A Fuzzy approach of the Competition on the air transport market

Souhir Charfeddine
UT2 &ENAC
10, Rue de la Briqueterie 31400 Toulouse France
Tel home: 0033 5 62 16 08 31, office :0033 5 62 17 46 95
souhir.charfeddine@enac.fr

Marc De Coligny
UT2- Toulouse
Tel: 0033 5 61 50 4609
Coligny@univ-tlse2.fr

Felix Mora Camino
ENAC, LAAS
7, Avenue Edouard Belin 31055 Toulouse- France
Tel : 0033 5 62 17 43 58
mora@laas.fr, felix.mora@enac.fr

Carlos Alberto Nunes Cosenza
UFRJ - Universidade Federal Do Rio De Janeiro
cosenza@pep.ufrj.br

Abstract

The aim of this communication is to study with a new scope the conditions of the equilibrium in an air transport market where two competitive airlines are operating. Each airline is supposed to adopt a strategy maximizing its profit while its estimation of the demand has a fuzzy nature. This leads each company to optimize a program of its proposed services (frequency of the flights and ticket prices) characterized by some fuzzy parameters. The case of monopoly is being taken as a benchmark. Classical convex optimization can be used to solve this decision problem. This approach provides the airline with a new decision tool where uncertainty can be taken into account explicitly. The confrontation of the strategies of the companies, in the case of duopoly, leads to the definition of a fuzzy equilibrium.

This concept of fuzzy equilibrium is more general and can be applied to several other domains. The formulation of the optimization problem and the methodological consideration adopted for its resolution are presented in their general theoretical aspect.

In the case of air transportation, where the conditions of management of operations are critical, this approach should offer to the manager elements needed to the consolidation of its decisions depending on the circumstances (ordinary, exceptional events,..) and to be prepared to face all possibilities.

Keywords: air transportation, competition equilibrium, convex optimization , fuzzy modeling,

I. INTRODUCTION

The study of the competition between operators in a transportation market has been done in several situations and under multiple hypotheses, the approach here treats this problem in the case where the estimation of demand is fuzzy.
The decision making and the choice of the strategies of an operator either when he is monopolistic or when there is a competition with other operator(s), need an estimation of his share of the market. This estimation of the demand is in general obtained through econometric regressions based on historic data and through statistical methods. In such a case, a crisp function depending on the explicative variables is obtained. Some authors[1] have recently proposed to use fuzzy modeling techniques to represent the uncertainty related with the demand. In this paper, the operators supply decision making process under a fuzzy estimation of demand is investigated and a fuzzy equilibrium situation is considered.
But before the study of the competition, and in the section II, the case of monopoly has been treated in both crisp and fuzzy estimation of demand function.

II. MONOPOLY: CHOICE OF PRICE AND SUPPLY

Let consider first a transport market where only one operator is acting. This operator has to choose the level of its supply Q and its price p, in order to maximize its profit π. The market is characterized by a demand function D(p) and the operations cost which is supposed to be a function of the level of operator's supply Q and is denoted C(Q). The satisfied demand is given by: $\min\{D(p), Q\}$ and the profit is equal to: $\pi = p.\min\{D(p), Q\} - C(Q)$. Here the demand is supposed independent of the supply level and is assumed to depend only on the price.
When an estimation $\hat{D}(p)$ of the demand function is available, the program of the operator is:

$$\underset{p,Q}{Maximize} \quad p.\min\{\hat{D}(p), Q\} - C(Q)$$

In this part, two cases are treated: first, the classical case, in which demand is considered as a crisp perfectly known function, is recalled. Then the analysis of the case of a fuzzy estimation of the demand function is developed

A. *Demand as a crisp function*

It is assumed, in this first part, that $\hat{D}(p)$ is a crisp function and that the cost function is also exactly known by the operator.
The illustration of the displayed concepts will be achieved in the linear case (linear demand and cost functions).

Illustration:
For simplicity, these demand and cost functions are assumed to be linear:
$D(p) = D_0 - \lambda.p$, for $0 < p_{\min} \le p \le p_{\max} \le D_0/\lambda$,
where D_0, λ, $p_{\min}$ and $p_{\max}$ are strictly positive parameters.
And $C(Q) = c_0 + c.Q$ for $0 < Q \le Q_{\max}$,
where c, c_0 and Q_{max} are strictly positive parameters, c_0 is the fixed cost, c is a constant marginal cost and Q_{max} is the supply capacity of the operator. It is supposed in this study that the lower and upper bounds of p and Q are never reached. The program of the company is:

$$\underset{p,Q}{Maximize}\ p.\min\ D_0 - [illegible]\ Q - c_0 - [illegible]$$

Two cases are considered, depending on the nature of the satisfied demand:

1) 1st case: D(p)≤Q

In this case, the demand is considered to be not limited by the level of supply but by the price level and the profit of the operator is given by: $\pi(p,Q) = pD(p) - C(Q)$, for a given price p.

