NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1405

STEADY NUCLEAR COMBUSTION IN ROCKETS

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Translation

"Stationäre Kernverbrennung in Raketen." Astronautica Acta, I, Fasc. 2, 1955.

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SUMMARY

The astrophysical theory of stationary nuclear reactions in stars is applied to the conditions that would be met in the practical engineering cases that would differ from the former, particularly with respect to the much lower combustion pressures, dimensions of the reacting volume, and burnup times.

This application yields maximum rates of heat production per unit volume of reacting gas occurring at about 10^8 °K in the cases of reactions between the hydrogen isotopes, but yields higher rates for heavier atoms. For the former, with chamber pressures of the order of 100 atmospheres, the energy production for nuclear combustion reaches values of about 10^4 kilcalories meter^{-3} second^{-1}, which approaches the magnitude for the familiar chemical fuels. The values are substantially lower for heavier atoms, and increase with the square of the combustion pressure. The half-life of the burnup in the fastest reactions may drop to values as low as those for chemical fuels so that, despite the high temperature, the radiated energy can remain smaller than the energy produced, particularly if an inefficiently radiating (i.e., easily completely ionized reacting material like hydrogen), is used.

On the other hand, the fraction of completely ionized particles in the gases undergoing nuclear combustion must not exceed a certain upper limit because the densities ($\sim 10^{-10}$ g cm^{-3}) lie in the range of high vacua and only for the previously mentioned fraction of nonionized particles can mean free paths be retained small enough so that the chamber diameters of several dozen meters will suffice.

Under these conditions it appears that continuously maintained stable nuclear reactions at practical pressures and dimensions are fundamentally possible and their application can be visualized as energy sources for power plants and propulsion units.

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[*]"Stationäre Kernverbrennung in Raketen." Astronautica Acta, I, Fasc. 2, 1955, pp. 61-88.

Three basic methods of utilizing the enthalpy of nuclear-reaction gases, of the order of 10^{10} kilocalories per kilogram, in jet-propulsion units, suggest themselves: (1) the direct expansion of these gases in a pure atomic rocket; (2) the admixture of the reacting gases with the surrounding air in turbojets, ram jets, or with other inert gases in thermal atomic rockets; and (3) the conversion of energy in photon gas in the photon rocket.

For the last two methods, the combustion pressures available in practice seem insufficient to attain the required chamber loading.

On the other hand, the technical conditions for the operation of stable nuclear reactions in central power stations, marine propulsion plants, or in pure atomic rockets of very large dimensions seem much more attainable.

I.   SOCIOLOGICAL AND TECHNICAL ASPECTS OF THE PROBLEM

Technical realization of space travel demands, on the part of humanity, an effort that may be represented by an eleven place figure of working hours. In the course of a lifetime, an individual can at best contribute a five place figure; therefore, several million people will have to occupy themselves principally with space travel. This is already the case if air travel is regarded as an initial stage of space travel.

This effort towards space travel can be made in two ways: (1) either by a vast, concentrated program; or (2), by many small contributions. These programs are similar except that the second effort presents different psychological problems.

Much strenuous effort would be required to achieve the technical development of manned space machines on the basis of presently existing chemical rocket motors. This method, while still in the initial stages has been followed under the pressure of military necessity. However, majority opinion hesitates at so concentrated an undertaking, since the investment is so high for so limited a goal and since the millions of people needed to work on space travel must include not only the professional people working on air travel but also those who could better serve in other occupational categories.

The less spectacular path of many smaller contributions to technical research, especially in the field of nuclear-powered rocket motors, promises to be less expensive, less risky, and more effective than the vast, consentrated program. In this research, the number of people required would be limited to those now working in air travel and in similar fields.

This second method of achieving space travel has already been fol-
lowed, since a major aspect of the problem (i.e., the method of obtaining
propulsive energy from the conversion of mass to energy) is the present
concern of the people working with power development, weapons, land and
marine transportation.

Both methods are in a healthy competition with each other, but the
former, which makes use of chemical reactions, attains a conversion of
but 10^{-10} of the mass into energy, whereas the second method strives for
the 100-percent conversion occuring in nature but presently achieves a
conversion of about 10^{-3} in nuclear combustion. When technical mastery
is achieved, this method of producing energy promises greater repercus-
sions on the history of humanity than were produced by the deed of the
mythical Prometheus.

According to the views of the astrophysicists, steady nuclear com-
bustions are the predominant source of energy release in the universe
and thus the source for all life on earth. The state of nuclear physical
research permits us to assume that the new Prometheus already lives among
us who will also steal fire from heaven and bestow it upon humanity for
immediate use as steady nuclear combustion.

The difficulties of technical nuclear combustion arose primarily
from the fact that combustion pressures and burnup times must be much
smaller than the values attained in the cosmic burnup at the centers of
the sun and stars. It is with these technical difficulties, in partic-
ular in the application of the steady nuclear combustion in rocket
motors, with which we will be concerned.

II.   THE NUCLEAR COMBUSTION PLASMA

A gas at rest contains, according to the Maxwell-Boltzmann statis-
tical distribution, a most probable thermal particle velocity
$c^2 = 2kT/AM$, determined by its temperature T with a distribution of ther-
mal velocities of the particles extending from zero and infinity.

The collisions between the gas particles, under the assumption of
normal temperature of the gases, will be predominantly of an elastic
nature (i.e., there is neither loss nor gain of translational kinetic
energy).

If, within the Maxwell distribution of the given gas we pass to
higher velocities, we soon encounter inelastic impacts in which energy
is lost by excitation of internally quantized degrees of freedom, namely,
rotation, vibration, or excitation of the gas molecules. Energy is
thereby removed so that the translational energy and the translational
temperature of the gas particles is reduced.

This energy transfer to the internal degrees of freedom can, in turn, give rise to the emission of photons which, if the gas volumes are assumed to be infinitely large compared with the photon mean free paths, form a photon gas in equilibrium with the corpuscles of the rest mass that constitutes the gas itself.

As we continue to pass to the more energetic but less frequent collisions, we find the dissociation of gas molecules into radicals and atoms. These in subsequent inelastic collisions, with suitable recombination partners, may loose their internal energy so that the translational energy and, therefore, the temperature of the gas, increased.

With still higher collision velocities complete ionization of the atoms is possible, which then opens the way for collisions between the nuclei themselves.

These nuclear collisions will at first again be predominantly elastic. Only in very rare cases the still relatively small collision velocities will lead, as a consequence of the wave mechanical tunnel effect, to nuclear reactions. These collisions will yield on the average up to 10^7 times higher energy release than the chemical recombination collisions. Only for those very rapid and, therefore, extraordinarily rare collisions whose energy is sufficient to overcome the classical Coulomb potential of the nucleus, will there be an appreciable probability for nuclear collisions that lead to nuclear reactions. These reactions will occur continuously in the same way as the chemical reactions of combustion gases, but will occur so rarely as to be negligible from the energy production standpoint.

With true adiabatic isolation of the gas under consideration, these very rare nuclear reactions must naturally also lead to a gradual increase in the temperature in accordance with the Maxwell distribution, which then leads to a more and more rapid increase in the frequency of all the processes described. Since the process accelerates at high collision velocities, an avalanche-like increase of the number of thermal nuclear reactions in the gas results, which tends to reach explosively an end temperature of the order of magnitude of 10^{10} degrees, if it consists of light elements.

This natural and self-supporting process does not occur in our more restricted environment because no gas masses of infinite volume exist and no boundary of a finite volume is completely impervious to heat.

If man causes this process artificially, then he will have to form boundary conditions to have as low a heat loss as possible and, at the same time, accelerate the process. This can be accomplished, for example, by reducing the long start-up period through artificial heating.

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This method has already succeeded in bringing about explosion-like thermal nuclear combustions of certain kinds of atoms to a technically useful scale.

The original form in which the energy appears in chemical reactions, and the mechanism according to which it is distributed over the combustion gas particles are as yet only very little known, and has only recently begun to be investigated.

In the case of nuclear chemical reactions, we are far better informed on the primary heating up process itself owing to the direct possibility of the observation of the paths and velocities of the nuclear reaction products as a result of their ionization action in the surrounding gases, a circumstance that is lacking in the case of the chemical reaction products because of their much smaller energies.

The nuclear chemical heat of reaction can readily be computed in advance, with sufficient accuracy, from the mass defect of the particles taking part in the reaction. This is impossible in the case of chemical reactions since the mass defects are too small to be weighed accurately.

Furthermore, the multiplicity of the degrees of freedom for receiving this released energy, in the case of chemical reactions (translation of the reaction products and possibly of photons, rotation, oscillation, different energy level excitation, and ionization), is considerably greater than for nuclear reactions where only the translation of the reaction products and of the photons are principally being considered.

The nuclear reaction proceeds in such a manner that the two reaction partners combine to form a compound nucleus of very short life, after which, depending on the degree of excitation and their individual properties, they divide approximately in half (fission), into very many smaller parts (spalling), or emit individual small particles (α-particles, protons, neutrons, electrons, positrons, and photons). The last process has been principally investigated in nuclear combustions at the present time.

The excess energy (heat of reaction) appears here as kinetic energy of translation, the newly formed particles disperse with great velocity, as we also have recently been more and more forced to assume for the case of chemical combustion reactions.

Since the nuclear chemical burnup process for readily attained pressures often lasts considerably longer than that of the chemical combustions of equal pressures, the newly formed particles experience very many elastic collisions with the remaining particles of the reaction plasma (i.e., with atomic nuclei and electrons), and with these particles the Maxwell-Boltzmann velocity distribution is immediately assumed.

For the temperatures and pressures that are of interest here, the mean free path of the particles is of the order of magnitude of 10^7 centimeters within a fully ionized plasma and 10^{-1} centimeters in a non-fully ionized plasma of the same temperature. In a surrounding cooler gas jacket in which nuclear reactions occur more rarely, the mean free path is many tenth powers less.

The particles, therefore, will instantly diffuse from a fully ionized plasma of steady pressure, even if the diameter of the combustion zone was of astronomical dimensions.

Only in incompletely ionized plasmas obtained (e.g., by the inclusion of heavy ions), can the dimensions of the reactions remain feasible even at the highest possible pressures.

III.   THERMAL NUCLEAR REACTION AND ENERGY RATE

The problems of the production of energy through thermal nuclear reactions have been clarified particularly through the work of the astrophysicists and we here follow essentially the known work of G. Gamow (ref. 1).

Nuclear reactions between fully ionized atoms (i.e., bare positively charged atomic nuclei), become very probable when the relative collision velocities of the particles are so high that the centers of the nuclei, during the collision, approach against the repulsion of the Coulomb field to a distance equal to the sum of the de Broglie wavelengths of the particles.

From considerations of wave mechanics the potential barrier in some cases can be penetrated by slower collisions, although with a probability which decreases exponentially with the collision velocity. The probability of the nuclear reaction by a particle which has penetrated into a nucleus is, in addition, determined by the resonance relations of the compound nucleus formed, and by the escape probability of the particle from this intermediate nucleus.

All these influences on the reaction probability are evident in the behavior of the so-called reaction cross-section σ, the variation of which is plotted schematically in figure 1, against the collision velocity, for reactions between charged particles.

