

# Regulation in the Air: Price-and-Frequency Caps

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## Abstract

Despite deregulation on the air-transportation markets, many connections are still operated by a single operator. Regulation is thus a central issue in this industry. There is however a great concern for the (possibly negative) consequences of price regulation on the quality of services. We argue that both aspects should be considered jointly and propose a mechanism that allows to decentralize the optimal structure of services in this industry. The regulatory procedure is robust to the introduction of heterogeneity in the travellers' valuation of the connections' frequency.

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## 1 Introduction

“Maintaining competition in deregulated airlines markets” is, in the words of Meyer and Menzies (2000), a key concern of the air-transportation industry. Despite the deregulation that occurred in the last years, there is still a very low level of competition on the European market. In 1997, namely five years after the adoption of the “third package”<sup>1</sup> by the European Commission, almost 85% of the 336 connections over the French territory were run by one operator and more than 12% by only two. This makes it clear that regulation is a perspective that cannot be ignored.

Regulation of air transportation services cannot escape the quality issue. Economies of scale leads naturally the monopolist to provide connections with a lower frequency than what would be optimal from a social welfare point of view. And in a regulated environment, it might be feared that firms concentrate their efforts on reducing costs at the expense of quality; frequency may thus be reduced and welfare further deteriorated. The specificity of the approach consists in addressing simultaneously both distortions: the distortion in terms of prices and the distortion in terms of frequency. It is shown in this paper that the socially optimal supply of services as characterised by the price and the frequency can be reached by the means of a simple regulatory mechanism: a price-cap constraint that depends on the frequency of services.

The air-transportation industry is made of a complex network of travel services. Within this network, each of the services is in interaction with the others in order to insure possible connections. Most companies are nevertheless organised according to a star network, that is, with transportation services that connect a central airport (hub) to the periphery (spoke). The generalisation of this “hub-and spokes” system makes the management of each of this services almost independent from the others. As a matter of

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<sup>1</sup>2409/92: Council Regulation of 23 July 1992 on fares and rates for air services (OJ L 240 of 28 August 1992). This is the last step in the european process that implemented full deregulation in the sector.

facts, a large fraction of the passengers may actually pursue its travel and have a connection. However, as long as there is not a unique final destination and passengers are distributed over several connections, there are no reasons to favour a specific arrival time. This is in particular true on the most important routes, for which the frequency of services is quite high. As a result, each of the transportation services can be considered as an independent market.

In the model, we thus focus on a single origin-destination pair. The (aggregate) demand for air transportation services is a function of the price and the frequency of services. Each (Air-)travel translates indeed in both monetary and (waiting-)time costs for the passengers. We first characterise the first best allocation that can be interpreted as a generalization of the marginal pricing rules. When compared to the standard monopoly regulation problem there is an additional trade-off. An increase in the frequency of services induces an increase in operational costs but also in improvement of consumers' welfare. These benefits are evaluated by using the well-known concept "value of time". It is shown that, if sustainability is not a concern, the optimal allocation is such that the "generalised price" for users is equal to the average transportation costs. A second step of the analysis consist in displaying the choices made by a non-regulated monopolist. The characteristics of the service supplied by the monopolist is compared to the optimal structure introduced before. The study goes on by considering the second-best optimum, a more realistic situation where social welfare is maximised taking into account the sustainability constraint. This is a modified Ramsey-Boiteux problem that takes into account the specificities of the air transportation sector. The model presented here bears several characteristics in common with the literature on quality regulation<sup>2</sup>. The model brings however several new insights that are not explicitly dealt with in this literature. In particular we address the implementation problem for the second-best allocation when quality is taken into account. Regarding the air-transportation

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<sup>2</sup>See Laffont and Tirole (1993, chap. 4).

sector, Panzar (1979) is the first to address these questions. The issues of air transportation regulation and public policy are discussed at length by Levine (1987). The closest model to our analysis is a recent contribution by Brueckner and Zhang (2001). Their much more ambitious study calls however for *a priori* assumptions on the demand function that we are able to avoid here.

## 2 The model

The supply of air-transportation services between two airports is characterised by the pair  $(p, f)$  where  $p$  denotes the ticket price and  $f$  the frequency of the flights. The company has to bear fixed costs  $F$  and operational costs. Production costs. The later are directly related to the frequency of connections and the nature of the planes. A one way flight with a plane of capacity  $K$  translates into operational costs  $C(K)$  on the link that is considered.