π decreases when Q increases (for a given p) so the couple (p,Q) achieving the maximum profit for this case takes place when Q is exactly equal to D(p). The problem reduces here to:

$$\begin{cases} \underset{p}{Max} & pD(p) - C(D(p)) \\ \text{such that} & D(p) \le Q \end{cases}$$

The resolution of such a program is more or less hard de pending on the respective expressions of the demand and cost functions.

Application to the linear case:
In this case, optimality is obtained from the first and second order Lagrange conditions.
The first order Lagrange's condition : $\partial\pi / \partial p = 0$
Leads to a unique solution: $p^* = c/2 + D_0/2\lambda$
and since the second order condition

$$\partial^2\pi / \partial p^2 = -2\lambda < 0$$

is always satisfied, this value of price p^* *is optimal for this program.*
The corresponding level of supply is equal to:

$$Q^* = (D_0 - \lambda c)/2$$

Observe that this equality supposes that $D_0 > \lambda c$. *The optimal profit can then be written as:*

$$\pi^* = ((D_0 - \lambda c)/2)^2 / \lambda - c_0 \qquad (1)$$

2) 2nd case: D(p) ≥Q

In this case, the satisfied demand is limited by the level of supply and the program of the company becomes:

$$\begin{cases} \underset{p,Q}{Max} & p.Q - C(Q) \\ & D(p) \ge Q \end{cases}$$

Application to the linear case: here the assumption $D(p) \ge Q$, *implies an upper limit for p:* $p \le (D_0 - Q)/\lambda$. *When p increases and Q stays unchanged, the profit increases so the optimal value for p is equal to its maximum allowed value that is to say:* $p^* = (D_0 - Q)/\lambda$.
Then the profit can be expressed as a function of Q, only; and it can be maximized with respect to the level of supply. The optimality conditions lead to the same expression for the expected profit as in the first case as expressed in (1).

B. Fuzzy estimation of demand:

In this subsection, the estimation of the demand adopted by the company is considered as fuzzy. For a given price, $\tilde{D}(p)$ is for simplicity assumed to be represented by a trapezoidal fuzzy number. Figure: fig.1 sketches such a function by showing for the interval $p_{\rm}$[redacted]p_{max} "level curves" of $\tilde{D}(p)$: D_2 and D_3 are the curves for which the degree of membership becomes equal to 1, D_1 and D_4 are the curves for which the grade of membership starts from zero (see fig1'). For consistency reasons, these four functions are supposed not to intersect on the domain p[redacted]p_{max} .

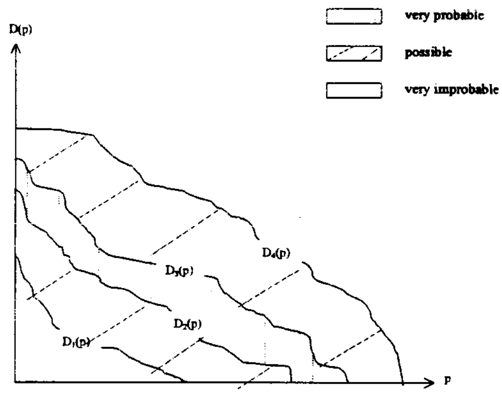


Fig1'. Fuzzy demand function

For every price p in the allowed domain, the membership function $\mu_{\tilde{D}}^{p}$ of the demand d, represented by fig. 2. is defined as follows:

$$
\begin{cases}
\dfrac{d - D_1(p)}{D_2(p) - D_1(p)} & \text{if } D_1(p) \le d \le D_2(p) \\
1 & \text{if } D_2(p) \le d \le D_3(p) \\
\dfrac{d - D_4(p)}{D_3(p) - D_4(p)} & \text{if } D_3(p) \le d \le D_4(p) \\
0 & \text{if } d \le D_1(p) \text{ or } d \ge D_4(p)
\end{cases}
$$

The fuzziness of demand propagates to the profit of the operator and when this latter chooses the couple (p,Q), he should get a fuzzy estimation of his profit $\tilde{\pi}$.