The cross section for thermal nuclear reactions between charged particles increases exponentially with the collision velocity w until the kinetic energy is of the same order of magnitude as the potential energy at the summit of the potential barrier; the increase then becomes

slower, the cross section reaches a maximum, and then, at very high collision velocities, it decreases with further increasing w because the duration of time that the colliding particles remain in the hit nucleus becomes increasingly shorter and less sufficient for a nuclear reaction.

Gamow gives the following quantitative expression for this relation:

$$\sigma = \text{\AA}^2\pi \, \exp(8\pi e\sqrt{2RAMZ_1Z_2} \, | h - 4\sqrt{2} \, \pi^2 e^2 Z_1 Z_2 | hw) \, 4\pi^2 \Gamma R^2 AM | h^2 \, [cm^2] \quad (1)$$

for which the following symbols are used:

$\text{\AA} = \lambda/2\pi$ scattering wavelength, cm

$\lambda = h/AMw$ de Broglie wavelength of material wave of particles, cm

$A = A_1 A_2/(A_1 + A_2)$ reduced atomic weights for energy absorption in plastic collision [-]

A_1, A_2 atomic weights of collision partners [-]

$M = 1.6604 \times 10^{-24}$ g   nucleone mass

$e = 4.805 \times 10^{-10}$ cm$^{3/2}$g$^{1/2}$sec^{-1} elementary charge

$R = 1.7 + 1.22(A_1 + A_2)^{1/3} \times 10^{-13}$ cm   radius of compound nucleus

Z_1, Z_2 atomic numbers of colliding particles [-]

$h = 6.624 \times 10^{-27}$ erg-sec   Planck's constant

Γ half width of nuclear resonance level [erg]

The three multiplied factors in equation (1) have the following physical meaning:

$\text{\AA}^2\pi$ denotes the effective scattering cross section; the exponential represents the transfer coefficient, which also contains the tunnel effect. For the collision velocities greater than $w' = e\pi/2 \sqrt{Z_1 Z_2/RAM}$ the exponential is greater than one, so that the reaction cross section σ can become fundamentally greater than the scattering cross section.

The maximum value of the transfer coefficient occurs for $w = \infty$ and is of the order of magnitude 10^1. The velocity w' is equal to that which is required for the nuclear centers to approach to a distance $8/\pi^2 R \approx R$.

Finally, the fraction appearing as the third factor is the ratio of the reaction probability $2\pi\Gamma/h$ to the eigen frequency $h/2\pi R^2 AM$ of the nucleus. This ratio for (p,α) reactions lies in the neighborhood of one, so that each penetration of the potential wall leads to the reaction. For (p,γ) reactions, the ratio is generally 10^4 to 10^5 times smaller, and for (p,β) reactions, up to 10^{21} times smaller. Only by experiment in certain cases can Γ be more accurately determined.

Equation (1) can be reduced to a somewhat more manageable form:

$$\sigma = R^2\pi \frac{\Gamma}{AMw^2} \exp\left[\frac{4\sqrt{2}\pi e\sqrt{Z_1 Z_2}}{h}\left(2\sqrt{RAM} - \pi e -/Z_1 Z_2/w\right)\right] \quad [\text{cm}^2] \qquad (1a)$$

The variation of the reaction cross section σ with the collision velocity w shows a maximum at the collision velocity

$$(w)_{\sigma_{max}} = 2\sqrt{2}\ \pi^2 e^2 Z_1 Z_2/h \quad [\text{cm s}^{-1}] \qquad (1b)$$

which, thus, has no connection with the velocity w' mentioned previously. The maximum σ corresponding to this velocity is

$$\sigma_{max} = R^2\pi \frac{h^2\Gamma}{8\pi^4 AMe^4 Z_1^2 Z_2^2} \exp\left(\frac{8\pi e -\sqrt{2RAMZ_1 Z_2}}{h} - 2\right) \qquad (1c)$$

The optimum collision velocity with respect to σ according to equation (1b) can be obtained from the following considerations.

The absorbed kinetic energy corresponding to $(w)_{\sigma_{max}}$ is given by

$$(E)_{\sigma_{max}} = \frac{AM(w^2)_{\sigma_{max}}}{2} = \frac{4\pi^4 AMe^4 Z_1^2 Z_2^2}{h^2}$$

and the corresponding de Broglie wavelength:

$$\lambda\!\!\!/ = \frac{\lambda}{2\pi} = h/2\pi\ AM(w)_{\sigma_{max}} = \frac{h^2}{4\sqrt{2}\ \pi^3 AMe^2 Z_1 Z_2}$$

Therefore,

$$(E)_{\sigma_{max}} = \frac{Z_1 Z_2 e^2}{\sqrt{2}/\pi\lambda\!\!\!/} \qquad (1d)$$

The absorbed kinetic energy is equal to the Coulomb potential energy at the mutual approach of the particles to a distance of $\sqrt{2}/\pi$ times the de Broglie scattered wavelength.

If the cross section variation in figure 1 is known, the number of nuclear reactions taking place in a given plasma may be computed by multiplying the probability curve of the reactions (σ) by the probability curve of the thermal velocities (dN/dw). The product indicated as a dotted curve in figure 1 has a sharp maximum at

$$w_{opt} = \left(\frac{4\sqrt{2}\ \pi^2 e^2 z_1 z_2 kT}{AMh}\right)^{1/3} \tag{2}$$

Gamow integrated the product versus w by replacing the actual curve with a Gaussian probability curve of the same height and width and thus obtained, for the energy production through thermal nuclear reactions per unit mass and unit time of the reaction plasma

$$\Delta E = E\rho c_1 c_2 k_1 \frac{\left(k_2 T^{-1/3}\right)^2}{\exp\left(k_2 T^{-1/3}\right)}, \text{ erg g}^{-1} \text{ sec}^{-1} \tag{3}$$

where

E   heat of a single reaction, ergs

ρ density of plasma, g cm^{-3}

c_1, c_2 relative weight concentrations of the reaction particles -

$k = 1.3805 \times 10^{-16}$ erg-deg^{-1} Boltzmann constant

$$k_1 = \frac{2}{3^{5/2}} \frac{\Gamma h R^2}{\pi M^3 AA_1 A_2 e^2 z_1 z_2} \exp 8\left(\frac{2\pi^2 RAMe^2 z_1 z_2}{h^2}\right)^{1/2}, \text{ cm}^3 \text{ g}^{-2} \text{ sec}^{-1} \tag{3a}$$

$$k_2 = 54\pi^4\left(\frac{AMe^4 z_1^2 z_2^2}{kh^2}\right)^{1/3}, \text{ deg}^{1/3} \tag{3b}$$

k_1 and k_2 are the constants determined by the nature of the reacting particles and may be computed once and for all.

Differentiating equation (3), there is also obtained a maximum of the energy rate with respect to the temperature at

$$T_{opt} = (k_2/2)^3 = \frac{54\pi^4 A M e^4 Z_1^2 Z_2^2}{8kh^2} \tag{3c}$$

and of an amount

$$\Delta E_{max} = (2/e)^2 E\rho c_1 c_2 k_1 = \tag{3d}$$

$$\frac{8}{3^{5/2}\pi} E c_1 c_2 \rho \frac{R^2 Th}{M^3 A A_1 A_2 e^2 Z_1 Z_2} \exp\left(\frac{8\pi e\sqrt{2RAMZ_1 Z_2}}{h} - 2\right)$$

The position of this optimum, which is independent of the density of the plasma, is approximately obtained from the consideration that in figure 1 the position of $(w)_{\sigma_{max}}$, according to equation (1b), is independent of the temperature, whereas the most probable velocity c increases with increasing temperature T and can be made to coincide with $(w)_{\sigma_{max}}$. If, therefore, the plasma temperature is so chosen that the velocity corresponding to the maximum of the reaction rate coincides with $(A\sum_i c_i/A_i)^{-1/2}$ times the most frequent velocity $c = (2kT\sum_i c_i/A_i/M)^{1/2}$ of the Maxwell distribution equation (1b), then this is approximately the temperature of the greatest reaction probability, thus

$$T_{opt} \sim \frac{4\pi^4 A M e^4 Z_1^2 Z_2^2}{kh^2} \tag{4}$$

Equation (3c) gives a more accurate value, about 69 percent higher, arising from the fact that the area under the $\sigma dN/dw$ curve does not accurately become the greatest when the maximum of this curve is the greatest.

If the maximum of the Maxwell curve dN/dw in figure 1 coincides with the maximum of the σ curve, then the $\sigma dN/dw$ curve must also have its maximum at the same point.

Finally, the very important optimum temperature of the thermo-nuclear combustion at constant density can, according to equation (3c), also be understood clearly from the following considerations.

In the inelastic collision between two particles of different weights the absorbed energy, according to the classical laws of impact, is obtained from the momentum and energy laws as

$$\frac{A_1 A_2 (Mw^2)}{A_1 + A_2\ 2} \simeq \frac{AMw^2}{2} \tag{5}$$

In order to make two charged particles approach each other in opposition to the Coulomb repulsion to within a distance of $8/\pi\sqrt{54}$ times the de Broglie scattering wavelength $\lambdabar = \lambda/2\pi = h/2\pi\ AMw$, there is an accompanying absorption of kinetic energy of an amount

$$\frac{Z_1 Z_2 e^2}{8/\pi\sqrt{54}\times\lambdabar} = \frac{\sqrt{54}\pi Z_1 Z_2 e^2 2\pi AMw}{8h} \simeq \frac{AMw^2}{2}$$

and, therefore, the collison velocity

$$w = \frac{\sqrt{54}\pi^2}{2}\frac{e^2 Z_1 Z_2}{h}$$

If this required collision velocity w becomes equal to the most frequent velocity $c^2 = 2kT/AM$ in the Maxwell-Boltzmann distribution, then the gas temperature is again T_{opt}, according to equation (3c).

In a manner similar to the method used to find the position of the maximum of the energy, production curve equation (3) we may try to estimate the value of this maximum.

According to Maxwell-Boltzmann statistics, the number of collisions between different particles in a mixture of two kinds of gas per unit of mass and time is

$$N = \frac{4c_1 c_2 \rho\sqrt{kT}}{M^{5/2}A_1 A_2 \sqrt{2\pi A}}\ \sigma s \tag{6}$$

hence, at the temperature of the peak of the energy production curve, that is, with the substitution of equation (3c)

$$N = \frac{\sqrt{54}\pi^{3/2}c_1 c_2 \rho e^2 Z_1 Z_2}{A_1 A_2 M^2 h}\ \sigma s \tag{7}$$

If the greater number of the collisions do not lead to scattering but lead to a reaction, there is substituted in place of the scattering

cross section σs the reaction cross section $\sim\sigma_{max}$ as given by equation (1c). To obtain the approximate value for the maximum of the energy rate curve,

$$\Delta E_{max} = \frac{\sqrt{54}}{8\pi^{3/2}} Ec_1c_2\rho \frac{R^2Th}{AA_1A_2M^3e^2Z_1Z_2} \exp\left(\frac{8\pi e\sqrt{2RAMZ_1Z_2}}{h} - 2\right)$$

(8)

which is thus about 2 percent less than the accurate value according to equation (3d).