In the long run, the company is assumed to adjust the capacity  $K$  of the planes to the total traffic observed  $X$ . The relation  $K = X/f$  is considered to hold all along the paper. The framework may however easily be adapted to situation where planes are not used at full capacity.

We also assume that there are increasing returns to scale: the average transportation cost  $C(K)/K$  is a decreasing function of the *aircraft* size  $K$ . This hypothesis is fully backed by empirical data. As an example, for Airbus A320 category, even when we ignore the fact that bigger planes usually allow to reach higher distances, the elasticity with respect to capacity of total consumption per passenger is almost constant at  $-0.84$ . From a theoretical point of view, this hypothesis brings an explanation to several facts. First, it explains why there are no competitors on the connection considered. Second, it also explains why it is less costly for to company to offer low-frequency services with large airplanes rather than numerous connections with small capacity aircrafts.

The demand in air-transportation services depends on the price  $p$  but also

on the frequency of connections  $f$ . Assuming the ideal departure time to be uniformly distributed along the time interval that *separates* two departures, the average waiting time is equal to  $1/2f$ . Denote  $\nu$  the value of time of the population that is considered. The average (waiting-)time costs for services of "quality"  $f$  amounts to  $\nu/2f$  for each passengers flying between the two cities.

Let  $S(\cdot)$  be the (gross) surplus of the representative travellers, a function of its travel consumption. The net surplus is obtained by taking off all the costs supported by the travellers: ticket price  $p$  and time costs  $\nu/2f$ . We can thus define the demand function as:

$$X(p, f) = \arg \max_X \left\{ S(X) - \left( p + \frac{\nu}{2f} \right) X \right\}. \quad (1)$$

Substitute the demand function into the net surplus to get the indirect utility function:

$$V(p, f) = S[X(p, f)] - \left( p + \frac{\nu}{2f} \right) X(p, f). \quad (2)$$

The identities (1) and (2) display the fact that the unitary costs of the commodity  $X$  (one travel) for the passengers amount to the "generalised price"  $\bar{p} = p + \nu/2f$ . In other words, demand is a function of the whole transportation costs and not the sole price  $p$ . This explains why the observed traffic is also a function of the value of time  $\nu$ .

### 3 Social Optimum

In this section, we analyse the first-best allocation, that is the allocation that maximizes the social welfare (the sum of consumers' surplus and firm's profits). At this stage the company is not required to break-even. We thus implicitly assume that fixed costs can be financed without efficiency cost through a subsidy financed from the general budget. Such a solution is usually not considered to be realistic. Nevertheless it provides us with an interesting benchmark.

Total surplus can be expressed as follows:

$$W_1(X, f) = S(X) - \frac{\nu}{2f}X - fC\left(\frac{X}{f}\right) - F \quad (3)$$

where fixed costs, operational costs but also passengers' time costs are subtracted from the gross surplus. Differentiating (3) with respect to  $X$  and  $f$ , and rearranging yields the following first-order conditions:

$$S'(X) = C'(K) + \frac{\nu}{2f}, \quad (4)$$

$$\frac{\nu}{2f} = \frac{C(K)}{K} - C'(K). \quad (5)$$

Equation (4) evidences the two components of the marginal cost of an additional passenger. On the one hand, the (standard) marginal cost of production  $C'(K)$  as supported by the firm. On the other hand, the time costs  $\nu/2f$  supported by this additional passenger. Equation (5) evidences the twofold effect of an increase in frequency. On the one hand, an increase of the operational costs that is proportional to the unit cost of a flight (thus the average transportation cost). This is a consequence of the marginal increase in the number of flights. On the other hand, a marginal decrease in the cost of each flight that follows from the decrease of the capacity  $K$ . As a result, the hypothesis of capacity adjustment yields to the conclusion that, the optimal (long run) allocation as characterised by  $X$  and  $f$  should be such that:

$$S'(X) = \frac{C(K)}{K}. \quad (6)$$

In words, the double *marginal* rule that should govern the choice of  $X$  and  $f$  results in a rule where the optimal capacity is defined by the *average* costs. Travellers' maximising behaviour implies  $S'(X) = p + \nu/2f$ . Substituting this expression into (4) and (6) leads to:

$$p = C'(K), \quad (7)$$

$$\bar{p} = \frac{C(K)}{K}. \quad (8)$$

Expressions (7) and (8) show respectively that first-best allocation can be decentralized through (i) marginal cost pricing and (ii) a frequency of connections such that the generalised price  $\bar{p}$  as supported by the passenger exactly equals the average transportation cost. Interestingly enough, this induces an efficient setting of the transportation services characteristics. The (only) travellers are those for which the transportation costs (including time costs) are smaller than the firms' operational costs.

