An Extension to the Kalman Filter for an Improved Detection of Unknown Behavior

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Abstract—The use of Kalman filter (KF) interferes with fault detection algorithms based on the residual between estimated and measured variables, since the measured values are used to update the estimates. This feedback results in the estimates being pulled closer to the measured values, influencing the residuals in the process. Here we present a fault detection scheme for systems that are being tracked by a KF. Our approach combines an open-loop prediction over an adaptive window and an information-based measure of the deviation of the Kalman estimate from the prediction to improve fault detection.

I. INTRODUCTION

A. Kalman Filter

Consider a discrete-time controlled process that is governed by a linear stochastic difference equation (1) and a measurement (2):

\[ x(t_i) = Ax(t_{i-1}) + Bu(t_i) + w(t_i) \]  
\[ z(t_i) = Hx(t_i) + v(t_i) \]

\( w(t_i), v(t_i) \) represent the process and measurement noise respectively and are assumed to be independent, white and Gaussian with probability distributions \( N(0, Q) \), \( N(0, R) \) respectively. Given the noise in the process and measurements, the KF [1] computes an unbiased estimate \( \hat{x} \) of the state \( x \) by providing an optimal solution of the least-squares method. This is achieved by recursively minimizing the a posteriori estimate error covariance

\[
\mathbb{E}[e(t_i)e^T(t_i)] = \mathbb{E}[(z(t_i) - H\hat{x}(t_i))^2]
\]

where \( e(t_i) = z(t_i) - H\hat{x}(t_i) \) is the a posteriori error between the true state \( x(t_i) \) and the a posteriori state estimate \( \hat{x}(t_i) \). First the state and error variance estimates are projected forward from time \( t_{i-1} \) to time \( t_i \) through the following equations:

\[
\hat{x}(t_i^-) = A\hat{x}(t_{i-1}) + Bu(t_i) \\
P(t_i^-) = AP(t_{i-1})A^T + Q
\]

where \( t_i^- \) indicates a priori values. An adaptive gain factor \( K \) minimizes (in the least-square sense) the error covariance. Noisy measurements of the process are then used to compute the a posteriori state estimate. Finally the a posteriori covariance estimate is computed. These three steps are summarized as:

\[
K(t_i) = P(t_i^-)H^T(HP(t_i^-)H^T + R)^{-1} \\
\hat{x}(t_i) = \hat{x}(t_i^-) + K(t_i)(z(t_i) - H\hat{x}(t_i^-)) \\
P(t_i) = (I - K(t_i)H)P(t_i^-)
\]

The KF has been the subject of extensive research and applications (2).

B. Fault Detection and the Kalman Filter

We argue that in several situations the KF is in cross-purposes with the fault detection. First, the KF is designed to filter any deviations in the measurements and predictions by using the measurement updates. As a result the magnitude of the residual \( e(t_i) = z(t_i) - H\hat{x}(t_i^-) \) is reduced, affecting the fault detection capability. Second, when the measurement noise is high the error covariance is so large that even a large residual falls well within its bounds. Furthermore, since the gain factor \( K \) is not dependent on the input matrix \( B \), the covariance minimization is not affected by any faults on the input (3).

II. PRELIMINARIES

A. n-step predictor

We define the n-step predictor of the state \( \tilde{x}_n(t_i) \) to be the n-step open-loop estimate of the state. \( \tilde{x}_n(t_i) \) is computed recursively by taking the KF state estimate at time \( t_{i-n} \) and then projecting it forward for \( n \) steps using equation (3). The covariance is also projected forward using equation (4).

\[
\tilde{x}_n(t_i) = A^n\hat{x}(t_{i-n}) + \sum_{j=1}^{n} A^{n-j}Bu(t_{i-n+j}) \\
\tilde{P}_n(t_i) = A^nP(t_{i-n})(A^T)^n + \sum_{j=0}^{n-1} A^jQ(A^T)^j
\]

Let \( \tilde{X}_n(t_i) \sim N(\tilde{x}_n(t_i), \tilde{P}_n(t_i)) \) be the random variable corresponding to the n-step prediction. We define \( D_n(t_i) = \tilde{X}_n(t_i) - \hat{X}(t_i) \), and \( D_n(t_i) \sim N(E[D_n(t_i)], \text{cov}(D_n(t_i))) \):

\[
E[D_n(t_i)] = d_n(t_i) = \tilde{x}_n(t_i) - \hat{x}(t_i) \\
\text{cov}(D_n(t_i)) = Ad_n(t_{i-1}) + K(t_i)(H(Bu(t_i) + A\hat{x}(t_{i-1})) - z(t_i))
\]