With the previous considerations it is now simple to investigate the technically more important energy rate of the nuclear combustion at constant pressure p instead of, as up to the present time, at constant density ρ.

If, using the gas equation for an expression of the density in terms of the

$$\frac{p}{\rho} = \frac{k}{AM} T \quad \text{or} \quad \rho \approx \frac{pM/kT}{\sum_i c_i/A_i}$$

(9)

pressure (where the Loschmidt number $L = 6.023 \times 10^{23}$ mol^{-1} was eliminated with the aid of the relation $LM = 1$ g mol^{-1}), a substitution is made in equation (3), the expression for the rate of energy production becomes

$$\Delta E' = Epc_1c_2 \frac{Mk_1}{k\sum_i c_i/A_i} \frac{\left(k_2T^{-1/3}\right)^2}{T\exp(k_2T^{-1/3})} \text{ erg g}^{-1}\text{ sec}^{-1}$$

(10)

The energy production per unit mass and time at constant pressure p has a maximum with respect to the temperature T which is independent of the pressure at a now lower temperature:

$$T_{opt}' = (k_2/5)^3$$

(10a)

and of the amount

$$\Delta E_{max}' = (5/e)^5 Epc_1c_2 \frac{Mk_1}{kk_2^3\sum_i c_i/A_i}$$

(10b)

The length of the nuclear thermal burnup is characterized by the half value time t of the reaction:

$$t = \frac{E}{\Delta E_{max}'} \ln 2 \tag{11}$$

The energy production per unit volume at a given combustion pressure is of greatest practical interest in achieving nuclear combustion. This value is obtained from equation (3) through multiplication by ρ and replacing the density ρ by p and T by means of the gas equation (9) to yield

$$\Delta E'' = E p^2 c_1 c_2 \frac{M^2 k_1}{(k \Sigma_i c_i / A_i)^2} \frac{(k_2 T^{-1/3})}{T^2 \exp(k_2 T^{-1/3})} \text{ erg cm}^{-3} \text{ sec}^{-1} \tag{12}$$

This energy production per unit of volume and time for constant pressure of combustion p has, with respect to the temperature T, a maximum at the now still smaller temperature and is again independent of the pressure:

$$T_{opt}'' = (k_2/8)^3 \tag{12a}$$

and of the amount

$$\Delta E_{max}'' = (8/e)^8 E p^2 c_1 c_2 \frac{M^2 k_1}{k_2^6 (k \Sigma_i c_i / A_i)^2} \tag{12b}$$

In table I, for fifteen chosen reactions between light, charged nuclei, characteristic numerical magnitudes of stoichiometric nuclear combustion computed from equations (1), (3a), (3b), (3c), (3d), (10a), (11), (12a), and (12b) are given.

It should be emphasized now, however, how extraordinarily arbitrary this choice is relative to the thousands of possible two-particle impact reactions between the elements of the entire periodic system, how many times greater the possibilities are for three-particle reactions or for a still greater number of particles, and how the recently developed field of meson physics makes our present ideas appear as mere childish efforts.

The first column of table I gives an index number to which the reaction is referred on later diagrams.

The second column defines the corresponding reaction in the usual notation. It is seen that several of the first elements of the periodic system are combined with protons, deuterons, or tritons. Practically, it is to be noted here, that in a nuclear combustion hardly a single one of these simple reactions takes place by itself, but there is generally an entire series of reactions, as for example, the familiar astrophysical carbon-nitrogen cycle of bright stars leading to the formation of helium from hydrogen (fig. 2(a)), the hydrogen chain of the darker stars similarly resulting in helium from hydrogen (fig. 2(b)), or the surmised hydrogen bomb process for forming helium from lithium hydride (fig. 2(c)).

The third column shows the heat liberation of the simple reaction twice, first in the usual unit of the nuclear physicists in MeV per single reaction, and then in the engineering unit calories per gram (i.e., per unit mass of the propulsive material). In the latter figure it is immediately evident that the energy releases are higher by six to seven tenth powers than the chemical heat release values.

The fourth column contains the measured half widths Γ of the nuclear resonance in eV. For the numbers put in parentheses no measured values are known and have been estimated partially on the basis of the considerations leading to equation (1).

The fifth and sixth columns show the material constants of the Gamow equation for the energy rate according to equations (3a) and (3b).

The first results of technical significance start with columns 7 and 8 where the maximal energy rate per unit mass $\Delta E_{max}/\rho$ at constant plasma density ρ and the corresponding optimum temperature T_{opt}, are given. Since the energy rate is proportional to the density it is possible to obtain the value for an arbitrary density from the table. The numbers shown represent the rate at the density $\rho = 1$ gram per cubic centimeter, which is close to the conditions as they may occur at the centers of the stars and in atom bombs. The half-value times of the reactions, which are not given in this particular instance, go as low as 10^{-10} second; the burnup under these conditions occurs in many reactions extremely rapidly and explosively; the optimum temperatures are always essentially higher than those at the centers of normal stars.

For continuous technical nuclear combustions, the succeeding columns are more important. Columns 9, 10, and 11 show the maximum energy rates at constant plasma pressure and again per unit mass, per unit pressure in atmospheres, so that the energy rates for any steady pressures can be directly obtained by multiplying the numerical values for the unit pressure by the corresponding pressure. The corresponding optimum temperatures are always lower here by more than a tenth power.

The half-value times for 1 atmosphere pressure are given; for other pressures they are obtained by dividing the given figures by the desired pressure. At the technically moderate pressure of 1 atmosphere, the half-value times of the reactions lie between a million times the age of our universe and fractions of a second, so that the choice of reactions for stationary technical nuclear combustions must primarily be made according to this column and not according to the heat of the reaction.

Finally, in columns 12 and 13, the maximum energy rates per unit volume are given at constant plasma pressure, again for the unit pressure 1 atmosphere. At other pressures, therefore, the energy rate per cubic centimeter is obtained through multiplication by the square of the corresponding pressure.

The pressure-independent value of the optimum temperature is observed to be noticeably smaller compared with the values of column 9.

The fastest reactions under these conditions are those of deuterium, tritium, and lithium at temperatures of the order of magnitude of 10^8 degrees for the lighter elements and 10^9 degrees for the heavier elements.

The energy production per unit volume remains moderate in all cases because of the extremely small gas density. Even at a 100 atmosphere pressure, the energy rate barely reaches the value of 10^4 kilocalories meter^{-3} second^{-1} for the economically practical nuclear materials and about 10 times more for the tritium combustion, as compared with the known combustion chamber loadings up to 10^6 kilocalories meter^{-3} second^{-1} of chemical rockets.

Here we encounter the first fundamental difficulty of continuous thermal nuclear combustion, the small burnup velocity and combustion chamber loading.

So far we have restricted ourselves to the quantitative energy productions at the corresponding optimum temperatures and will now investigate the dependence on the temperature numerically.

In figure 3 the energy rates per unit density of the nuclear reactions of table I are plotted against the plasma temperature T according to equation (3). As stated previously, the energy rate per unit mass, because of the dependence on the number of collisions per second upon the mean free path, is proportional to the density.

At constant density, the most favorable temperatures for energy production lie, as shown in table I, between about 10^9 and 10^{12} degrees in all cases, increasing with the atomic number of the reacting particles, and independent of the plasma pressure.

The highest point of the production curves is essentially determined by k_1 or by the maximum reaction cross section σ_{max} as given by equation (lc), and by E, ρ, c_1 and c_2. For the (p,α), (p,n), (d,α), (d,n), (t,n), and (n,α) processes, the highest point is determined by the energy production at a temperature such that the corresponding most frequent velocity is sufficient for penetrating the Coulomb barrier. In the case of many reactions, the reaction cross section is larger than the scattering cross section.

An example of the considerably less probable (p,β) reactions is the proton-proton reaction, indicated in the figure, with its extremely small cross section.

At still higher temperatures, the duration of stay of the colliding particles in the hit nucleus is less and, therefore, the cross section again decreases so that the production curves run through the maximum under discussion.

The slope of the energy production curves, on the other hand, is determined by the relation

$$k_2 = 42.7(Z_1 Z_2)^{2/3} \left[A_1 A_2/(A_1 + A_2)^{1/3} \right]$$

that is, by the atomic number and atomic weight of the reacting particles. For this reason, a steeper slope is observed in figure 3 for large k_2, that is, when one or both of the reaction partners has a large atomic number.

This circumstance leads, for example, to the result that the energy rate of the hydrogen reaction chain in the colder stars at 2 to 6×10^7 °K is proportional to T^4, whereas the energy rate of the carbon-nitrogen reaction cycle in the hotter stars is proportional to T^{16}, as can also be observed in figure 3.

At lower temperatures k_2 thus chiefly determined the energy production so that materials with small k_2 (like hydrogen) burn faster at low temperatures.

Within a wide temperature range, the deuterium-tritium, deuterium-deuterium, or tritium-tritium reactions, or either of these with normal hydrogen, are the fastest. Heavier elements become somewhat faster only at extremely high temperatures.

The numerical values shown in figure 3 hold for stoichiometric mixtures of bare nuclei for which, therefore, the relation $c_1 + c_2 = 1$ always holds. This means that they hold for the first instant of the

reaction initiated in the fresh gas at the temperature T, while with progressive combustion naturally the c_1 and c_2 decrease and, therefore, the energy production per unit time decreases.

The condition $c_1 + c_2 = 1$, however, is also in the first instant satisfied only for complete ionization of all reaction partners, but this is never the case at the lower plasma temperatures and thus the low-temperature region of the energy curves of figure 3 may be appreciably less favorable than shown.

At the temperatures that are here of special interest above 10^7 °K for the light elements, the fractions of nonfully ionized particles which are, therefore, not capable of reaction, are generally already negligibly small (i.e., smaller than is desirable from considerations of the mean free paths of the rest mass particles).

Finally, it should again be recalled that figure 3 is drawn for fixed combustion gas densities of $\rho = 1$ gram per cubic centimeter in order to simplify the computations. To this density there would, for example, correspond for the deuteron-deuteron fresh gas of 10^8 °K, a pressure of 4.22×10^9 atmospheres, that is, it is necessary to reduce the value of the energy rates of figure 3 by about 9 powers of 10 in order to have technically controllable steady-state pressure relations.

For clarifying these relations at commercially feasible, steady nuclear combustion conditions, the energy rate per unit volume is plotted against the temperature in figure 4 (according to equation (12)), for all the reactions previously considered.

The numerical values for the unit of pressure in atmospheres are obtained from those of figure 3 by multiplication with the factor

$$\frac{\Delta E''/p^2}{\Delta E/\rho} \approx \left(\frac{0.981 \times 10^6 M}{kT \sum_i c_i/A_i}\right)^2 \tag{13}$$

The maxima of the energy production curves per unit volume are considerably sharper than those per unit mass, also the variation between the different gases becomes larger, since the molecular weights enter quadratically. In spite of this, the advantage of the heavy hydrogens remains very clear; the reactions of the deuterium and tritium are again by far the fastest. The characteristic differences and difficulties of the technical as compared with the cosmical steady nuclear combustions clearly appear, which arise from the smaller pressures by about 15 powers of 10 and from the shorter times available for the technical combustions. The latter requirement makes necessary the application of higher temperatures than are used by nature herself at the centers of normal stars.