A consequence of this (optimal) pricing policy is however that the company does not break-even. More precisely, sales will only cover marginal costs and the deficit will amount at least to the fixed costs  $F$ . Profits may indeed be written as:

$$\Pi = pX - fC(K) - F = fK \left[ C'(K) - \frac{C(K)}{K} \right] - F$$

where  $C'(K) < C(K)/K$  from the increasing returns to scale assumption. Remark that, the higher the value of time  $\nu$  and the higher the traffic level  $X$ , the bigger the losses. By using (5), the profits of the firm at the first-best optimum can be rewritten as:

$$\Pi = -\frac{\nu}{2f}X - F.$$

As a result, the first-best solution is not feasible if the operator faces a break-even constraint. One has then to consider a second-best solution where prices are set above marginal cost in order to recover all the costs. This question is addressed below.

#### 4 Transportation services with a profit maximizing monopolist

The first-best allocation has been computed by considering social welfare and fully ignoring the issue of profitability. We now turn to the converse

situation by considering the choices made by a profit-maximising monopolist. The price  $p$  and the frequency  $f$  will be such that the profit

$$\Pi(p, f) = pX(p, f) - fC(X/f) - F$$

is maximum. This gives rise to the following first-order conditions:

$$\begin{aligned} \frac{\partial \Pi}{\partial p} &= X(p, f) + (p - C'(K)) \frac{\partial X}{\partial p} = 0 \\ \frac{\partial \Pi}{\partial f} &= (p - C'(K)) \frac{\partial X}{\partial f} - C(K) + \frac{X}{f} C'(K) = 0. \end{aligned}$$

In order to interpret these expressions, it is useful to introduce the price-elasticity of the demand function (in absolute value):

$$\epsilon_{X_p} = -\frac{p}{X(p, f)} \frac{\partial X(p, f)}{\partial p}.$$

This value measures the rate of demand decrease that follows from a one point increase in the price. Note that this parameter depends *a priori* on the price  $p$  and the frequency  $f$ . Since the link between price and frequency are at the center of the questions addressed in this paper, it is useful to study the impact on demand of changes in both parameters. For this purpose, we use equation (1) describing travellers' behaviour to get:

$$\frac{\partial X(p, f)}{\partial f} = -\frac{\nu}{2f^2} \frac{\partial X(p, f)}{\partial p}$$

We can now re-write the FOC to obtain the following characterization of the services supplied by a profit-maximising monopolist:

$$\frac{p - C'(K)}{p} = \frac{1}{\epsilon_{X_p}} \quad (9)$$

$$\frac{\nu}{2f} = \frac{C(K)}{K} - C'(K). \quad (10)$$

Equation (9) shows that the mark-up made by the monopolist is inversely related with the price-elasticity of demand. In words, the more captive the



travellers are, (*i.e.* the less alternatives they have so that they are constrained to pay their ticket "whatever the price"), the higher the profits of the company. Interestingly, this well-known monopoly pricing formula is not modified by the possibility of choosing the frequency of connections. Note however that this does not mean that the price  $p$  is independent from the frequency  $f$ : the elasticity  $\epsilon_{X,p}$  is indeed a function of both parameters.

It may appear surprising that equation (10) does not differ from the equation (5) that defines optimal frequency at the first-best. Again, the unchanged *rule* does not mean that *value* will be the same in both cases. While the average waiting time should always be equal to the difference between the mean cost and the marginal cost, these costs are evaluated for different values of the capacity  $K = X/f$ . It is nevertheless remarkable to find unchanged the rule that governs the choice of  $f$ . Even the unregulated monopolist sets  $f$  by taking into account, not only the impact of  $f$  on its own costs but also the impact of  $f$  on travellers surplus (because of its effect on the travel demand).

