Summary of Research for the Project
"Dynamical Evolution of Ring-Satellite Systems"
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Introduction

The goal of this research was to understand dynamical processes related to the evolution of size distribution of particles in planetary rings and application of theoretical results to explain features in the present rings of giant planets. We studied velocity evolution and accretion rates of ring particles in the Roche zone. We developed a new numerical code for the evolution of ring particle size distribution, which takes into account the above results for particle velocity evolution and accretion rates. We also studied radial diffusion rate of ring particles due to inelastic collisions and gravitational encounters. Many of these results can be also applied to dynamical evolution of a planetesimal disk. Finally, we studied rotation rates of moonlets and particles in planetary rings, which would influence the accretional evolution of these bodies. We describe our key accomplishments during the past three years in more detail in the following.

Key Accomplishments

Particle Velocity Evolution

We developed a numerical method of calculating equilibrium velocity of particles with a broad size distribution, and applied it to a planetesimal disk (Ohtsuki et al. 2002). We obtained the viscous stirring and dynamical friction rates of planetesimals with a Rayleigh distribution of eccentricities and inclinations, using three-body orbital integration and the procedure described by Ohtsuki (1999), who evaluated these rates for ring particles. We found that these rates based on orbital integrations agree quite well with the analytic results of Stewart and Ida (2000) in high velocity cases. In low velocity cases where Kepler shear dominates the relative velocity, however, the three-body calculations showed significant deviation from the formulas of Stewart and Ida, who did not investigate the rates for low velocities in detail but just presented a simple interpolation formula between their high velocity formula and the numerical results for circular orbits. We derived semi-analytic formulas for the stirring and dynamical friction rates based on our numerical results, and confirmed that they reproduce the results of N-body simulations with sufficient accuracy. Using these formulas, we calculated equilibrium velocities of planetesimals with given size distributions. In the late stage of planetary accretion where a small number of large bodies formed as a result of runway growth, we found that the inclinations of small collisional fragments calculated by our new formulas can be much smaller than those obtained by Stewart and Ida's formulas,

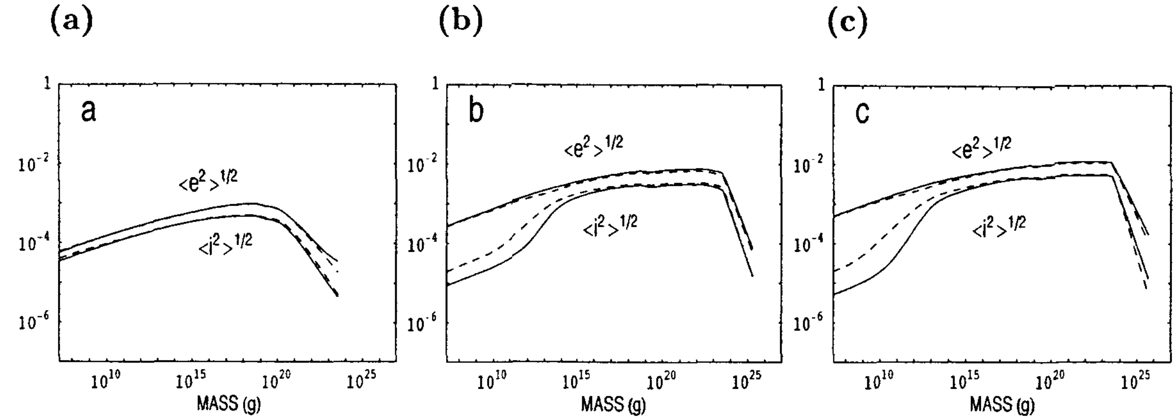


Fig. 1: Distribution of r.m.s. eccentricities and inclinations as a function of mass for the three size distributions (Ohtsuki et al. 2002). We consider the size distributions at three different stages of planetary accretion: (a) Before the onset of runaway growth; (b) Early stage of runaway growth, where accretion of planetesimals ($m > 10^{18}$ g) mostly contributes to the growth of runaway bodies; and (c) Later stage of runaway growth, where accretion of collisional fragments ($m < 10^{18}$g) signiTcantly contributes to the growth of runaway bodiesS he solid lines are the results obtained by using our new formulas (Ohtsuki et al. 2002), and the dashed lines are produced by using the analytic formulas oS I tewart anC da (2000).