Let $\tilde{s}^{p,Q}$ be the fuzzy estimation of the satisfied demand corresponding to (p,Q): $\tilde{s}^{p,Q} = \min(\tilde{D}(p), Q)$; the membership function of $\tilde{s}^{p,Q}$ is denoted $\mu_{s}^{p,Q}$, this membership function is deduced from the one of $\tilde{D}(p)$:

$\mu_s^{p,Q}$ (s) = $\mu_{\tilde{D}}^{p}$ (s) if s < Q ,

$$\mu_s^{p,Q}(Q) = \left[\min_{d \ge Q} \mu_{\tilde{D}}^{p}(d), \max_{d \ge Q} \mu_{\tilde{D}}^{p}(d) \right]$$

and $\mu_s^{p,Q}$ (s) = 0 if s> Q

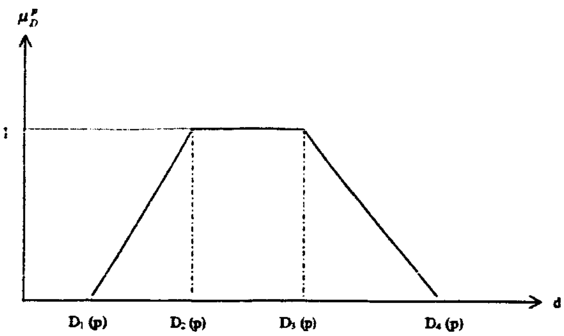


Fig.2: Trapezoidal Membership function of the demand

Then the fuzzy estimation of the profit of the company corresponding to the couple (p,Q) is given by:

$$\tilde{\pi}(p,Q) = p\tilde{s}^{p,Q} - C(Q)$$

And the membership function $\mu_\pi^{p,Q}(\pi)$ of $\tilde{\pi}(p,Q)$ is obtained from the one of s as follows:

$$\mu_\pi^{p,Q}(\pi) = \mu_s^{p,Q}[(\pi + C(Q))/p]$$

To every feasible couple (p,Q), corresponds a fuzzy set representative of the distribution of the estimate of the corresponding profit. To solve its decision problem, the company has to choose a couple (p, Q). It is not possible to compare directly fuzzy numbers, however, since demand is expected to be represented by a convex fuzzy set, it will be the case also for the profit and different possibilities appear to rank convex fuzzy sets: ranking according to the barycenter of the fuzzy set, or by more sophisticated methods as in [3] . In a simpler consideration, and when the fuzzy numbers are normalized, which is the case here (in fact, here for every (p,Q), there exists at least a π such as $\mu_\pi^{p,Q}(\pi)=1$ ($\pi / \mu_\pi^{p,Q}(\pi)=1 \neq \phi$) (as shown by figures 3), fuzzy numbers can be ranked according to the barycenter of the values whose membership is equal to1.

In this case, the calculation of the expected profit imposes the consideration of five different profit subsets configurations: depending on the shape of the membership function of $\tilde{s}$ as it is sketched by the five figures 3.

- 1st case: $Q \geq D_4(p)$; (a trapeze, fig.3a)
- 2nd case: $D_3(p) \leq Q \leq D_4(p)$; (a pentagon, fig.3b)
- 3rd case: $D_2(p) \leq Q \leq D_3(p)$, (rectangular trapeze, fig.3c)
- 4th case: $D_1(p) \leq Q \leq D_2(p)$; (a union of a triangle with a vertical segment, fig.3d)
- 5th case: $Q \leq D_1(p)$, (a vertical segment, fig.3e).

The expected profit is here taken as the barycenter of the fuzzy base of π.

Let π_e(p,Q) be the surrogate value adopted to rank the expected profits. Once a couple (p*,Q*) such as:

$$\pi_e(p^*, Q^*) = \underset{p,Q}{Max}\ \pi_e(p,Q)$$

has been found, a fuzzy estimation of the best expected profit is given by the membership function $\mu_\pi^{p^*,Q^*}(\pi)$.

An alternate approach, a conservative one, could be, instead of trying to maximize the profit, to minimize the possible loss, according to fig.4. It is possible to assign too, to each couple (p, Q), a measure of this risk.

Application to the linear case: for simplicity, these four functions are assumed to be linear:

$$D(p) = D_{i0} - \lambda_i p \,,\; i \in 1,2,3,4$$ [redacted] $p_{min} \le p \le p_{max}$ *(see fig. 5)*

here D_{i0} *and* λ_i *are positive parameters.* $(D_{i0})_{1 \le i \le 4}$ *and* $(D_{i0} / \lambda_i)_{1 \le i \le 4}$ *are taken as increasing sequences (these functions do not intersect on* p [redacted] p_{max} *).*

