid truthfully, then the final allocation is $\hat{H}$. This mechanism is also efficient.

B Interpretation of airlines' bids in the example

The first relevant slot for airline 1 is in fact for several links. Two flights from sectors s_1 and s_2 go to the airline's hub in sector s_4. From this point, two flights go to sectors s_3 and s_6. Airline 1's ability to pay for those four air links is 70 monetary units. The airline does not try to get slots separately, because a part of them could miss it and then effects of its hub would be reduced. Alternative slots for those air links are delayed. Abilities to pay decrease as flight hours are put back.

Airlines 1 and 2 are in competition for a part of their first air link: sector s_1, s_6 and s_4. If its first choice is not satisfied, airline 2 prefers to bypass the congestioned sector, by crossing s_5. Due to the over-cost with this route, the ability to pay for this second choice is equal to 5 monetary units less than to the first one.

Airline 3's bid is for several flights between two airports. One takes off during the period t_1 for the air link s_4-s_5, and then it comes back. The same slot takes into account the two air links, because if the first does not begin at t_1, the second air link is necessary delayed. The second choice for this round-trip is postponed. For the third choice, the slot is so delayed that the return is canceled. It is the reason why the ability to pay for this slot is so low compared to the others: 20 monetary units instead of 52.

Airlines 1 and 3 compete for their second air link: sectors s_2 at t_1 and s_1 at t_2. Airline 3's value is the highest.

Sectors s_3 and s_6 at t_1 is common for air links of airlines 2 and 4. Each slot is made of a different complementary component for the two. For the whole slot airline 2's value is the highest, with 20 monetary units.

Sector s_4 at t_1 is asked four times. But sector capacity is equal to two flights. This over-demand will lead to allocation of slots which are not "favorite" slot.

References

[1] L.M. AUSUBEL. An efficient ascending-bid auction for multiple objects. *Working Paper*, juin 1997.

[2] B.D. BERNHEIM et M.D. WHINSTON. Menu auctions, resource allocation, and economic influence. *Quaterly Journal of Economics*, pages 1–31, février 1986.

[3] A-K. BJORKLUND. Legalisation of European slot trading? *The Avmark Aviation Economist*, pages 13–15, septembre 2000.

[4] S. BORENSTEIN. On the efficiency of competitive markets for operating licenses. *Quarterly Journal of Economics*, pages 357–385, 1988.

[5] E. CLARKE. Multipart pricing of public goods. *Public Choice*, 8:19–33, 1971.

[6] P. CRAMTON. Money out of thin air: The nationwide narrowband PCS auction. *Journal of Economics and Management Strategy*, 4:267–343, 1995.

[7] P. CRAMTON. The FCC spectrum auctions: an early assessment. *Journal of Economics and Management Strategy*, pages 431–495, 1997.

[8] P. CRAMTON, J. MCMILLAN, P. MILGROM, B. MILLER, B. MITCHELL, D. VINCENT et R. WILSON. *Simultaneous ascending with Package Bidding*. Charles River Associates and Market Design Inc., 1997. Report 2.

[9] P. CRAMTON, J. MCMILLAN, P. MILGROM, B. MILLER, B. MITCHELL, D. VINCENT et R. WILSON. *Auction Design Enhancements for Non-Combinatorial Auctions*. Charles River Associates and Market Design Inc., 1997a. Report 1a.

[10] P. CRAMTON, J. MCMILLAN, P. MILGROM, B. MILLER, B. MITCHELL, D. VINCENT et R. WILSON. *Package Bidding for Spectrum Licences*. Charles River Associates and Market Design Inc., 1997b. Report 1b.

[11] P. DASGUPTA et E. MASKIN. Efficient auctions. *Quaterly Journal of Economics*, 115, 2000.

[12] R. DEB et L. RAZZOLINI. Auction-like mechanisms for pricing excludable public goods. *Journal of Economic Theory*, 88:340–368, 1999.

[13] DOTECON. Auctioning airport slots, janvier 2001. A report for HM Treasury and the Department of the Environment, Transport and the Regions.

[14] R. ENGELBRECHT-WIGGANS. Revenue equivalence in multi-object auctions. *Economics letters*, 1988.

[15] P. ESO et E. MASKIN. Multi-good efficient auctions with multidimensional information. *Working Paper*, mars 2000.

[16] J.R. GREEN, A. MAS-COLLEL et M.D. WHINSTON. *Microeconomic Theory*. Oxford University Press, 1995.

[17] T. GROVES. Incentives in teams. *Econometrica*, 41:617–631, 1973.

[18] M. HARRIS et A. RAVIV. A theory of monopoly pricing schemes with demand uncertainty. *The American Economic Review*, 71(3):347–365, juin 1981.

[19] B. HOLMSTROM. Groves' scheme on restricted domains. *Econometrica*, 47, 1979.

[20] P. JEHIEL et B. MOLDOVANU. Efficient design with interdependent valuations. *Econometrica*, 69(5):1237–1259, septembre 2001.

[21] P.D. KLEMPERER. What really matters in auction design ? *Working Paper*, octobre 2000.

[22] E. MASKIN. *Privatization*, chapitre *Auctions and Privatization*, pages 115–136. Horst Siebert, 1992.

[23] R.P. MCAFEE et J. MCMILLAN. Analyzing the airwaves auction. *Journal of Economic Perspectives*, 10:159–176, 1996.

[24] P. MILGROM. Putting auction theory to work: The simultaneous ascending auction. *Journal of Political Economy*, 2000.

[25] P. MILGROM et R.J. WEBER. A theory of auctions and competitive bidding. *Econometrica*, 50(5), 1982.

[26] M. MOUGEOT. UMTS : fallait-il renégocier ? *Revue d'Économie Politique*, 1(112), 2002.

[27] F. NAEGELEN. *Les mécanismes d'enchères*. Economica, 1988.

[28] OECD. Competition policy and international airport services, mai 1998.

[29] R. PRÉGET. *Les adjudications des valeurs du Trésor : enchères discriminatoires versus enchères à prix uniforme*. Thèse de Doctorat, Université de Paris-I – Panthéon-Sorbonne, 2001.

[30] E. VAN DAMME. UMTS-auctions in Europe. *Working Paper*, novembre 2000.