so that they are more easily accreted by larger bodies in our case (Fig.1). The above method of calculating the velocity evolution for particles with a broad size distribution gives a fast and accurate calculation of ring particle velocity distribution.

Gravitational apture Probability

We obtained the gravitational capture probability of colliding ring particles using three-body orbital integration (Ohtsuki 2005b). We expanded our previous numerical calculations presented in Ohtsuki (1993) to cases involving a wide range of eccentricities and inclinations, and surface friction and rotation of particles are also taken into account. Figure 2(a) shows the plots of the capture probability for $= {}_e$ (and are particles' random velocity and escape velocity, respectively) and various values of $_H$ ($_H$ is the ratio of the sum of the radii of colliding particles to their mutual ill radius), as a function of the normal coefficient of restitution (the tangential coefficient of restitution is . 0 999). In the cases of small values of $_H$, we found that the capture probability decreases abruptly near a certain critical value of , as expected from the functional form of the analytic expression, so that the critical coefficient of restitution is well-defined. On the other hand, in the case of $_H$ 1, the dependence of the capture probability on is clearly different from the above small $_H$ cases, especially when C 0 5. In this case of strong tidal force, the capture probability decreases gradually as increases up to unity, and there is no critical coefficient of restitution above which the capture probability vanishes. This is in contrast to the simple capture criteria adopted by Canup and sposito (1995), who defined the critical coefficient of restitution even in the case of strong tidal force ($_H$ 1) on the analogy of Ohtsuki's

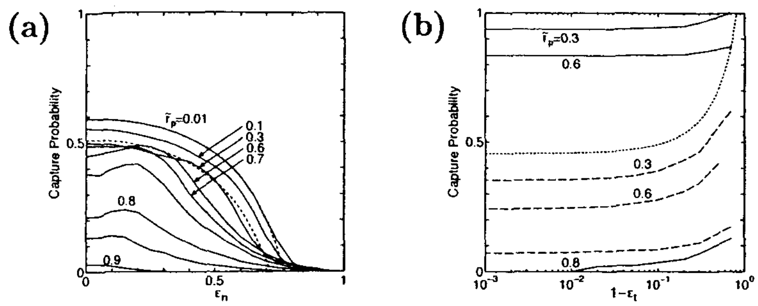


Fig. 2: (a) Plots of the capture probability for v (the random elocity of particles is e ual to their escape elocity) and various alues of $\tilde{r}_p$, as a function of (θ - -) he solid lines are the numerical results of three body orbital integration, and the dashed lines show the analytic results for $\tilde{r}_p$ θ 01 and 0.1. (b) Dependence of the capture probability on the tangential coe cient of restitution. he solid lines show the results for = 0 (circular orbits), while the dashed lines show the cases with $_n$ $_n$ he dotted line represents the analytic result for $\tilde{r}_p$ θ 01 with

(1993) analytic expression derived for $\tilde{r}_p \ll$ 1. Figure 2(b) shows the dependence of the capture probability on the tangential coefficient of restitution. Note that 1 $\rightarrow$ S 0 in the limit of no friction. We find that the dependence of the capture probability on is relatively weak when 0 , although it can be significantly enhanced if is much smaller. We also obtained the probability of gravitational accretion averaged over a Rayleigh distribution of eccentricities and inclinations both analytically and numerically. These results can be used in a statistical simulation of size distribution evolution of particles in planetary rings with low optical depth (Ohtsuki 2002).