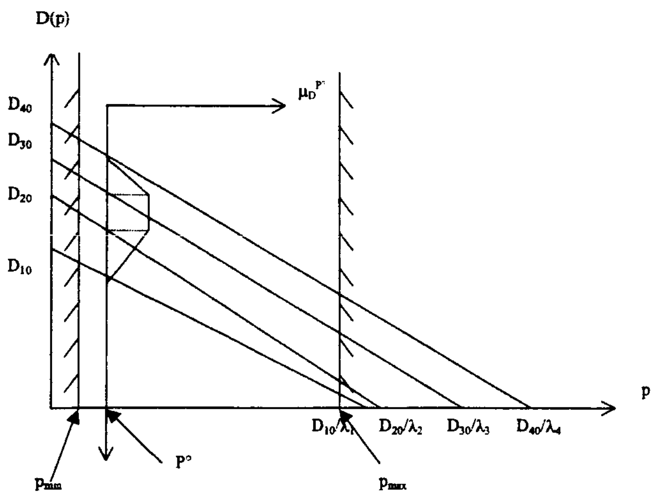


Fig5 : fuzzy demand function. linear case

III. COMPETITION UNDER DUOPOLY

Here, it is supposed that two companies are operating on the same market. Each company i ($i \in \{1,2\}$), attracts a demand D_i depending on the prices p_1 and p_2 of both companies and produces a supply level denoted Q_i which costs to it $C_i(Q_i)$. These operators are supposed not to co-operate but to compete playing a Cournot game. The 'Cournot' equilibrium of a such game is studied in this section. The case of crisp demand functions of the two operators is revisited in a first part and then the case of fuzzy estimation of the demands is treated in the second part

Every firm i ($i \in \{1,2\}$)will suppose that the parameters $p_{j \neq i}$ and $Q_{j \neq i}$ are known, and will choose its price p_i and supply level Q_i that maximize its profit π_i depending on these values.

When an estimation of the demand function $\tilde{D}_i(p_1, p_2)$ is given, the program of the company i is then

$$\max_{p_i, Q_i} \; p_i \cdot \min\left(\tilde{D}_i(\text{[redacted]}), Q_i\right) - C_i(\text{[redacted]})$$

$p_{j \neq i}$ and $Q_{j \neq i}$ are taken as known.

the case of crisp demand functions of the two operators is revisited in a first part and then the case of fuzzy estimation of the demands is treated in the second part.

A. *Demand as a crisp function:*

It is assumed in this first part that $\tilde{D}_1(p_1, p_2)$ and $\tilde{D}_2(p_1, p_2)$ are crisp functions and the cost functions $C_1(Q_1)$ and $C_2(Q_2)$ are exactly known by the operators

Application: demand and cost as linear functions:

Here again, the demand and cost functions of both companies are assumed to be linear:

$$D_i(p_i, p_j) = D^i{}_0 - \lambda_i . p_i + \mu_i p_j, \quad \text{for} \quad 0 < p^i_{\min}(p_j) \leq p_i \leq p^i_{\max}(p_j) \leq D_{i0}/\lambda_i + (\mu_i/\lambda_i) p_j, \quad i \in \{1,2\}$$

where $D_{i0}, \lambda_i, \mu_i (< \lambda_i)$, $p^i_{\min}$ *and* $p^i_{\max}$ *are strictly positive parameters for* $i \in \{1,2\}$.

$$C_i(Q_i) = c_{i0} + c_i . Q_i \ \text{for} \ 0 < Q_i \leq Q^i_{\max}, \quad i \in \{1,2\}.$$

where c_i, c_{i0} *and* $Q^i_{\max}$ *are strictly positive parameters,* c_{i0} *is the fixed cost,* c_i *is a constant marginal cost for the firm i and* $Q^i_{\max}$ *is its supply capacity. It is supposed here that the lower and upper bounds of* p_i *and* Q_i *are never reached. The program of the ith company* ($i \in \{1,2\}$) *becomes:*

$$\begin{cases} \underset{p_i, Q_i}{Maximize} \ p_i . \min \left(D_{i0} + [illegible] p_{j \neq i} - \lambda_i . p_i , Q_i \right) - c_{i0} - [illegible] Q_i \\ \text{where } (p_j, Q_j) \text{ is the solution of the program of the other company } (j \neq i). \end{cases}$$

Two cases are considered, depending on the nature of the satisfied demand by firm i:

1st case: $D_i(p_i, p_j) \leq Q_i$

In this case, the demand is considered to be not limited by the level of supply but by the price level. The profit of the company i is given by:

$$\pi_i((p_i, Q_i) / p_j) = p_i . D_i(p_i, p_{j \neq i}) - C_i(Q_i).$$

π_i decreases when Q_i increases (for a given p_i) so the couple (p_i, Q_i) achieving the maximum profit for this case takes place when Q_i is exactly equal to $D_i(p_i, p_j)$.