## 5 Traffic and frequency complementarity

Since Spence (1975) we know that a monopolist may under- or over- supply quality (with respect to what would be socially optimum) depending on the complementarity or substitutability of the quantity and the quality. By definition,  $X$  and  $f$  will be complement if the social benefits of quality increase with the number of travellers or \_ this is equivalent \_ if the marginal benefits of one travel increase with the frequency. There will be substitutability otherwise. Formally complementarity is defined by:

$$\frac{\partial^2 W}{\partial X \partial f} = \frac{\partial^2 W}{\partial f \partial X} = \frac{\partial}{\partial f} [p - C'(K)] \geq 0 \quad (11)$$

In other words,  $X$  and  $f$  are complements (*resp.* substitutes) if the difference between the marginal willingness to pay for a ticket and the marginal cost of a travel is increasing with the frequency (*resp.* decreasing). The behaviour of

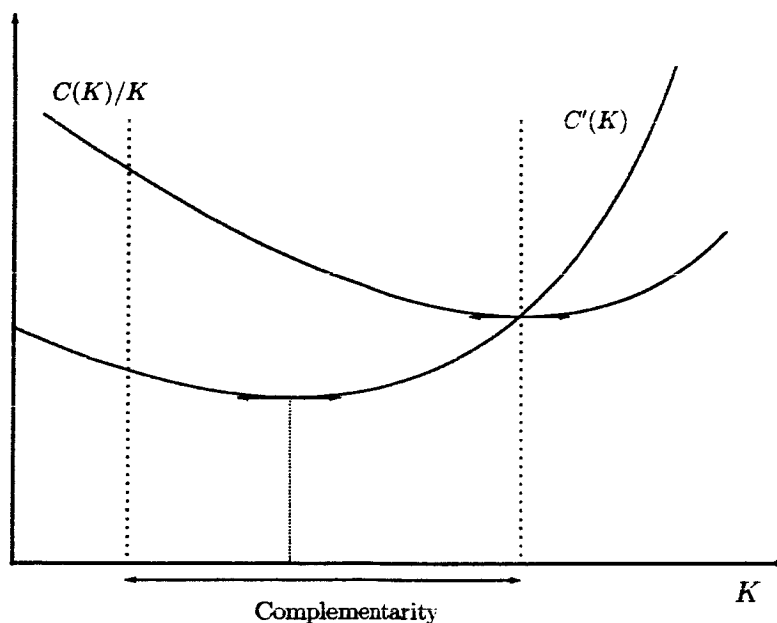
the firm will however depend on its capacity to extract the consumer surplus rather than its value. This leads us to rewrite equation (11) by using equation (5) that characterises frequency to obtain:

$$\frac{\partial^2 W}{\partial f \partial X} = -\frac{K}{f} \frac{d}{dK} \left[ \frac{C(K)}{K} - C'(K) \right].$$

This equation makes it clear that demand and frequency are complements if the difference between the mean cost and the marginal cost is decreasing with the plane capacity. Observe that this same difference governs the frequency choice both at the social optimum and at the profit maximising equilibrium. Given a frequency of connections  $f$ , the demand  $X$  and thus the capacity of the aircrafts  $K$  will be lower in the monopoly case than at the first-best. In case of complementarity, the average waiting time is decreasing with  $K$ . Thus the monopolist will set a *lower* frequency than what would be socially optimum. In case of substitutability, the frequency would be *higher*. Note that complementarity of  $X$  and  $f$  is actually a fair assumption since

$$\begin{aligned} \frac{d}{dK} \left[ \frac{C(K)}{K} - C'(K) \right] &= \frac{1}{K} \left( C'(K) - \frac{C(K)}{K} \right) - C''(K) \\ &= -\frac{\nu}{2X} - C''(K). \end{aligned}$$

Thus, in order to have substitutability, one should have a (strongly) decreasing marginal cost:  $C''(K) < -\nu/(2X)$ . As soon as it is not the case, traffic and frequency will be complement and the monopolist will set a frequency  $f$  below what would be socially optimum. This is the assumption made in the remaining part of the paper.



## 6 Second-best

We now turn to the so-called second-best, where social welfare

$$W_2(p, f) = V(p, f) + \Pi(p, f) = S(X) - \frac{\nu}{2f}X - fC\left(\frac{X}{f}\right) - F$$

is maximised under the constraint that the firm may break-even. Observe that, even without any fixed costs  $F$ , the assumption made earlier according to which the mean cost  $C(K)/K$  is decreasing implies that the price should be higher than the marginal cost for the firm to break-even.