Evolution of Ring Particle Size Distribution Due to Accretion

We studied the evolution of particle size distribution due to accretion. We developed a new numerical code to solve a coagulation equation. Then, using the velocity stirring and collision rates of ring particles based on three-body orbital integrations (Ohtsuki 1999, 2000, 2005b), we performed statistical simulations of evolution of particle size distribution due to accretion. First, we assumed that particles accrete whenever they collide. In this case, we confirmed good agreement with the result of N-body simulation for the evolution of particle size distribution (Fig.3). Next, we take account of tidal effects on the gravitational accretion of particles in the statistical simulation, based on our results of three-body orbital integration (Ohtsuki 2005b). We found a significantly rapid relative growth of the largest body compared to the mean value, in contrast to the perfect accretion case, where we found orderly growth (Ohtsuki 2002).

Radial Diffusion Rate of Particles

A new formulation for calculating the radial diffusion rate of ring particles due to collisions and gravitational encounters was obtained (Tanaka et al. 2003), and the viscosity of a particle

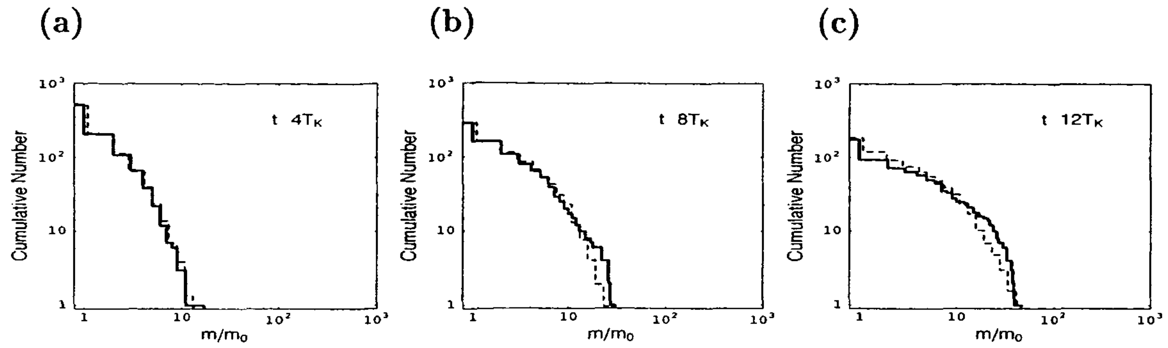


Fig. 3: lots of cumulatNe number distribution at three different times, calculated bE N body simulation (thick lines) and statistical simulation (thin lines).– n the statistical simulation, we incorporated the collision rate anP elocity e olution rate of ring particles based on threHbody orbital integrations (Ohtsuki 1-- , 2000). n the N body simulation, we used the code de eloped by Ohtsuki and Emori (2000)E n both cases, we simulated the size distribution e olution for a system of initiallC^3 10Rparticles witE 1m at the location of IaturF s ρ ring, with their material density being ⁻ θ g cG R θ 1, and the optical deptED θ 01.

disk due to gravitational encounters was calculated using three-body orbital integrations (Ohtsuki and Tanaka 2003). In the previous studies of the viscosity in self-gravitating rings, the viscosity was expressed as a sum of three components, and each of them was defined separately in different ways (aisaka et al. 2001). We derived a formulation in which the viscosity is expressed in terms of changes of orbital elements of particles due to collisions and/or gravitational encounters (Tanaka et al. 2003). We also derived a formulation for the viscosity in ring with low optical depth, which can be evaluated using three-body orbital intergrations. In the case of non-gravitating rings, we confirmed that the viscosity calculated from the above formulation agrees with the results of N-body simulations. We also calculated the viscosity of a particle disk due to mutual gravitational encounters (Ohtsuki and Tanaka 2003), and found that previous results based on the two-body approximation (Stewart and Kaula 1980M ornung et al. 1985) significantly underestimate the viscosity for encounters with low random velocities (Fig.4).

