

A NOVEL POROSITY MODEL FOR USE IN HYDROCODE SIMULATIONS K. Wünnemann¹, G. S. Collins², and H. J. Melosh¹, Lunar and Planetary Laboratory, University of Arizona, Tucson, AZ 85721, USA, wunnema@lpl.arizona.edu, ²Department of Earth Science and Engineering, Imperial College London, London SW7 2AZ, UK, g.collins@imperial.ac.uk.

Introduction: Numerical modeling of impact cratering has reached a high degree of sophistication; however, the treatment of porous materials still poses a large problem in hydrocode calculations. Porosity plays only a minor role in the formation of large craters on most planetary objects, but impacts on comets are believed to be highly affected by the presence of porosity, which may be as much as 80%. The upcoming Deep Impact Mission (launched January 2005) will provide more detailed data about the composition of a comet (Tempel 1) by shooting a ~370 kg projectile onto the surface of its nucleus. The numerical simulations of such impact events requires an appropriate model for how pore space in the comet is crushed out during the violent initial stage of the impact event.

Most hydrocodes compute the pressure explicitly using an “equation of state” (EOS) for each material, which relates changes in density and internal energy to changes in pressure. The added complication introduced by porosity is that changes in a material’s density are due to both the closing of pore space (*compaction*) and *compression* of the matrix. The amount of resistance to volume change and the amount of irreversible work done during these two processes is very different; it is far easier to compact a porous material sample than to compress a non-porous sample of the same material.

As an alternative to existing porosity models, like the P- α model [1], we present a novel approach for dealing with the compaction of porosity in hydrocode calculations.

Porosity Model: Any initial porosity in the target results in an increase in irreversible work during the early stages of the impact due to the collapse of pore space and the corresponding increase in density at relatively low shock pressures. Thus higher temperatures occur in the material after the shock wave has passed and, consequently, the Hugoniot curve of pressure versus specific volume is much less steep for the porous sample (see Fig 2). As the pore space is reduced, the slope of the pressure-specific volume curve steepens; the slope becomes identical to that of the non-porous material when all pores are closed. Once all pores are removed, however, the Hugoniot curves of the porous and non-porous sample of the same material do not lie on top of one another. This is because a greater amount of irreversible work has been performed on the porous sample, raising its internal en-

ergy to a higher level and hence raising the pressure at an equivalent specific volume.

Previous models, like the often used P- α model [1], account for the collapse of pores by assuming a pressure-dependent compaction function of the porosity ϕ . Instead of porosity, a distention parameter $\alpha \geq 1$ is often used, with $\alpha = 1/(1-\phi)$, $\alpha = V/V_s$ (V specific volume, V_s specific volume of the matrix). The pressure P is then calculated as follows [2]:

$$P = (E, \rho, \alpha) = \frac{1}{\alpha} P_s(\alpha\rho, E) = \frac{1}{\alpha} P_s(\rho_s, E) \quad (1)$$

where P_s is the pressure in the solid matrix, ρ_s the density of the matrix, ρ bulk density, and E is the internal energy. This equation causes some problems when implemented into a hydrocode: For a given time step t , α_{t+1} must be known to derive P_{t+1} , but $\alpha_{t+1} = f(P_{t+1})$. A common solution is to use small sub-cycles to iterate the new P_{t+1} value. This method requires extra computation time and is numerically not very stable.

As an alternative, we propose to use a volumetric strain dependency for the compaction function $\alpha = f(\varepsilon_V)$, instead of pressure. ε_V is more directly related to the crushing of pore space than pressure and a maximum compaction rate can be readily derived from the definition of volumetric strain:

$$\varepsilon_V = \frac{\Delta V}{V_0} \rightarrow \int_{V_0}^V \frac{dV}{V} = \ln\left(\frac{V}{V_0}\right) \quad (2)$$

If one assumes that no compression of the matrix occurs during the crushing of pores ($V_s = \text{constant}$) eq. 2 yields:

$$\varepsilon_V = \ln\left(\frac{V}{V_0}\right) = \ln\left(\frac{V}{V_s} \frac{V_s}{V_0}\right) = \ln\left(\frac{\alpha}{\alpha_0}\right) \quad (3)$$

$$\Rightarrow \alpha = f(\varepsilon_V) = \alpha_0 e^{\varepsilon_V}$$

In reality the crushing-out and closure of pores does not begin until a critical volumetric strain is reached ε_e . This defines a kind of plastic yield threshold below which volume changes are elastic (reversible) and above which compaction (permanent pore collapse) occurs (see Fig. 1). Moreover, pore collapse may not occur at the maximum rate and some compression of the matrix accompanies the compaction. Hence, we introduce a parameter $\kappa \leq 1$ to control the rate of pore closure in the compaction regime. The exponential compaction (eq. 2) works well for the early stage of compaction, but does not give a smooth transition as α approaches 1 (complete compaction).