The problem reduces here to:

$$\begin{cases} \max_{p_i} \ p_i . D_i(p_i, p_{j \neq i}) - C_i(D_i(p_i, p_{j \neq i})) \\ p_j \ \text{is as given} \end{cases}$$

2nd case: $D_i(p_i, p_{j \neq i}) \geq Q_i$

In this case, the satisfied demand is limited by the level of supply and the program of the company becomes:

$$\begin{cases} \underset{p_i, Q_i}{Max} \ p_i . Q_i - C_i(Q_i) \\ D_i(p_i, p_j) \geq Q_i \\ p_j \ \text{is as given} \end{cases}$$

In both cases, the resolution of the associated program and the study of the existence of an equilibrium are more or less difficult, depending on the respective expressions of the demand and cost functions. The relationship between p_i and p_j could be studied for different levels of market share ($MS_i = D_i / (D_i + D_j)$)(respectively for different levels of profit π_i). Isomarketshare(resp. isoprofit) curves could be dressed.

Application to the linear case:
As it has been shown in the first section, when the functions are linear, both cases lead to the same solution:

$$p_i{}^* = c_i / 2 + (Di_0 + \mu_i p_j) / 2\lambda_i$$
$$Q_i{}^* = (D_{i0} + \mu_i p_j - \lambda_i c_i) / 2$$

for $i, j \in 1{,}2$ [redacted]

The ith optimal profit can then be written as:

$$\pi_i{}^* = ((D_{i0} + \mu_i p_j - \lambda_i c_i)/2)^2 / \lambda_i - c_{i0} \qquad (1')$$

In conclusion, the ith optimal program is such as:

$$p_i{}^* = (c_i + D_{i0} / \lambda_i + (\mu_i / \lambda_i) p_j) / 2 \quad , \qquad Q_i{}^* = (D_{i0} + \mu_i p_j - \lambda_i c_i) / 2$$

where $j \in 1{,}2$ [redacted]

It corresponds to a Cournot equilibrium which is also here a Nash equilibrium.

$$p_1^* = (c_1 + D_{10} / \lambda_1 + (\mu_1 / \lambda_1) p_2) / 2 \quad \textit{and} \quad p_2^* = (c_2 + D_{20} / \lambda_2 + (\mu_2 / \lambda_2) p_1) / 2$$

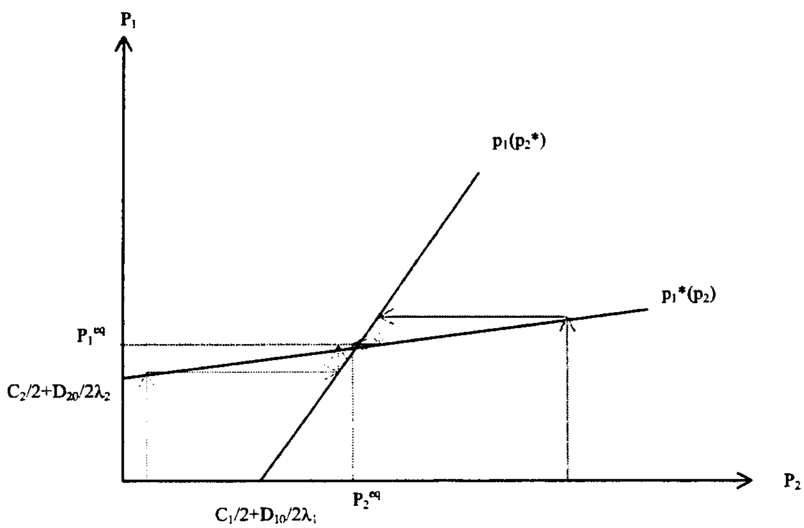
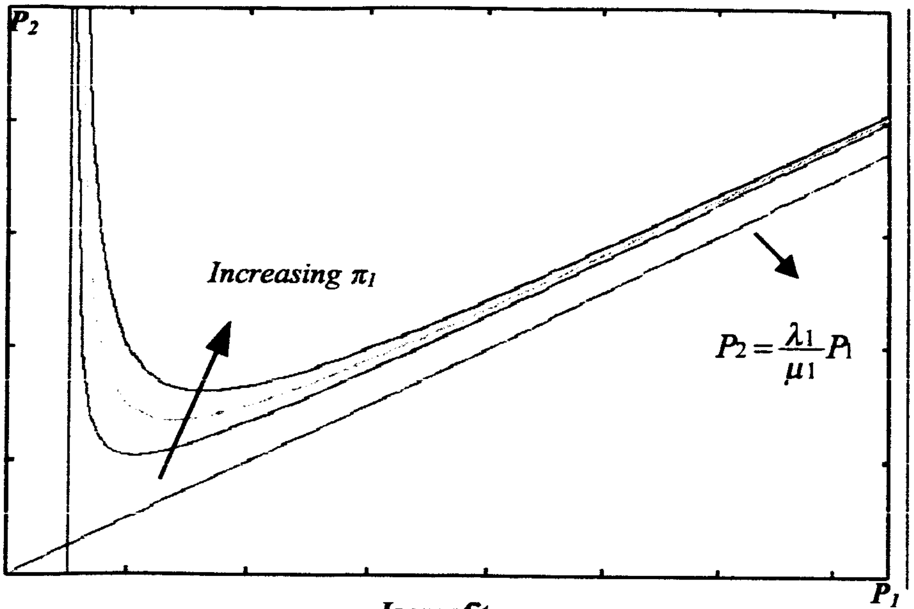