Only at the optimal temperatures and at the same time at the highest possible practically attainable pressures do the half-value times of the burnup and, therefore, the combustion chamber loading become of comparable magnitude with those of chemical combustions.

IV. RADIATION

We have already established that every gas adiabatically shut off from its surroundings experiences thermal nuclear reactions which increase exponentially with time so that, in this case, the ignition temperature would lie at an arbitrarily low value.

Under actual man-made conditions, the nuclear ignition temperatures will depend on the ratio of the energy production in the gas volume to the heat simultaneously given off to the surroundings.

When this ratio permanently becomes greater than 1, the temperature continues to rise by itself and "ignition" occurs.

The energy radiated can take place fundamentally both through the diffusion of rest mass particles and through the diffusion of photons, where the ratio of the geometric dimensions of the reaction plasma to the mean free paths of the previously mentioned types of entities is important.

In this way, we come to the second fundamental difficulty of the practical steady nuclear combustions as compared with the cosmic steady combustions. The mean free path of the particles with rest mass is

$$\Lambda = \frac{0.177}{n\sigma_s} \tag{14}$$

(where $n = p/kT$ is the number of particles per cm^3, and σ_s the scattering cross section in cm^2). For the most favorable reaction temperature $T \sim 10^8$ °K and for the best nuclear fuel at an atmosphere combustion pressure the particle density is $n = 7.13 \times 10^{13}$ centimeters^{-3}, which gives a mean free path $\Lambda_1 = 2.5 \times 10^1$ centimeters, if we have nonfully ionized particles for which $\sigma_{S1} \sim 10^{-16}$ square centimeters and $\Lambda_2 = 2.5 \times 10^9$ centimeters, if we have fully ionized particles where $\sigma_{S2} \sim 10^{-24}$ square centimeters.

The corresponding number of collisions per second of a particle is obtained from

$$Z = \sqrt{\frac{8kT\sum_i c_i/A_i}{\pi M \Lambda^2}}$$

(15)

which for a fresh deuterium-deuterium gas has the respective values $Z_1 = 4.11 \times 10^6$ second^{-1} atmosphere^{-1} and $Z_2 = 4.11 \times 10^{-2}$ second^{-1} atmosphere^{-1} for the two states of ionization under consideration.

Actually, the reaction plasma always consists of mixtures of particles in the two states of ionization so that the actual free path Λ will lie between the two extremes Λ_1 and Λ_2 and are given by

$$\Lambda \sim \frac{0.177}{n_1 \sigma_{S1} + n_2 \sigma_{S2}} \sim \frac{0.177}{n \sigma_{S1}} \frac{1 + n_1/n_2}{10^{-8} + n_1/n_2}$$

(16)

This means that for ratios $n_1/n_2 > 10^{-3}$ of nonfully ionized particles, the mean free path Λ will temporarily be proportional to n_2/n_1, which is the case for amounts of heavier and therefore nonfully ionized elements, such as are unavoidable as natural impurities of the fuels, and such as occur in the purest hydrogens in consequence of the statistical ionization process.

The mean free path in the reaction plasma should not, therefore, be expected to be larger than

$$\Lambda = \frac{0.177 \times 10^{16}}{n} \frac{1 + n_1/n_2}{n_1/n_2}$$

(14a)

If the geometric dimension r of the volume of the reaction plasma is large compared with these mean free paths, then the number of the diffusing rest mass particles within the burnup time of the plasma will be small.

The product pr for the combustion gas must always be at least of the order of magnitude

$$pr \sim 100 p\Lambda \sim 2.5 \times 10^3 \frac{1 + n_1/n_2}{n_1/n_2} \text{ atm cm}$$

therefore,

(17)

$$n_1/n_2 = 10^{-3}, \quad pr \geqslant 2.5 \times 10^6 \text{ atm cm}$$

The relations will be still less favorable for the diffusing particles without rest mass (i.e., photons). The photon cross sections of the nuclei are of the order of 10^{-30} to 10^{-40} square centimeters, while the scattering cross section of photons on electrons is of the order of magnitude 10^{-24} square centimeters. Thus, in a fully ionized reaction plasma the mean free path of the photon gas component would be many orders of magnitude larger than that of the gas component constituted by the rest mass particles. This statement is true also for the nonfully ionized plasma. For a gas volume whose geometric dimensions are large compared with the mean free path of the rest mass particles, photons will always diffuse and energy will be lost mainly through optical radiation, if the density of the photon component of the gas is of appreciable magnitude.

At the high plasma temperatures which are under consideration here, the number of the processes which lead to photon production is very high. Without going into their detailed description, we shall mention here only nuclear gamma radiation, bremsstrahlung, recombination radiation, radiation from transitions in nonfully ionized atoms, plasma vibrations and so forth, so that the loss by radiation will be practically continuous.

If the dimensions of the plasma volume (e.g., of spherical shape), were also large compared with the free path of the photon gas, then an equilibrium condition would exist and the laws of black body radiation would hold. The energy radiated away would, in the usual way, be proportional to the area of the plasma volume and the fourth power of the plasma temperature.

The energy production, on the contrary is, according to equation (12), proportional to the content of the plasma volume. Setting the two equal to each other a relation is obtained for the ignition temperature, or for the steady temperature, as a function of the radius r of the plasma sphere, if the plasma pressure p is held constant:

$$r = 3aT^6 \frac{\exp(k_2 T^{-1/3})}{(k_2 T^{-1/3})^2} \frac{\left(k \underset{i}{\Sigma} c_i / A_i\right)^2}{M^2 E c_1 c_2 p^2 k_1} \tag{18}$$

where $a = 5.68 \times 10^{-5}$ erg centimeter^{-2} degree^{-4} second^{-1} is the Stefan-Boltzmann constant.

The plasma radius has a minimum at the point where, in figure 4, the tangent of the T^4 law touches the energy rate curves, which for deuteron-deuteron reaction is at about 10^7 °K. The magnitude of this minimum radius in this case and at a plasma pressure of 100 atmospheres is about $r \sim 10^{17}$ centimeters, therefore, of astronomical order of magnitude.

For a particle density of $n \sim 10^{16}$ centimeters^{-3} and an optical cross section of $\sigma \sim 10^{-33}$ square centimeters, the mean free path of the photon gas becomes $\Lambda = 0.177/n\sigma$ or the thickness of an optically infinite layer of the plasma is $r_\infty \sim 4.6/n\sigma$ which is of the order of 10^{17} centimeters. Therefore, in stationary plasmas of practical dimensions, no black radiation can occur in spite of the continuous radiation because the optically infinite layer thicknesses required for this purpose cannot be attained.

It will nevertheless, in this case, be necessary to consider approximately gray radiation whose intensity ratio to black radiation is given by the relation

$$I/I_0 = 1 - e^{-rn\sigma} \sim rn\sigma \qquad (19)$$

The dependence of the effective optical cross section σ on ρ and T in the absorption constant $n\sigma$ we shall assume, in the absence of better data, to be the expression given by Kramers, Eddington, and Gaust (ref. 2) for the plasma at the interior of the sun:

$$\sigma \sim 6.60(1 + c_H)(1 - c_H - c_{He}) \frac{\sum_i c_i z_i^2/A_i}{\sum_i c_i/A_i} \rho^{0.75} T^{-3.5} =$$

$$6.60(1 + c_H)(1 - c_H - c_{He}) \frac{\sum_i c_i z_i^2/A_i}{(k/M \times \sum_i c_i/A_i)^{1.75}} p^{1.75} T^{-1.25} \qquad (20)$$

so that the radiation intensity of the plasma is

$$I \sim I_0 rn\sigma =$$

$$3.97 \times 10^{24} \, ar(1 + c_H)(1 - c_H - c_{He}) \frac{\sum_i c_i z_i^2/A_i}{(k/M \times \sum_i c_i/A_i)^{1.75}} p^{1.75} T^{-1.25} \qquad (21)$$

At constant plasma pressure we thus arrive at the surprising result that the radiation intensity temporarily drops with increasing temperature even somewhat more sharply than the plasma density, until at a higher temperature, with the onset of intense bremsstrahlung, mass radiation, and finally the vanishing of the boundaries between matter and energy, the radiation intensity again rises.

From equations (12) and (21) is now obtained the very important ratio of the energy radiation loss to the energy production for a plasma sphere of radius r:

$$\zeta = \frac{4r^2\pi I}{4/3 \times r^3 \pi \Delta E''} = 3n\sigma I_0 / \Delta E'' = \frac{1.192 \times 10^{25} \; \alpha(1 + c_H)(1 - c_H - c_{He})}{Ec_1 c_2 k_1 k_2^2} \times$$

$$\left(\frac{k}{M} \sum_i c_i / A_i \right)^{0.25} \sum_i c_i z_i^2 / A_i \times p^{-0.25} \; T^{1.417} \; \exp(k_2 T^{-1/3}) \tag{22}$$

which is independent of the dimension r of the reaction plasma, depends little on the plasma pressure p and is essentially determined by the temperature T.

The ratio ζ for constant pressure p, therefore, has a minimum with respect to T at

$$T = \left(\frac{k_2}{4.251}\right)^3 \tag{22a}$$

a value which is again independent of the pressure and remains somewhat above the optimum temperature of the energy production as shown by equation (12a).

The magnitude of the minimum ratio ζ is

$$\zeta_{min} = \frac{1.77 \times 10^{24} \; \alpha(1 + c_H)(1 - c_H - c_{He})}{Ec_1 c_2 k_1 k_2^{-2.251}} \sum_i c_i z_i^2 / A_i \left(\frac{k}{M} \sum_i c_i / A_i \right)^{0.25} p^{-0.25} \tag{22b}$$

The Kramers relation (eq. 20) assumes, perhaps somewhat optimistically, that plasmas which consist of only fully ionized particles like hydrogen and helium do not radiate, so that the actual radiation is caused by the unavoidable impurities of the nuclear fuels with heavier elements, or to the 10^{-3} fraction of nonfully ionized particles as shown by equation (16), to limit the mean free path length.

In this way, the factor $(1 - c_H - c_{He})$ in the preceding equations, becomes equal to 10^{-3}. With this somewhat arbitrarily chosen number, the radiation ratio ζ is plotted in figure 5 as given by equation (22); also given are the optical cross section by equation (20), and the absolute radiation intensity I by equation (21) at the deuterium-deuterium plasma pressure of 100 atmospheres, where for the computation of I, the radius of the plasma sphere was assumed to be r = 1 meter.

The plot of the number of particles n per cubic centimeter and per atmospheric pressure against the temperature shows that for nuclear reaction plasmas, the densities considered are such as would otherwise be denoted as a high vacuum.