As a result we define a secondary compaction regime for volumetric strains less than the transition volumetric strain ϵ_X ($\epsilon_V < \epsilon_X$). We use a power-law expression for the compaction rate (as is frequently used in implementations of the P- α model):

$$\alpha = f(\epsilon_V) = \alpha_0 e^{\kappa(\epsilon_V - \epsilon_c)} \quad \left| \begin{array}{l} \kappa \leq 1 \\ \epsilon_e > \epsilon_V > \epsilon_X \end{array} \right.$$

$$\alpha = F(\epsilon_V) = 1 + \left(\alpha_0 e^{\kappa(\epsilon_X - \epsilon_c)} - 1 \right) \left(\frac{\epsilon_c - \epsilon_V}{\epsilon_c - \epsilon_X} \right)^2 \quad \left| \begin{array}{l} \epsilon_X > \epsilon_V > \epsilon_c \end{array} \right. \quad (4)$$

Eq. 4 applies for volumetric strains greater than ϵ_c , the volumetric strain where all pore space is crushed out; further volumetric strains are purely compressive. Note, that ϵ_c can be derived for a given ϵ_X to achieve a continuous transition at $\alpha=f(\epsilon_X)=F(\epsilon_X)$; thus, only ϵ_e , ϵ_X and κ are free parameters.

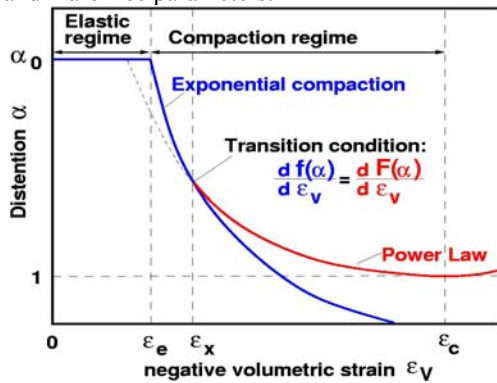


Fig 1: Compaction model showing the volumetric strain dependence of the distention α . Note that in this figure positive strain is compressive whereas in all other equations in this paper $\epsilon_V > 0$ is tensile.

Verification of the model: The present approach was incorporated in the SALE-3MAT hydrocode [3], which is based on the SALE hydrocode [4] and is basically very similar to SALEB by Ivanov et al. [5]. To verify the model we first (i) reproduced data from shock experiments on porous Aluminum for both Eulerian and Lagrangian mode. Fig 2 shows modeled Hugoniot curves in comparison to experimental data [6] with porosities $\phi=0$ ($\alpha=1$) and $\phi=0.5$ ($\alpha=2$). A good fit was obtained by adjusting the free parameters for the porosity model. We used a Tillotson equation of state to calculate pressure from density and internal energy (see eq. 1). (ii) Additionally, we modeled a simple one-dimensional impact experiment (reported by Butcher et al [7]) in which a porous Aluminum sample ($\alpha=1.3$) was impacted by a thick Aluminum plate, with a velocity of 126 m/s, and buffered by another Aluminum plate. A velocity interferometer (VISAR) was used to measure the velocity at the free surface of the buffer plate as a function of time. Fig 3

shows the results of the numerical model in comparison to the experimental data.

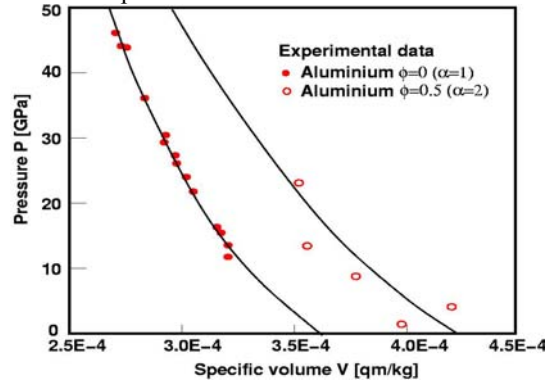


Fig 2: Calculated Hugoniot curves for Aluminum with $\alpha=1$ and $\alpha=2$ in comparison to experimental results [2].

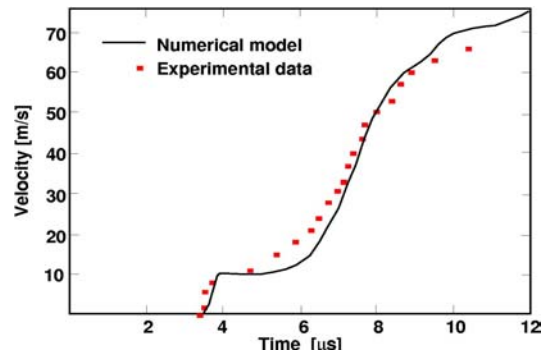


Fig 3: Velocity-time history at the free surface of the impact experiment as described in the text.

Conclusion: We present a simple and intuitive porosity model for use in hydrocodes that reproduces experimental data well. The model requires only three material-specific input parameters and each has a physical basis. Thus, although these values should be determined for a given material by comparing model results with experimental data, it is possible to make sensible estimates for the parameters where no data for a porous material exists. For future work we intend to transform the two-regime compaction definition (eq. 4) into one equation that fulfills the given boundary conditions.

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