

## **Identification of Cyclically Symmetric Resonance in Experimental**

### **Data for Engine Failure Analysis**

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Many structural components are rotationally symmetric. For a perfectly manufactured structure, the modal wave shapes are exactly sinusoidal. These structures can be excited by similarly-shaped forcing functions such as those found frequently in or near turbomachinery. Due to orthogonality, resonance occurs only when the integer wave number of the structure equals the integer wave number of the excitation as well as when the excitation frequency is close to the natural frequency of the mode. This resonance can lead to high-cycle fatigue (HCF) cracking of the structure. To help understand this phenomenon, it is desirable to determine the modal composition of the response and excitation. Once the modal composition is known, either the excitation or the structure can be modified to reduce the response and therefore prevent HCF cracking.

This response phenomenon has been exhibited in the flowliner component of the Space Shuttle Main Engine, where a small crack was noticed during a routine inspection of an orbiter in mid 2002 (prior to the Columbia accident). This crack sparked an evaluation of the entire Shuttle fleet main propulsion feedlines which led to an investigation to determine the cause of the small cracks and to come up with a repair that could return the Shuttle fleet back to operation safely. During this investigation, it was determined that backward traveling wakes and acoustics initiated by flow fluctuations of the Space Shuttle Main Engine low pressure fuel turbopump were at a frequency and shape close to the modes of the

flowliner. Additionally, there is a high modal stress in these modes at the crack location.

Therefore, it was concluded that these pump induced wakes and acoustics caused the liner to vibrate in the complex diametric modal shapes and induce the crack.

The first step in solving this problem was to characterize the excitation shapes. Fluid physics personnel completed this task using in-house software and provided the structural dynamics group with the amplitude, frequency, and shape of the excitation.

The next step is the analysis of the response shapes for which the innovative three-part method described in this paper was developed. The first part in the methodology is to use Fourier series decomposition to break down the structural modes into their constituent elements in order to determine which wave modes are most prevalent. The data used to analyze the response shapes came from two sources: strain gauge readings from engine hot-fire tests and flowliner modal test data. The analysis was done using two sets of algorithms written in Matlab. One set decomposes full-range sets of flowliner data whose measurements span 360°, and the other set decomposes half-range sets of data spanning 180°. The half-range equations are as follows, where  $L$  is the half-circumference of the flowliner,  $n$  is the number of wave modes, and  $x$  is in the range  $(0 < x < L)^1$ :

Cosine Series

$$f(x) \approx a_0 + \sum a_n \cos(n\pi x / L)$$

$$a_0 = \frac{1}{L} \int_0^L f(x) dx$$

$$a_n = \frac{2}{L} \int_0^L f(x) \cos(n\pi x / L) dx$$

Sine Series

$$f(x) \approx \sum b_n \sin(n\pi x / L)$$

$$b_n = \frac{2}{L} \int_0^L f(x) \sin(n\pi x / L) dx$$



The full-range equations are as follows, with  $x$  in the range  $(-L < x < L)$ :

$$f(x) \approx a_0 + \sum_{n=1}^{\infty} a_n \cos(n\pi x / L) + b_n \sin(n\pi x / L)$$

$$a_0 = \frac{1}{2L} \int_{-L}^L f(x) dx$$

$$a_n = \frac{1}{L} \int_{-L}^L f(x) \cos(n\pi x / L) dx$$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin(n\pi x / L) dx$$

In both the half-range and full-range sets of equations,  $a_0$  is the initial value of the Fourier series function,  $a_n$  is the cosine component, and  $b_n$  is the sine component. The wave numbers from 1 to 20 of both sine and cosine are then summed, and the percent contribution from each mode shape to the resulting response shape are calculated by dividing each wave number value by the total summation. This normalization is useful for determining the mode shapes that contribute most to the response shape. An example of one such Fourier Decomposition can be seen in figure 1 below.