The plot of the optical cross sections σ (referred to the relative mass concentration of the particles which are neither hydrogen nor helium) shows the extremely strong drop of this average optical cross section with increasing temperature and its relatively small dependence on the pressure. In the region of the most favorable reaction temperatures of good nuclear fuels σ lies at about 10^{-33} square centimeters. Correspondingly, the optical absorption constant $n\sigma$ likewise drops approximately with $T^{-5.25}$ and results in the decrease of the radiation intensities I decrease approximately as $T^{-1.25}$ when the optical layer densities $rn\sigma$ become less than 4.6. For $rn\sigma > 4.6$, the intensity is practically the same as that for black radiation, also plotted as the asymptote to all the radiation curves.

Finally, the most important plot of the radiation ratio ζ shows that under steady, realizable conditions at temperatures below about 10^7 °K the radiation always remains larger than the energy production, so that below this temperature a steady nuclear combustion cannot be obtained. At higher temperatures, the energy production may become larger than the radiation if the proportion of heavy or nonfully ionized particles in the plasma is sufficiently small, of the order of magnitude 10^{-3} to 10^{-7}.

The other curves in figure 5, for various values of p, r, and $(1 - c_H - c_{He})$, require no detailed discussion but serve to complete essentially this section on the radiation relations of steady technical nuclear combustion plasmas.

These fundamental considerations show that with decreasing content in the plasma of the nonfully ionized particles the radiation of photons rapidly decreases; also, the radiation from the massive particles increases so that a minimum of the energy discharged, with respect to the content in nonfully ionized particles, is obtained.

In this way, the conditions under which it may be possible to avoid having the energy production of the nuclear combustion be initially lost to the surroundings, instead of being used for the preparation of the fresh gas, are approximately outlined, which evidently, is an essential condition for steady nuclear combustion.

V. MIXTURE PREPARATION

The mixture preparation in steady nuclear combustions embraces the same phases as those in chemical combustions (ref. 3), in particular, the fuel injection, the jet atomization, the droplet formation, the droplet evaporation, the macromixing, the micromixing, the heating up to ignition, and, finally, the ignition itself.

We confine ourselves here, owing to its extraordinary energy takeup, to the particularly important partial process of the heating up to the ignition, which includes also the total ionization processes of the fresh gases, since in the case of nuclear reactions, the ignition temperature lies above the ionization temperature.

As previously mentioned, chemical and nuclear chemical reactions take place continuously in every gas of our environment and consequently in the combustion gases of our jet engines although this occurrence is so rare it is generally unobserved and is of little importance from the energy standpoint. The great majority of the collisions of the gas particles, due to their thermal motion, concerns only the electron envelopes of the colliding atoms without any occurrence of direct collisions between nuclei.

This condition of thermal nuclear reactions is satisfied, to a great extent, only when combustion gas temperatures are so high that the greater number of the gas atoms is fully ionized and are completely freed of their electron envelopes. Investigations at these high combustion temperatures, particularly of electrons, must, strictly speaking, include the methods of relativistic thermodynamics, but we shall be satisfied here with several approximate considerations on the basis of classical thermodynamics.

To have an equilibrium of the ionization processes requires an equilibrium between the collision ionizations occurring in the first instant due to collision between nonionized atoms and the ions and electrons taking part in further collision ionizations. There must also be a regeneration of the high velocity end of the Maxwell distribution that was consumed in the first stages of ionization. Accompanying the increased ionization there is increased recombination, thereby acting to reduce the degree of ionization. Equilibria between all of these processes require about 10^9 collisions. Thus, equilibrium will not be set up during the burnup of fast nuclear chemical combustions as experienced in the case for chemical combustions.

During fast nuclear combustions, the pressure-dependent ionization equilibrium (such as is attempted to be described by the Eggert-Saha equation) does not have to be considered, but is actually maintained in the slowly burning plasmas of the cosmic nuclear combustions. During a

considerable part of the burnup, we shall mainly have to consider only the pressure-independent primary spontaneous ionization which, depending on the combustion gas pressure under consideration, can lead to smaller, equal, or the same degrees of ionization as the pressure-dependent ionization equilibrium. We shall next consider the primary ionization in more detail.

Collision ionization is to be expected in the case of thermal collisions between particles of different mass A_1M and A_2M if the energy $A_1A_2/(A_1 + A_2) \times Mw^2/2 = AMw^2/2$, which vanishes in the inelastic collision in the collision system, is equal to the ionization energy I.

The average required collision velocity w_i required for ionization is

$$w_i = \sqrt{2I/AM} \tag{23}$$

For equal masses of the colliding particles, this means that the relative kinetic energy of a particle $A_1Mw^2/2$ must, on the average, be at least twice as large as the ionization energy which vanishes in the collision system so that the remaining half of the kinetic initial energy is still able to satisfy the requirements of conservation of momentum.

Within the Maxwell-Boltzmann distribution of the thermal particle velocities it is thus necessary to have the most frequent velocity

$$c = \sqrt{2kT(\sum_i c_i/A_i)/M} \tag{24}$$

attain at least the required collision velocity w_i, and from this the required temperature of the gas for full ionization is obtained as

$$T = \frac{I}{kA\sum_i c_i/A_i} \tag{25}$$

The energy I required for the complete ionization of an atom depends on the atomic number Z, approximately, according to the relation

$$I = 2.18 \times 10^{-11} \, Z^{2.418} \, [\text{erg}] \tag{26}$$

Substituting this in equation (25) there is finally obtained

$$T = \frac{2.18 \times 10^{-11} \sum_i c_i Z_i^{2.418}}{kA \sum_i c_i/A_i} \tag{27}$$

Below a gas temperature of 3×10^5 °K and from consideration of ionization there is, therefore, no expectation that any gas or any type of hydrogen would produce a reaction yield of any technical importance when dealing with a fast, short time combustion process.

At the temperature of the centers of normal stars (from $2°$ to $6° \times 10^7$) only the first eight elements of the periodic system up to oxygen can be expected to give thermal nuclear reactions, a choice which holds, in quite the same way for the chemical reactions in chemical rockets.

Below the full ionization temperatures represented by equation (27), only small fractions of the gas mass can take part in the nuclear reactions. According to the Maxwell distribution curve, the number of reactions become smaller when further removed from the actual gas temperatures below the values of equation (27).

The numerical fraction of the gas particles N'/N, which are faster than w_i, from which, therefore, complete spontaneous ionization is to be expected, is approximately

$$\frac{N'}{N} = \frac{2}{\sqrt{\pi}}\left[\sqrt{\frac{E}{kT}}\ e^{-E/kT} + \int_{x=\sqrt{E/kT}}^{\infty} e^{-x^2}\ dx\right] \sim \frac{2}{\sqrt{\pi}}\left[\frac{w_i}{c}\ e^{-(w_i/c)^2} + \int_{x=w_i/c}^{\infty} e^{-x^2}\ dx\right] \tag{28}$$

In accordance with the approximate nonrelativistic treatment, $N'/N = 1$ only for $c \approx \infty$, that is, for infinitely high temperature.

For $w_i/c = 1$ there is found $N'/N \approx 0.58$, that is, at the temperatures represented by equation (27), a majority of 58 percent of the particles always attain the collision velocity required for complete ionization, at half the temperature the number is still 25 percent, at $T/4$ the number is 4 percent, and at $T/8$ considerably below 1 percent.

The fraction of completely ionized particles of $N'/N \approx (1 - 10^{-7})$ in figure 5 is, according to equation (2ɔ), attained for a temperature which is about 25 times as great as that according to equation (27). Only at these extremely high temperatures, especially for the heavier elements, does the amount of complete ionization rise and the radiation therefore drop to an extent that makes continuous nuclear combustions possible.

The next problem concerns the energies required to heat the fresh gas to these high combustion temperatures.

We first assume that the heating process is adiabatic, that is, neglect energy losses to the outside due to diffusing photons or electrons or due to convective heat transfers to the boundary walls by atomic, ionic, nuclear, electron, or photon collisions against the wall.

This assumption we are justified in making under certain conditions according to the preceding section.

This assumption will again closely apply for very rapid heating processes for which the partial pressures of the previously mentioned five most essential combustion gas components also do not have time for establishing their own equilibria.

In spite of these simplifying assumptions, the problem of the determination of the specific heats of the combustion gas is also of a static character although, because of the predominating effects of the level excitations and their decay, the specific heats become functions of the time.

The Boltzmann equipartition law, as a consequence of the classical Boltzmann statistics, still possesses within the framework of the quantum theory, the significance of a limiting law which is valid for sufficiently high temperatures, as under the conditions we are considering. It does not hold, however, for the photon gas which leads to the Rayleigh-Jeans radiation law, valid only for long wavelengths.

The Boltzmann equipartition law states that each degree of freedom of a mechanical system, whose energy depends quadratically on the velocity or coordinate has, on the average, an associated energy $kT/2$. The mean energy of a gas atom, with its three spatial translational degrees of freedom, therefore, amounts to $3kT/2$.

If an atom of this energy is supplied with the ionization energy I_1, an electron is split off which, in accordance with experimental results, after only a few collisions, comes into translational energy equilibrium with the remaining gas, that is, adjusts to the Maxwell-Boltzmann velocity distribution and, therefore, in turn takes up the mean energy content of $3kT/2$ so that, referred to the original atom, a total energy of $(3kT/2 + I_1 + 3kT/2)$ must have been made available. If the atom ionizes multiply, for example Z times, the total mean energy increases to

$$U_Z = 3kT/2 + \sum_1^Z I + 3ZkT/2 = \sum_1^Z I + (Z + 1)\, 3kT/2 \qquad (29)$$

The specific heat c_p at constant pressure and per unit mass approaches asymptotically, therefore, with infinitely increasing temperature, the limiting value

$$c_p = \sum_i c_i \frac{Z_i + 1}{A_i} \frac{5k}{2M} \qquad (30)$$

while for very low temperatures it corresponds to the value

$$c_p = \sum_i \frac{c_i}{A_i} \frac{5k}{2M} \qquad (31)$$

The transition curve between these two values depends on the rapidity of the heating. For the short time heating here assumed, the transition curve can be given with the aid of equation (28) for intermediate temperatures. For medium temperatures, there is generally obtained a weak maximum, due to the ionization energy of the mean specific heats. The maximum may easily lie far above the limiting value as shown by equation (30).

While the initial values of the specific heats per unit mass thus decrease monotonically and hyperbolically to zero with increasing atomic weight, the final values at high temperatures approach somewhat less regularly the final value $c_p \sim 2$ calorie gram^{-1} degree^{-1}, since the value $(Z + 1)/A$ drops toward 0.4 with increasing atomic weight.

The largest changes in the specific heats occur for the lightest elements, especially the various hydrogens, as shown in figure 6.

If we are not considering short time processes, but assume that a thermal equilibrium is established, then the horizontal branches of the curves of the mean specific heats remain unchanged. Only the S-shaped transition portion, which represents the ionization processes, shifts in a manner that depends on the temperature. At very high temperatures, we also have, in this case, the same enthalpy as in short-time processes if the level excitations and reradiations are not considered.

With these assumptions and with the aid of figure 6, we can construct an enthalpy-temperature diagram, as shown in figure 7 for example, for stoichiometric fresh gas mixtures of hydrogen and deuterium with the first elements of the periodic system.

