Studies in Forced and Time Varying Turbulent Flows

Final Report

Principal Investigator - Chester E. Grosch

Old Dominion University, Norfolk, Virginia 23529

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Abstract

The reasearch focused on two areas; (a) the dynamics of forced turbulent flows and (b) time filtered Large Eddy Simulations (TLES). The dynamics of turbulent flows arising from external forcing of the turbulence are poorly understood. In particular, here are many unanswered questions relating the basic dynamical balances and the existence or nonexistence of statistical equilibrium of forced turbulent flows. This research used rapid distortion theory and direct numerical simulations to explore these questions. The properties of the temporally filtered Navier-Stokes equations were also studied

Forced Isotropic Turbulence

The physics of turbulent flows arising from time varying mean flow or forcing is very complex. There is no accepted theory which can be applied to these types of flows. Experiments on these types of flows are problematical because of the uncertainty in what is statistical equilibrium and whether or not such a state exists or can exist for these classes of flows. Thus there is a lack of understanding as to which quantities should be measured and interpreted.

Rapid Distortion Theory allows one to study the dynamics of homogenous turbulence undergoing distortion driven by homogenous shear. The theory has been extended to general time-varying homogenous shear shear, *i.e.* a forcing field of the form

$$\vec{U} = (S_o\, F(t)\, x_2)\, \hat{e_1} + (0)\, \hat{e_2} + (0)\, \hat{e_3}$$

with $F(t)$ an arbitrary function of time. A few examples of $F(t)$ are:

1. Oscillating Homogenous Shear; $F(t) = \cos(\omega\, t)$,
2. Modulatted Homogenous Shear; $F(t) = \left(\frac{1}{1+\delta}\right)\, [1 + \delta \cos(\omega\, t)]$,
3. Accelerating Homogenous Shear; $F(t) = (\beta t)^n$.

Calculations of the statistical moments for these examples are proceeding.

Direct numerical simulation (DNS) is a well known technique for studying the dynamics of turbulent flows at low and moderate Reynolds numbers. Isotropic turbulence is in a statistical equilibrium at each instant of time, albeit that the statistical moments, spectrum, etc. are slowly varying functions of time.

Here we consider both decaying isotropic turbulence and isotropic turbulence with forcing. The forcing in the present study was applied to a DNS of a turbulent flow in statistical equilibrium. The DNS code is pseudo-spectral in two directions (x_1, x_3) and uses fourth order finite differences in the other (x_2) direction. The time advancement is done by using a third-order, low storage Runge-Kutta method. The boundary conditions are periodic in all directions in a cube of size L. The effect of the forcing must depend on the "size" of the forcing function relative to the "size" of the turbulent structures. To be more precise, consider the spectrum of, say, isotropic turbulence.

In statistical equilibrium the shape of the spectrum is invariant and contains (1) an energy containing range, (2) an energy transfer range wherein the energy from the energy containing range is transferred to (3) the energy dissipation range. These three ranges have different "size" turbulent structures with the "size" decreasing from (1) thru (2) to (3). Therefore the effect of forcing will depend on in which range the "size" of the forcing function matches the "size" of the turbulent structures.

The most natural way to match the "size" of the forcing function to the "size" of the turbulent structures is to do so in wavenumber (Fourier) space rather than physical space. This arises because, in wavenumber space, the "size" of the turbulent structure is precisely defined by the inverse of the corresponding wavenumber. Thus the forcing was done in wavenumber space and was applied in each of the three significant dynamical ranges; the energy containing range, the energy transfer range and the energy dissipation range. These correspond, in wavenumber space, to the small wavenumbers, the intermediate wavenumbers and the large wavenumbers.

In applying the forcing the objective is fix the energy in the particular range chosen while permitting these Fourier modes to interact with themselves and all of the other modes. To see how this can be done, one notes that the energy of a particular Fourier mode is proportional to the absolute value of the complex Fourier amplitude of that mode while mode-mode interactions are essentially controlled by phase interactions. Thus to fix the energy in the particular range of wavenumbers the *amplitudes* of the Fourier modes in this range must be held fixed and to permit these Fourier modes to interact with themselves and all of the other modes requires that the *phases* of these Fourier modes be allowed to evolve in accordance with the underlying dynamics, that is, be governed by the Navier-Stokes equations.

An algorithm to accomplish this requires an initialization and implementation at each time step. The initialization consists of first choosing a range of wave numbers, second Fourier analyzing the velocity field and third, storing the amplitude of every Fourier mode in the appropriate range, perhaps after having multiplied each of them by some wavenumber forcing function. Then, at every subsequent time step, the velocity field must be Fourier analyzed and the amplitude of the Fourier modes in the forcing range be set equal to the stored amplitude while retaining the phase unchanged. Finally, this field is inverse Fourier transformed to produce a velocity field with the prescribed forcing amplitude and appropriate phase distribution. No changes need be made to the pressure field because it is calculated using the gradients of the velocity field and so will be consistent with the velocity field.

The results of these DNSs are being analyzed in terms standard dynamical quantities, energy, dissipation rate, two-point correlations, spectra and so on, in order to determine the effects of forcing in each of these ranges.

Time Filtered Large Eddy Simulations (TLES)

The properties of the residual (subgrid-scale) stress of the temporally filtered Navier-stokes equations were studied. These properties include the frame invariance properties of the filtered equations and the residual stress. Causal time domain filters, parametrized by a temporal filter width $0 < \Delta < \infty$, were considered. The differential forms of the filters are preferred to the corresponding integral forms. The behavior of the residual stress in the limits of vanishing and infinite filter widths were studied. It was shown analytically that, in the limit $\Delta \to 0$, the residual stress vanishes and the unfiltered Navier-Stokes equations are recovered. In the limit $\Delta \to \infty$, the residual stress was shown to be equivalent to the long-time averaged stress and the Reynolds-averaged Navier-Stokes equations were recovered from the temporally filtered equations. The predicted behavior at these asymptotic limits of the filter width was validated by numerical simulation of the temporally filtered, forced, viscous Burger's equation. Finite, non-zero filter widths were used in the simulations and both *a priori* and *a posteriori* analyses of temporal similarity and temporal approximate deconvolution models of the residual stress were carried out for the model problem.