Abstract for SIAM Computational Science and Engineering Conference, Orlando, FL, Feb. 11-15, 2005

A Streaming Language Implementation of the Discontinuous Galerkin Method

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We present a Brook streaming language implementation of the 3-D discontinuous Galerkin method for compressible fluid flow on tetrahedral meshes. Efficient implementation of the discontinuous Galerkin method using the streaming model of computation introduces several algorithmic design challenges. Using a cycle-accurate simulator, performance characteristics have been obtained for the Stanford Merrimac stream processor. The current Merrimac design achieves 128 Gflops per chip and the desktop board is populated with 16 chips yielding a peak performance of 2 Teraflops. Total parts cost for the desktop board is less than $20K. Current cycle-accurate simulations for discretizations of the 3-D compressible flow equations yield approximately 40-50% of the peak performance of the Merrimac streaming processor chip. Ongoing work includes the assessment of the performance of the same algorithm on the 2 Teraflop desktop board with a target goal of achieving 1 Teraflop performance.

StreamFEM

A Streaming Language Implementation of the Discontinuous Galerkin Finite Element Method

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A DG Finite Element Method for Conservation Laws

StreamFEM implements the Discontinuous Galerkin (DG) finite element method for systems of nonlinear conservation laws in divergence form in 2-D or 3-D:

$$u_t + \mathrm{div}(\vec{f}) = 0$$

The DG Finite Element Variational Statement

Find $u \in V_h$ such that $\forall w \in V_h$

$$\sum_{\text{elements}} \left(\int_K w\, u_t \, d\vec{x} - \int_K \vec{f}(u) \cdot \nabla w \, d\vec{x} + \int_{\partial K} w_-\, h(n; u_-, u_+) \, d\vec{x} + \int_K \epsilon_K(u) \, \nabla w \cdot \nabla u \, d\vec{x} \right) = 0$$

with

$$\epsilon_K(u) = h^{2-\beta} \| u_t + \mathrm{div} f(u) \|_K \ , \quad \beta \geq 0 \ .$$

Variable Arithmetic Intensity

StreamFEM includes discontinuous Galerkin models of several representative nonlinear partial differential equation (PDE) systems of increasing complexity in 3-D:

- Scalar Advection (1 PDE)
- Euler Equations (5 PDEs)
- Magnetohydrodynamics (8 PDEs)

StreamFEM also includes various piecewise polynomial representations with an increasing number of degrees of freedom (dofs) ranging from piecewise constant to piecewise cubic polynomial approximation in 3-D

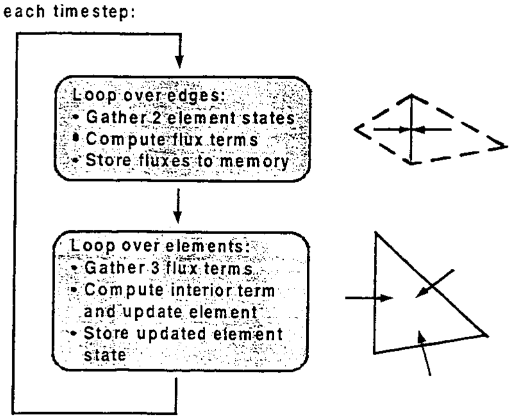
- Piecewise constant elements (1 dof / (element-equation))
- Piecewise linear elements (4 dofs / (element-equation))
- Piecewise quadratic elements (10 dofs / (element-equation))
- Piecewise cubic elements (20 dofs / (element-equation))

By increasing the number of PDEs and the number of degrees of freedom per element, it is possible to alter the overall arithmetic intensity of the computation by 10x or more.

StreamFEM Flow Chart

StreamFEM has been implemented in the Brook stream language and later translated into StreamC/KernelC. The current algorithm utilizes a simple Runge-Kutta(1) time stepping algorithm:

For each timestep:



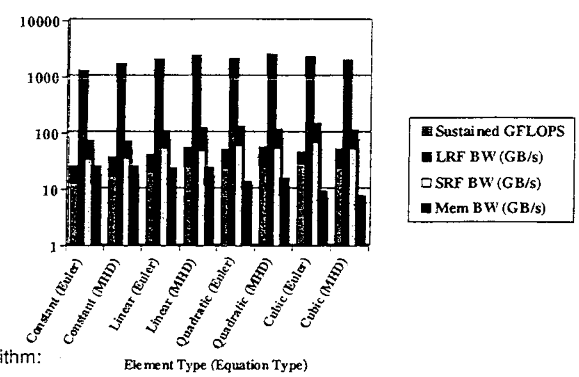
Brook Functional Simulation of Flow Over a 2-D Forward Facing Step

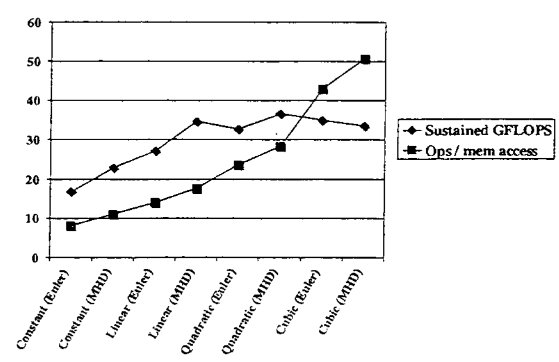
M=3 →  t=0.2

M=3 →  t=0.8

M=3 →  t=1.6

StreamFEM-2D Hardware Simulated Performance





Future Directions

- Optimized simulations of StreamFEM-3D (in progress)
- Multiple node performance simulation and optimization
- Streaming language implementation of sparse linear algebra kernels

Streaming Languages
Compilation
System Architecture
Node Architecture
StreamFlo
StreamMD
StreamFEM