Denote  $L$  the Lagrangian expression associated with this problem while  $\lambda$  is the multiplier of the break-even constraint. We obtain the following first order conditions:

$$\frac{\partial L}{\partial p} = \left[ S'(X) - \left( p + \frac{\nu}{2f} \right) \right] \frac{\partial X}{\partial p} - X(p, f)$$

$$\begin{aligned}
& + (1 + \lambda) \left[ X(p, f) + (p - C'(K)) \frac{\partial X}{\partial p} \right] = 0, \\
\frac{\partial L}{\partial f} = & \left[ S'(X) - \left( p + \frac{\nu}{2f} \right) \right] \frac{\partial X}{\partial f} - \frac{\nu}{2f^2} X(p, f) \\
& + (1 + \lambda) \left[ (p - C'(K)) \frac{\partial X}{\partial f} - C(K) + \frac{X}{f} C'(K) \right] = 0.
\end{aligned}$$

By using equation (1) that describes travellers behaviour and the various notations introduced above, this system can be simplified to get

$$\frac{p - C'(K)}{p} = \frac{\lambda}{1 + \lambda} \frac{1}{\epsilon_{X_p}}, \quad (12)$$

$$\frac{\nu}{2f} = \frac{C(K)}{K} - C'(K). \quad (13)$$

Equation (12) shows that the rule that governs the setting of prices at the second-best is not modified when time costs are taken into account. This is the standard Ramsey formula. Since the distortion that follows from a price set above the marginal cost increases with the elasticity of demand, the mark-up should be inversely related to this price elasticity. It is set in such a way that the overall distortion is minimised and the firm can recover all its costs which importance is measured by the shadow price  $\lambda$ .

Equation (13) is unchanged with respect to equation (5) obtained for the first-best. This does not come as a surprise since the equation governing the choice of frequency was already the same for the un-regulated monopolist. Remind that this does not mean that the frequency will be identical. In particular, if  $X$  and  $f$  are complements, the frequency set by a regulated monopolist should be higher than the frequency that would be chosen by a profit-maximising firm. (and lower than the first-best level).

A last remark should be made regarding equation (13). That the multiplier  $\lambda$  is not part of it does not mean that the frequency  $f$  is independent of the fixed costs  $F$ . If costs increase, then  $\lambda$  increases since the mark-up increases in order to cover this additional costs. Thus the price  $p$  (at second-best) is

an increasing function of  $F$ . More precisely, the price  $p$  and the frequency  $f$  at the second-best optimum less and less differ from the values set by a profit-maximising firm (thus more and more differ from the first-best values). In other words, a state-owned firm which maximizes social welfare subject to the break-even constraint but which is relatively inefficient (and thus has to finance high fixed costs) does not really differ from a profit maximising firm.<sup>3</sup>

## 7 Implementation: price-and-frequency cap

The implementation of the (second-best) optimal allocation raises several difficulties. On the one hand the regulator does not usually have a sufficient knowledge of the market in order to decide what should be the characteristics (price and frequency) of each city-pair connection. On the other hand, without any competition and control, the air-transportation company is expected to offer services with (too) high tariffs at a (too) low frequency. This will push down the welfare of inhabitants and the profits of the firms in the concerned cities; and thus hinder the economy of an entire region. Note that a publicly owned firm would not solve for this problem. As soon as managers' reward is linked with the firm performance \_which appear to be desirable feature\_ the company will adopt a strategy that aims to maximise profits. The problem is thus fundamentally linked with the working of markets, or, in the words of Spence, with the "divergence between private and social benefits". Again, facts speaks from themselves. On the Paris-Toulouse connection, for example, in the years that follow deregulation in Europe, the number of passengers raised by one half, the average capacity of the flights has been halved and the number of moves tripled. The aim of the mechanism proposed here is

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<sup>3</sup>If in addition to fixed costs  $F$ , operational costs  $C(K)$  are also higher for a public owned firm, it might be more convenient for the consumers to face a profit-maximising firm.

precisely to give the “right incentives” to the firms where deregulation has not allowed markets to escape a monopolistic situation. However, as already mentioned, the regulator should not (and cannot) substitute herself to the company because of obvious asymmetric information problems. We show that it is nevertheless possible to decentralise the optimal (second-best) solution by the means of a price-cap conjugated with a suitable “quality reward”.