The enthalpies, up to the corresponding value of the heat of the nuclear reaction, are drawn as a solid line; beyond that the line is dotted. In addition, the optimum reaction temperatures shown in column 12 of table I are plotted.

The heats of reaction for the hydrogens are quite sufficient for heating to the most favorable reaction temperatures, but for the reactions of the heavier elements, this is not always the case. Since it is also for the heavy elements that the radiation during the reaction is considerably higher than for the hydrogens, the fundamental possibility of continuous nuclear combustion for these heavier elements is very questionable.

VI. APPLICATION TO JET PROPULSION

From the preceding investigations, which undoubtedly are associated with a number of uncertainties of considerable importance, the following most important conclusions for the achievement of continuous nuclear combustions can be drawn:

(1) The fuel of primary consideration appears to be deuterium of the highest purity, with respect to heavier elements. The reasons for these choices are: the small half-value time of burnup, high energy production per second per unit volume of the combustion chamber, small radiation intensity of the reaction plasma and high heat of reaction in comparison with the enthalpy of the fresh gas at the optimum reaction temperature and economical operation.

(2) The combustion chamber loading increases as the square of the pressure employed, but even for combustion pressures of 100 atmospheres, attains only values of the order of magnitude of 10^4 kilocalories meter^{-3} second^{-1}, and, therefore, is less than that of chemical high pressure combustions.

(3) The minimum dimensions, even of high-pressure combustions, are of the order of many meters since the mean free paths of the high-pressure reaction plasma are of the order of centimeters.

(4) For a cooling of the burned up plasma considerable below the most favorable combustion temperature of 10^8 °K there occurs considerable recombination and radiation of extraordinary intensity which, at about 10^5 °K, may cause the optical radiation to amount to a considerable part of the total energy production.

From these circumstances it is evident that nuclear combustions for stationary units or those used in ships are more readily realizable than those employed in aircraft or spacecraft, where in the former, the minimum outputs required are about 200,000 horsepower (several milligrams of fuel consumption per second), which can more easily be exceeded.

For jet propulsion units, three fundamental processes for utilizing the combustion gas enthalpy of 10^{10} kilocalories per kilogram have been considered:

(1) The direct release of the combustion gas such as that used in the familiar chemical rockets (i.e., the so-called pure atomic rockets). The exhaust velocity would be of the order of magnitude 10^7 meters per second, and the specific fuel consumption about 10^{-3} kilogram per second, so that a combustible layer of protective gas or protective liquids for the furnace and nozzle walls appears tolerable. The ignition of the reaction plasma with the cooling layer, which would occur if the evaporation proceeded more rapidly than recombination, could be restricted to a possible mixing process of the plasma with the protective gas. This circumstance could make the pure atomic rockets appear in a new and not altogether hopeless light. The radiation of the reaction plasma itself which, as shown in figure 5 is 1000 calories per square centimeter second, could be further controlled in the combustion chamber walls except when the penetrating x-ray radiation predominates.

The convection product $H\rho c$ that determines the convective heat transfer and has a value of about 4×10^{11} kilcalories per square meter is, in spite of the small gas densities, about 10^4 times larger than that used in chemical high-pressure rockets and does not permit any anticipation of controlling the heat transfers at an unprotected wall by simple wall cooling. On the other hand, because of the unusual kinematic viscosity $c\Lambda/3$ of the plasma flow and its correspondingly low Reynolds number $r/\Lambda \sim 10^3$, there are favorable prospects for the mixing zones of protective gas and plasma which suggest a compromise solution between pure and thermal atomic rockets as considered by H. J. Kaeppeler (ref. 4).

(2) The next method for utilizing the nuclear plasma enthalpy consists in completely mixing the hot plasma with the surrounding air (atomic air jet units) or with other inert working gases (thermal atomic rockets), thereby heating these gases to a temperature lying only one order of magnitude above that of chemical propulsion units. Thermal atomic rockets of this kind evidently differ only in degree from the previously mentioned pure atomic rockets having a protective gas layer, although for both admixture applications, it is preferable to have the lowest plasma pressures at which the energy production is just sufficient. Moreover, in both cases during the mixing, the temperature range of extremely high radiation intensity must be run through so that no general conclusions can be drawn without a detailed study of these special relations, which have been taken into consideration by I. Sänger-Bredt and H. J. Kaeppeler (ref. 4).

In any case, it may be said that the energy radiation cannot become larger than the energy production and the value for a combustion pressure

of 10 atmospheres and for a radius r(m) of a spherical chamber would be
about $I = 23$ r kilocalories meter^{-2} second^{-1}, so that, even for a 10
meter combustion chamber radius, the radiation is less than 0.023 kilo-
calories centimeter^{-2} second^{-1}, which is feasible to handle and may per-
haps be of interest for stationary nuclear combustion units.

(3) Finally, a propulsive method utilizing the high radiation
intensities that were the cause of the cooling reaction plasma in the
other two methods will be considered (the so-called photon rocket)
(ref. 5).

If this cooling of the reaction plasma occurs in a closed space
with, for example, central reaction zone and surrounding cool gas jacket,
that is, without appreciable gas flow, it should be easily possible to
arrange for the greatest part of the plasma enthalpy to be converted
into photon gas and radiated.

The radiation intensity again depends linearly on the lamp radius
r(m) and on the square of the combustion pressure p(atm) and for the
triton-deuteron reactions is supposedly

$$I = 3.7rp^2, \text{ kcal m}^{-2} \text{ sec}^{-1} \tag{32}$$

corresponding to a radiation pressure of

$$p_s = 5.6\times10^{-10} \text{ } rp^2, \text{ atm} \tag{33}$$

In order, therefore, to attain radiation pressures of an order of
magnitude that is of technical interest (i.e., at least 0.01 atm), the
combustion pressures for a lamp radius of r = 1 meter must amount to
p = 4.2×10^3 atmospheres.

For photon rockets, therefore, in the same way as for atomic air
jet propulsion units, it will still be necessary to seek energy sources
of higher reaction velocity in order to reduce the technical difficulties.

REFERENCES

1. Gamow, G., and Critchfield, C. L.: Atomic Nucleus and Nuclear Energy
      Sources. Clarendon Press (Oxford), 1950.

2. Gamow, G., and Critchfield, C. L.: l. c., p. 270.

3. Sänger, E.: Brennstoff-Chem., 32, 1, 33, 1951; 35, 41, 1954.

4. Sänger-Bredt, I.:   Probleme der Weltraumforschung. Biel, Laubscher,
      1955.

   Kaeppeler, H. J.:   Probleme der Weltraumforschung. Biel, Laubscher,
      1955.

5. Sänger, E.:   Probleme der Weltraumforschung. Beil, Laubscher, 1955.

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TABLE I. - CHARACTERISTIC VALUES FOR STOICHIOMETRIC TWO-PARTICLE NUCLEAR COMBUSTION OF CHARGED LIGHT PARTICLES

Number	Reaction	Heat of reaction, E, Mev (cal g^{-1})	Resonance width, Γ, ev	Material constant of Gamow equation k_1, cm^3 g^{-2} sec^{-1}	k_2, $deg^{1/3}$	Maximum energy rate per unit mass at constant plasma density T_{opt}, °K	$\Delta E_{max}/\rho$, cal g^{-1} sec^{-1} g cm^{-3}	Maximum energy rate per unit mass at constant plasma pressure T'_{opt}, °K	$\Delta E'_{max}/p$, cal g^{-1} sec^{-1} atm	Half value time, T'_{opt}, sec atm	Maximum energy rate per unit volume at constant plasma pressure T''_{opt}, °K	$\Delta E''_{max}/p^2$, cal cm^{-3} sec^{-1}/atm^2
1	$H^1(p,e^+)H^2$	1.42 (1.64×10^{10})	(2×10^{-19})	~10^8	3.39×10^3	4.86×10^9	7×10^{-7}	3.12×10^8	2.95×10^{-14}	3.8×10^{23}	7.61×10^7	7.00×10^{-28}
2	$H^2(p,\gamma)He^3$	5 (3.84×10^{10})	1	2.48×10^{26}	3.73×10^3	6.50×10^9	5.74×10^{12}	4.23×10^8	2.87×10^5	9.27×10^4	1.02×10^8	6.95×10^{-9}
3	$H^3(p,\gamma)He^4$	20 (1.15×10^{11})	(10)	~2.80×10^{27}	3.88×10^3	7.31×10^9	2.18×10^{14}	4.73×10^8	1.33×10^7	5.98×10^3	1.15×10^8	3.70×10^{-7}
4	$He^3(p,e^+)He^4$	19.8 (1.14×10^{11})	10	1.87×10^{27}	6.16×10^3	2.93×10^{10}	1.44×10^{14}	1.89×10^9	3.50×10^6	2.26×10^4	4.60×10^8	1.51×10^{-8}
5	$Li^6(p,\alpha)He^3$	3.8 (1.25×10^{10})	5×10^5	8.70×10^{31}	8.44×10^3	7.52×10^{10}	8.45×10^{16}	4.80×10^9	1.91×10^{10}	4.57×10^{-1}	1.18×10^9	4.11×10^{-5}
6	$Li^7(p,\gamma)2He^4$	17.2 (4.95×10^{10})	4×10^4	6.69×10^{30}	8.50×10^3	7.71×10^{10}	2.61×10^{17}	5.00×10^9	6.67×10^9	5.14×10^0	1.21×10^9	1.60×10^{-5}
7	$Be^9(p,\alpha)Li^6$	2.2 (5.07×10^9)	10^6	2.07×10^{32}	1.04×10^4	1.40×10^{11}	8.48×10^{17}	9.10×10^9	1.78×10^{10}	1.97×10^{-1}	2.19×10^9	2.44×10^{-5}
8	$B^{10}(p,\gamma)C^{11}$	8.6 (1.80×10^{10})	10	2.67×10^{27}	1.21×10^4	2.24×10^{11}	3.94×10^{13}	1.45×10^{10}	6.86×10^5	1.82×10^4	3.48×10^9	5.51×10^{-10}
9	$B^{11}(p,\alpha)2He^4$	8.8 (1.69×10^{10})	10^6	2.62×10^{32}	1.22×10^4	2.25×10^{11}	3.64×10^{18}	1.45×10^{10}	6.80×10^{10}	1.73×10^{-1}	3.52×10^9	5.78×10^{-5}
10	$N^{15}(p,\alpha)C^{12}$	4.84 (6.97×10^9)	10^7	4.54×10^{33}	1.54×10^4	4.53×10^{11}	3.06×10^{19}	2.93×10^{10}	4.17×10^{11}	1.16×10^{-2}	7.08×10^9	1.86×10^{-4}
11	$H^2(d,n)He^3$	3.2 (1.84×10^{10})	3×10^5	4.21×10^{31}	4.28×10^3	9.81×10^9	6.98×10^{17}	6.34×10^8	3.53×10^{10}	3.61×10^{-1}	1.53×10^8	6.60×10^{-4}
12	$H^3(d,n)He^4$	17.5 (8.07×10^{10})	(10^6)	~1.07×10^{32}	4.54×10^3	1.18×10^{10}	9.10×10^{18}	7.58×10^8	5.23×10^{11}	1.07×10^{-1}	1.83×10^8	1.08×10^{-2}
13	$Li^6(d,\alpha)He^4$	22.1 (6.37×10^{10})	(10^6)	~1.68×10^{32}	1.02×10^4	1.33×10^{11}	1.44×10^{19}	8.56×10^9	2.57×10^{11}	1.72×10^{-1}	2.07×10^9	2.97×10^{-4}
14	$Li^7(d,\alpha)He^5$	14 (3.59×10^{10})	(10^6)	~1.68×10^{32}	1.03×10^4	1.36×10^{11}	8.92×10^{18}	8.76×10^9	1.64×10^{11}	1.52×10^{-1}	2.12×10^9	2.05×10^{-4}
15	$H^3(t,2n)He^4$	11 (4.22×10^{10})	(10^6)	~7.5×10^{31}	4.90×10^3	1.48×10^{10}	4.25×10^{18}	9.55×10^8	2.45×10^{11}	1.20×10^{-1}	2.31×10^8	4.02×10^{-3}
1	2	3	4	5	6	7	8	9	10	11	12	13