oonlet Rotation Rate

Our study on gravitational capture probability showed that it depends on the rotation rate of colliding bodies (Ohtsuki 2005b). Therefore, we also examined the rotation of a moonlet embedded in planetary rings caused by impacts of ring particles, using analytic calculation and three-body orbital integration (Ohtsuki 2004a, b). First, we evaluated systematic components of rotation arising from an average of a number of impacts. Calculations for parameter values corresponding to Saturn's C ring show that a moonlet would spin slowly in the prograde direction if most of impactors are small particles whose velocity dispersion is much smaller than the moonlet's escape velocity. We also studied the case with the Rayleigh distribution of orbital eccentricities and inclinations of particles, and found similar results (Fig.5a).ʺ owever, we also found that the effect of the random component (i.e., contribution

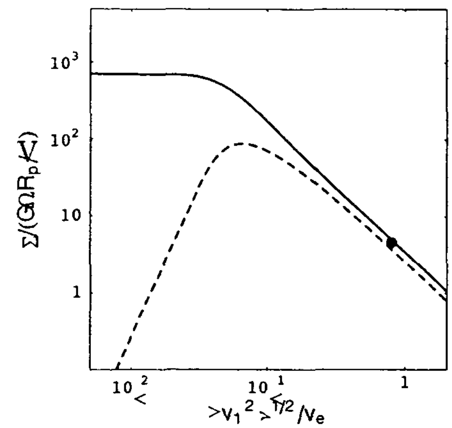


Fig. 4: Giscosity of a planetesimal disk scaled by $\Sigma \acute{M}$ Ω for the case of m $\times$ ^{r} ^{2} 10$\$$ g, v 3 0km, and 2 ^{2} 1$\bar{0}$ se$\triangleright$ ^{1} (Ohtsuki and anaka 200)v he solid line is obtained from the semi-analytic formula (Ohtsuki and anaka 200), while the dashed line is the result calculated by the analytic expression of tewart K aula (1- 30) H he value calculated from the analytic result of ornung et al. (1- 3R) is also shown by the solid circle.

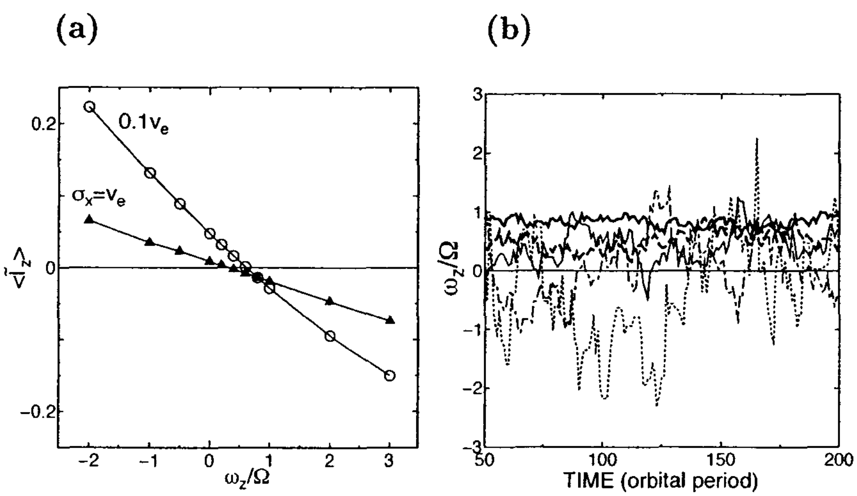
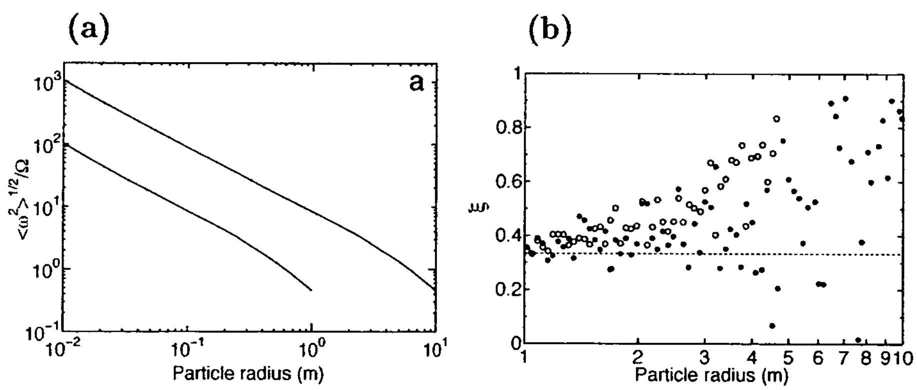