Assume that the firm maximizes its profits subject to the following “price-and-frequency” cap constraint:

$$\tilde{p} = p + \frac{\nu}{2f} \leq \bar{p}. \quad (14)$$

The company is free to use its knowledge of demand in order to choose the price  $p$  and the frequency  $f$  on the considered market provided that the generalised price that travellers have to support does not exceed an upper bound  $\bar{p}$ . As a result, the quality of services dispose the upper limit for prices. Put it the other way: tariff setting determines a minimum frequency level. The first-order conditions of this maximisation program can be written as:

$$\begin{aligned} X(p, f) + (p - C'(K)) \frac{\partial X}{\partial p} - \mu &= 0 \\ (p - C'(K)) \frac{\partial X}{\partial f} - C(K) + \frac{X}{f} C'(K) + \mu \frac{\nu}{2f^2} &= 0 \end{aligned}$$

where  $\mu$  is the Lagrange multiplier associated to the “price-and frequency” cap constraint (14). Assume that the regulator fix the upper bound  $\bar{p}$  in such a way that

$$\mu = X^* / (1 + \lambda)$$

where  $X^*$  is the demand at the second-best optimum. The monopolist will find it profitable to fix  $p$  and  $f$  such that:

$$\begin{aligned} \frac{p - C'(K)}{p} &= \left(1 - \frac{\mu}{X}\right) \frac{1}{\epsilon_{X_p}} = \frac{\lambda}{1 + \lambda} \frac{1}{\epsilon_{X_p}} \\ \frac{\nu}{2f} &= \frac{C(K)}{K} - C'(K). \end{aligned}$$

In order to implement the optimal solution, it is thus sufficient to compel the firm to offer services such that their "generalised prices" do not exceed their second-best optimal values.

Such a regulatory mechanism could appear to be an artificial (and useless) rewriting of the problem if the regulatory body would not have the necessary information to fix  $\bar{p}$ . Despite its simplicity, the mechanism proposed here appears to be perfectly implementable. The optimal solution can be reached by the means of an iterative mechanism inspired by Vogelsang and Finsinger (1979) that is based on the sole book-keeping data.<sup>4</sup> In the remaining part of the paper, we study how it extends to an heterogeneous population.

## 8 Heterogeneity of characteristics and regulation

The outlined regulatory process ability to work when travellers are heterogeneous is the focus of the present section. To do this we consider a population of travellers with value of time  $\nu$  distributed over  $[0, +\infty[$ , according to the density function  $g(\nu)$  and the cumulative distribution function  $G(\nu)$ . The aggregate demand is thus given by

$$X(p, f) = \int_0^{+\infty} x_\nu(p, f) g(\nu) d\nu$$

where  $x_\nu(p, f)$  is the individual demand of a traveller with a value of time  $\nu$ , as defined by equation (1). Note that, at this stage, the value of time  $\nu$  is the only characteristic that differs across individuals.

Taking into account the very fact that the company cannot offer different connection frequencies to the passengers, the computation of the social

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<sup>4</sup>More precisely, one need to observe traffic, prices and profits at each period and for each connection. The principle consists in adjusting the coefficient of the constraint at each period and one can show that this coefficient will converge to their optimal values. On this mechanisms and their limits, see Laffond and Tirole (1993).

optimum as defined by the first-best leads to almost unchanged conclusions. Indeed, the optimal allocation is now defined by the system

$$S'(x_\nu) = C'(K) + \frac{\nu}{2f} \quad \text{all } \nu, \quad (15)$$

$$\frac{\bar{\nu}}{2f} = \frac{C(K)}{K} - C'(K). \quad (16)$$

where  $\bar{\nu} = \int_0^{+\infty} \nu (x_\nu/X) g(\nu) d\nu$  is a weighted average of the value of time. In words equation (15) states that *all travellers*<sup>5</sup> should see the marginal benefits of their travel to equate the sum of the marginal cost of production  $C'(K)$  and their own (waiting-) time costs  $\nu/2f$ . Equation (16) substitutes for equation (5) in defining the optimal frequency  $f$ . It states that the mean value of time  $\bar{\nu}$  to be considered is an average that weights the value of time proportionally to the relative number of travel  $x_\nu/X$ . In other words, the more people travel, the more their value of time impact on the value  $\bar{\nu}$  considered by the social planner.