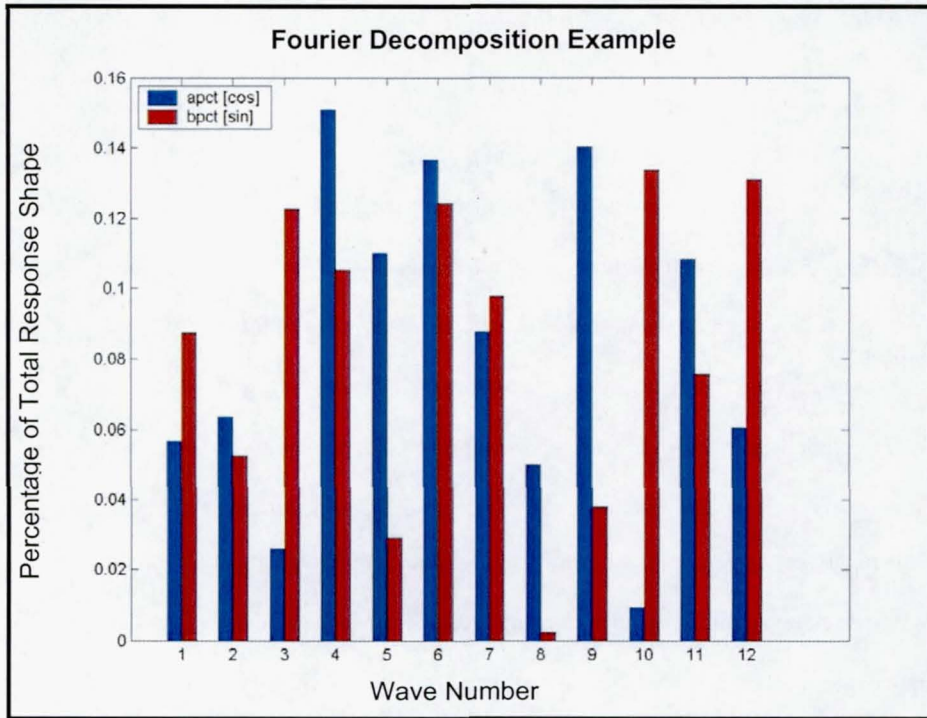


Figure 1.

The second part of this methodology is to animate the hot-fire strain gauge data and visually inspect the resonance shape. This is done by first passing the strain gauge data through a band pass filter centered on a resonant peak for time slices of interest. Typically, a 20 Hz band pass filter is used, though the range varies with signal bandwidth. Each strain gauge filtered time history is then normalized and given a DC offset. The normalized-offset data is then interpolated spatially through the strain gauge's respective clocking positions ranging from 0 to 180° around the flowliner. This interpolated data is then plotted in a polar plot making up a half circle with a radius equal to the DC offset. The peak response of the plot is equal to the normalization value. This plot is then stepped through time, creating an



animation of the flowliner resonance response. . From the animations, the number of circumferential waves (nodal diameters) can be counted over the 180° section. The band pass filter is applied to the strain gauge time history data in the code PC Signal. The normalization/offset, interpolation and animation is performed in Matlab.

The third part of the methodology was to make a comparison between the Fourier decomposition, resonance animation, modal test results, and excitation characteristics. The modal test of the flowliner was performed in air. Using a knock-down factor for liquid hydrogen on the modal frequencies, the authors were able to narrow down the number of possible flowliner mode shapes to a small family. A comparison was then made between all the previous elements to pinpoint the suspected mode being excited.

One problem that presented itself when analyzing these data sets was the lack of data points. For best results using this Fourier decomposition method, it is best to use as many data points as possible. With 16 gauges ranging over 180° and the most common angular separation between gauges being 9.5°, the hot fire data had an effective spatial Nyquist number of 19 nodal diameters. With this fidelity, the authors found that any shape with a complexity greater than 9 nodal diameters was difficult to identify. This identification of some of the higher order mode shapes was made difficult because some test responses had multiple modal participation at the same frequency, elevated noise floors introduced noise into the resonant signals, and rotating forcing functions complicated the animations and Fourier decompositions. The quality of the results, therefore, are somewhat mixed. Some resonant shapes compared quite well between the animations, modal tests and Fourier decomposition, while others did not. In conclusion, though, the new procedure provided a

very useful new tool in the failure investigation and helped to identify potential fixes for the problem.

#### *References*

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- <sup>1</sup> Powers, David L., Boundary Value Problems, 2<sup>nd</sup> Ed., Academic Press, Inc., New York, 1979, pp. 30-57.