Fig. 5: (a) $_t$ ayleigh distribution a erages of the scaled mean angular momentum delivered to a moonlet, as a function of the moonlet s initial rotation rate ($_n$ 0 , . ϕ)S$^{r}_{\varepsilon\sigma}$ 1)$_x$ τ_e and are the particles radial elocity dispersion and the moonlet s escape elocity, respectively. wo cases with different elocity dispersions of particles are shown$_x$ he marks represent the alues of the initial rotation rate for which the orbital integration was performed. (b) Numerical results of N body simulation for the e olution of the component of the rotation rate of a moonlet embedded in planetary rings of a thousand, 1m radius, e ual-sized particles5 hey are assumed to be located at the radial distance of 4 1 0$\triangleright$^{5} ^{2} 10 km from I aturn7 7 ϕ)5 $_n$ 0 , and the optical depth of the ring is 0.0R ρ $\acute{M}$e cases with different sizes of the moonlet are shown: m 0 0R (thick solid line), 0.1 (thick dashed line), 0.2 (thin solid line), 0.R(thin dot dashed line), and 1 (thin dotted line)S he plots are shown for R0-200 orbital periods from the beginning of the simulation, to demonstrate the uasi-e uilibrium behavior of the system.

from a small number of large impacts) can be significant> in this case, slow rotation in both prograde and retrograde directions would be expected (Fig.5b).

Rotation Rate of Ring Particles

Rotational states of particles in Saturn's rings are not directly observable, but have been inferred from spacecraft and ground-based observations of their thermal emission. A temperature contrast between night and day sides of particles in Saturn's C ring during crossing of the planetary shadow has been detected and interpreted as indicating particles' slow rotation. Previous numerical simulations showed slow rotation when the observed particle size distribution was not taken into account. Faster rotation of smaller particles has been suggested, but rotation of ring particles with a broad size distribution has been poorly understood. We derived a new evolution equation for rotational energy of ring particles with an arbitrary size distribution, and show calculations of rotation rates of particles with a broad size distribution from centimeters to ten meters (Ohtsuki 2005a). Numerical results show that 10cm-sized or smaller particles spin more than several tens to one hundred times in one orbital period, while large ones spin slowly (Fig. a). Our N-body simulation shows that the spin axes of slowly rotating large particles have a tendency to be nearly aligned in the direction normal to the ring plane, while rapidly rotating small particles have random spin orientation (Ohtsuki and Toyama 2005 Fig. b). In optically thin rings such as Saturn's C ring, slowly rotating large particles, as well as small particles with a spin axis nearly pointing toward the planet, are likely to be responsible for the observed temperature contrast. Rapidly rotating small particles have larger orbital inclinations than slowly rotating large particles thus ring particles' rotational states have vertical heterogeneity.



FigC : (a) E uilibrium distributions of rotation rates of ring particles with a broad size distrik bution at the location of thCm ring, for the two cases with the largest particle radius ($_{\varepsilon a\ x}$) of 1m and 10m, respectNely (4 θ R anḁ a θ). (b) Distribution of ≃ which represents the spin orientation of particles. ≃ 1Ω (shown by the dotted line) for a random orientation, while ≈F 1 when thẽ component dominates particles spin angular momentum5 he solid and the open circles correspond to each of the two cases with different size of largest particles.

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