Interestingly enough, this optimal allocation can still be implemented by the means of a marginal pricing rule. More precisely, the travellers' behaviour as defined by (1) implies that  $S'(x_\nu) = p + \nu/(2f)$  all  $\nu$ . Thus  $p = C'(K)$  and  $f$  defined by equation (16) will exactly decentralise the optimum allocation. Consequently, under the assumption of this model the heterogeneity does not introduce any source of inefficiency in the setting of the transportation services. Whatever its value of time  $\nu$ , each traveller use transportation services up to (and no more than) a level such that her marginal benefits exactly equates the total (*i.e.* production and time) marginal cost. Note that, in contrast to the representative agent case, each traveller will now support a different generalised price,  $\tilde{p}_\nu$  :

$$\tilde{p}_\nu = p + \frac{\nu}{2f} = \frac{C(K)}{K} + \frac{\nu - \bar{\nu}}{2f}.$$

In words, the generalised price is equal to the *average* cost plus the difference between the their own (waiting-) time and the average one.

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<sup>5</sup>Whatever their value of time  $\nu$ .



As long as the company sticks to linear tariffs, the profit-maximising structure of services is defined by the pair  $(p, f)$  that solves for the system:

$$\begin{aligned} X(p, f) + (p - C'(K)) \frac{\partial X}{\partial p} &= 0 \\ (p - C'(K)) \frac{\partial X}{\partial f} - C(K) + \frac{X}{f} C'(K) &= 0 \end{aligned}$$

Price is thus defined by the standard formula

$$\frac{p - C'(K)}{p} = \frac{1}{\epsilon_{X_p}},$$

while the frequency obeys an equation unchanged in its form:

$$\frac{\hat{\nu}}{2f} = \frac{C(K)}{K} - C'(K).$$

In contrast to what happens when the regulator sets for  $f$ , the average value of time considered is not proportional to frequency of use of air-transportation services. The firm rather consider the profitability of each type so that  $\hat{\nu}$  is defined by:

$$\begin{aligned} \hat{\nu} &= \int_0^{+\infty} \nu \frac{\partial x_\nu / \partial p}{\partial X / \partial p} g(\nu) d\nu \\ &= \int_0^{+\infty} \nu \frac{x_\nu}{X} \frac{\epsilon_\nu}{\epsilon_{X_p}} g(\nu) d\nu, \end{aligned}$$

where

$$\epsilon_\nu = \frac{p}{x_\nu} \left( \frac{-\partial x_\nu}{\partial p} \right).$$

This does mean that the value of time is biased "toward" the more sensitive types, *i.e.* the types with the higher price elasticity  $\epsilon_\nu$ .

Not surprisingly, the second-best allocation corresponds to a solution the lies in-between the first-best allocation and the profit-maximising one. More precisely, price and frequency are defined by the system:

$$\begin{aligned} \frac{p - C_X}{p} &= \frac{\lambda}{1 + \lambda \epsilon_{X_p}}, \\ \frac{\hat{\nu}}{2f} &= \frac{C(K)}{K} - C'(K). \end{aligned}$$

where  $\lambda$  is the “usual” Lagrange multiplier associated with the break-even constraint and the value of time  $\tilde{\nu}$  is a weighted sum of the value already introduced  $\hat{\nu}$  and  $\bar{\nu}$ :

$$\tilde{\nu} = \frac{\bar{\nu} + \lambda\hat{\nu}}{1 + \lambda}.$$

Such a result sheds *a priori* a very negative light on the applicability of the regulatory mechanism proposed above. Information considerations makes it obvious that such a value cannot be assumed to be known by the regulator. It makes thus more striking the following result: the second-best allocation will be (exactly) implemented by a monopolist submitted to the regulatory constraint:

$$\tilde{p} = p + \frac{\bar{\nu}}{2f} \leq \bar{p}$$

where  $\bar{\nu}$  is the mean value of time over the plane passengers. The proposed mechanism does not require the regulator to have more knowledge than the “social” or “average” value of time.

## 9 CONCLUSION

The optimal tariffs and optimal frequency of air transportation services is determined. Despite the complex interaction between price and quality (frequency), the optimal price is exactly defined by the Ramsey-Boiteux rule. The optimal frequency should be such that the time costs equate the difference between the average and the marginal costs. If quantity and frequency are strategic complements, a monopoly will setup the prices above the socially optimal value and the frequency below the socially optimal level.

In order for the optimal structure of services to be set up by the monopolist, incentives should be given both to decrease price and to increase frequency. This is possible, if the transportation company is submitted to a regulatory constraint that bears on the generalised transportation costs, that this the sum of the ticket price and (the monetary value of) the time

costs. Implementation requires only book-keeping data and the knowledge of the social or average value of time. It appears thus possible to propose a regulatory scheme that deal with both price and quality aspects.

## 10 References

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