Fig6' : static stability of the equilibrium

$$MS_i = D_i(p_i, p_j) / (D_i(p_i, p_j) + D_j(p_i, p_j))$$
$$= (D_{i0} - \lambda_i . p_i + \mu_i p_j) / (D_{i0} + D_{j0} - \lambda_i . p_i + \mu_j p_i - \lambda_j . p_j + \mu_i p_j)$$
$$\Rightarrow (MS_i(\mu_i - \lambda_j) - \mu_i) p_j = (MS_i(\mu_j - \lambda_i) - \lambda_j) p_i + D_{i0} - MS_i(D_{i0} + D_{j0})$$

$$\pi i = p_i D_i(p_i, p_j) - C(Q_i)$$
$$\Rightarrow \pi_i = (p_i - c_i)(D_{10} - \lambda_i p_i + \mu_i p_j) - c_{i0}$$

$\Rightarrow$ *for* $p_i \neq c_i$, $p_j = (\lambda_i p_i^2 + (D_{i0} - \lambda_i c_i) p_i + \pi_i + c_{i0} + c_i D_{i0}) / \mu_i (p_i - c_i)$, *for a given level of profit* π_i. *The figure below sketches the isoprofit cruves giving relationship between p2 and p1.*



Isoprofit curves

B. fuzzy Demand functions

In this subsection, the estimation of the demand adopted by each company is considered as fuzzy. For a given couple of prices (p_1, p_2), $\tilde{D}_i(p_1, p_2)$ is assumed to be represented by a trapezoidal fuzzy number. On the domain p [redacted] p^i_{max} X p [redacted] p^j_{max} , some "level mappings" of $\tilde{D}_i(p_1, p_2)$ can be pointed out:

- D_i^b and D_i^c are the surfaces where the degree of membership of d_i becomes equal to 1.
- D_i^a and D_i^d are the surfaces where the grade of membership of d_i starts from zero.
- For coherency, these sets cannot intersect.

Application: to the linear case:
Here the level mappings are such as:

$$D_i^k(p_i, p_j) = D_i^k - \lambda_i^k p_i + \mu_i p_j, \qquad (i, j \neq i) \in 1,2^{2}, \qquad k \in a, b, c$$ [redacted]

$(p_i, p_j) \in p$ [redacted] p^i_{max} x p [redacted] p^j_{max}

here D_i^k, λ_i^k and μ_i^k ($< \lambda_i^k$) are positive parameters. $(D_i^k)_{k \in a,b,c}$ [redacted] *and $(D_i^k / \lambda_i^k)_{k \in a,b,c}$* [redacted] *are taken as increasing sequences so that these functions do not intersect, it is also assumed that for every $k \in a, b, c$* [redacted]*, the rate λ_i^k / μ_i^k is a constant equal to a real α_i (>1). The firm i will take the price and the supply level of the firm j as known and it will face a program analogous to the one treated in the example given in the case of monopoly. And it is the same for the firm j. Does this situation have an equilibrium?*

A first approach of this problem consists to fuzzify solutions found in the crisp case(see fig.7):

$p_1^* = (c_1 + \tilde{D}_{10} / \tilde{\lambda}_1 + (\tilde{\mu}_1 / \tilde{\lambda}_1) p_2) / 2$, $p_2^* = (c_2 + \tilde{D}_{20} / \tilde{\lambda}_2 + (\tilde{\mu}_2 / \tilde{\lambda}_2) p_1) / 2$