If \( n = 1 \), then \( \tilde{x}_1(t_{i-1}) = \hat{x}(t_{i-1}) \) and \( d_1(t_i) = -K(t_i)e(t_i) \) as \( d_0(t_i) = 0 \).
Due to the potentially large variance $S_n(t_t)$, $(z(t_t), x_n(t_t))$ may not be sufficient for quick detection. The \textit{a posteriori} $n$-steps prediction likelihood $L(x_n(t_t), x(t_t))$ assesses the distance between $x_n(t_t)$ and $x(t_t)$. We examine the Kullback-Leibler (KL) divergence \cite{4} between $x_n(t_t)$ and $x(t_t)$ which measures how different the two distributions are. $KL(x_n(t_t), x(t_t))$ can be understood as the average number of bits that are wasted by encoding events from the predicted distribution (over $n$-steps) with a code based on the estimated distribution. Therefore, the less bits are wasted, the more it is likely the system behavior is nominal. We thus note $L(x_n(t_t), x(t_t)) = 1/(2\pi n^{n/2})P(t_t)^{1/2}$. Due to the potentially large variance $S_n(t_t)$, $L(z(t_t), x_n(t_t))$ may not be sufficient for quick detection.

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diagnosis articulates the tracking and the consistency-based engine for particle filter hybrid diagnosis engine that combines a Rao-blackwellized

Algorithm 1 presents the filter loop at time step \( t_i \). It is initialized with \( \hat{x}(0) = \tilde{x}(0) = x_0 \), \( P(0) = \tilde{P}(0) = P_0 \) and \( n = n_{\text{min}} \). The implementation requires storing or recomputing several values and matrices: \( e(t_{i-n}), K(t_{i-n}), P(t_{i-n}) \). This is consistent with modern diagnosis engines that work on a fixed temporal window [5], although increasing the computational complexity of the KF. The following results help in mitigating the computational effort:

\[
\begin{align*}
&d_{n+1}(t_i) - d_n(t_i) = -A^nK(t_{i-n})(e(t_{i-n}) \\
&\tilde{P}_{n+1}(t_i) - \tilde{P}_n(t_i) = A^nK(t_{i-n})HP(t_{i-n})(A^n)^T
\end{align*}
\]

These relations appear on steps 2 and 3:

1: Standard Kalman filter prediction and update.
2: \( d_n(t_i) \) computation:
\[
\begin{align*}
&d_{n-1}(t_{i-1}) = d_n(t_{i-1}) \\
&\quad + A^nK(t_{i-n+1})e(t_{i-n+1}) \\
&d_n(t_i) = Ad_{n-1}(t_{i-1}) + K(t_i)(HBu(t_i)) \\
&\quad + HA\hat{z}(t_i) - z(t_i) \\
\end{align*}
\]

3: \( n \)-steps prediction:
\[
\begin{align*}
&\tilde{x}_n(t_i) = d_n(t_i) + \tilde{x}(t_i) \\
&\tilde{P}_{n-1}(t_{i-1}) = \tilde{P}_n(t_{i-1}) - A^{n-1}K(t_{i-n+1}) \\
&\quad HP(t_{i-n+1})(A^{n-1})^T \\
&\tilde{P}_n(t_i) = A\tilde{P}_{n-1}(t_{i-1}) + Q
\end{align*}
\]

4: Fault detection: \( \text{sign}(\delta_n(z(t_i), \tilde{X}_n(t_i), \tilde{X}(t_i))) \).
5: Adapt \( n \) according to \( \theta \) and \( \delta_{n+1,n}(z(t_i), \tilde{X}_n(t_i), \tilde{X}(t_i)) \).

Algorithm 1: KF with \( n \)-steps open-loop and fault detection

IV. APPLICATION

The application that motivated this research was that of a hybrid diagnosis engine that combines a Rao-blackwellized particle filter (RBPF) [6] with a logical approach to the diagnosis [7]. An efficient fault detector was needed to articulate the tracking and the consistency-based engine for logical diagnosis, i.e. for deciding when to trigger the latter, or returning to the former.

As a preliminary test, we plugged the fault detector into the RBPF and tracked a simulated noisy thermostat. The RBPF tracks multi-modal linear systems with Gaussian noise. The belief state is a mixture of Gaussians whose statistics are propagated with a KF. The particle weight is computed as the observation probability \( p(z(t_i) | \tilde{X}(t_i^-)) \). Our strategy uses the fault detector to assert the quality the estimate and lowers the weight of particles that are not in the correct mode. Figure 2 shows a run on a faulty thermostat \( (n \leq 50) \): the number of alarming particles rises at each mode change. Our version of the filter detects wrong modes and faults almost instantly. Identification however depends on the modes sampling.

Unfortunately, on large multi-dimensional continuous spaces, the computational weight of the detector is very heavy due to the MC calls for the KL computation. Moreover, results are deceiving on systems with uncertain parameters (high process noise) and precise sensing (low observation noise). For these reasons, we are not using this detector in our current diagnosis engines.

REFERENCES