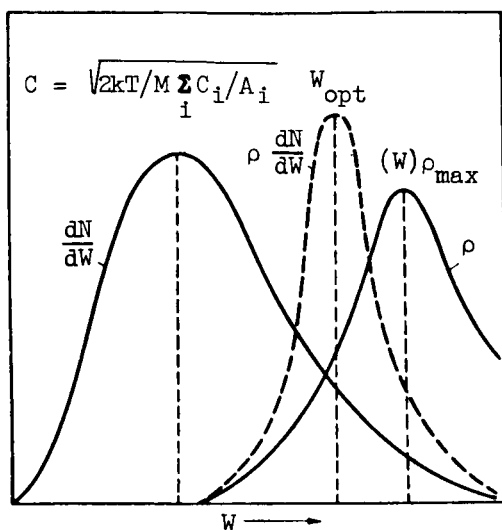$$C = \sqrt{2kT/M \sum_i c_i/A_i}$$

Figure 1. - Maxwell-Boltzmann distri-
   bution dN/dw of particle ve-
   locities in nuclear combustion
   plasma; variation of cross section
   σ of thermal nuclear reactions
   with particle velocity w and
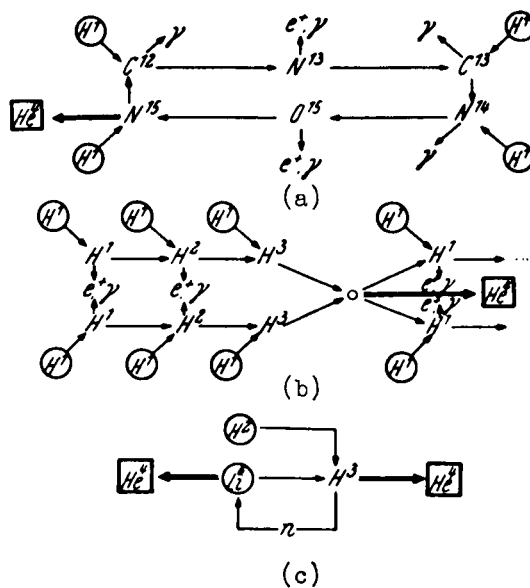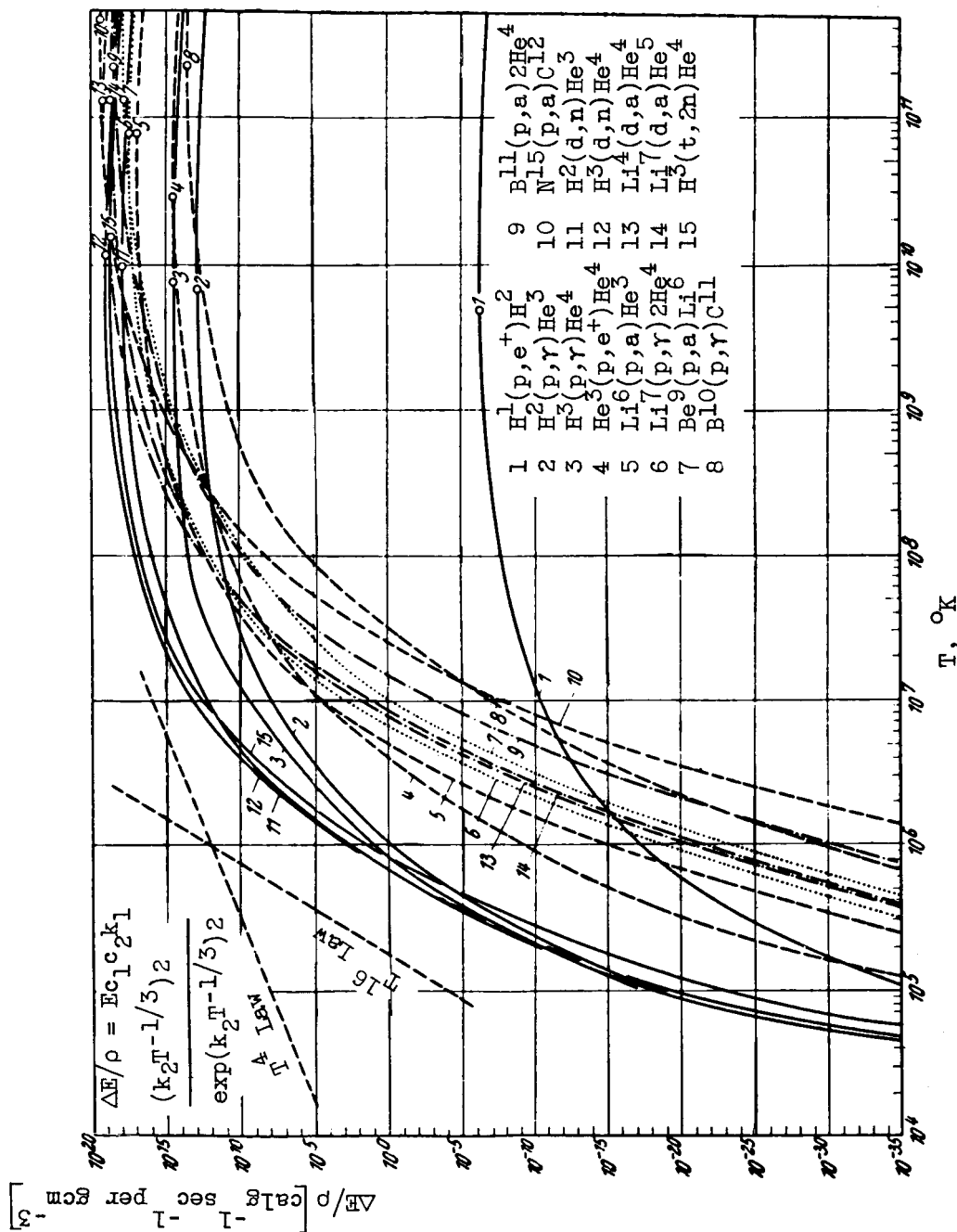   curve of reaction frequency,
   σ dN/dw.

(a)

(b)

(c)

Figure 2.

Figure 3. - Energy production per unit mass of stoichiometric thermal nuclear combustions at constant plasma density, ρ = 1 gram centimeter^{-3}. (In reaction 13 read Li^6 in place of Li^4.)
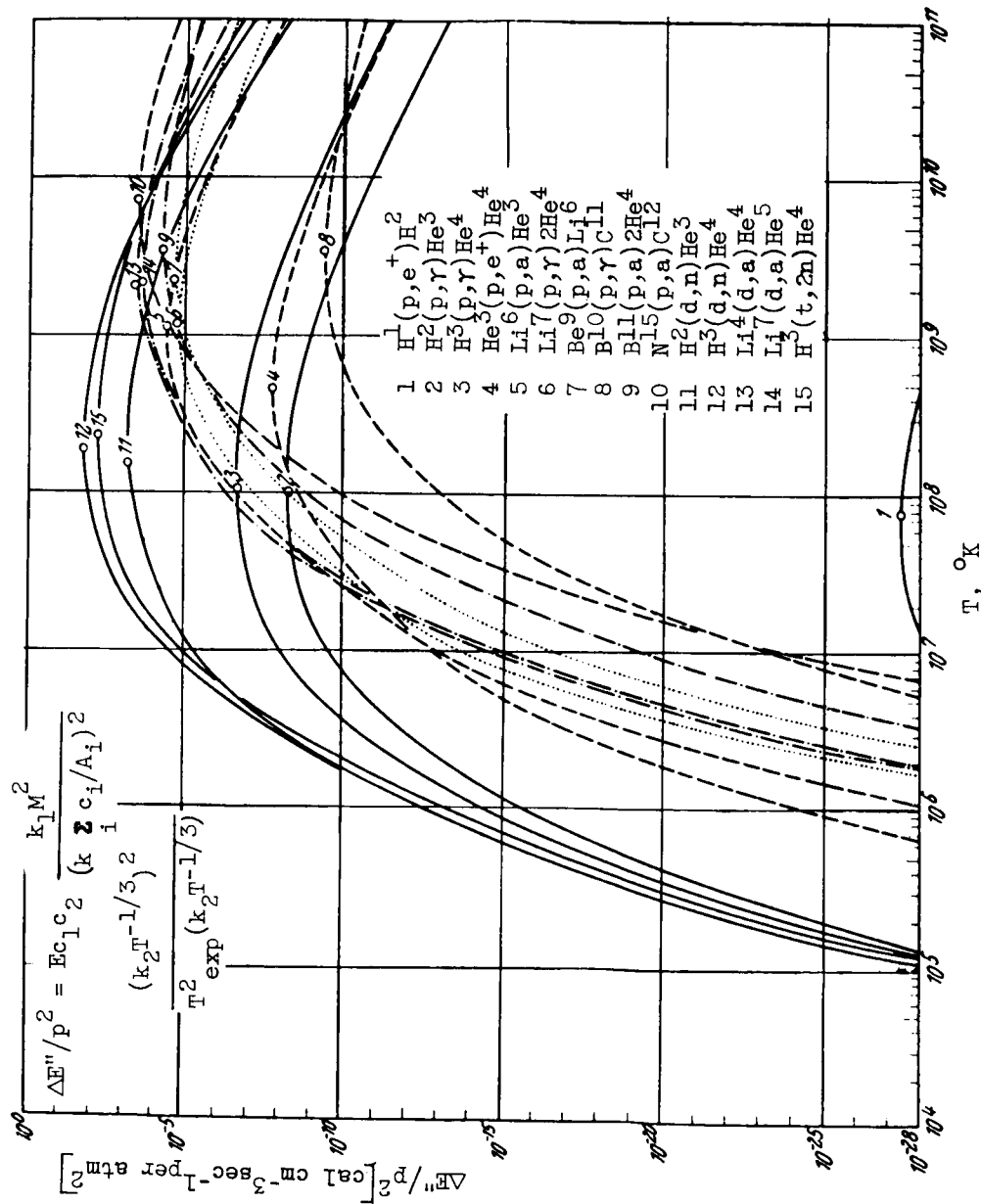
Figure 4. - Energy production of stoichiometric thermal nuclear combustions
per unit volume at constant plasma pressure, p = 1 atmosphere. (In reaction
13 read Li⁶ in place of Li⁴.)