$Q_1^* = (\tilde{D}_{10} + \tilde{\mu}_1 p_2 - \tilde{\lambda}_1 c_1) / 2$, $Q_2^* = (\tilde{D}_{20} + \tilde{\mu}_2 p_1 - \tilde{\lambda}_2 c_2) / 2$

with $\tilde{D}_{i0}, \tilde{\lambda}_i$ and $\tilde{\mu}_i$ are fuzzy parameters (as described here D_i^k and λ_i^k are positive parameters. $(D_i^k)_{k\in a,b,c,d}$ and $(D_i^k / \lambda_i^k)_{k\in a,b,c,d}$ are taken as increasing sequences)

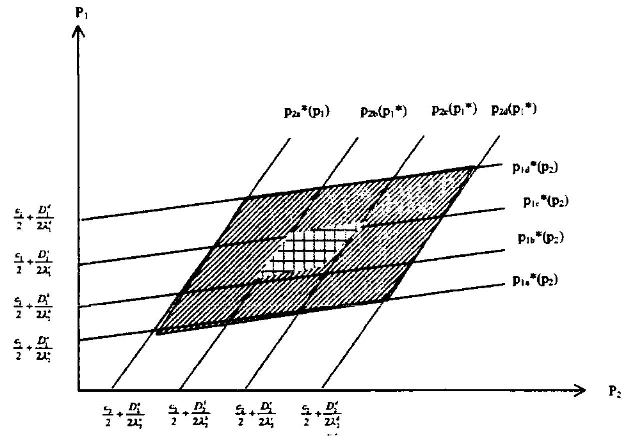


Fig7

Another approach is to consider the problem as in the first part of the paper (case of monopoly) where a fuzzy profit is considered. Through defuzzyfication (for example as a barycenter depending on the shape of the membership function of the satisfied demand,..) one expected profit can be obtained for each firm i associated with the pairs (p_i, Q_i) and (p_j, Q_j), i≠j : $\pi_i^e(p_i, Q_i, p_{j\neq i}, Q_{j\neq i})$. It can be then maximized with respect to p_i, Q_i, an optimized profit will be obtained : $\pi_i^{e*}(p_{j\neq i}, Q_{j\neq i})$ and the couple $(p_i^*(p_{j\neq i}, Q_{j\neq i}), Q_i^*(p_{j\neq i}, Q_{j\neq i}))$ realizes this maximum and then the firm will expect a fuzzy profit $\tilde{\pi}_i(p_i^*(p_{j\neq i}, Q_{j\neq i}), Q_i^*(p_{j\neq i}, Q_{j\neq i}))$.

An eventual equilibrium could be defined by the confrontation of these expressions of solutions:

$$\begin{cases} (p_1^*(p_2, Q_2), Q_1^*(p_2, Q_2)) \\ (p_2^*(p_1, Q_1), Q_2^*(p_1, Q_1)) \end{cases}$$

In this approach the values of the prices and the levels of supplies are defined in a crisp way and to them are associated fuzzy profits as in the case of monopoly. But in the first approach, for every couple of prices correspond two degrees of membership and then for each company a fuzzy profit is associated .

IV. CONCLUSION

A new approach of the resolution of the decision problem of firms has been introduced. Several domains can use it especially airlines to choose their frequency and ticket prices. The main advantage of this 'fuzzy' approach is to let the firm be prepared to all possible events and to take into account the optimistic as the pessimistic attitudes when estimating the expected demand addressed to the firm.

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Figures:

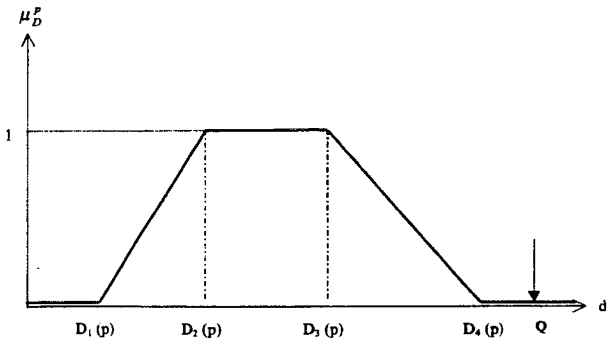

Fig.3a: Membership function of the satisfied demand ($Q>=D_4$ (p))

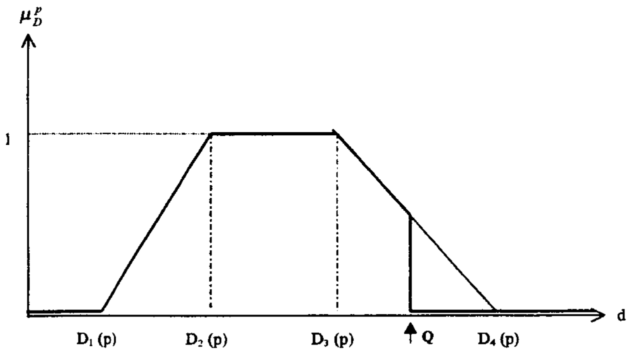

Fig.3b: Membership function of the satisfied demand (D_3 (p)<=Q<D_4 (p))

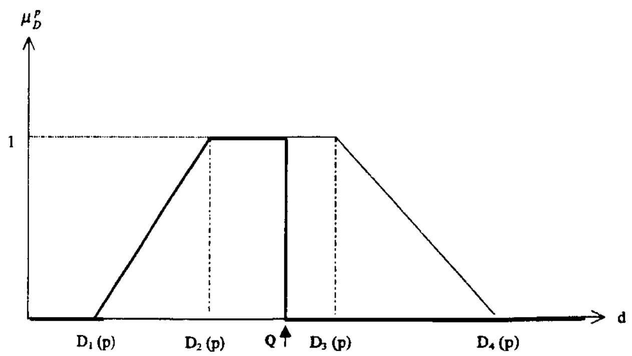

Fig.3c: Membership function of the satisfied demand (D_2 (p)<=Q<D_3 (p))

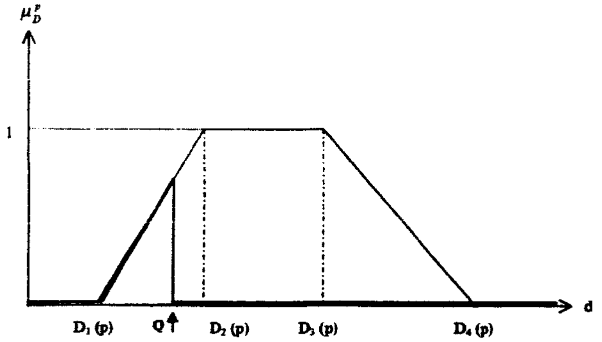

Fig.3d: Membership function of the satisfied demand (D_1 (p)<=Q<D_2 (p))

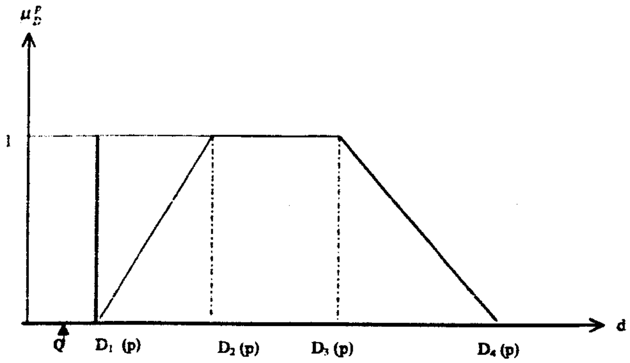

Fig.3e: Membership function of the satisfied demand (Q<D_1 (p))

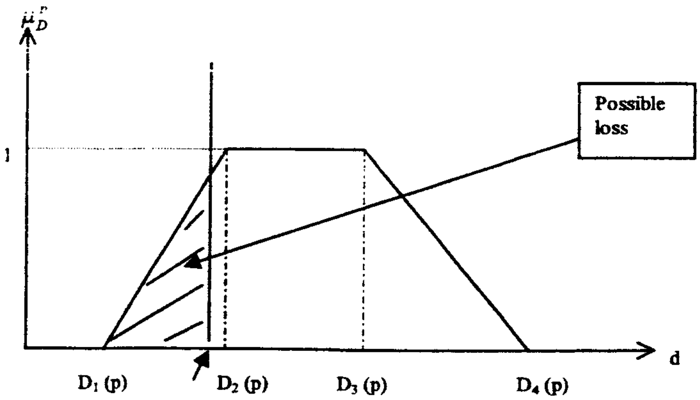

Fig.4

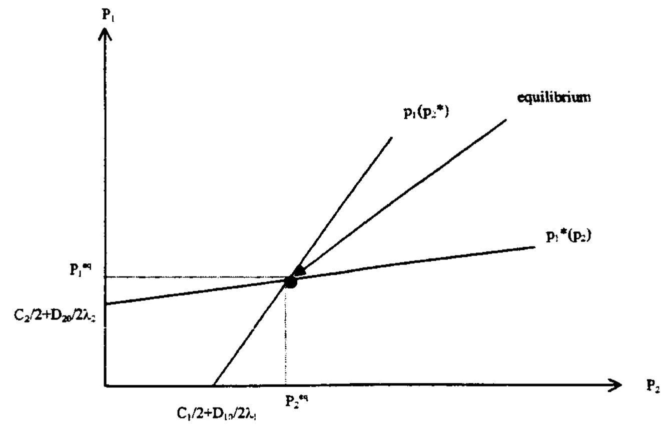

Fig6