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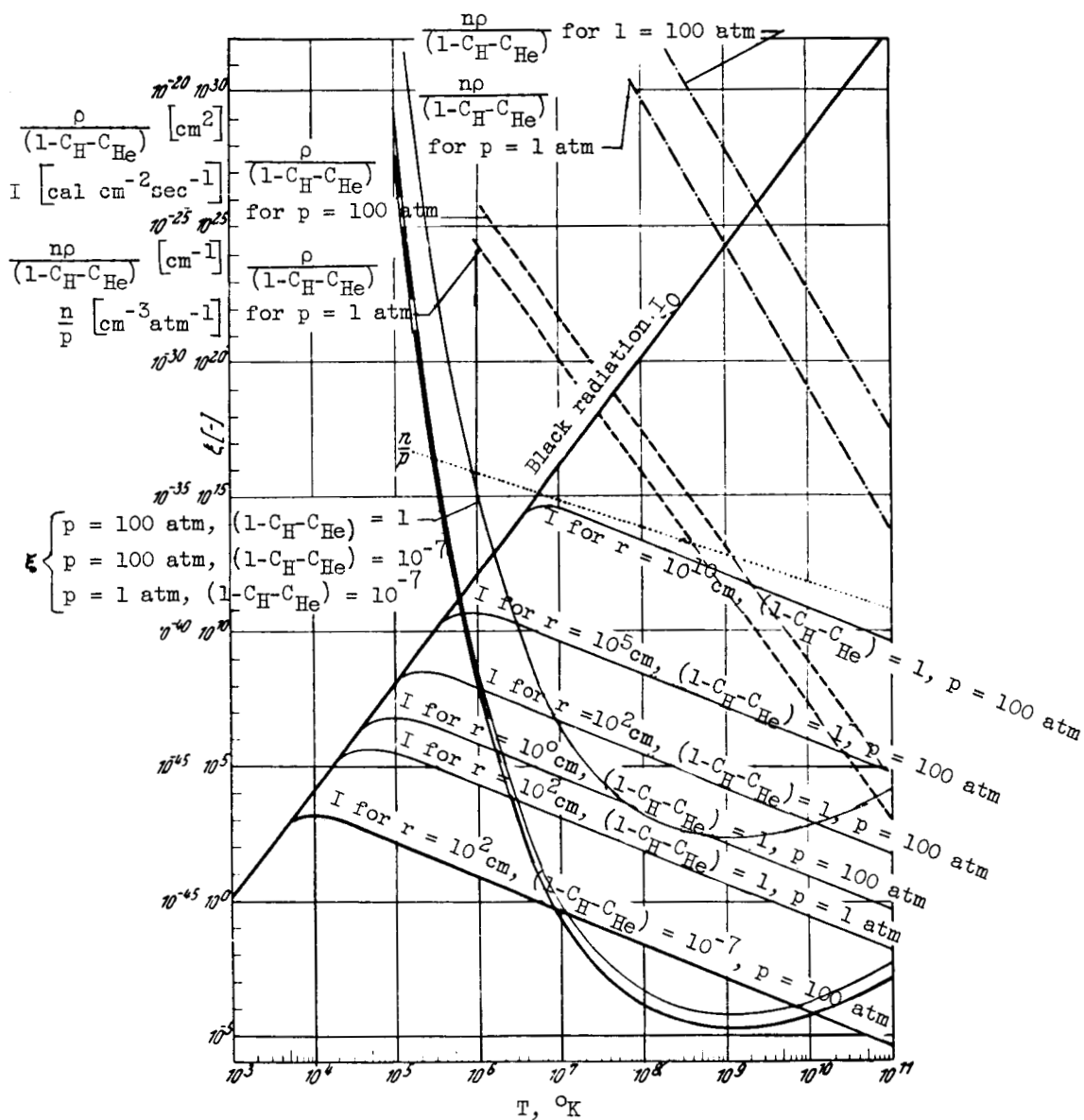

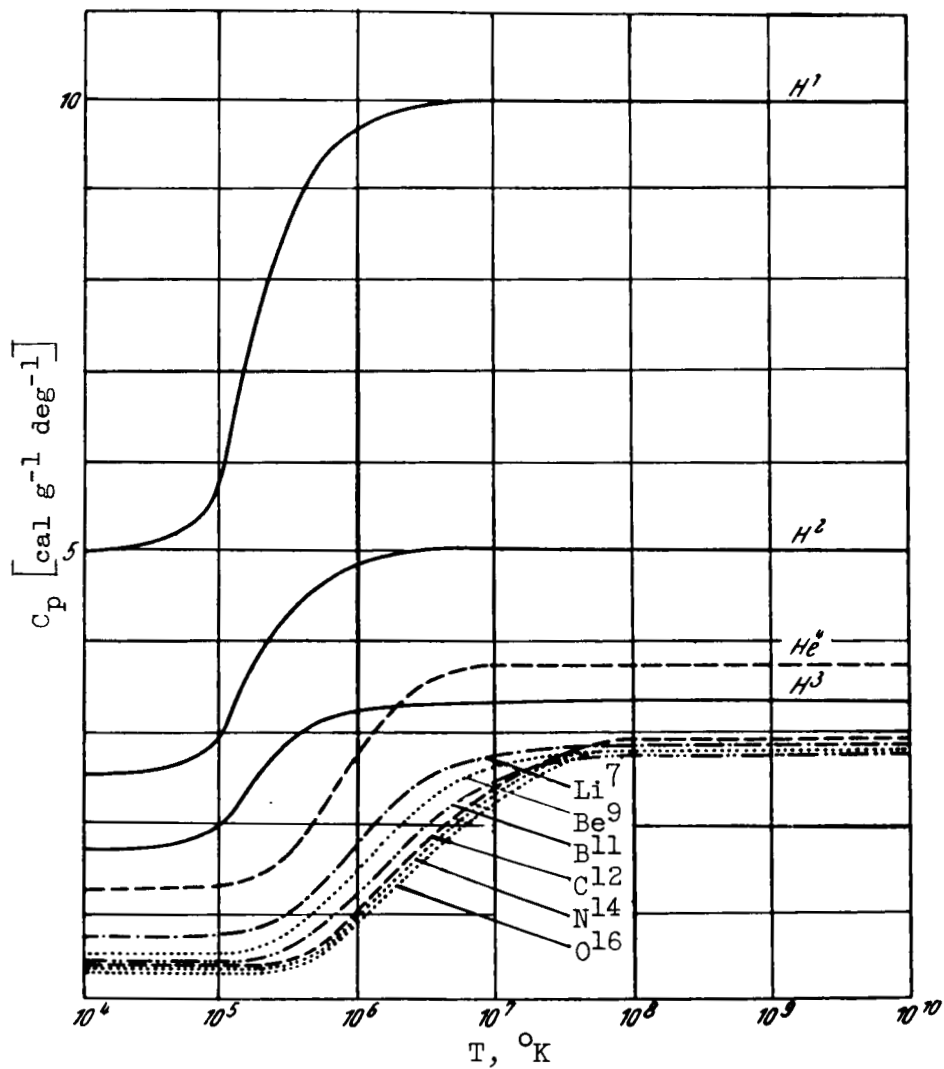Figure 5. - Radiation of nuclear combustion plasma.

Figure 6. - Mean specific heats of ionizing gases
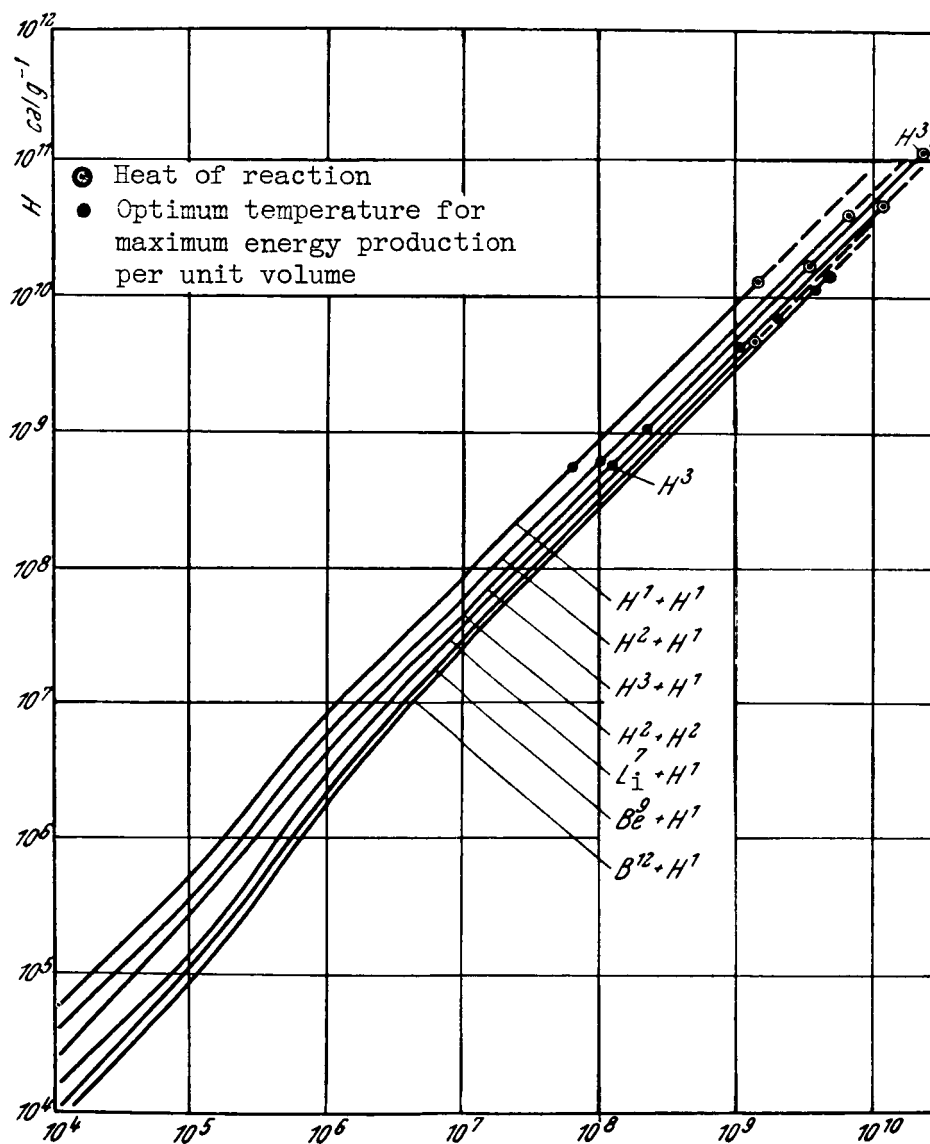in short time processes.

Figure 7. - Enthalpy temperature diagram of stoichiometric
    fresh gases in short time processes with reaction heats
    and optimum combustion temperatures.